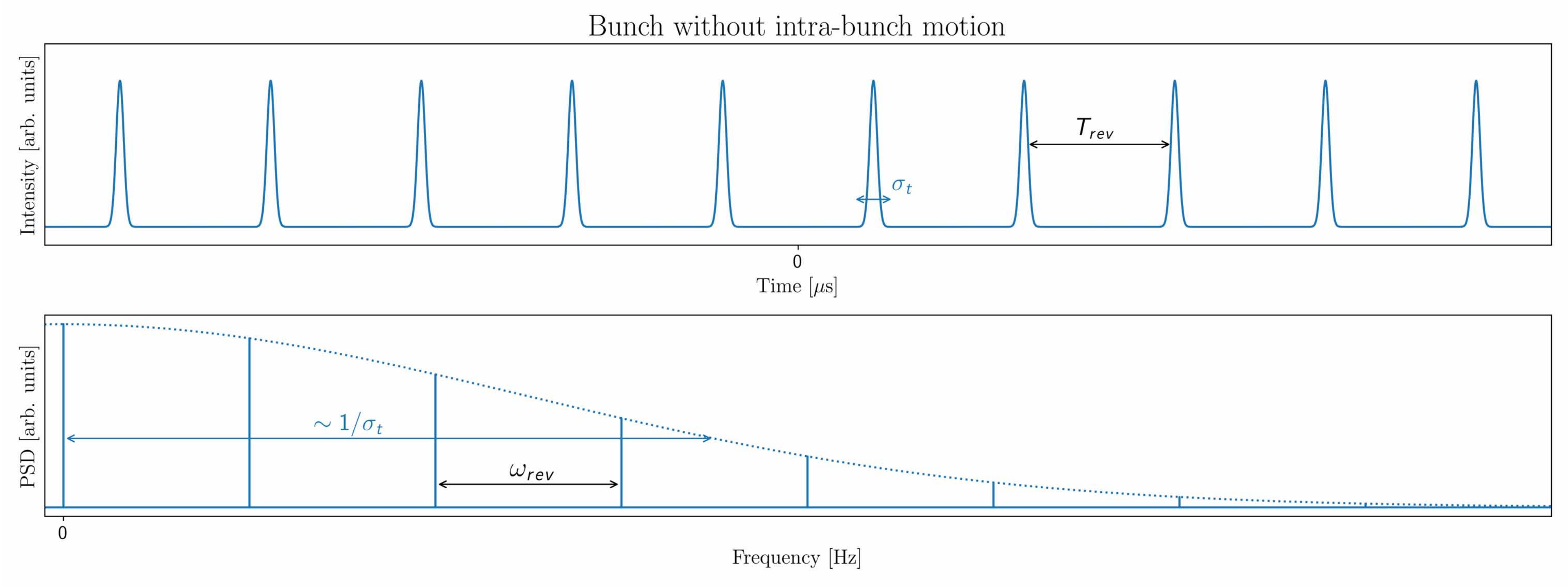
Analysis of the Transverse Schottky Signals in the LHC

Kacper Łasocha, Diogo Alves, CERN Beam Instrumentation Group

14.09.2023, Saskatoon, IBIC'23

Schottky signals

- Fluctuations of the macroscopic beam signal due to **discrete motion (synchrotron or betatron)** of individual particles within the bunch



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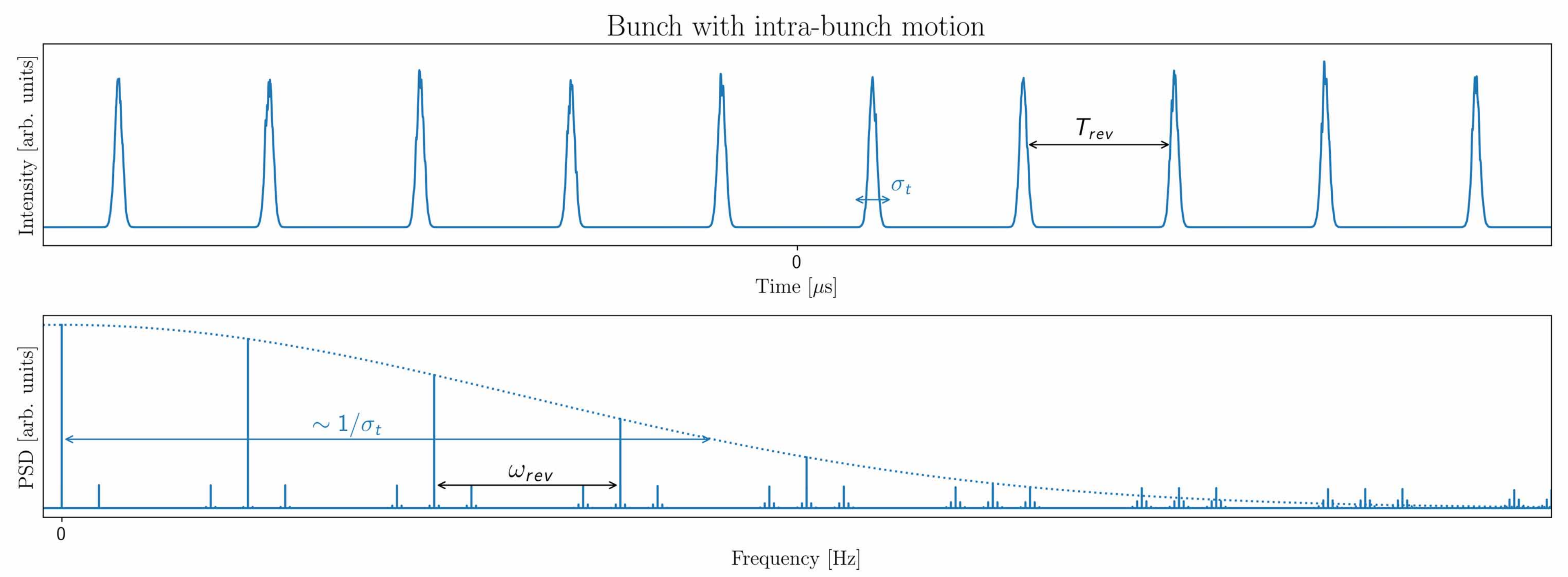

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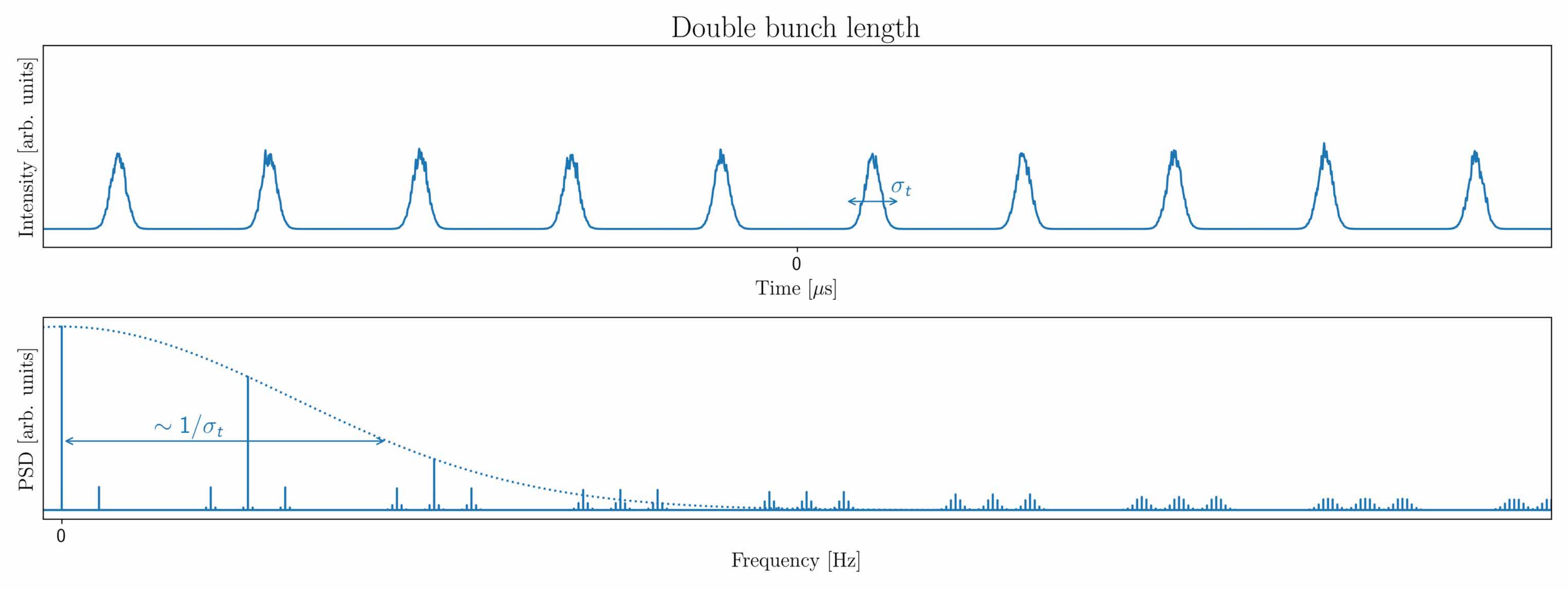

