Diagnostics with Quadrupole Pick-Ups

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IBIC'22, Krakow, Poland
13 September 2022

Abstract

The beam quadrupole moment of stored beams can be measured with a four plate quadrupole pick-up. The frequency spectrum of the quadrupole moment contains not only the usual first-order dipole modes (the betatron tunes) but also the second-order coherent modes, comprising of

(1.) *(even) normal envelope modes,*
(2.) *odd (skew) envelope modes and*
(3.) *dispersion modes.*

As a novel diagnostic tool, the measured frequencies and amplitudes provide direct access to transverse space charge strength (through the tune shift) as well as linear coupling (and mismatch thereof), at the benefit of a non-invasive beam-based measurement. Technically, quadrupole moment measurements require a pick-up with non-linear positions sensitivity function. We discuss recent developments and depict measurements at the GSI SIS18 heavy-ion synchrotron.

Motivation

Measure direct space charge strength through frequency shift of beam size oscillations about matched $\sigma_{x,y}$:

1^{st} order

rigid dipolar centroid oscillation:

- Newton's third law, actio = reactio

⟶ no direct space charge (SC)
[except higher-order projections]

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rigid dipolar centroid oscillation:

- Newton's third law, actio = reactio

→ no direct space charge (SC) [except higher-order projections]

2nd order

quadrupolar envelope oscillation:

- defocused by transverse SC force

→ frequency of envelope oscillation decreases with SC

Content

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II. Newly Observed 2$^{\text{nd}}$ Order Modes

- Skew Envelope Mode
- Dispersion Mode

III. GSI SIS18 Measurements

- Quadrupole Beam Transfer Function
- Chromaticity vs. Envelope Mode

I. Overview

Quadrupole Pick-up (QPU) Schematics

Induced voltage on electrodes:

$$U_{right} \propto I_{\text{beam}} \left(1 + z_{1x} + z_2 + \ldots\right)$$
$$U_{left} \propto I_{\text{beam}} \left(1 - z_{1x} + z_2 + \ldots\right)$$
$$U_{top} \propto I_{\text{beam}} \left(1 + z_{1y} - z_2 + \ldots\right)$$
$$U_{bottom} \propto I_{\text{beam}} \left(1 - z_{1y} - z_2 + \ldots\right)$$

where

$$z_{1x} \propto \frac{\langle x \rangle}{d},$$

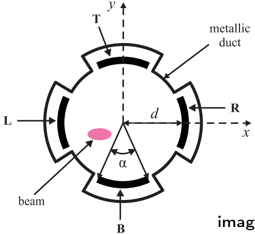

image from Ref. [1]

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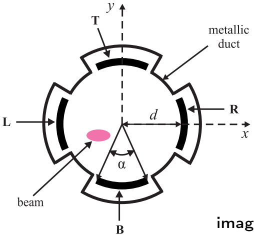

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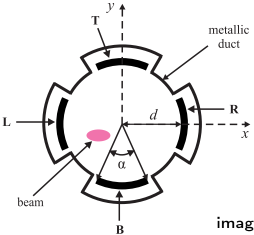

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$$z_{1x} \propto \frac{\langle x\rangle}{d},$$
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$$z_2 \propto \frac{\langle x^2\rangle - \langle y^2\rangle}{d^2} = \frac{\sigma_x^2 - \sigma_y^2 + \langle x\rangle^2 - \langle y\rangle^2}{d^2} \quad \text{(neglecting dispersion)}$$

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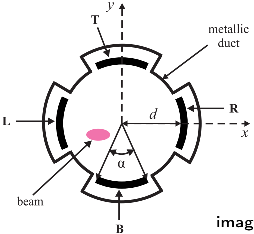

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$\Longrightarrow$ combine voltages to measure **dipolar** beam moments (usual BPM):

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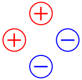

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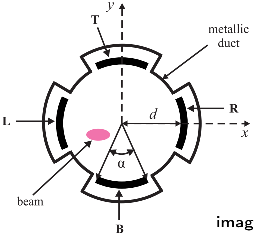

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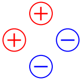
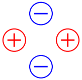


