LHC Schottky Spectrum from Macro-particle Simulations

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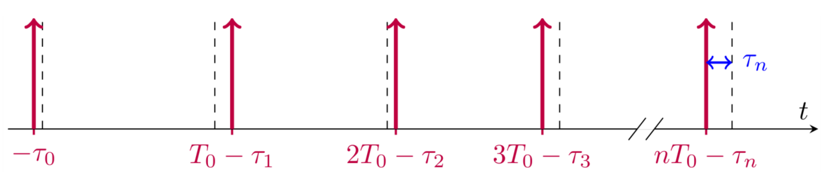
I. Introduction

We introduce a method for building Schottky spectra from macro-particle simulations applied to LHC beam conditions. In this case, the use of a standard Fast Fourier Transform (FFT) algorithm to recover the spectral content of the beam becomes computationally intractable memory-wise, because of the relatively short bunch length compared to the large revolution period. To circumvent this difficulty, a semi-analytical method was developed to compute efficiently the Fourier transform . The spectral content of the beam is calculated on the fly along with the macro-particle simulation and stored in a compact manner, independently from the number of particles, thus allowing the processing of one million macro-particles in the LHC, over 10'000 revolutions, in a few hours, on a regular computer. The study presented herein is based on simulations performed with PyHEADTAIL [1, 2], a macro-particle code that can be used to track turn-by-turn the six-dimensional phase space evolution of a bunch, possibly including the effects of direct space charge and beam-coupling impedances (although this capability is not yet used for this study). The simulated Schottky spectrum is then compared against theoretical formulas and measurements of Schottky signals previously obtained with lead ion beams in the LHC.

II. Method

The Schottky spectrum is the power spectral density of the beam current in the longitudinal plane and the dipole moment in the transverse planes.

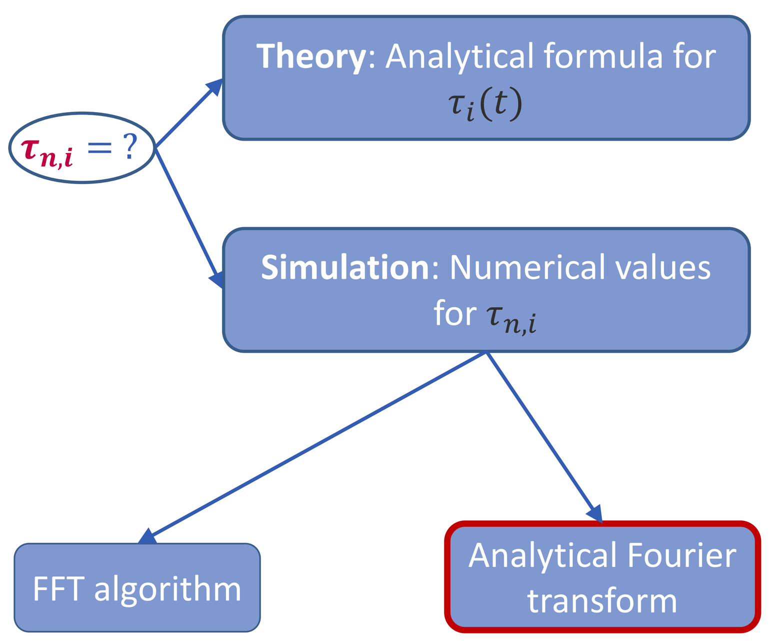
- The current of a single particle can be expressed as:



$$i_i(t) = q\sum_{n=-\infty}^{\infty}\delta\Big(t-(nT_0-\tau_{n,i})\Big) \quad (1)$$

With:
- $\tau_{n,i}$: time difference between particle i and the synchronous particle at turn n.
- q : charge of the particle.
- T_0 : revolution period.

- To compute the frequency components of the current signal we need to know the $\tau_{n,i}$, several paths can be considered:

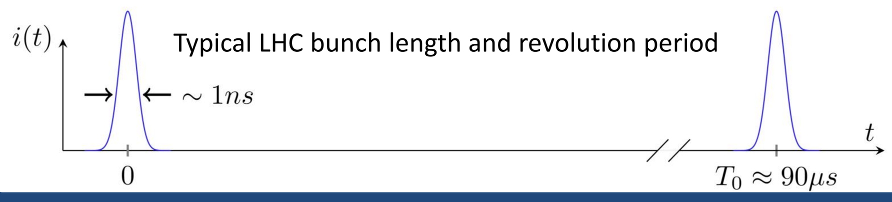


A. Theory

Theoretical spectra can be obtained by substituting simple analytical expression for $\tau_{n,i}$ in Eq. (1) and transforming the time signal to the frequency domain [3, 4]. The developed theory is however limited only to simplified beam dynamics and does not include, e.g., collective effects and beam interaction with the vacuum chamber through impedance. Due to the complex theoretical description of such effects, it is most suitable to study their impact on Schottky spectra using multi-particle simulations as done in Ref. [5].

B. Simulation (FFT)

Substituting numerical values of $\tau_{n,i}$ in Eq. (1) (calculated from multi-particle simulations) gives a current signal that we can discretise in time and on which we can apply the FFT algorithm to retrieve the Schottky spectra. For the case of the CERN Large Hadron Collider (LHC), this method is particularly challenging computationally due to the highly sparse characteristic of the current signal produced by a single bunch.



Sample each passage of the bunch with 100 points over 10'000 turns ➔ Array of 10^{11} samples or 1 TB of data.
While manageable for smaller accelerator like the PS, the FFT is too complex memory wise for the LHC.

