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MODELLING AND OPTIMIZATION OF BEAMS DYNAMICS IN LINAC

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Abstract

Problems of acceleration and focusing in linear acceleratirs are considered. A general mathematical problem of charged particle beam control is formulated. Methods and algorithms of solving these problems are developed. Problems of mathematical simulation of beam dynamics are discussed in detail. Some beam quality functionals depending on all particle tracks are proposed. Mathematical methods are used for choosing parameters of forming systems. Designed codes allow to simulate and optimize beam dynamics.

This report is devoted to the realization of general approach to problem of dynamical system trajectories control in accelerating and focusing structures.

Let us consider the system of differential equatios

$$\dot{X} = f(t, X, u), \qquad (1)$$

where t is time, X is R^n vectir of phase coordinates, u is R^r control vector and f is vector function. We assume that system (1) has the solution $X = X(t, t_o, X_o)$ with initial conditions $X(t_o, t_o, X_o) = X_o$ for $X_o \in M_o$, where M_o is the set of initial values. Let us denote $M_{t,u}$ the shift of set M_o through trajectories of system (1). Let us suppose that the function $\rho(t,X) \geq 0$ is the system (1) integral invariant and functions $\Phi(t,X,\rho)$ and $G(X,\rho)$ are given and non-negativ.

The main problem is to find the control $u = u(t)$, $t \in [t\ ,T]$, that gives infimum to the functional

$$I = \int_{t_o}^{T} \int_{M_{t,u}} \Phi(t, X_t, \rho(t, X_t))\, dX_t\, dt + \int_{M_{T,u}} G(X_T \rho(T, X_T))\, dX_T. \qquad (2)$$

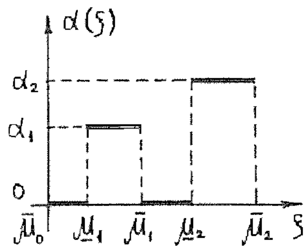
This general approach is used for the charged particles beam control in LINAC [1,2].

Let us consider formulations of cotrol problems related with formating of required accelerating, bunching and focusing regimes for charged particles beams.

The longitudional dynamics of the beam in accelerators with drift tubes may be described by well known equations

$$\frac{d\gamma}{d\xi} = \alpha(\xi)\cos\varphi, \quad \frac{d\varphi}{d\xi} = 2\pi\gamma/\sqrt{\gamma^2 - 1}, \qquad (3)$$

where γ is energy and φ is particle phase, $\xi \in [0, L]$, is longitudional coordinate, L is the length of the structure. In the equation (3) piece-wise constant function $\alpha(\xi)$ is defined by formulas



$$\alpha(\xi) = \begin{cases} 0, & \xi \in [\bar{\mu}_{i-1}, \mu_i], \\ \alpha_i, & \xi \in [\mu_i, \bar{\mu}_i]; \end{cases}$$

$$i = 1, 2, \dots, N; \quad \mu_0 = 0; \ \mu_N = L.$$

and proportional to intensity of accelerating field. Let us suppose that energy $\hat{\gamma}$ and phase $\hat{\varphi}$ of particles at the end of acceleratir are given or equal to average particles energy and phase correspondingly. The minimization of functional

$$I = \int_{M_{L,\alpha}} \left[a\left(\frac{\gamma_L}{\hat{\gamma}} - 1\right)^2 + b\left(\varphi_L - \hat{\varphi}\right)^2 \right] d\varphi_L\, d\gamma_L \qquad (4)$$

that characterizes the beam at the end of accelerator, provides optimal parameters.

Let us consider the radial motion now. Let variables η and $\varkappa$ are reduced radial coordinate and velocity of a charged particle correspondingly. Then equetions (3) are copled with system

$$\frac{d\eta}{d\xi} = \varkappa;$$
$$\frac{d\varkappa}{d\xi} = -\alpha(\xi)\left(\cos\varphi \frac{\gamma\varkappa}{\gamma^2 - 1} - \frac{\eta \sin\varphi}{\sqrt{\gamma^2 - 1}}\right) - U(\xi)\frac{\eta}{\gamma^2 - 1} \qquad (5)$$

where piece-wise function $U(\xi)$ is intensity of solenoid longitudional magnetic field. To

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take into account radial dynamics we add integral

$$I_2 = \int_0^L \int_{M_{\xi,\alpha,u}} \left[c\eta^2 + \alpha æ^2\right] d\gamma_\xi \, d\varphi_\xi \, d\eta_\xi \, dæ_\xi \, d\xi \qquad (6)$$

to functional (4).

This approach gives us opportunity to solve more complicated problems. For example, we can optimize the lattices with beam's space charge taking into account. Let us consider one of such problems. Suppose that particles beam has circle cross-section and almost homogeneous charge density distribution Then the force acting on the particle may be defined by formula

$$F_\eta = \frac{K(\gamma)\eta}{R^2}, \qquad (7)$$

where R is an effective radius of beam cross section. Then the beam's dynamics can be obtained by adding the function F_η to right hand side of second equation system (5).

The choice of the functional (2) allows to define the prelimenary structure of an accelerator and to optimize it to obtain parameters needed.

In monographs [1,2] there is a vast survey of methods of solving different beam's optimal problems. The technique, proposed for solving these problems, allow to construct directed methods of choosing optimal parameters. The analitical formulas for the gradient of control parameters are proposed in these books. In particulary formulas for gradients

$$\frac{\partial I}{\partial \alpha_i}, \quad \frac{\partial I}{\partial \mu_i}, \quad \frac{\partial I}{\partial \bar{\mu}_i}, \quad i = 1,2,\dots,N$$

for above mentioned examples are developed.