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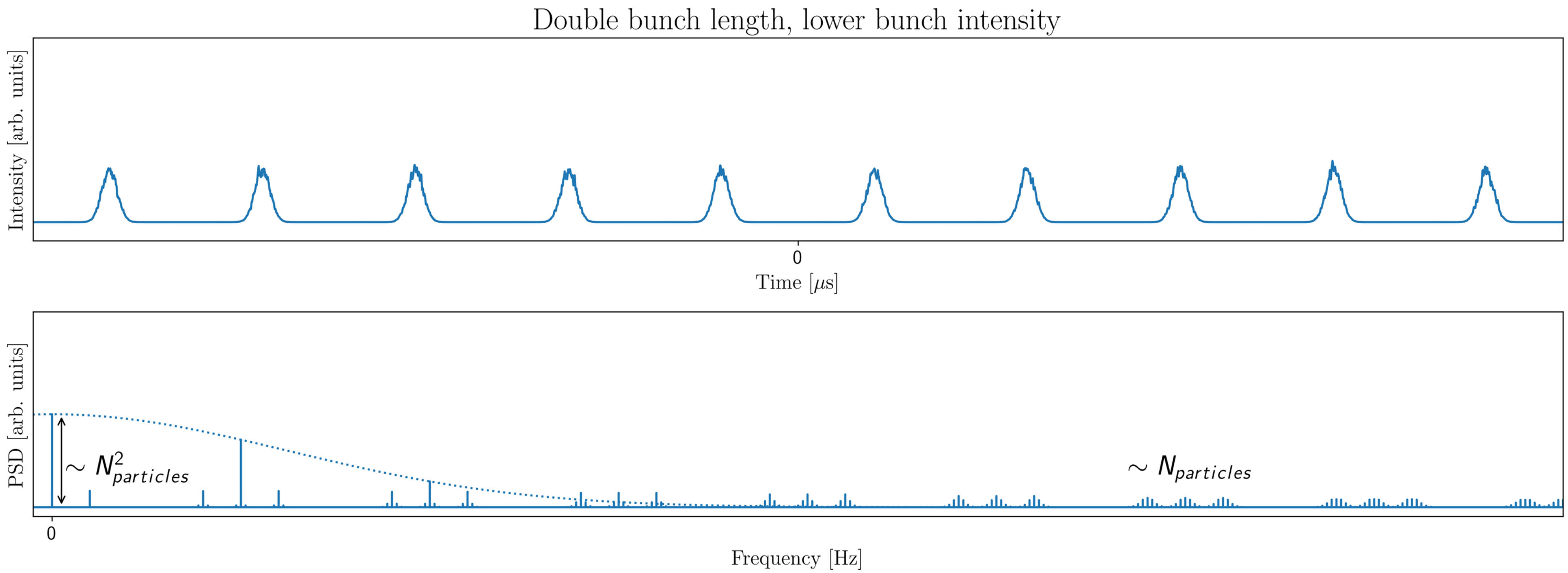
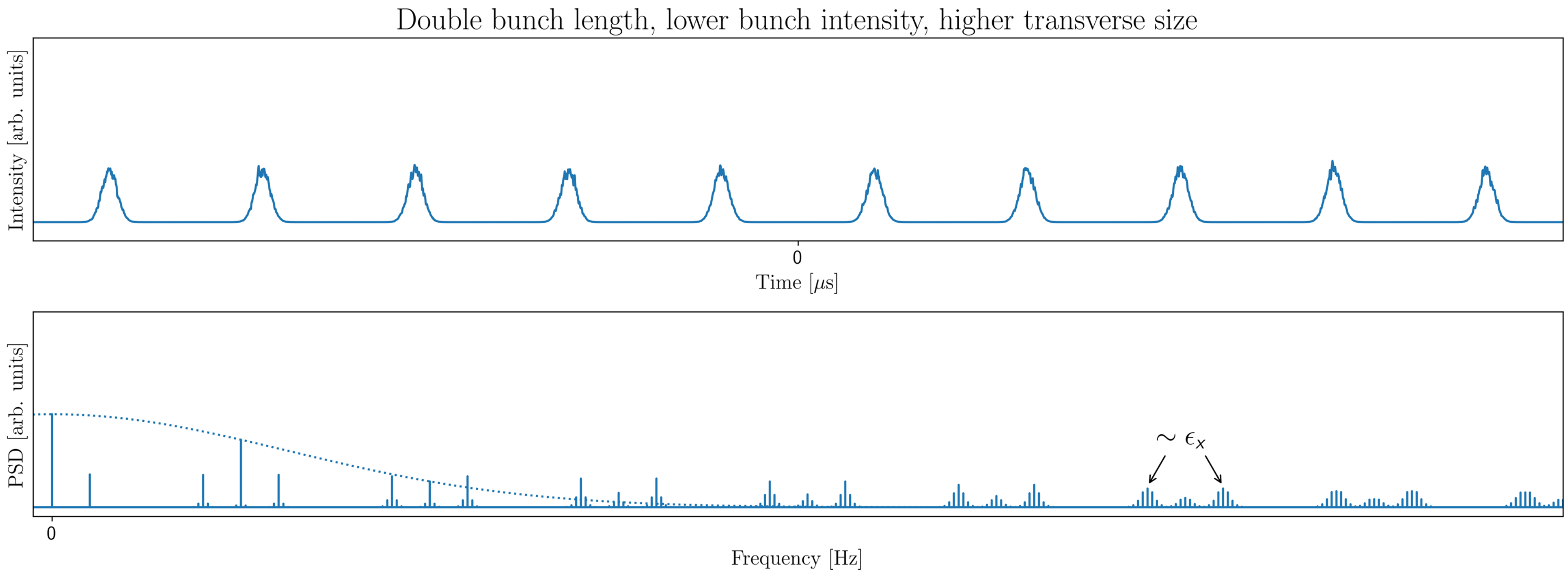
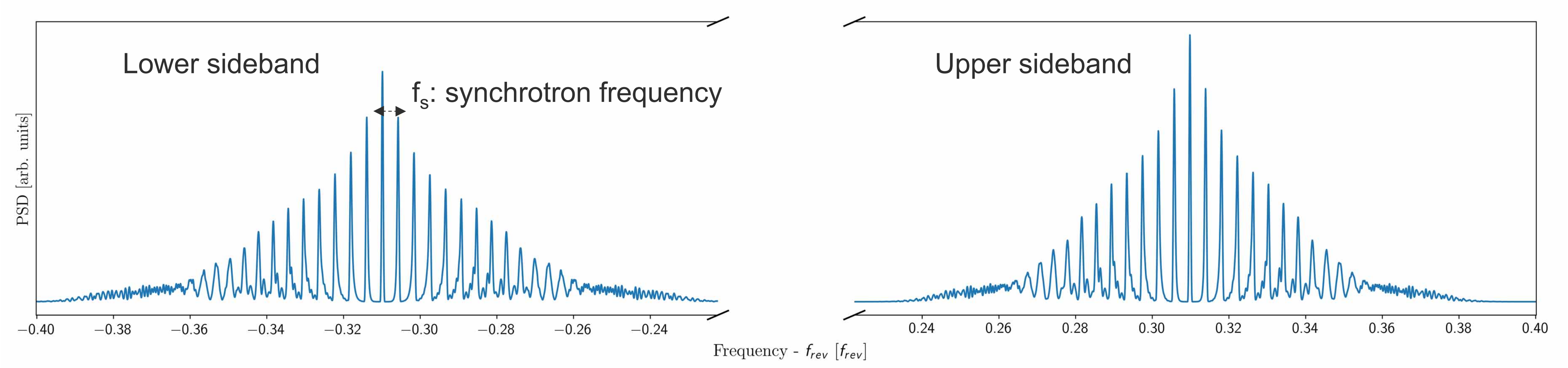


Schottky signals

- Fluctuations of the macroscopic beam signal due to **discrete motion (synchrotron or betatron)** of individual particles within the bunch
- Most pronounce for **long, low intensity, transversly large** bunches



Transverse Schottky signals



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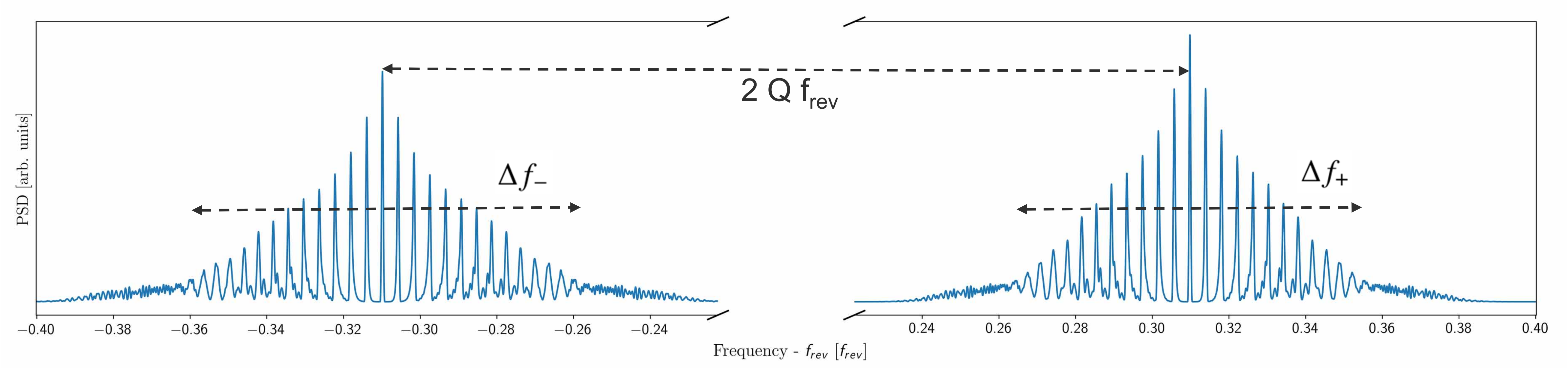


Betatron tune

Mirrored difference method, minimize the cost function:

$$C_{MD}(k) = \sum_{i=1}^{i=N} \left| P_T^{\pm}(\omega_{k-i}) - P_T^{\pm}(\omega_{k+i}) \right|$$

Chromaticity

$$Q\xi = -\eta \left(n \frac{\Delta f_- - \Delta f_+}{\Delta f_- + \Delta f_+} - Q_I \right)$$

$\Delta f_\pm$: RMS width of upper/lower sideband

In certain conditions +/- signs flip, see
K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 25, 062801

Transverse Schottky signals: assumptions

Synchrotron and betatron motion

Synchrotron motion – harmonic, with amplitude dependent frequency:

$$\tau_i(t) = \widehat{\tau}_i \sin\left(\Omega_{s_i} t + \varphi_{s_i}\right)$$

$$\Omega_{s_i} = \frac{\pi}{2\mathcal{K}\left[\sin\left(\frac{h\omega_0\widehat{\tau}_i}{2}\right)\right]}\Omega_{s_0}$$

Betatron motion – harmonic, with frequency modulated wrt to the momentum:

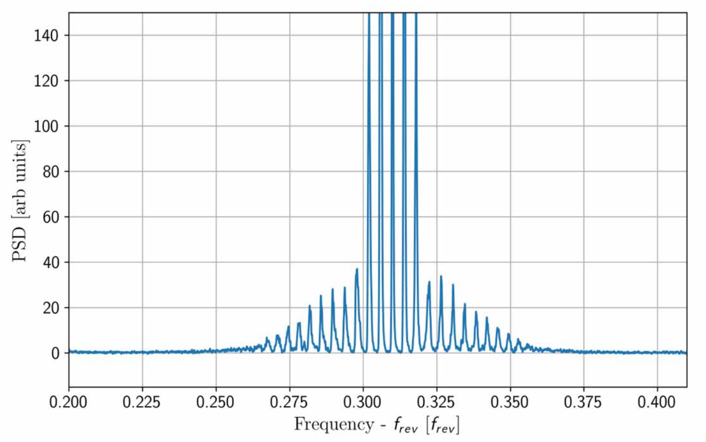
$$x_i(t) = \widehat{x}_i \cos\left[Q\omega_0 t + \frac{\widehat{Q}_i\omega_0}{\Omega_{s_i}}\sin\left(\Omega_{s_i} t + \varphi_{s_i}\right) + \varphi_{\beta_i}\right]$$

$$\widehat{Q}_i = Q\xi\frac{\widehat{p}_i}{p_0}$$

Uniform distribution of phases; no "coherent" components

Uniform distribution of φ_{s_i} and φ_{β_i} implies PSD proportional to the number of particles N.

