Beam-Based Calibration of Sextupole Magnet Displacement with Betatron Tune Shift

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Outline

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III. Feasibility studies on the SPring-8 storage ring
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Introduction

- The alignment of sextupole magnets is one of the critical issues for the upcoming 4th generation light sources and future colliders to ensure enough dynamic aperture for stable operation and minimize deterioration of beam quality.

- A sextupole magnet horizontally/vertically displaced from the beam exert a normal/skew quadrupole (Q) field.

- We propose a beam-based calibration (BBC) method for the sextupole magnet displacement by observing the betatron tune shift.

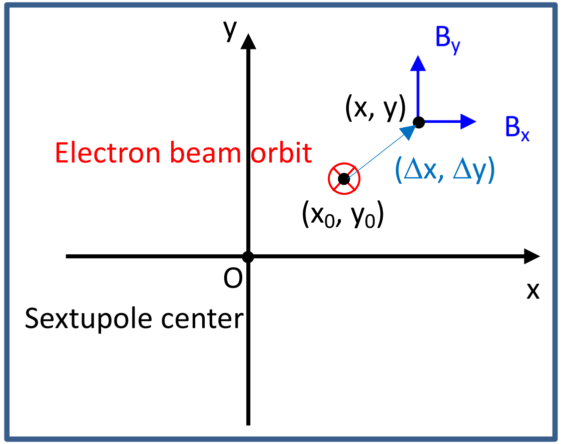
- The beam position that makes the horizontal and vertical betatron tunes invariant to the sextupole strength marks the magnet center.

Theoretical background: Sextupole magnetic field

$$B_x = B''xy \qquad B_y = \frac{B''}{2}(x^2 - y^2)$$

$$x = x_0 + \Delta x \qquad (x_0, y_0) \text{ : beam offset from the center}$$

$$y = y_0 + \Delta y \qquad (\Delta x, \Delta y)\text{: beam displacement}$$



$$B_x = \boxed{B''x_0y_0} + \boxed{B''x_0}\Delta y + \boxed{B''y_0}\Delta x + \boxed{B''}\Delta x\Delta y$$

Dipole Normal Q Skew Q Normal S

$$B_y = \boxed{\frac{B''}{2}(x_0^2 - y_0^2)} + \boxed{B''x_0}\Delta x \boxed{- B''y_0}\Delta y + \boxed{\frac{B''}{2}}\left((\Delta x)^2 - (\Delta y)^2\right)$$

- Normal Q field by horizontal offset, $x_0 \neq 0$
- Skew Q field by vertical offset, $y_0 \neq 0$ → Linear betatron coupling δC

$$\delta C = \frac{1}{2\pi}\frac{B''L}{[B\rho]}y_0\sqrt{\beta_x(s)\beta_y(s)}e^{i[\phi_x(s)-\phi_y(s)-2\pi\Delta\cdot s/L_0]}$$

L: Effective length of Sextupole

L_0: ring circumference

Betatron tune shift by a displaced sextupole

Tune shift due to the sextupole strength change, $B'' \rightarrow B'' + \Delta B''$

$$\Delta\nu_x = \boxed{+\frac{\beta_x(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0}\boxed{-\frac{1}{2}\sqrt{\Delta^2 + |C_0 + \delta C|^2} + \frac{1}{2}\sqrt{\Delta^2 + |C_0|^2}}$$

$$\Delta\nu_y = \boxed{-\frac{\beta_y(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0}\boxed{+\frac{1}{2}\sqrt{\Delta^2 + |C_0 + \delta C|^2} - \frac{1}{2}\sqrt{\Delta^2 + |C_0|^2}}$$

Normal Q contribution

Skew Q contribution

- Tune shift due to a change in the total coupling driving term.

cf. M. Takao, PRST-AB **9,** 084002 (2006).