We produced codes for IBM PC compatible computers for solving these problems. One of these codes provides optimal parameters for solenoids, quadrupols and gaps with drift tubes. Authors wanted to make the program allowing in interactive regime

- to simulate charge particles beam's dynamics;
- to formulate conditions for the beam configuration in the space of coordinates and speeds at the end of structure;
- to calculate feasible structure parameters to satisfy formulated conditions.

Main assumptions are

a) the particle interaction isn't taken into consideration;

b) electromagnetic field amplitudes are piece-wise constant.

Under this hypothesis equations of beam's dynamics allow the analytical solution everywhere exept accelerating gaps. The aim is to place particles into domain, bounded by curve on phase plane. Let vector Y is (W,φ) or (z,z') or (x,x') or (y,y') and $S(Y)=C$ is equation for boundary. Let the function $F(Y)$ is defined by formulas

$$F(Y) = 0, \quad \text{if} \quad S(Y) < C$$
$$\text{else} \quad F(Y) = (S(Y) - C)^2.$$

Let the minimizating functional is the integral of function $F(Y)$ by the particle set. We shall minimize it by varying elements lengths and values of electric and magnetic fields amplitudes. The functional's gradient can be obtained with the help of conjugate system that is solved analitically. The restrictions on control are considered during optimization The phase variables $(t, x, y, \dot{x}, \dot{y}, \dot{z})$ values at the end of every structure element are computed and saved. These data is used for construction of beam's dynamics vizualization that is made with the help of graphics and tables. The user can enter the accelerating and focusing structure, particle's type and initial beam phase configuration.The program made it possible to watch the 2D & 6D beam dynamics and to optimize the structure's parameters. If the user want to optimize the structure he should set the beam phase configuration at the end of the structure and put restrictions of the control. User can control the process of optimization. The codes can be installed on IBM PC/AT 286.

The initial and optimized beams cross-section are depictured on figures 1,2,3,4.

The beam's projections on the plane(W,φ) at the end of initial and optimized structures are depictured on fig.1 & 2. The ellipse bounds the set desired.

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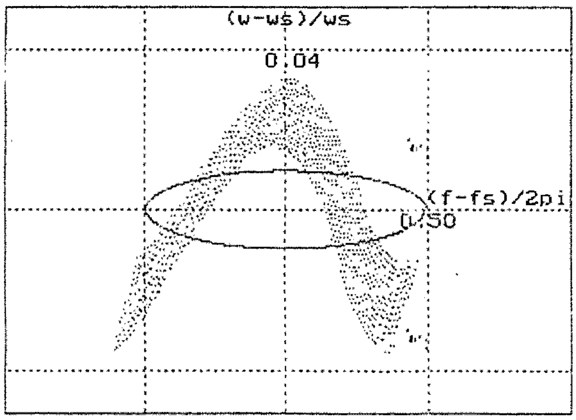

Fig. 1

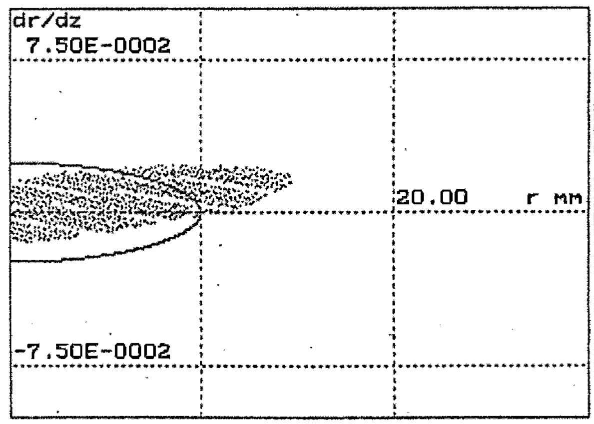

Fig. 3

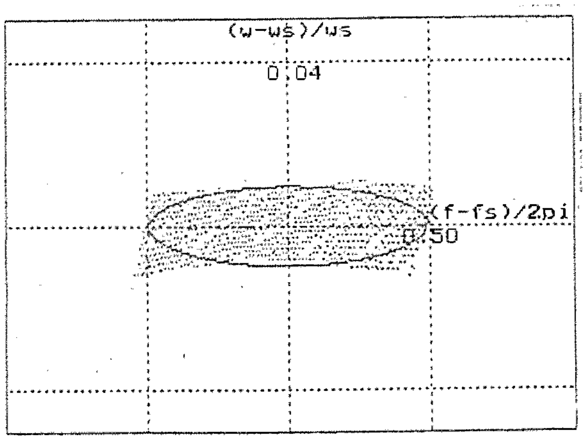

Fig. 2

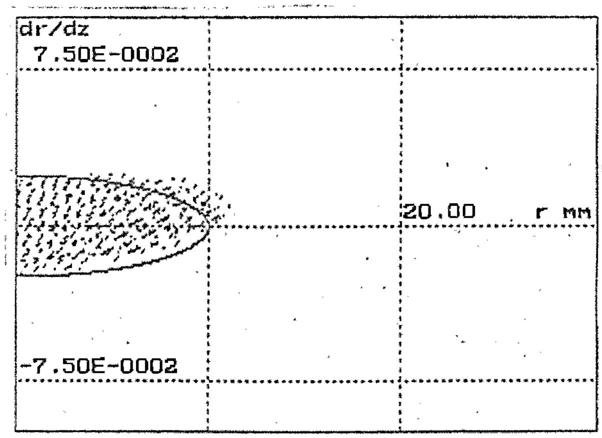

Fig. 4

The beam's projections on the plane (τ,τ') at the end of initial and optimized structures are depicted on fig.3 & 4 correspondingly. The ellipse bounds the set desired.

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2. Ovsyannikov D.A. Simulation and optimization of charged particle beam dynamics.L.: Leningrad University, 1990 (in Russian).

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