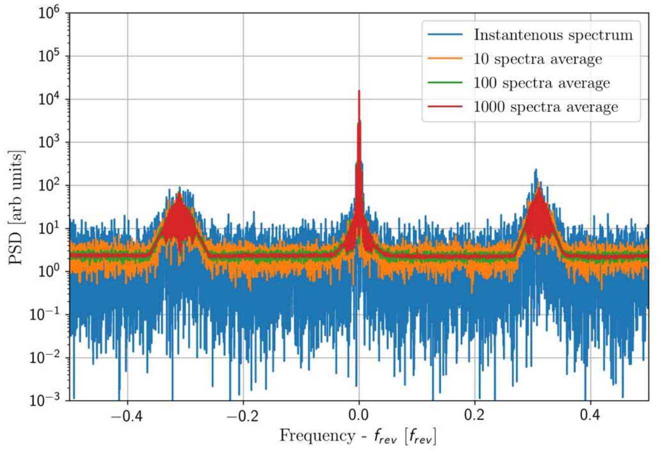
Otherwise the power can be proportional to N^2:



Sufficiently long time averaging

The theory predicts only the expected, ensemble averaged spectrum. Time averaging required to have a correspondence.

Analyzed LHC spectra are averaged for 100 s.



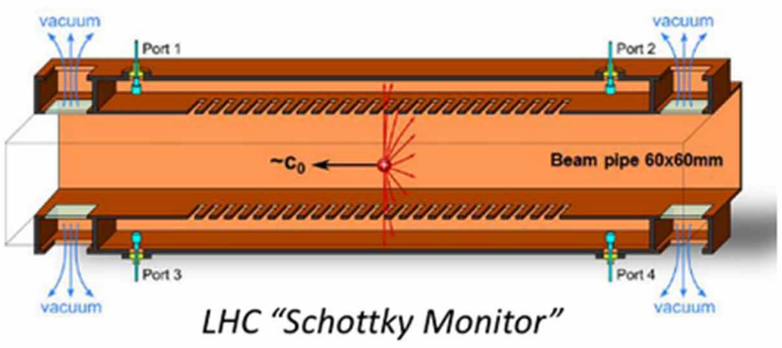
See C. Lannoy et al., WEP035, this conference.

Schottky signals in LHC

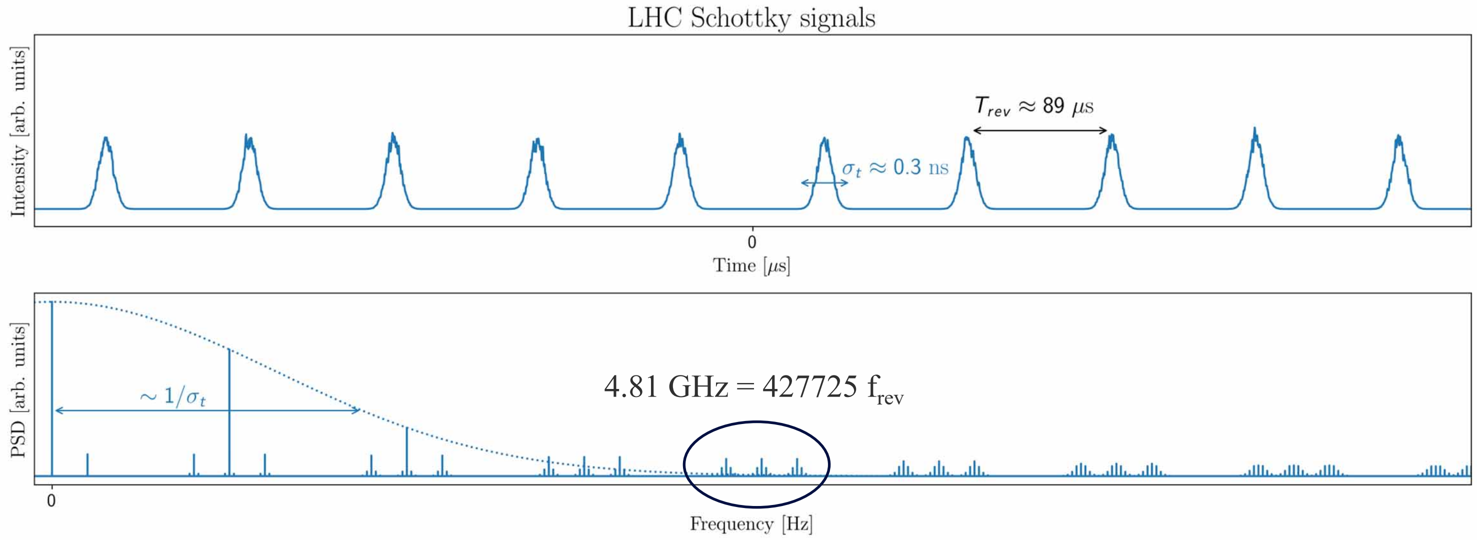
- One system for two particle species: protons and Pb^{82+} ions, one device per beam and plane
- Pair of slotted waveguides, probing beam field at 4.81 GHz, filtering and downmixing signal to 11.2 kHz
- Gating system enables observation of single bunches
- **The only instrument measuring the LHC chromaticity in the non-invasive way**

Typical LHC beam parameters:

	p^+	Pb^{82+}
$N_{particles}$ (per bunch)	10^{11}	10^{8}
Bunch length (4σ)	1-1.4 ns	
Normalized transverse emittance	1.5-2.5 μm	
Energy Inj/Flattop (per nucleon)	0.45 - 6.8 TeV	0.18 - 2.6 TeV

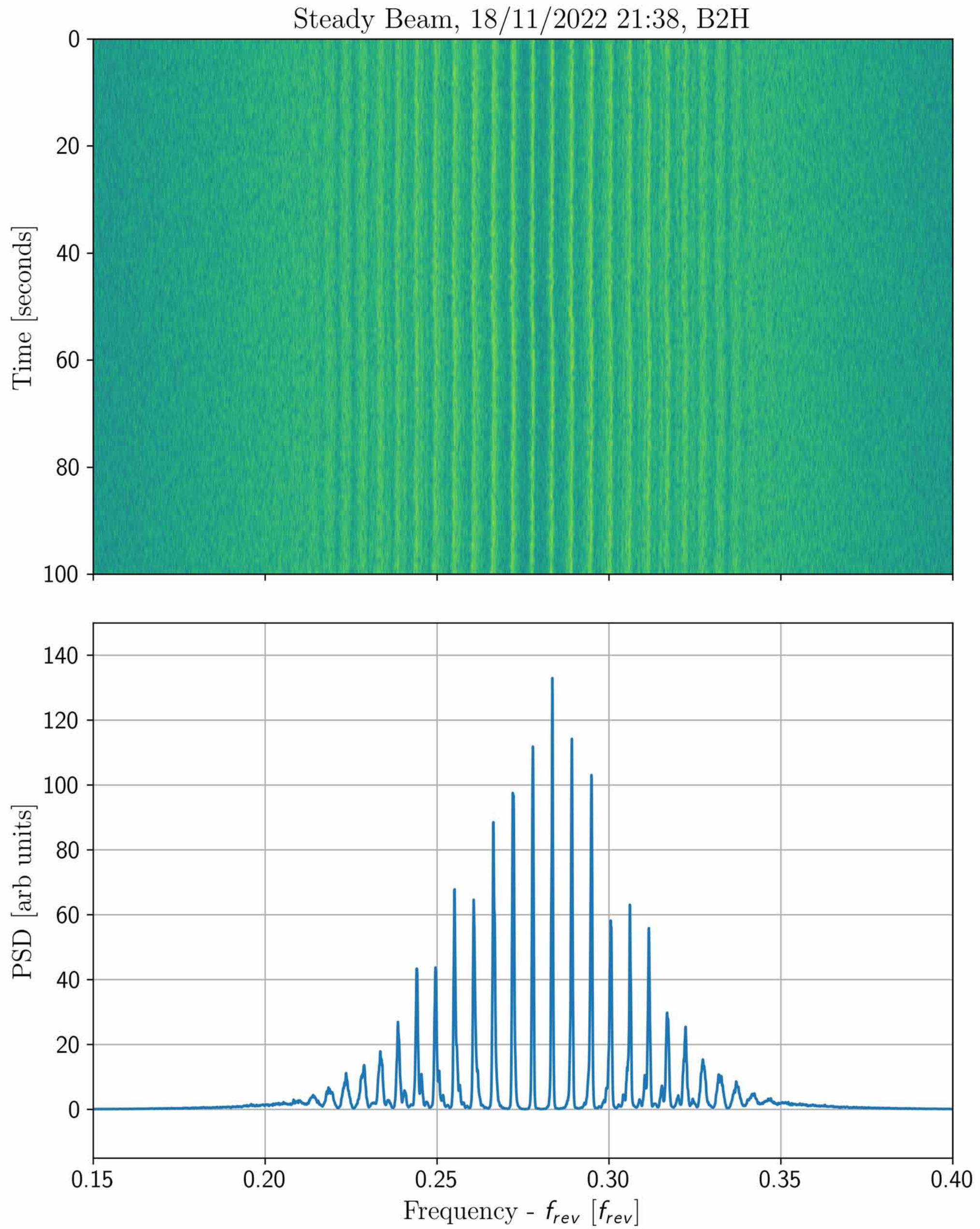


Details on the LHC Schottky system in
M. Betz et al., NIM, vol. 874, pp 113-126, 2017



Schottky spectra examples

I. **Steady beam conditions:**
  - Mostly at flattop energy of ion fills, shorter periods at flatbottom
  - Easy to analyse: just use the theory