$\Longrightarrow$ or combine voltages to measure **quadrupolar** beam moments:

$$z_2 \propto \sigma_x^2 - \sigma_y^2 + \langle x\rangle^2 - \langle y\rangle^2$$
$$\propto U_{right} + U_{left} - U_{top} - U_{bottom}$$



Some Historical Perspective

Time domain for emittance measurements:

- 1983, R. H. Miller et al. at SLAC [2]
- 2002, A. Jansson at CERN in PS [3]

challenge: remove dipole parts in $z_2 \sim \langle x\rangle^2 - \langle y\rangle^2$

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Frequency domain for emittance measurements:

- 2007, C. Y. Tang at Fermilab [4]

Frequency domain for space charge measurements:

- 1996, M. Chanel at CERN in LEAR [5]
- 1999, T. Uesugi et al. at NIRS in HIMAC [6]
- 2000, R. Bär at GSI in SIS18 [7]
- 2014, R. Singh et al. at GSI in SIS18 [8]

$\Longrightarrow$ all studies for coasting beams

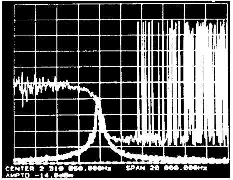

Figure: Quad-BTF [5]

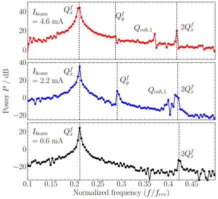

Figure: Injection Oscillations at SIS18 [8]

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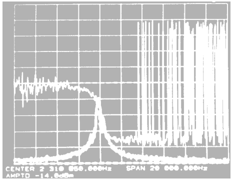

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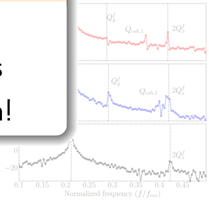

Figure: Injection Oscillations at SIS18 [8]

… but …

What about bunched beams?

- QPU as diagnostic tool: most useful in critical situations
- synchrotrons limited by space charge $\Longleftrightarrow$ bunched beam!

Bunched Beam

QPU studies with bunched beams:

- HB2018 contribution ↗
  - ⟶ quadrupole BTF measurements at CERN PS
  - ⟶ study space charge shifted envelope *bands*
  - ⟹ impact of **chromaticity** on resonance width!
  - ⟹ discovery of strong **coherent dispersion mode**!

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  - ⟹ impact of **chromaticity** on resonance width!
  - ⟹ discovery of strong **coherent dispersion mode**!
- MCBI2019 contribution ↗
  - ⟶ injection mismatch studies at CERN PS
  - ⟹ coherent dispersion mode grows with **head-tail instability**
  - ⟹ discovery of **skew envelope modes** (linear coupling)!

II. Newly Observed 2nd Order Modes

Skew Envelope Mode ↔ Linear Coupling

The beam features 2 odd (skew) eigenmodes: **2$^{\text{nd}}$ order** resonances due to linear coupling between the transverse planes (Chernin, Ref. [9, 10]).

1. low-frequency eigenmode: **difference resonance** $Q_x - Q_y$
2. high-frequency eigenmode: **sum resonance** $Q_x + Q_y$

⟶ driving terms for these originate from

  a. skew quadrupole component in optics
  b. space charge coupling in case of unequal beam sizes (e.g. $\epsilon_x \neq \epsilon_y$!)

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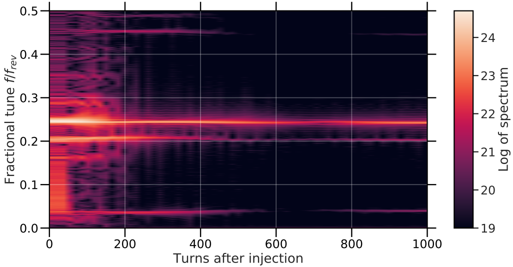
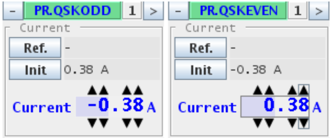
Example at CERN PS injection, $Q_x = 6.24$ and $Q_y = 6.21$:





PR.QSKODD

Ref. -

Init 0.38 A

Current -0.38 A

PR.QSKEVEN

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Figure: skew quadrupoles providing maximum coupling

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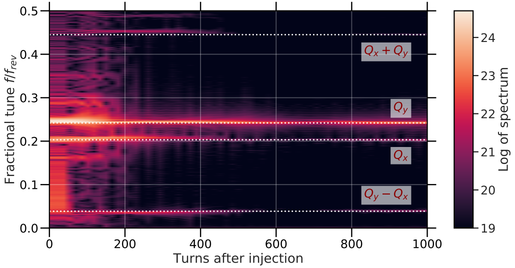

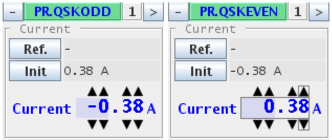

PR.QSKODD 1
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Ref. -
Init 0.38 A
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1. low-frequ
2. high-frec

Example at C

Key point!

Can be used as diagnostic tool:

- beam-based measurement of injection mismatch in case of linear coupling
- minimise amplitude in $Q_x \pm Q_y$ lines via skew quadrupoles
- in contrast to dipole BPM based methods (closest-tune approach $|C^-|$), this method includes space charge

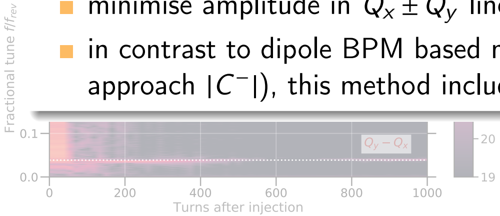
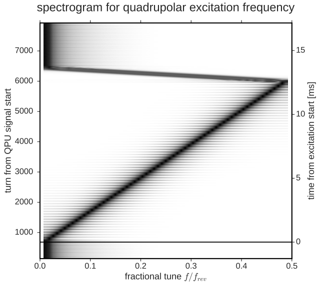


Figure: skew quadrupoles providing maximum coupling

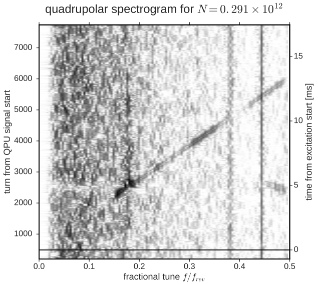
Quadrupolar BTF

CERN PS measurements with quadrupole exciter and QPU (at $\Delta Q_y^{KV} = 0.02$):

excitation signal



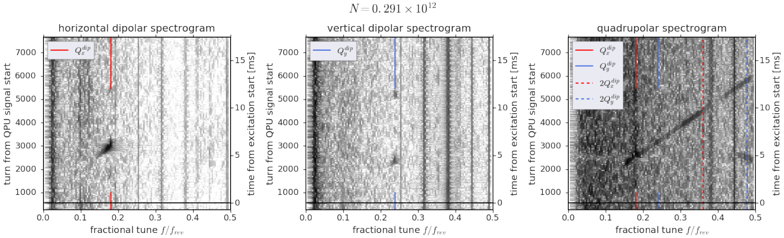
beam response (via QPU)



Observations:

- distinct peaks around machine tunes $f < 0.25 f_{\text{rev}}$
- frequency bands around twice the machine tunes
- (disregard the constant frequencies, due to instrumentation)

Measured Quadrupole BTF

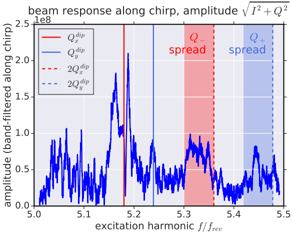


Observations:

- dipole spectra: only machine tunes
- distinct envelope bands below dashed $2Q_{x,y}$ lines

⟶ demodulation of QPU signal with excitation

⟹ distinct peak below Q_x line



Dispersion Mode

Simulations of eigenmodes present in QPU spectrum (for CERN PS, $\Delta Q_y^{KV} = 0.02$):