C_0: Initial coupling driving term for the whole

$\Delta \equiv \nu_x - \nu_y - q$:

difference between the fractional part of ν_x and ν_y with q an integer

$$\delta C = \frac{1}{2\pi}\frac{\Delta B''L}{[B\rho]}y_0\sqrt{\beta_x(s)\beta_y(s)}e^{i[\phi_x(s)-\phi_y(s)-2\pi\Delta\cdot s/L_0]}$$

To lowest order of $|\delta C|$

$$\Delta\nu_x \fallingdotseq +\frac{\beta_x(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 - \frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|\cos(\phi_0 - \phi_1)}{\sqrt{\Delta^2 + |C_0|^2}}y_0$$

$$\Delta\nu_y \fallingdotseq -\frac{\beta_y(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 + \frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|\cos(\phi_0 - \phi_1)}{\sqrt{\Delta^2 + |C_0|^2}}y_0$$

$$\phi_0 = arg(C_0)$$

$$\phi_1 = arg(\delta C) = \phi_x(s) - \phi_y(s) - 2\pi\Delta \cdot s/L_0$$

Betatron tune-based sextupole center calibration

Beam position (x_0, y_0) fixing the tune under sextupole change $\Delta B''$

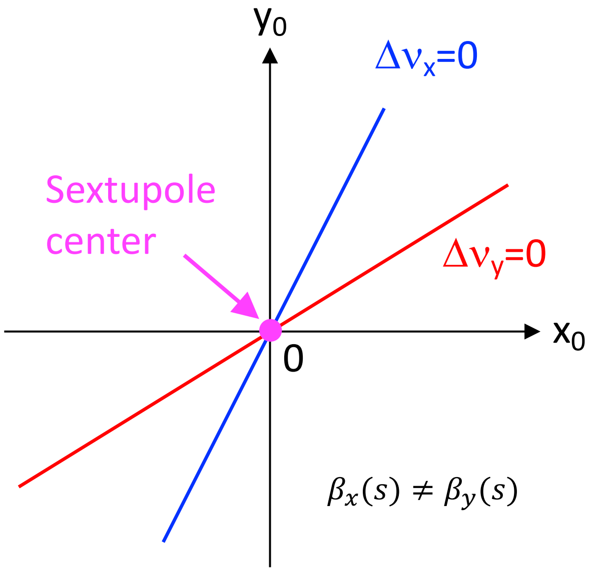
$$\Delta\nu_x = 0 \rightarrow \qquad x_0 = \left\{\frac{1}{2}\sqrt{\Delta^2 + |C_0 + \delta C|^2} - \frac{1}{2}\sqrt{\Delta^2 + |C_0|^2}\right\} \Big/ \left(\frac{\beta_x(s)}{4\pi}\frac{\Delta B'' L}{[B\rho]}\right)$$

$$\Delta\nu_y = 0 \rightarrow \qquad x_0 = \left\{\frac{1}{2}\sqrt{\Delta^2 + |C_0 + \delta C|^2} - \frac{1}{2}\sqrt{\Delta^2 + |C_0|^2}\right\} \Big/ \left(\frac{\beta_y(s)}{4\pi}\frac{\Delta B'' L}{[B\rho]}\right)$$

Intersetion point of the two loci for $\Delta\nu_x = \Delta\nu_y = 0$

||

Magnet center of the target sextupole



Some remarks about the betatron coupling

Small coupling limit, $|C_0| \to 0$

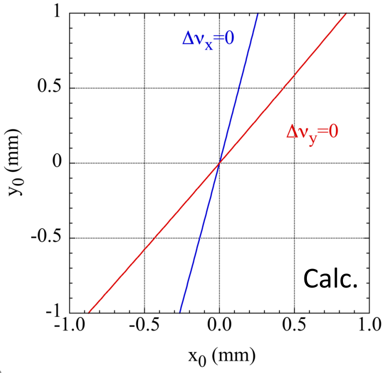
$$\Delta\nu_{x,y} \fallingdotseq \pm \frac{\beta_{x,y}(s)}{4\pi} \frac{\Delta B'' L}{[B\rho]} x_0$$

- Missing information on the vertical center
- Sextupole horizontal center knowable by either the horizontal or vertical tune

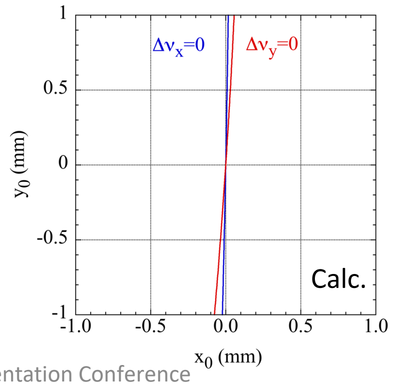
Key to successful sextupole BBC on real rings with a finite coupling

- Betatron coupling control to enhance sensitivity to the vertical displacement.