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   - Caused by residual coherence, beam parameter mesurements, beam interaction with the surrounding
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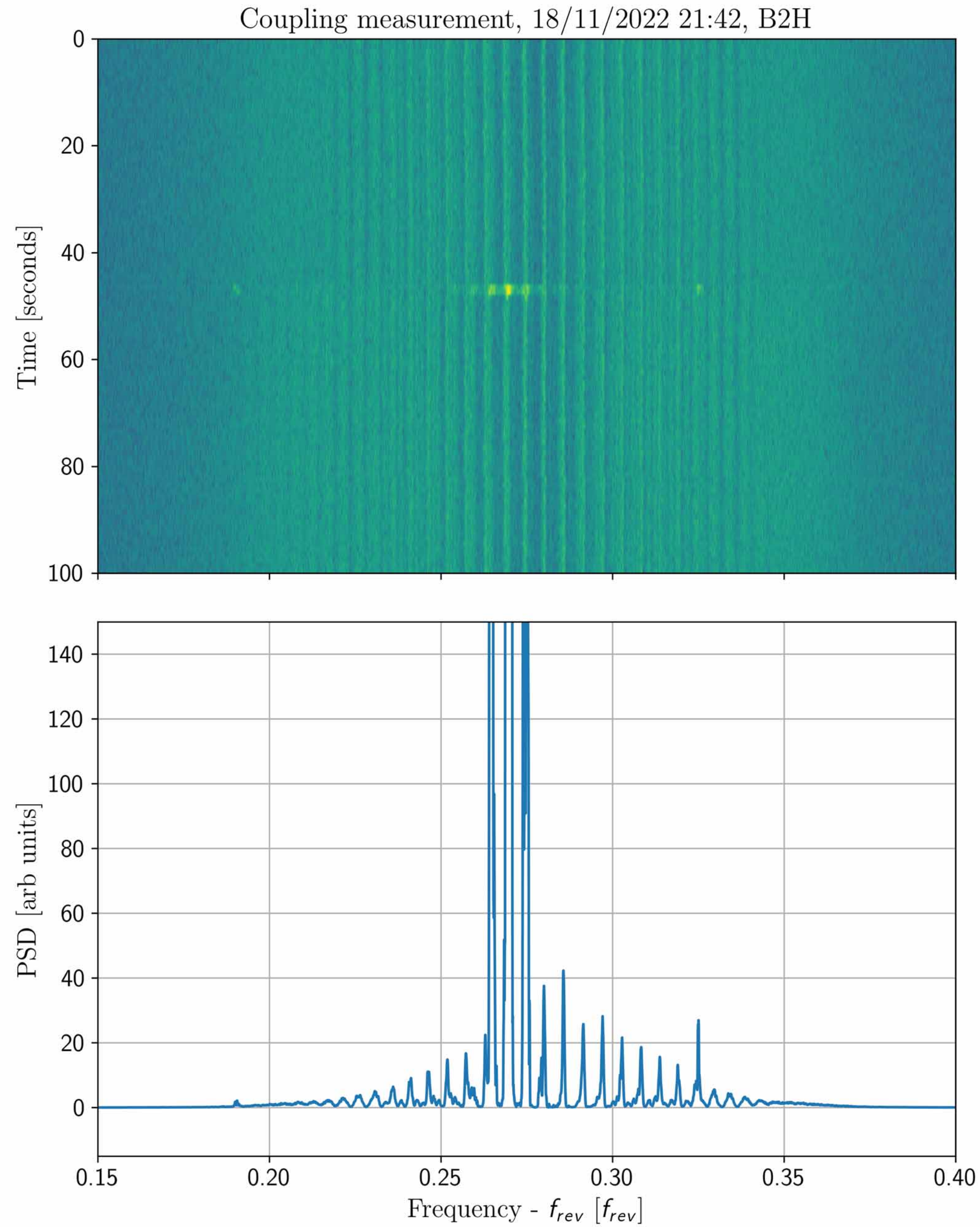

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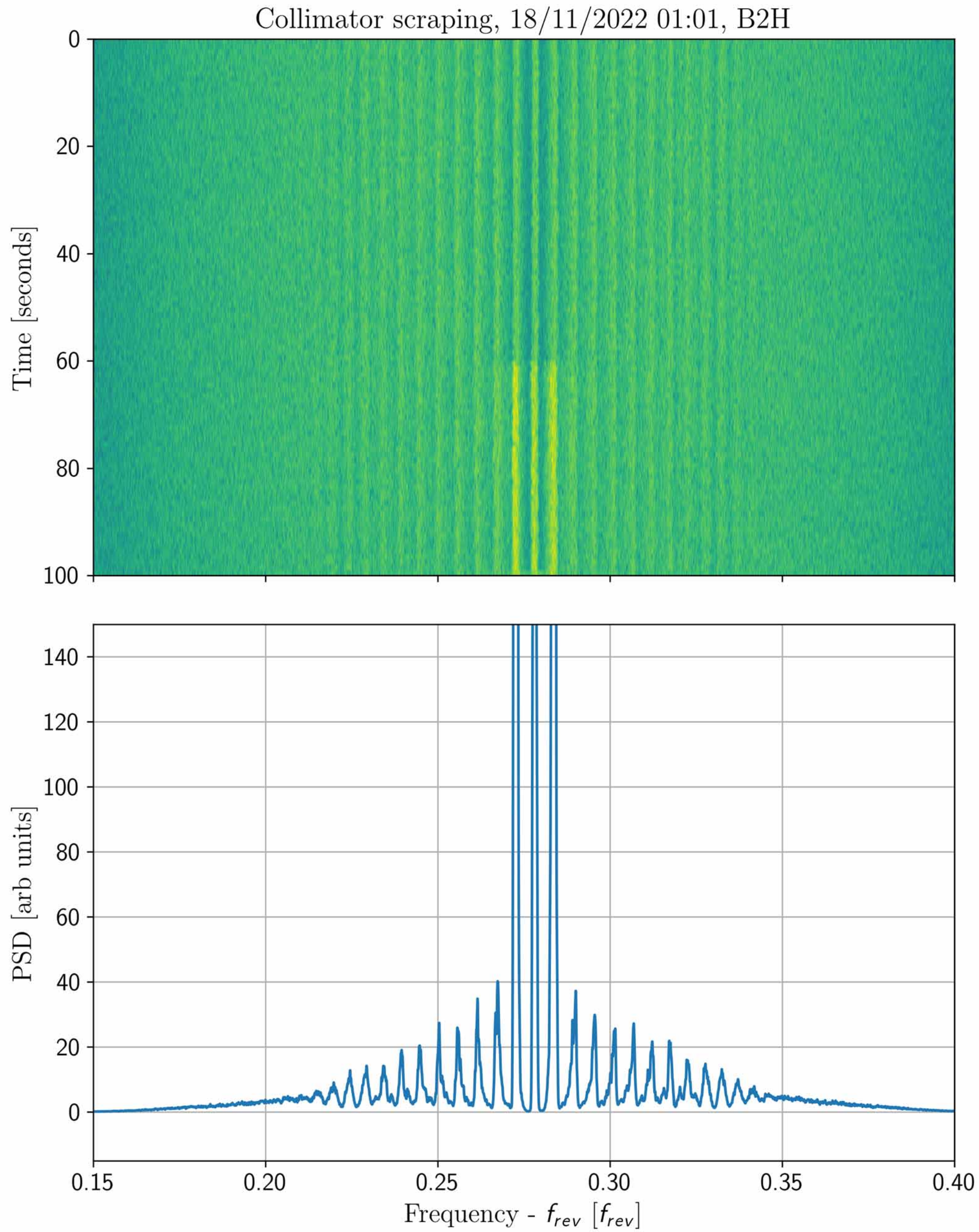

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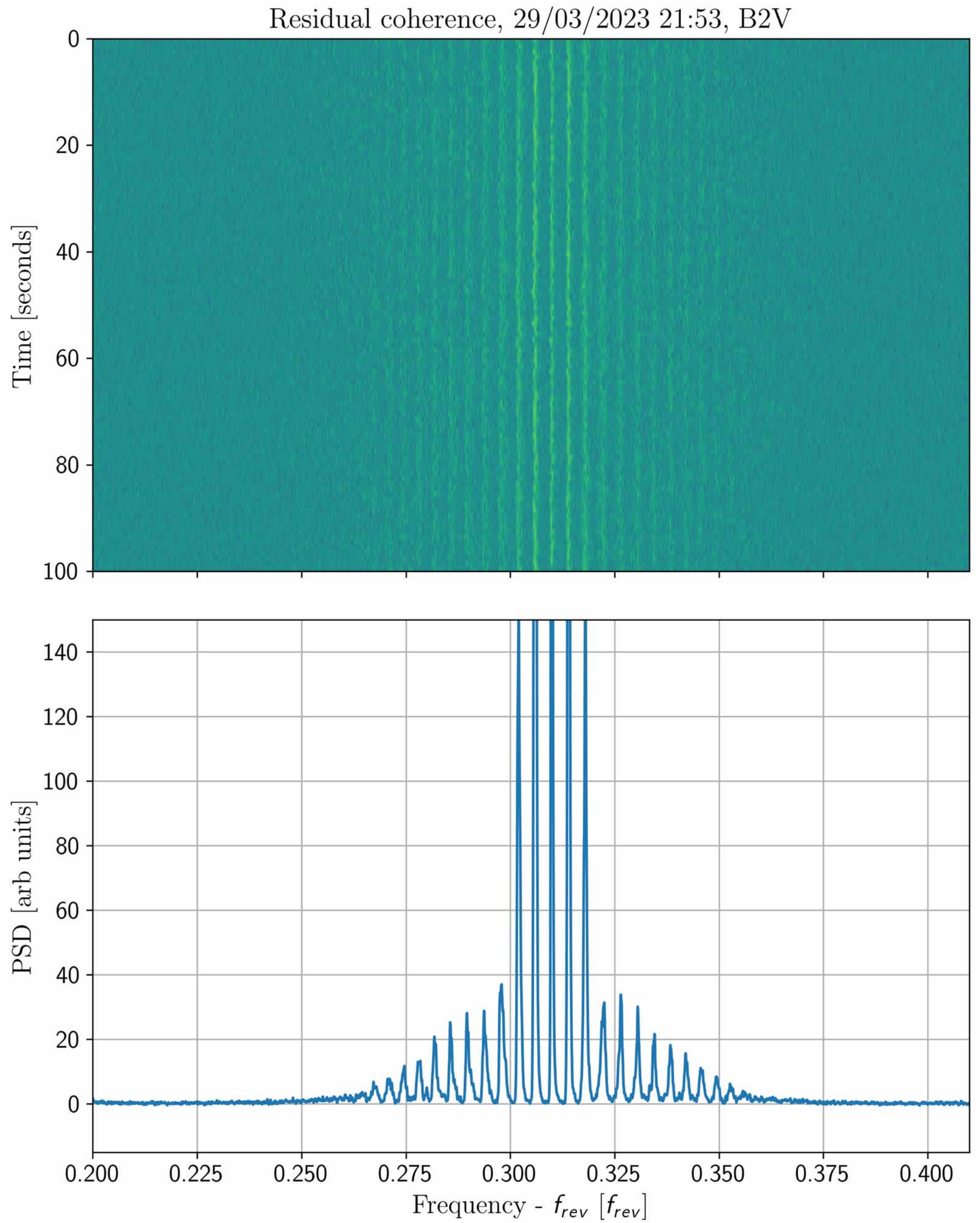