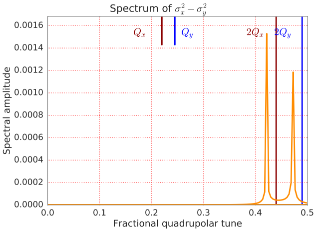


Figure: coasting, no dispersion

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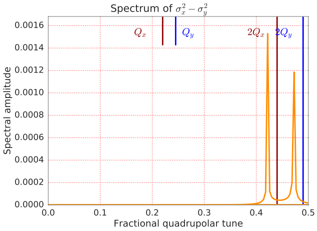

Figure: coasting, no dispersion

include

$\Longrightarrow$

dispersion

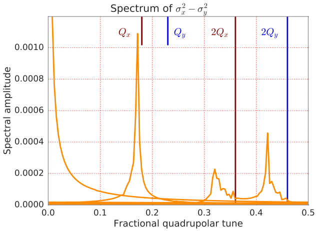

Figure: bunched, with dispersion

Coherent dispersion mode:

- oscillation about matched dispersion, mode measures correlation $\left\langle x \frac{\Delta p}{p_0} \right\rangle$, $\left\langle y \frac{\Delta p}{p_0} \right\rangle$
- negative tune shift with space charge (down from Q_x)

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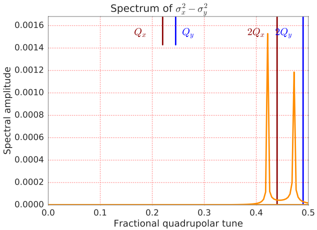

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include
$\Longrightarrow$
dispersion

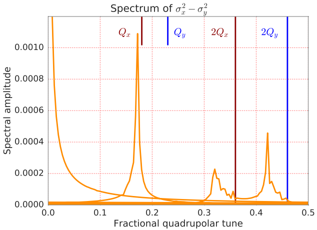

Figure: bunched, with dispersion

include
$\Longrightarrow$
chromaticity

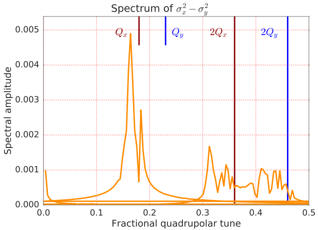

Figure: also with chromaticity

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Dispersion Mode in Q-BTF

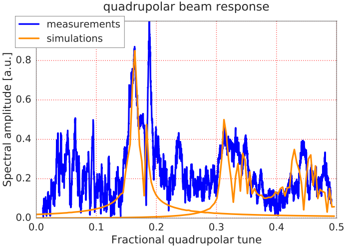
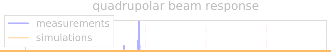


Figure: Q-BTF experiment vs. simulated eigenmodes

- envelope tune range: mix of space charge and chromaticity! (here Q' adds factor 2)
  $\Longrightarrow$ challenging to extract space charge tune shift ΔQ^{KV} from bunched beam Q-BTF at *natural* chromaticity!
- coherent dispersion mode discovered in CERN PS experiment

Dispersion Mode in Q-BTF


quadrupolar beam response

Key point!

Again, QPU can be used as diagnostic tool:

- beam-based measurement of dispersion injection mismatch
- minimise amplitude in SC shifted dispersion lines via dispersion matching
- in contrast to dipole BPM based methods, this method includes space charge

- envelope actor 2)
  $\Longrightarrow$ challenging to extract space charge tune shift ΔQ^{KV} from bunched beam Q-BTF at *natural* chromaticity!
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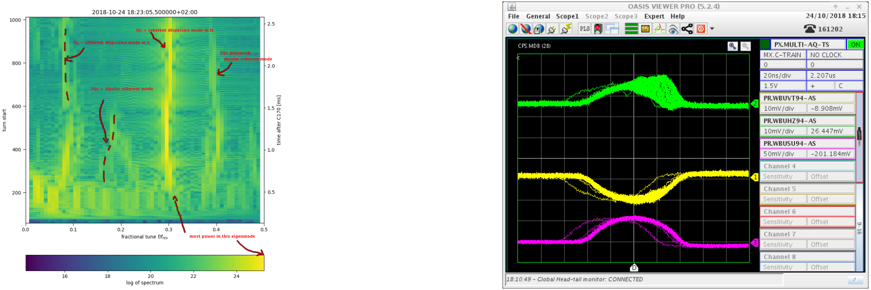
Dispersion Mode ↔ Head-tail Motion!