✓ Feasibility studies with increased coupling planned at SPring-8.



$|C_0|$=0.058
(ν_x, ν_y) =
(41.140, 19.225)

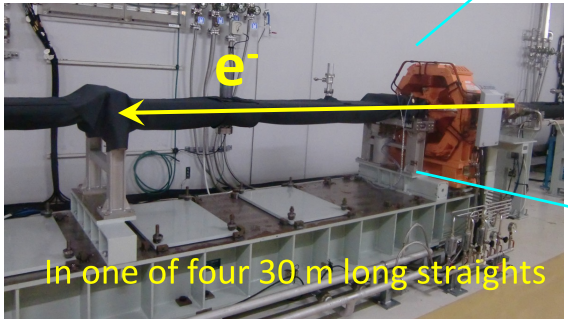


Normal operation:

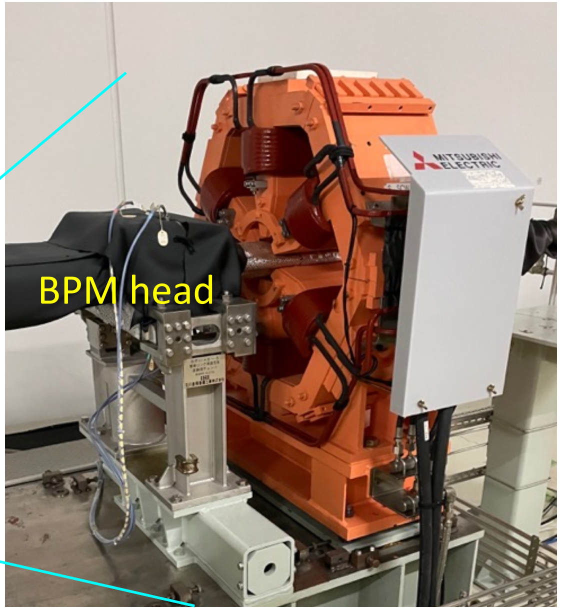
$|C_0|$=0.008
(ν_x, ν_y) =
(41.140, 19.325)