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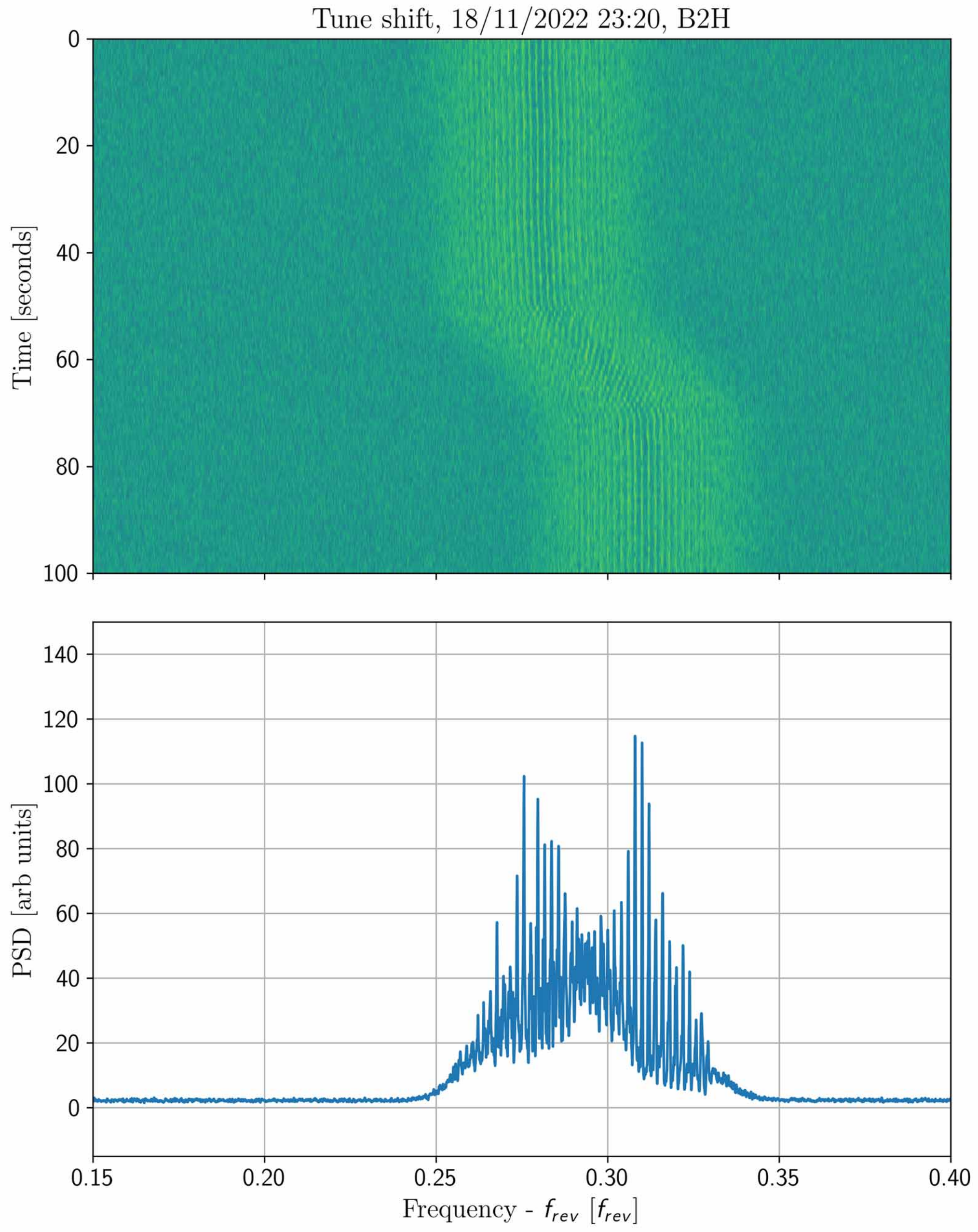
III. **Transient effects:**
   - Tune shifts, RF modulation, energy ramp
   - Theory cannot be directly used to averaged spectra, but spectograms are easy to read



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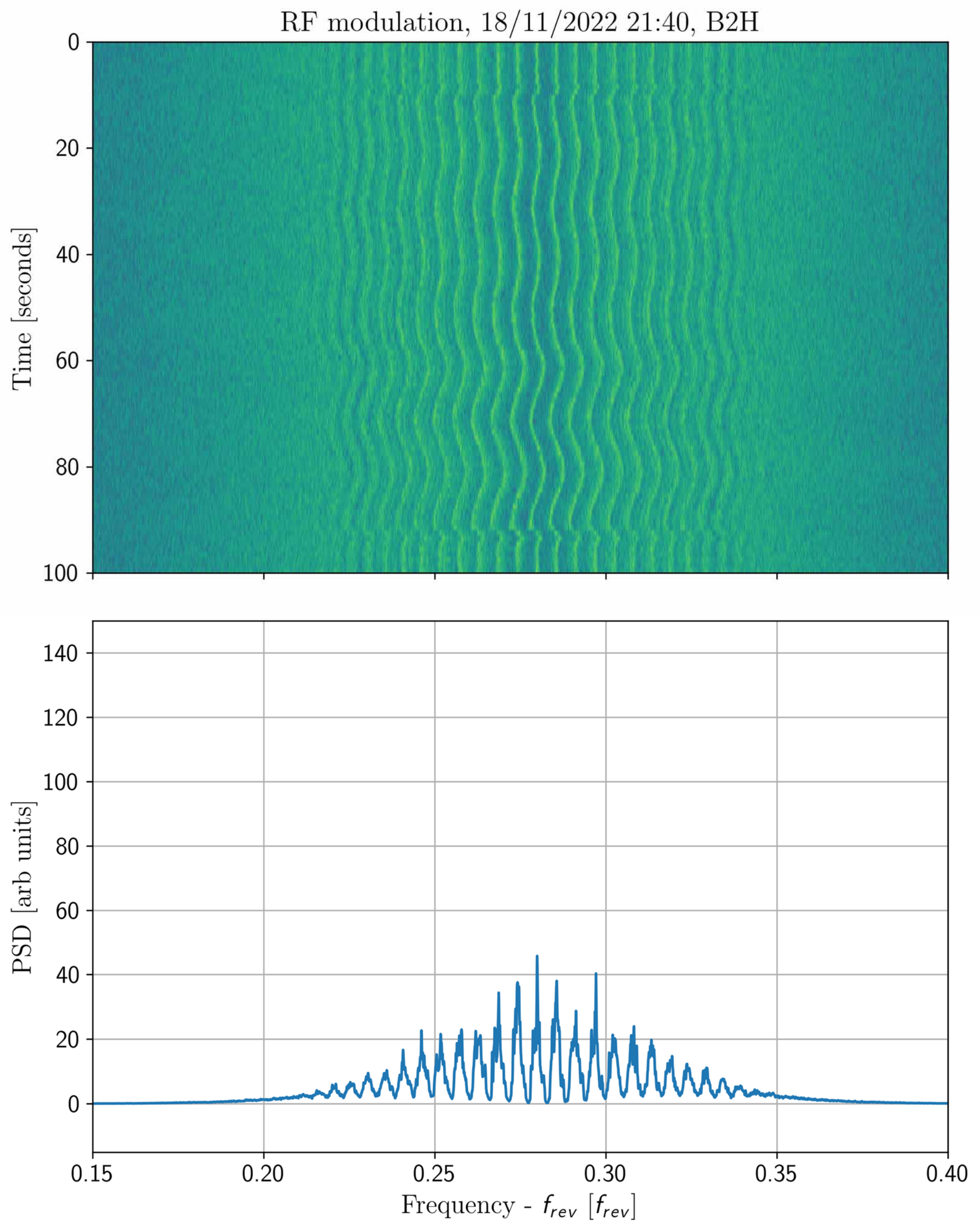

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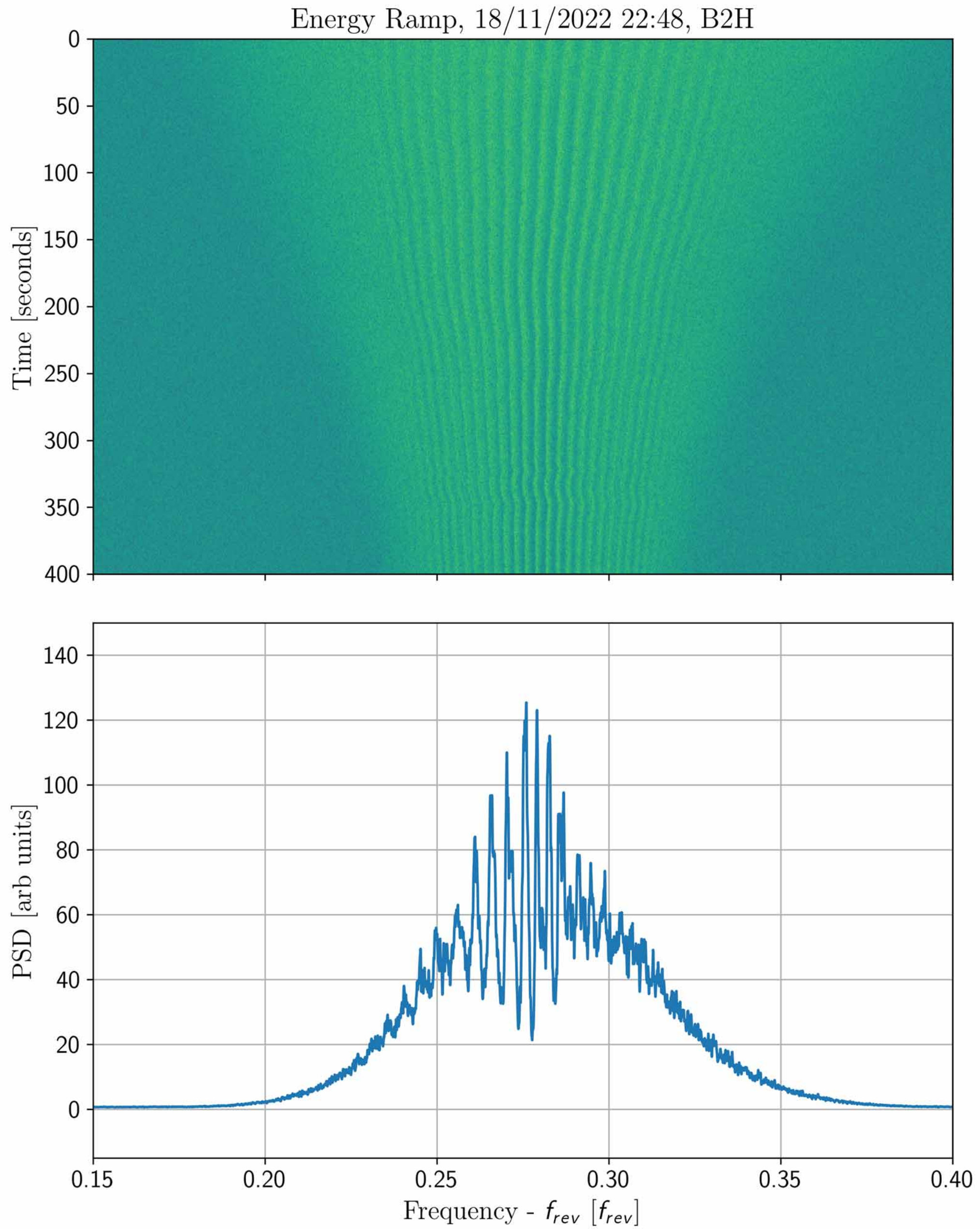