CERN PS at natural chroma ⇝ (higher-order) horizontal instability

- intrinsically unstable beams at typical intensities
- can be cured with transverse feedback



(a) quadrupolar spectrum

(b) wideband pick-up

Figure: without transverse feed-back

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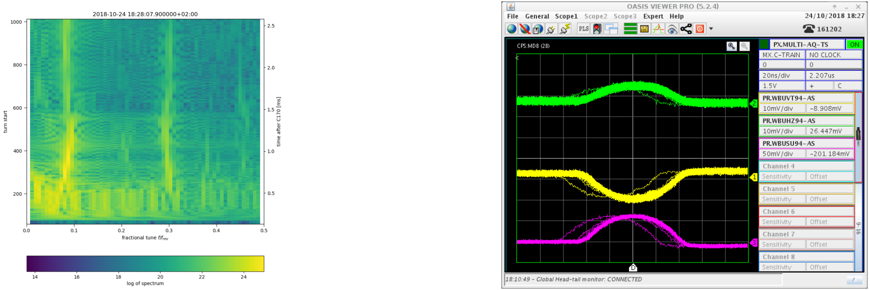

24/10/2018 18:27

161202

PX.MULTI-AQ-TS

MX.C-TRAIN | NO CLOCK

0 | 0

20ns/div | 2.207us

1.5V | + | C

PR.WBUVT94-AS: 10mV/div | -8.908mV

PR.WBUHZ94-AS: 10mV/div | 26.447mV

PR.WBUSU94-AS: 50mV/div | -201.184mV

18:10:49 - Global Head-tail monitor: CONNECTED

(a) quadrupolar spectrum

(b) wideband pick-up

Figure: with transverse feed-back switched on

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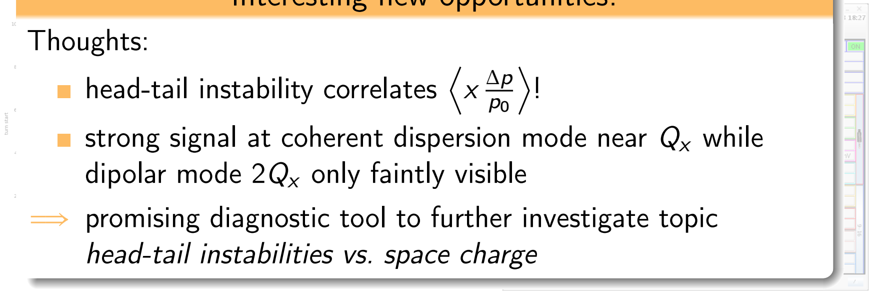


Interesting new opportunities!

Thoughts:

- head-tail instability correlates $\left\langle x\,\frac{\Delta p}{p_0}\right\rangle$!
- strong signal at coherent dispersion mode near Q_x while dipolar mode $2Q_x$ only faintly visible

⟹ promising diagnostic tool to further investigate topic *head-tail instabilities vs. space charge*

(a) quadrupolar spectrum

(b) wideband pick-up

Figure: with transverse feed-back switched on

III. GSI SIS18 Measurements

(Chromaticity Impact on Envelope)

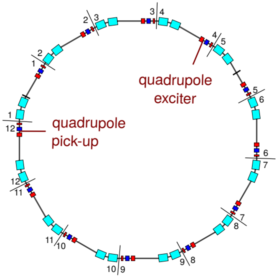
Setup

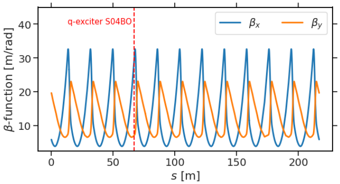
SIS18 setup of Q-BTF experiment:

- quadrupole exciter in sector 4
  - power max. 400 W
  - peak voltage 3 kVpp
  - frequency range 100 kHz to 2 MHz
- quadrupole pick-up in sector 12
- flat-top at $B\rho = 4.2$ Tm (with Pb^{65+} ions)