Feasibility studies on the SPring-8 storage ring

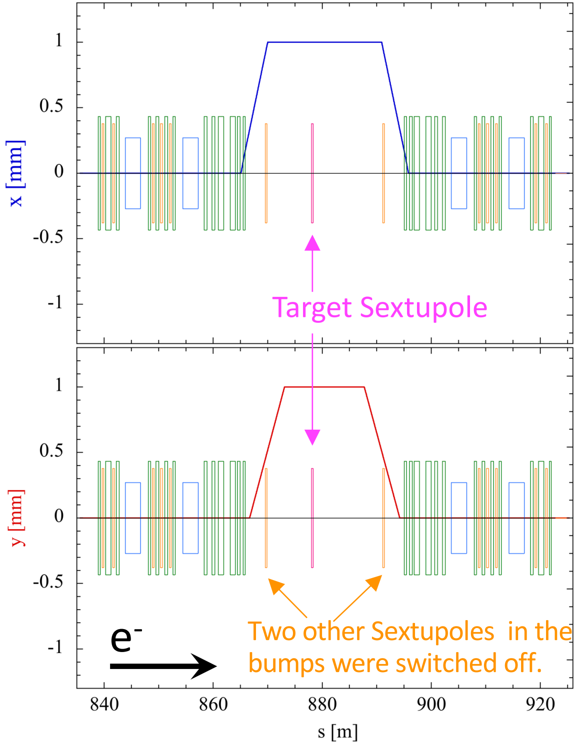
The SPring-8 electron storage ring



Target sextupole magnet

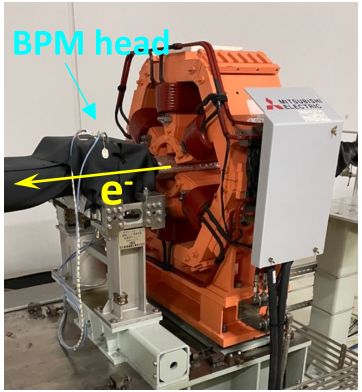


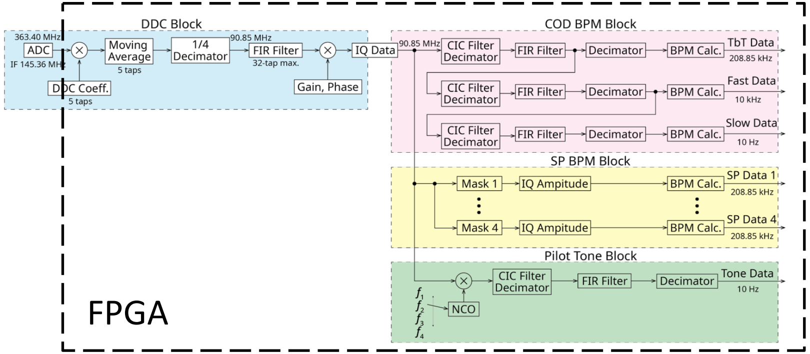
Local orbit bumps



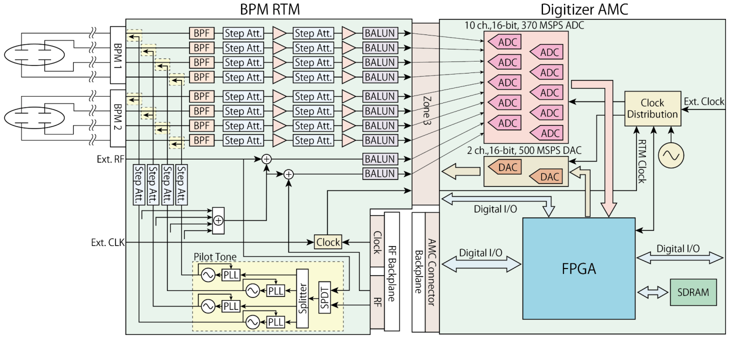
Beam position measurement

MicroTCA.4-based electronics for fast BPM readout

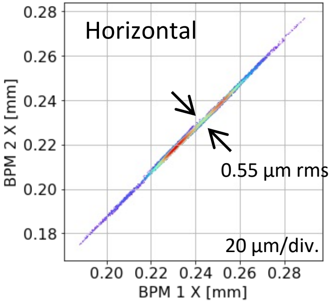


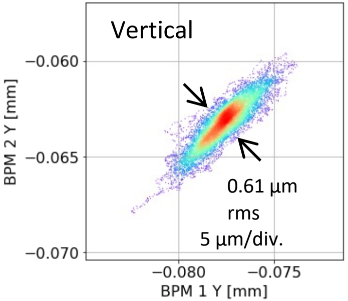


H. Maesaka, et al.,IBIC2019 WEBO03.



✓ Fast COD data sampled at 10kHz with sub-micron resolution.