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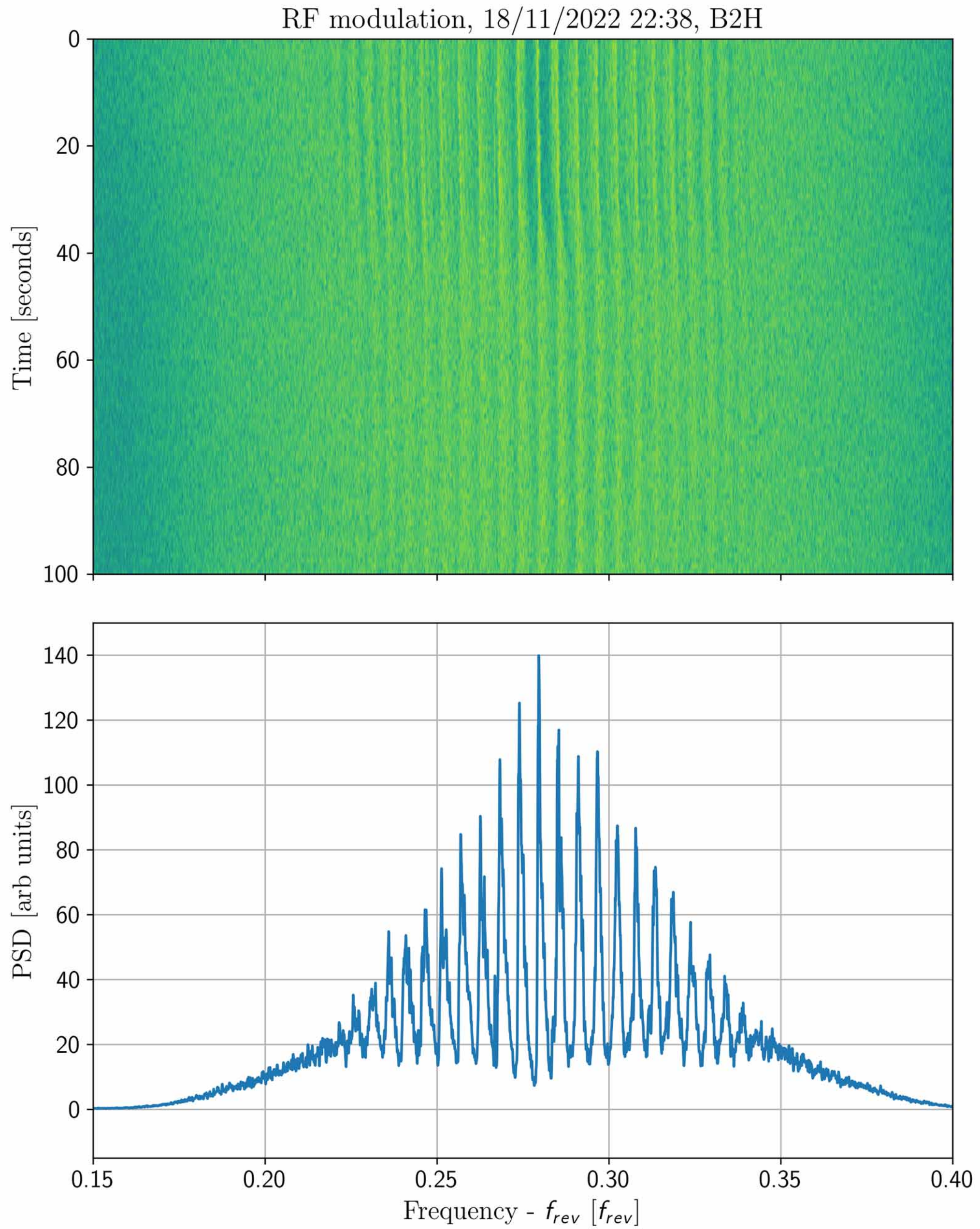
IV. Beyond current theory effects:

- Octupole magnets, betatron coupling, impedance, all intrument problems
- Theory to analyse such spectra is still to be developed



Schottky spectra examples

I. Steady beam conditions: **Analysis techniques derived**

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II. Local distortions:

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III. Transient effects: **Under development**

- Tune shifts, RF modulation, energy ramp
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IV. Beyond current theory effects:

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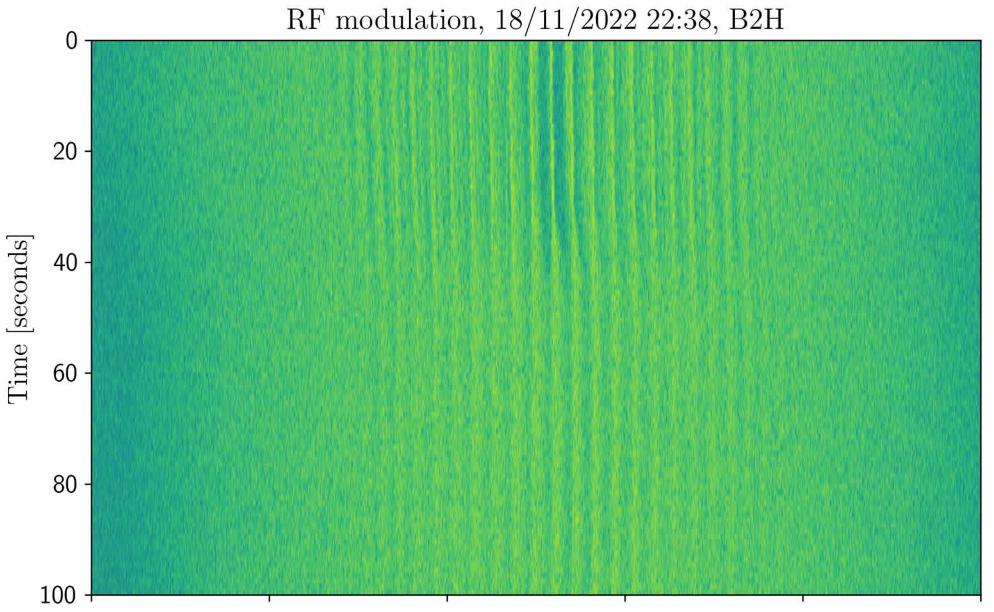
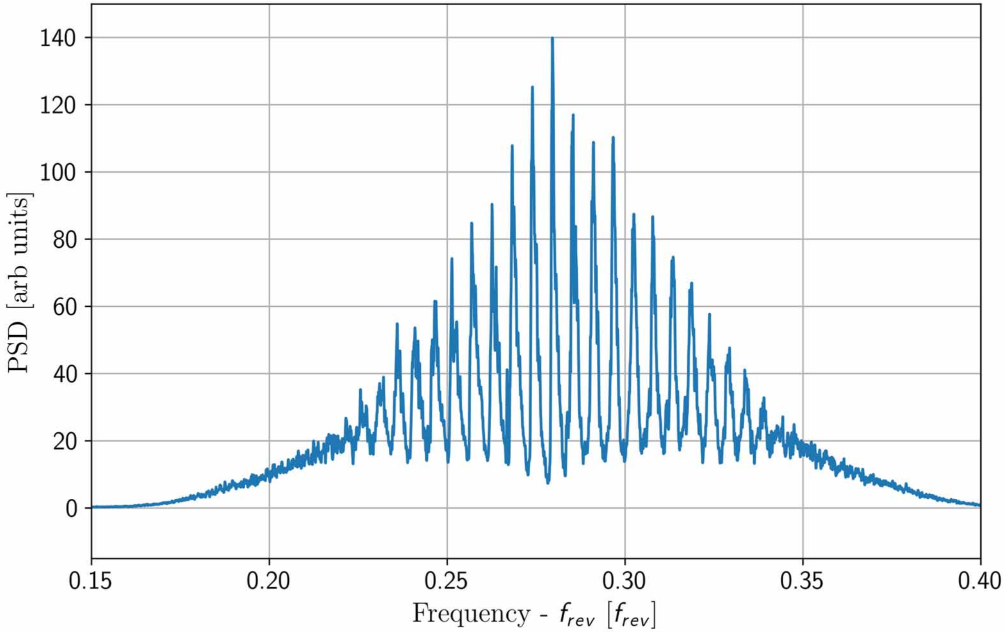
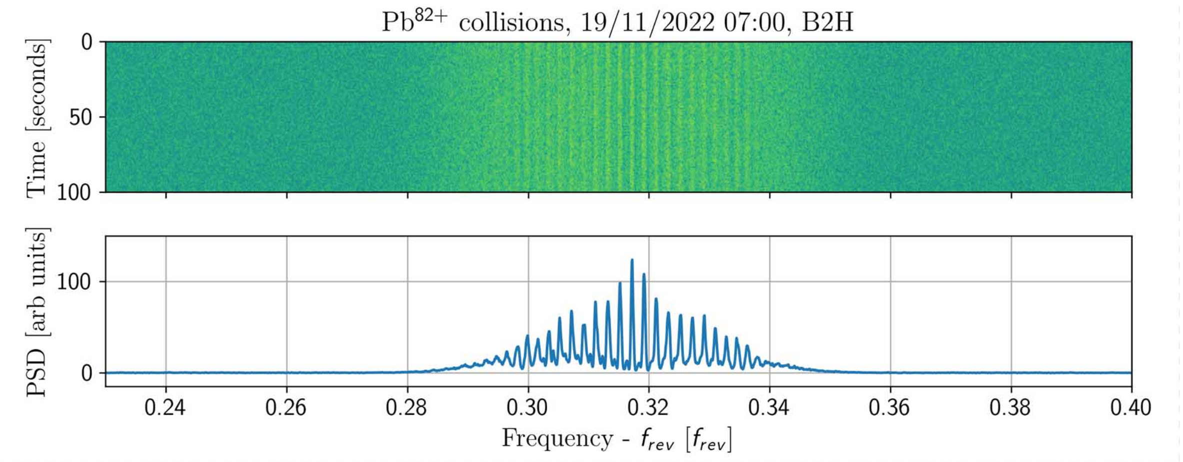