⟹ almost **vanishing space charge**: $\Delta Q_y^{KV} = 0.0005$

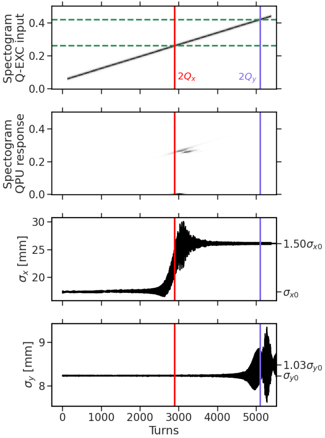
- set tunes: $Q_x = 4.1$ and $Q_y = 3.22$
  - ⟶ expect envelope tunes around $2q_x = 0.2$ and $2q_y = 0.44$
- quadrupole excitation sweep: 0.05 to 0.45 tune
- no head-tail instabilities with ions at SIS18 intensities!
  - ⟹ zero chromaticity without problems (in contrast to CERN PS)





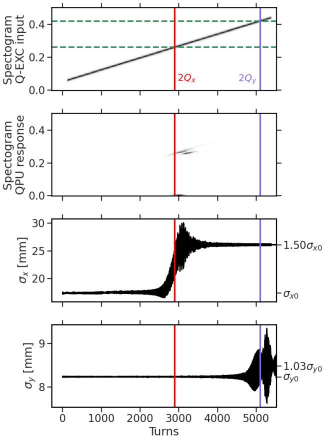
Simulations of Q-BTF

Natural Chromaticity

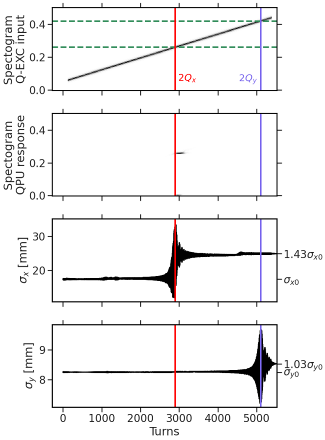


Simulations of Q-BTF

Natural Chromaticity



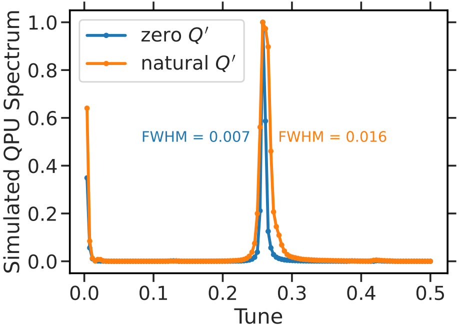
Zero Chromaticity



Simulations of 10 ms sweep:

- → main response around $2Q_x$, i.e. horizontal envelope tune
- → indeed, zero chromaticity (corrected with sextupoles) features narrower response
- → beam size growth mainly in horizontal plane by up to 50%

Simulation Comparison



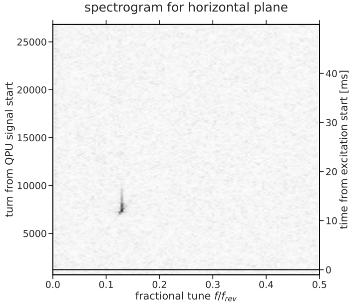
$\Longrightarrow$ zero chromaticity Q-BTF response ≈ 2.3 narrower than for natural chromaticity!

Measurements

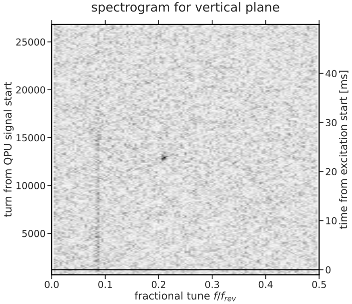
Measurements with Q-BTF of 50ms sweep time (at 80 W), natural chromaticity:

⟶ observe dipolar feed-down of quadrupolar excitation $\Longrightarrow$ coherent tunes $Q_{x,y}$

horizontal spectrogram



vertical spectrogram



Measurements

Measurements with Q-BTF of 50ms sweep time (at 80 W), natural chromaticity:

⟶ observe dipolar feed-down of quadrupolar excitation $\implies$ coherent tunes $Q_{x,y}$

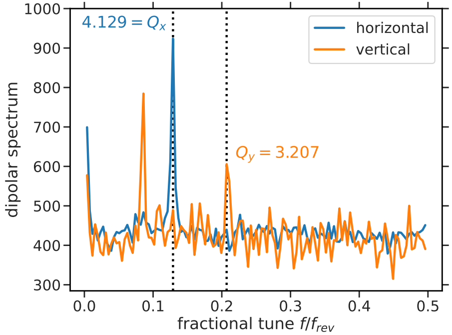


Figure: dipole spectrum

Evaluation of 50 ms Sweep

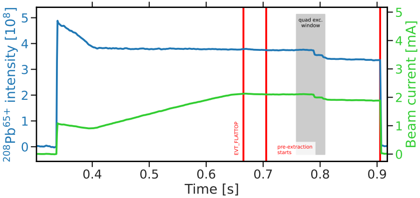


Figure: DC current transformer

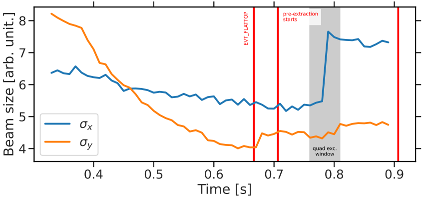


Figure: Ionisation profile monitor

Evaluation of 50 ms Sweep

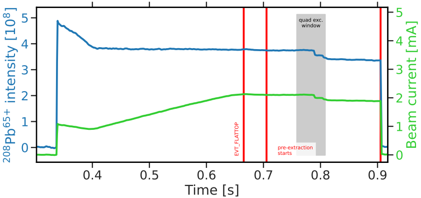


Figure: DC current transformer

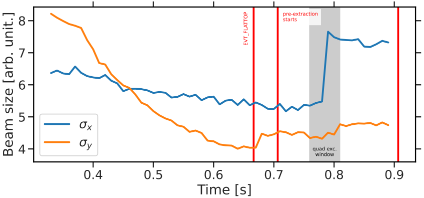
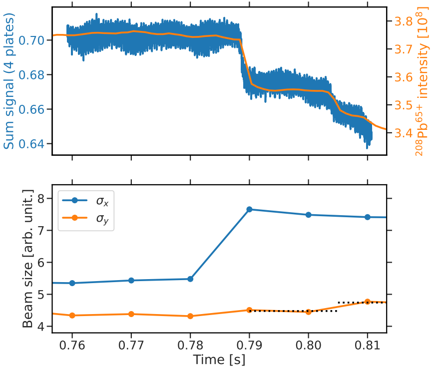


Figure: Ionisation profile monitor

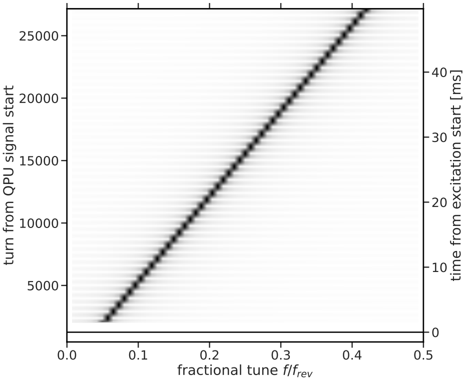


- first loss ↔ horizontal beam size increase
- second loss ↔ vertical beam size increase