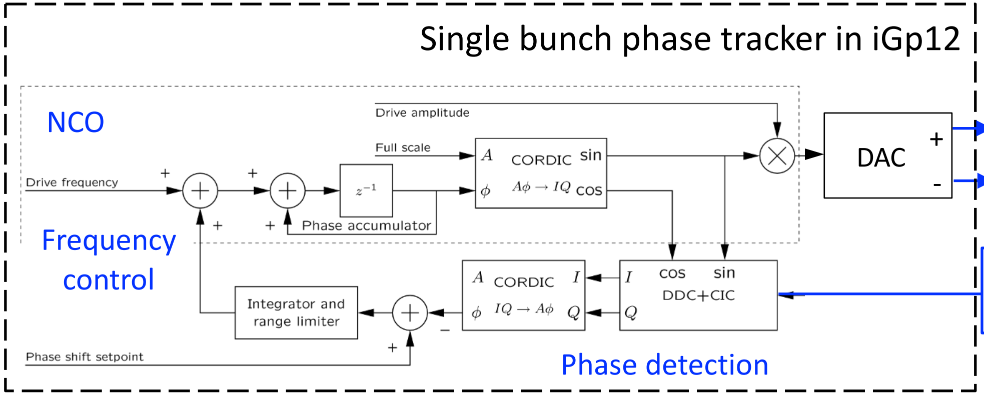


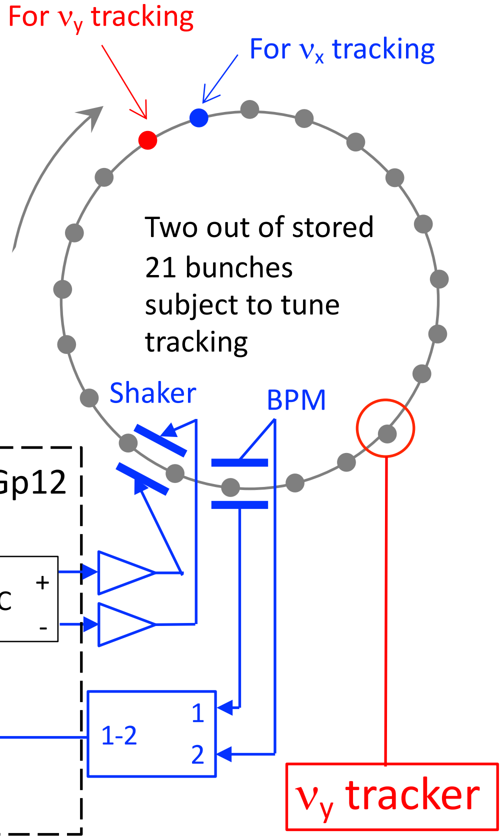
Real-time tune measurement

Bunch-by-bunch feedback and diagnostics signal processor: iGp12

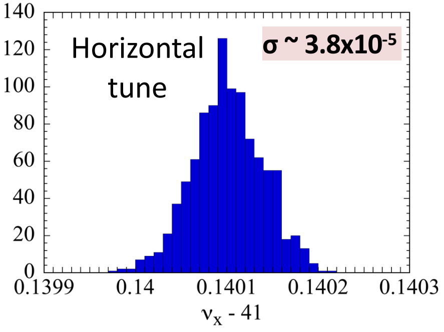
https://www.dimtel.com

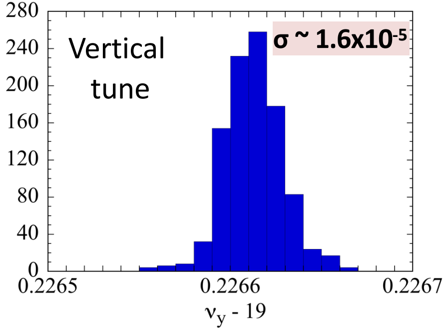
ν_x tracker





✓ ~ 10^{-5} precisions with 5 Hz sampling





Coupling control with skew Q magnets

$$C_T = C_0 + \Delta C_{SkQ}$$

$C_0 \equiv |C_0|e^{i\phi_0}$: original coupling driving term for the whole ring

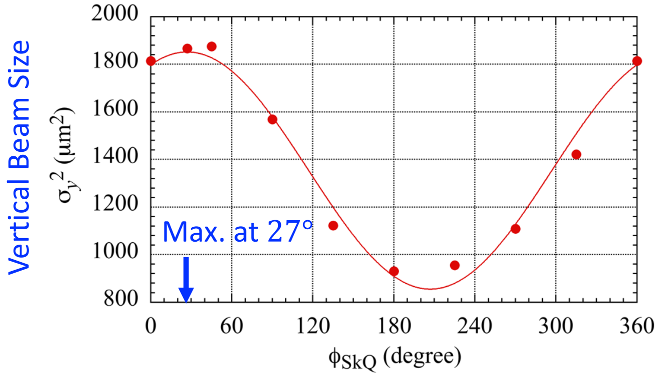
$\Delta C_{SkQ} \equiv |\Delta C_{SkQ}|e^{i\phi_{SkQ}}$: coupling driving term added by skew Q magnets