Signal analysis in stable conditions

Example: 8 hour long ion collisions in Nov 2022



Betatron tune

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Chromaticity

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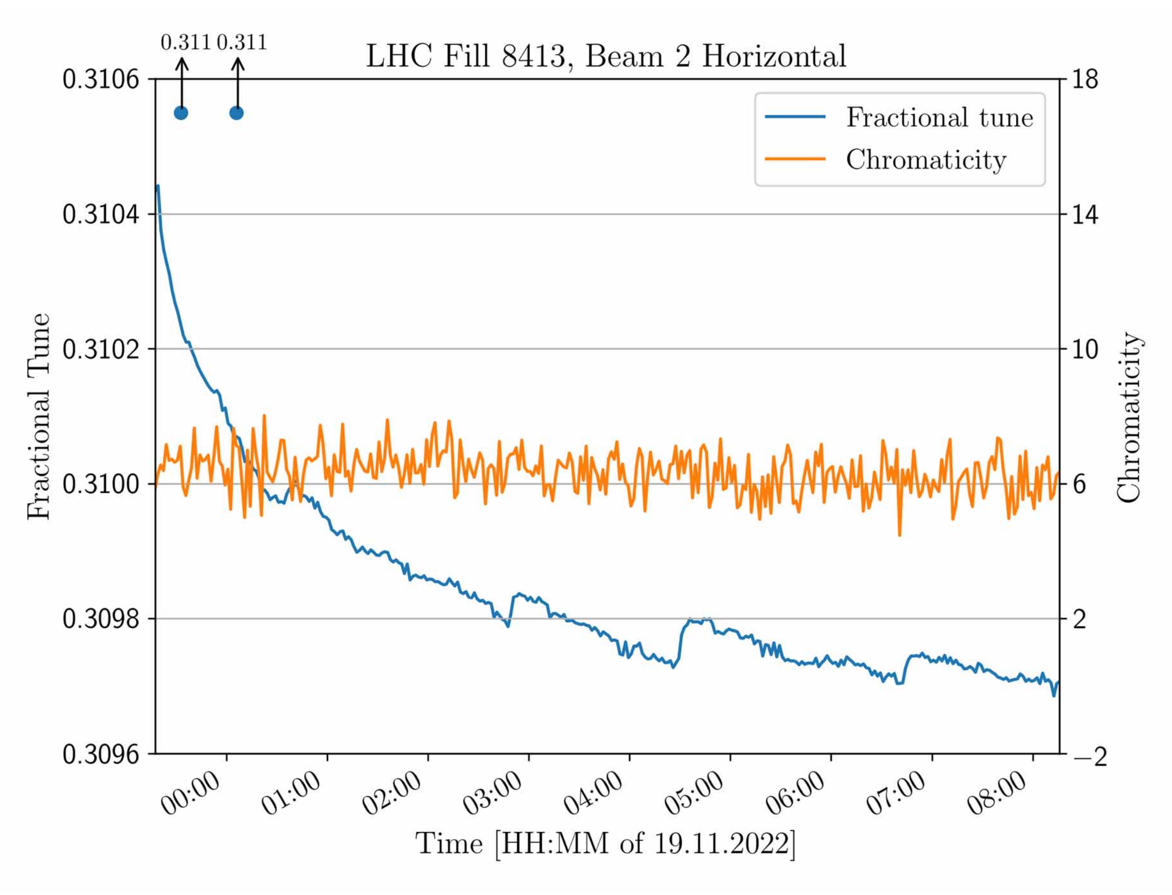
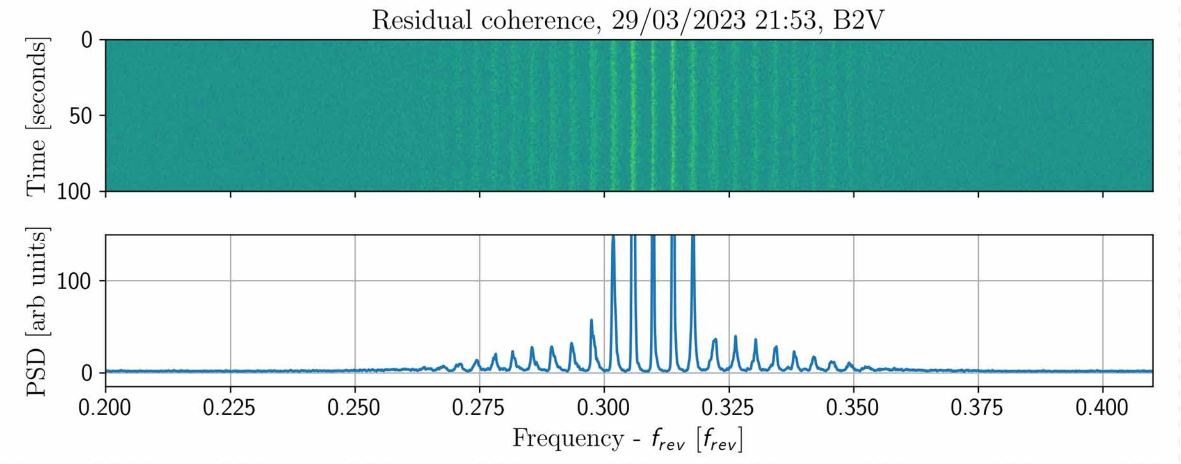


Signal analysis in presence of local distortions

Example: Early proton fill in March 2023



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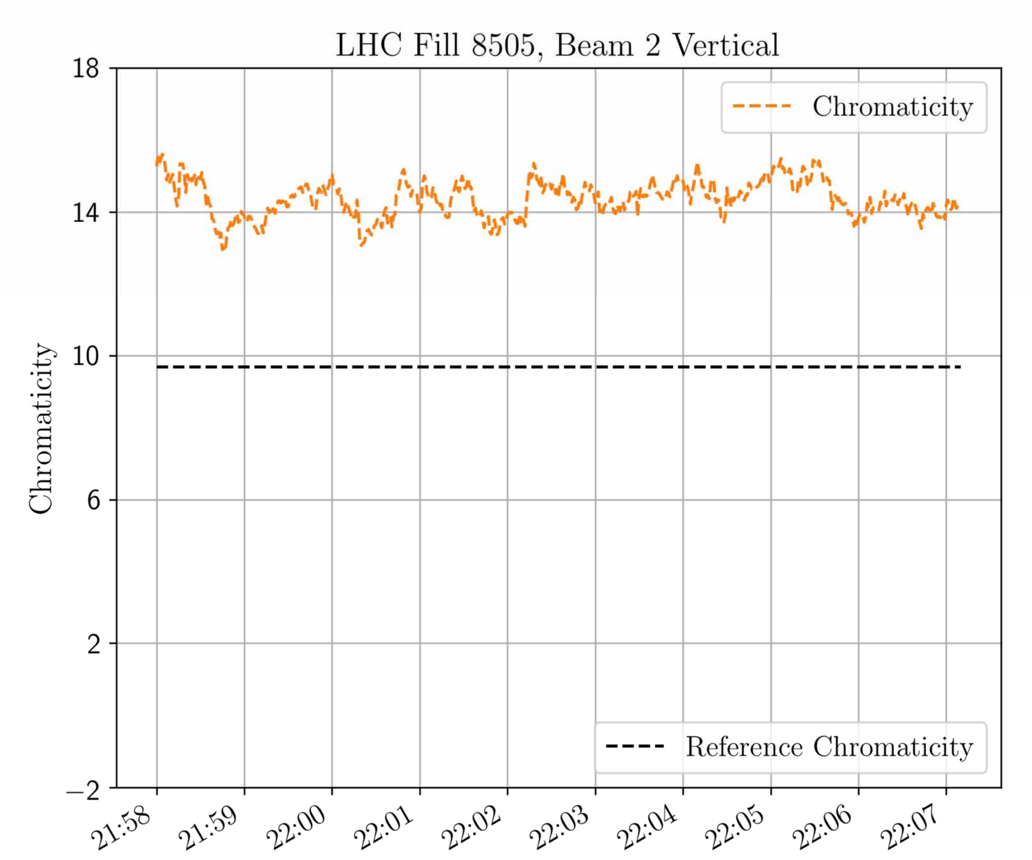


Offset of over 4 units...

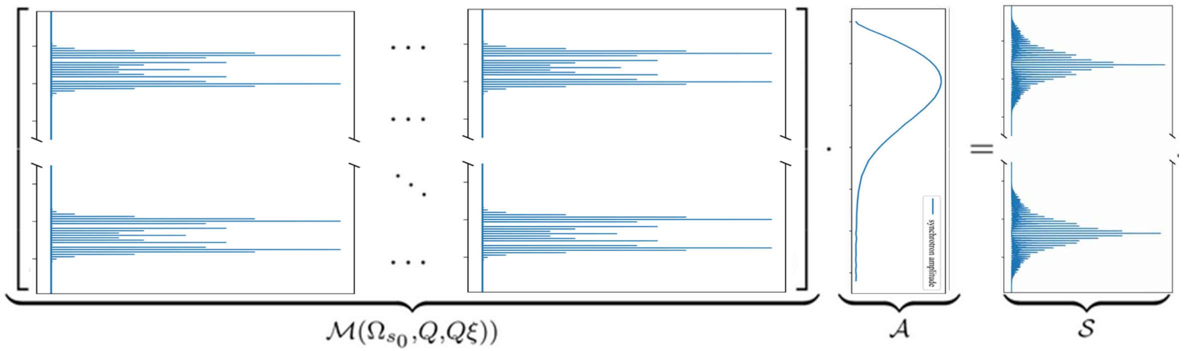
Matrix formalism for Schottky spectra

Mathematically, Schottky spectra are given as a function of:

- Synchrotron amplitude distribution - 2 parameters
- Nominal synchrotron frequency - 1 parameter
- Betatron tune - 1 parameter
- Chromaticity - 1 parameter

For given parameters the spectrum can be calculated with a simple matrix transform.

$$\underbrace{\begin{bmatrix} P_T^{\pm}(\omega_1,\hat{\tau}_1,\Omega_{s_0},Q,Q\xi) & \cdots & P_T^{\pm}(\omega_1,\hat{\tau}_n,\Omega_{s_0},Q,Q\xi) \\ P_T^{\pm}(\omega_2,\hat{\tau}_1,\Omega_{s_0},Q,Q\xi) & \cdots & P_T^{\pm}(\omega_2,\hat{\tau}_n,\Omega_{s_0},Q,Q\xi) \\ \vdots & \ddots & \vdots \\ P_T^{\pm}(\omega_m,\hat{\tau}_1,\Omega_{s_0},Q,Q\xi) & \cdots & P_T^{\pm}(\omega_m,\hat{\tau}_n,\Omega_{s_0},Q,Q\xi) \end{bmatrix}}_{\mathcal{M}(\Omega_{s_0},Q,Q\xi))} \cdot \underbrace{\begin{bmatrix} \tilde{g}(\hat{\tau}_1) \\ \tilde{g}(\hat{\tau}_2) \\ \vdots \\ \tilde{g}(\hat{\tau}_n) \end{bmatrix}}_{\mathcal{A}} = \underbrace{\begin{bmatrix} P_T^{\pm}(\omega_1) \\ P_T^{\pm}(\omega_2) \\ \vdots \\ P_T^{\pm}(\omega_m) \end{bmatrix}}_{\mathcal{S}} .$$



Details in: K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 23, 062803
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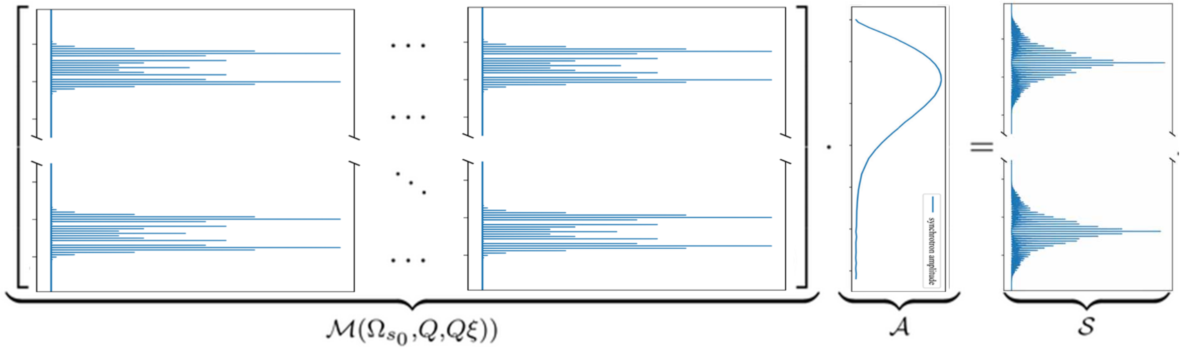
Use case 1: fast Schottky spectra simulation



Use case 2 (spectra fitting): given an experimentally measured spectrum, true parameters would minimize the cost function:

$$C\left(\Omega_{s0},Q,Q\xi,\mathcal{A}\right) = \left|\mathcal{M}\left(\Omega_{s0},Q,Q\xi\right)\cdot\mathcal{A} - \left[\mathcal{S}_{exp}\right]\right|^2$$

Minimizing routines iteratively simulate Schottky spectra and compare them with the measurement.