Excitation and Response

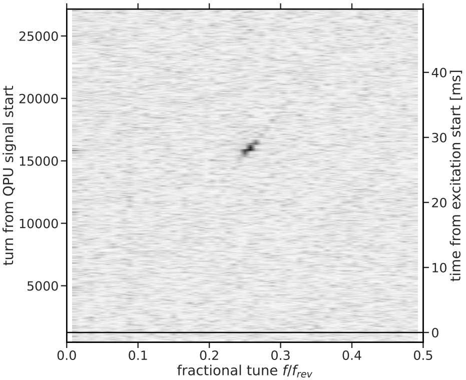
excitation

spectrogram for quadrupolar excitation frequency



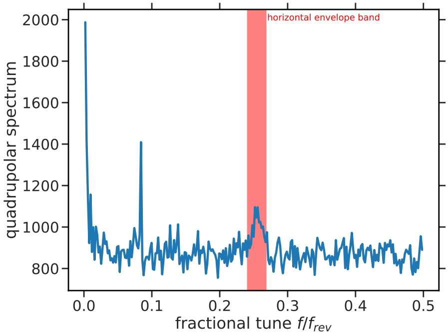
response in QPU

spectrogram for quadrupole pick-up

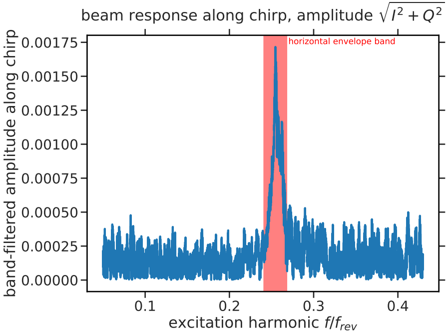


Measured Quadrupole BTF

FFT



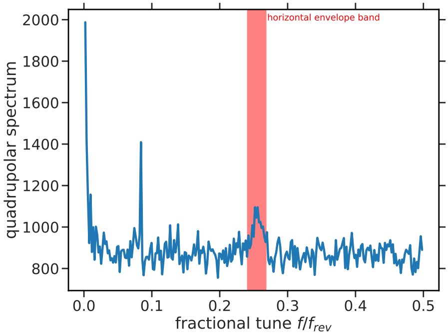
demodulation



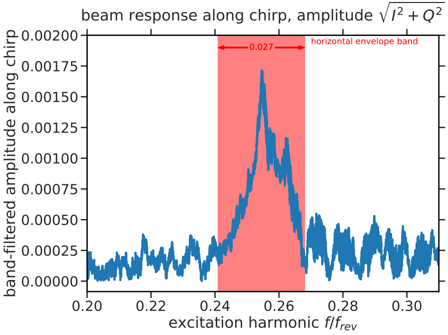
$\Longrightarrow$ horizontal envelope band of 0.027 width (FWHM 0.012)

Measured Quadrupole BTF

FFT



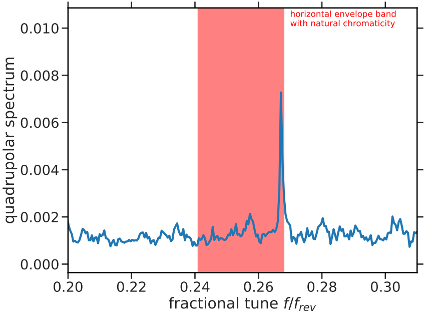
demodulation



⟹ horizontal envelope band of 0.027 width (FWHM 0.012)

Zero Chromaticity (10 ms Sweep)

Correcting the chromaticity to $Q'_{x,y} = 0$ via 2 sextupole families:



- ⟹ chromaticity is indeed the main factor behind broad envelope resonance
- ⟹ measuring space charge at zero chromaticity eliminates this influence!

Conclusion

Quadrupole pick-ups can provide non-invasive beam measurements:

- require non-linear pick-up structure
- frequency domain works better than time domain
- useful mismatch information in quadrupole spectrum:
  - skew envelope modes: linear coupling
  - dispersion modes: transverse-longitudinal correlation

    ⟶ dispersion mismatch, head-tail instability
  - even envelope modes: betatron mismatch
- typically 2nd order modes modify with space charge (as opposed to dipole order):
  - frequencies detune
  - mismatch oscillation amplitudes include space charge mismatch

  ⟹ measure space charge strength (at zero chromaticity)
- confirmed chromaticity broadening envelope mode width (with new GSI SIS18 setup)

Thank you for your attention!

Acknowledgements:

Rahul Singh

and

Oleksandr Chorniy, Peter Forck, Björn Galnander, Wolfgang Kaufmann, Christoph Krüger, Philipp Niedermayer, Dmitrii Rabusov, Thomas Sieber

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