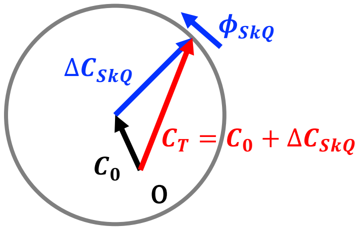
$$\frac{\sigma_y^2}{\beta_y} = \frac{\frac{1}{2}|C_T|^2}{\Delta^2 + |C_T|^2}\varepsilon_0 \approx \frac{\frac{1}{2}\left(|C_0|^2 + 2|C_0||\Delta C_{SkQ}|\cos(\phi_{SkQ} - \phi_0) + |\Delta C_{SkQ}|^2\right)}{\Delta^2}\varepsilon_0$$

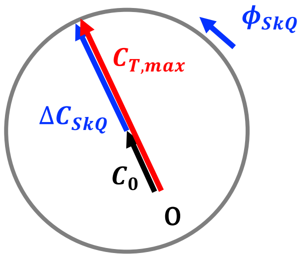
σ_y: projected vertical beam size, ε_0: unperturved emittance.

STEP1: Setting argument ϕ_{SkQ} of ΔC_{SkQ}

- Keeping $|\Delta C_{skQ}|$ const. (= 0.03).
- Maximum $|C_T|$ found at $\phi_{SkQ} = 27°(= \phi_0)$.
- ϕ_{SkQ} set at $27°(= \phi_0)$.

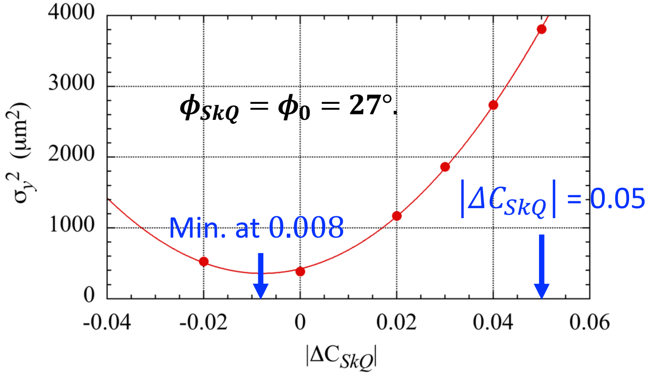






STEP2: Setting magnitude $|\Delta C_{skQ}|$

- $|C_0|$ evaluated to be 0.008
- $|\Delta C_{SkQ}|$ set at 0.05
- Maximum $|C_T| = |C_0 + \Delta C_{SkQ}| =$ 0.058



Coupling control with skew Q magnets (cont.)

Extra gain for the sensitivity to the vertical sextupole displacement obtained by manipulating the tune difference

Normal operation:

$(\nu_x, \nu_y) = (41.14, 19.325)$

$\Delta = 41.14 - 19.325 - 22 = -0.185$

$|C_0| = 0.008$

Feasibility studies:

$(\nu_x, \nu_y) = (41.14, 19.225)$

$\Delta = 41.14 - 19.225 - 22 = -0.085$

$|C_0| = 0.058$

$$\Delta\nu_x \fallingdotseq +\frac{\beta_x(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 - \boxed{\frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|\cos(\phi_0-\phi_1)}{\sqrt{\Delta^2+|C_0|^2}}y_0}$$

$$\Delta\nu_y \fallingdotseq -\frac{\beta_y(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0 + \boxed{\frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|\cos(\phi_0-\phi_1)}{\sqrt{\Delta^2+|C_0|^2}}y_0}$$

$$\frac{0.058}{\sqrt{0.085^2+0.058^2}}\Big/\frac{0.008}{\sqrt{0.185^2+0.008^2}} = 13.0$$

✓ Overall sensitivity improved by a factor more than 10 for the proof-of-principle experiment at SPring-8.