Details in: K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 23, 062803
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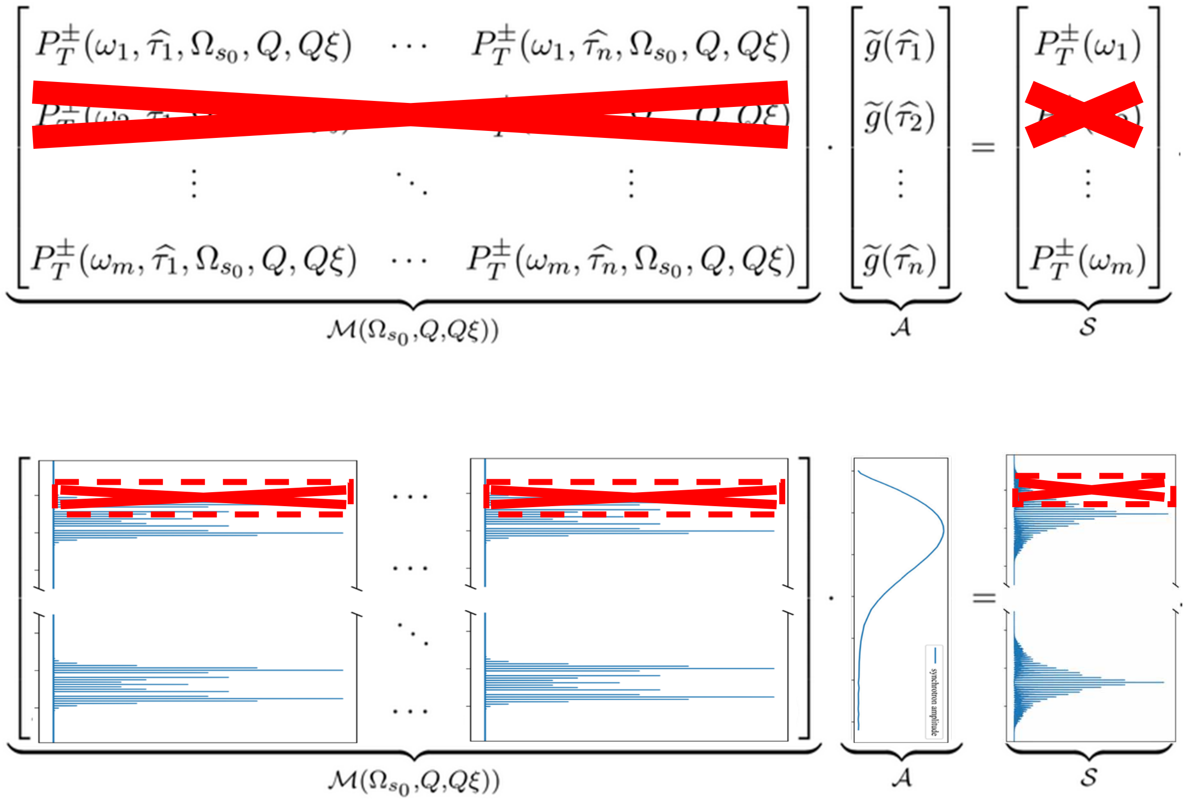
Matrix formalism: excluding frequency bins

Contrary to previous methods, fitting procedure allow to exclude the spectral regions with undesired components.



Signal analysis in presence of local distortions

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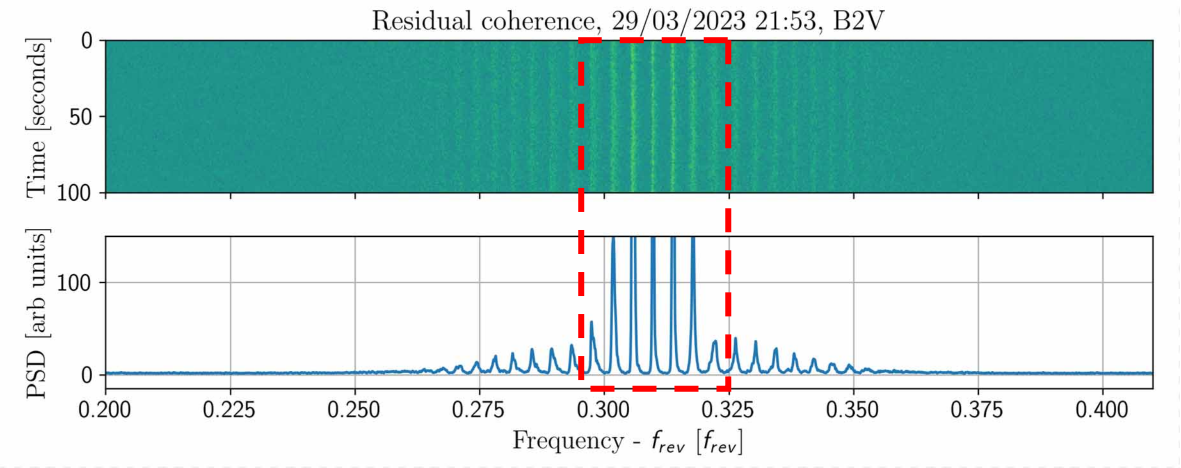

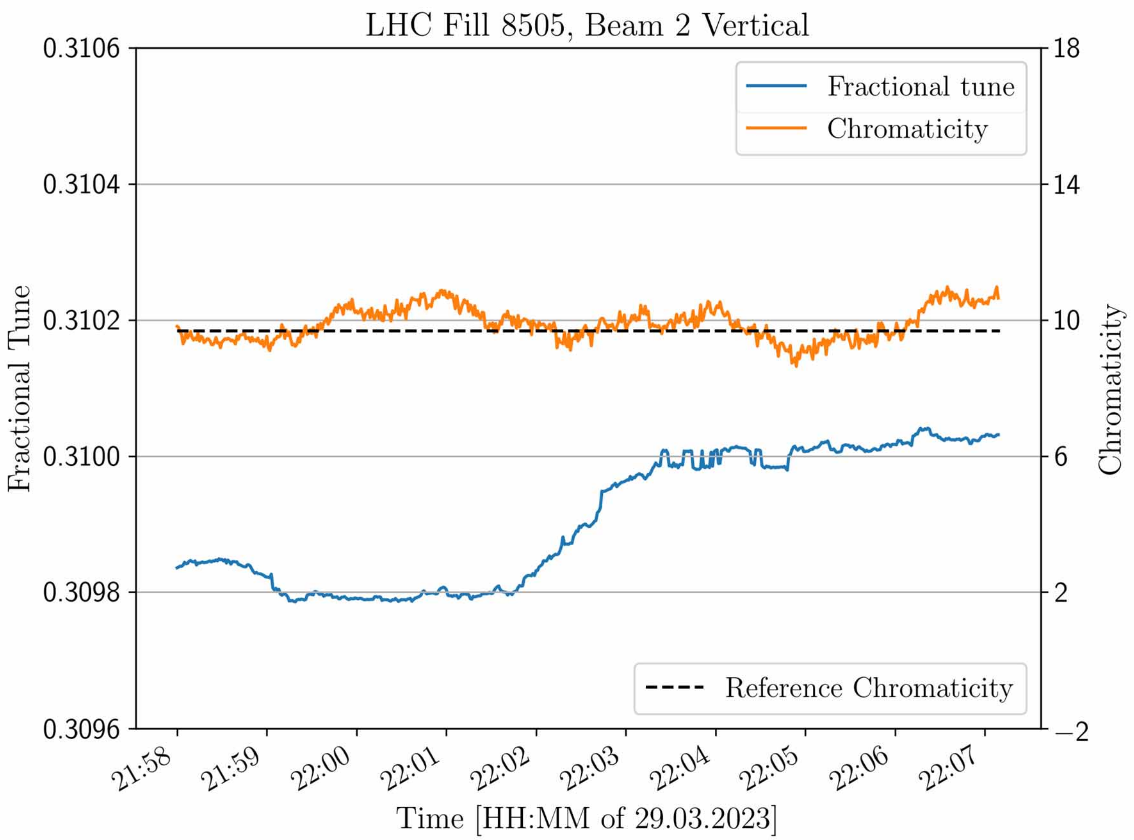

Betatron tune

$$C_{MD}(k) = \sum_{i=1}^{i=N} \left| P_T^{\pm}(\omega_{k-i}) - P_T^{\pm}(\omega_{k+i}) \right|$$

Only "valid" frequencies taken into sum

Chromaticity

$$C(\mathcal{A}, Q\xi) = |\mathcal{M}(Q\xi) \cdot \mathcal{A} - \mathcal{S}_{exp}|^2$$

Nominal synchrotron tune calculated independently,
Cost function minimization using Differential Evolution algorithm.

LHC Schottky online signal analysis pipeline

1. Obtain a Schottky spectrum
2. Get the distribution of **synchrotron amplitudes** from longitudinal bunch profile — *K. Lasocha, D. Alves MOCO03, IBIC'19*
3. Determine "valid" frequencies: — *C. Lannoy et al., WEP035, IBIC'23*
   1) Coefficient of variation
   2) Correlation between planes
4. Calculate nominal $\mathbf{f_s}$ based on RF voltage and longitudinal Schottky spectrum — *K. Lasocha, D. Alves MOCO03, IBIC'19*
5. Calculate **betatron tune** using MD algorithm
6. Determine **chromaticity** by fitting the transverse spectrum

Implementation in the final stage of development, planned be in use in the end of 2023

Conclusions

Summary of the talk:

- Theory of transverse Schottky spectra reviewed and succesfully applied to **stable beams**,
- **Local spectral distortions** mitigation technique proposed and demonstrated on proton spectra,
- Automation of the analysis will be tested in the coming days.

Possible next steps:

- Handling of **transient effects**: 1D ---> 2D analysis; image recognition techniques?
- Expanding the theory of Schottky spectra: **impedance**, **octupoles**, ... **See C. Lannoy et al., WEP034, IBIC'23**

Thanks for your attention!