Proof-of-principle experiment

Procedure:

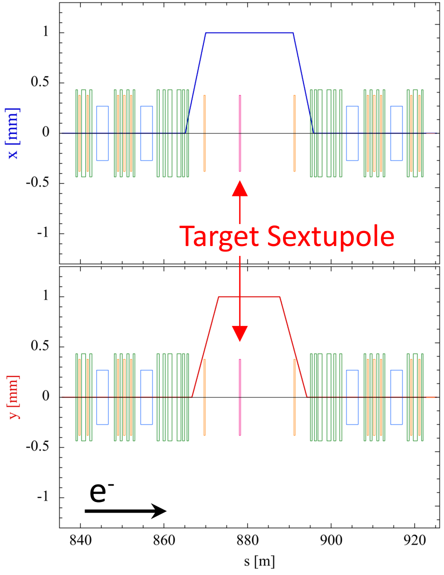
i. Set a vertical orbit bump.

ii. Sweep the beam horizontally monitoring the beam position and the betatron tune with the target sextupole OFF and ON.

iii. Set other vertical orbit bumps. Repeat step ii.

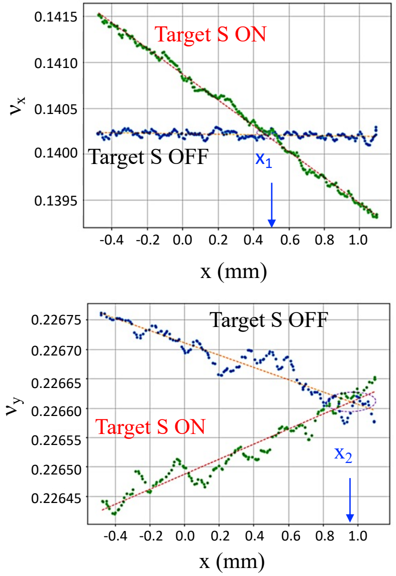
iv. Find beam positions (x_1, y_1) and (x_2, y_2) fixing the horizontal and vertical betatron tunes, respectively.

Orbit bumps



Examples of data

Y_bump = +0.1 mm



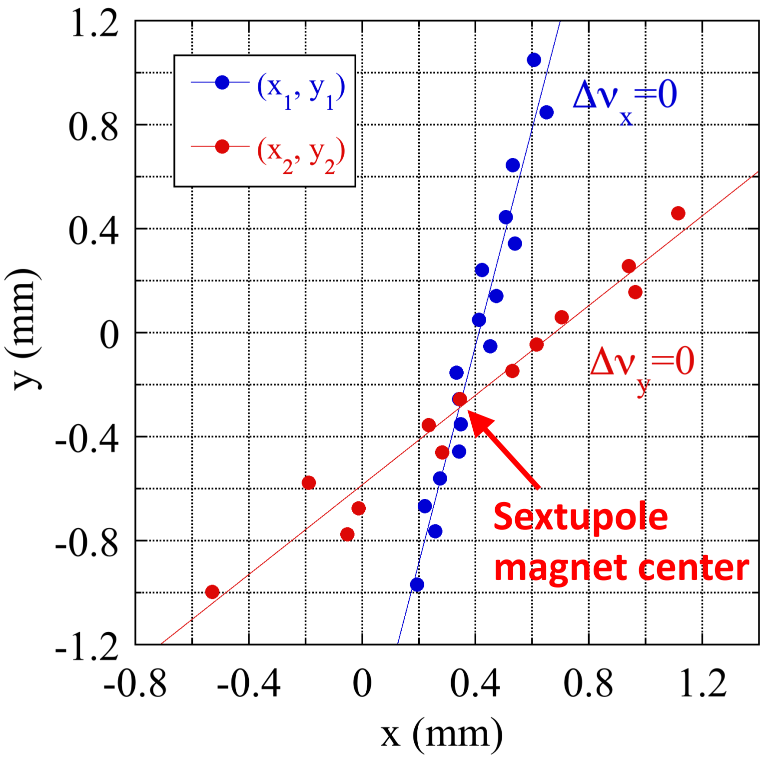
Result

- ✓ The center of the target sextupole magnet successfully marked by the beam response.

  X = 0.347 ± 0.014 mm
  Y = –0.288 ± 0.034 mm

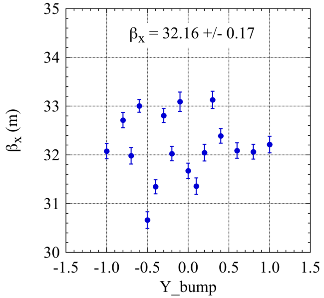
- ✓ Precisions of a few tens of microns evaluated from the fitting residuals.

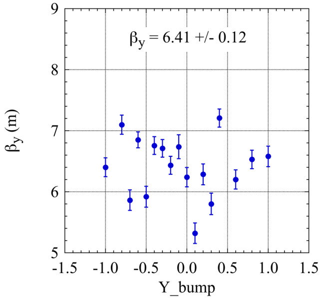
- ✓ BPM offset to the target sextupole also successfully figured simultaneously.



Comparison with the theory

- Calibrated betatron functions at the target sextupole:
  β_x = 32.2 m, β_y = 6.4 m.



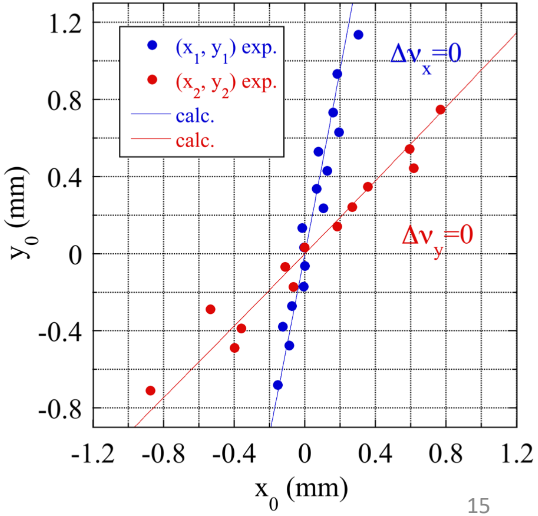


$$\Delta\nu_{x,y} \fallingdotseq \boxed{\pm\frac{\beta_{x,y}(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}x_0} \mp \frac{\sqrt{\beta_x(s)\beta_y(s)}}{4\pi}\frac{\Delta B''L}{[B\rho]}\frac{|C_0|\cos(\phi_0-\phi_1)}{\sqrt{\Delta^2+|C_0|^2}}y_0$$

normal Q contribution

- Theoretical loci of betatron tune fixing points

$$\Delta\nu_{x,y} = 0 \rightarrow \quad x_0 = \left\{\frac{1}{2}\sqrt{\Delta^2+|C_0+\delta C|^2} - \frac{1}{2}\sqrt{\Delta^2+|C_0|^2}\right\} \Big/ \left(\frac{\beta_{x,y}(s)}{4\pi}\frac{\Delta B''L}{[B\rho]}\right)$$

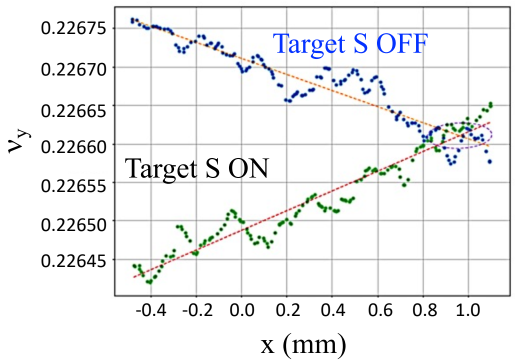


✓ Principle of betatron tune-based sextupole BBC demonstrated in quantitative agreement with the theory.

Issues and challenge

Issues in the feasibility studies

- Vertical tune for sextupole OFF depending on x
  - Residual field in the sextupoles in the orbit bumps
  - Orbit bump leakages
- Smaller β_y (6.4 m) than design (10.5 m)
  - Betatron function distortions over the storage ring



Challenge

- Practical application to the entire sextupole magnets of the storage ring
  - Studies employing faster beam orbit manipulation, i.e. by AC exciting the steering magnets, and faster betatron tune tracking to complete the measurement in a short time.

Summary

- A beam-based calibration (BBC) method with the betatron tune shift for the sextupole magnet displacement is proposed.

- The feasibility studies at SPring-8 successfully demonstrated the principle for both horizontal and vertical sextupole displacements in quantitative agreement with the theory.

- The measurement resolution of 10 μm will be feasible with further improvements.

- To apply to the entire sextupole magnets in a storage ring for practical use, it is necessary to complete the measurement in a short time. Future studies will be considered employing faster beam orbit manipulation and betatron tune tracking.

Dziękujemy za uwagę !

Thank you for your attention !