C. Simulation (analytical Fourier transform)

Given the high computational resource requirements of traditional FFT algorithms, we developed an alternative approach to compute the Fourier transform (FT). The main steps are:
- Write analytically the FT.
- The integral is trivial since the time signal of the current is a sum of delta Dirac functions.
- Write the second exponential as its Taylor expansion around the frequency of the Schottky monitor ω_c.
- Invert the summation over the particles and the Taylor coefficients.

Typical values for the number of turns and frequencies are:
$N_t \sim N_f \sim 10^4$ (Acquisition time and spectral resolution of the LHC Schottky Monitor).
Number of macro-particles: $N_p \sim 10^6$ (Realistic PyHEADTAIL simulation)

Longitudinal Schottky Spectrum

$$\widetilde{i(\omega)} = \int_{-\infty}^{\infty} i(t)e^{j\omega t}dt = q\sum_{n=0}^{N_t}\sum_{i=1}^{N_p} e^{j\omega(nT_0+\tau_{n,i})}$$

Computational requirements:
- Evaluate $O(N_t N_p N_f) \sim 10^{14}$ exponentials.
- Store in memory $N_t \times N_p$ number $\tau_{n,i}$ ~100 Gb.

$$= q\sum_{n=0}^{N_t} e^{j\omega nT_0}\sum_{i=1}^{N_p} e^{j\omega\tau_{n,i}}$$

$$e^{j\omega\tau_{n,i}} \approx \sum_{l=0}^{N_l}\frac{e^{j\omega_c\tau_{n,i}}}{l!}(j\omega\tau_{n,i}-j\omega_c\tau_{n,i})^l$$

$$= q\sum_{n=0}^{N_t} e^{j\omega nT_0}\sum_{l=0}^{N_l}\frac{j^l(\omega-\omega_c)^l}{l!}\sum_{i=1}^{N_p} e^{j\omega_c\tau_{n,i}}(\tau_{n,i})^l$$

$$= q\sum_{n=0}^{N_t} e^{j\omega nT_0}\sum_{l=0}^{N_l}\alpha_l(\omega)\,\mathcal{L}_{n,l}$$

Computational requirements:
- Evaluate $O(N_t N_p) \sim 10^{10}$ exponentials.
- No need to store in memory the $\tau_{n,i}$ as the $\mathcal{L}_{n,l}$ coefficients are calculated on the fly along with the PyHT simulation.

Transverse Schottky Spectrum

A similar development can be conducted with the dipole moment of the bunch:

$$d(t) = \sum_{i=0}^{N_p} x_i(t)\, i_i(t) = q\sum_{i=1}^{N_p}\sum_{n=0}^{N_t} x_{n,i}\,\delta\Big(t-(nT_0+\tau_{n,i})\Big)$$

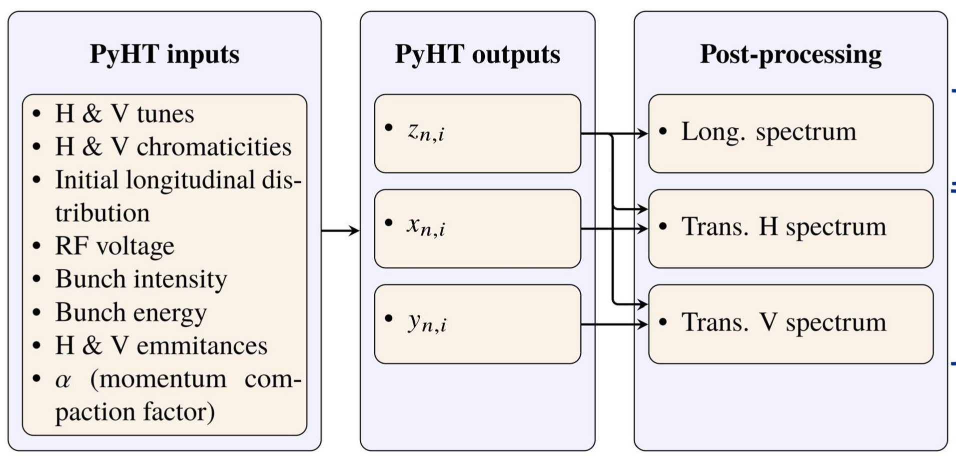
Numerical values from PyHT simulation

$$\widetilde{d(\omega)} = \int_{-\infty}^{\infty} d(t)e^{j\omega t}dt$$

$$= q\sum_{n=0}^{N_t}\sum_{i=1}^{N_p} x_{n,i}\, e^{j\omega(nT_0+\tau_{n,i})}$$

$$= q\sum_{n=0}^{N_t} e^{j\omega nT}\sum_{l=0}^{N_l}\alpha_l(\omega)\,\mathcal{T}_{n,l}, \qquad \mathcal{T}_{n,l} = \sum_{i=0}^{N_p} x_{n,i}\, e^{j\omega_c\tau_{n,i}}(\tau_{n,i})^l$$

Schottky spectrum simulation procedure



$$\widetilde{i(\omega)} = q\sum_{n=0}^{N_t} e^{j\omega nT_0}\sum_{l=0}^{N_l}\alpha_l(\omega)\,\mathcal{L}_{n,l}$$

$$\widetilde{d(\omega)} = q\sum_{n=0}^{N_t} e^{j\omega nT}\sum_{l=0}^{N_l}\alpha_l(\omega)\,\mathcal{T}_{n,l}$$

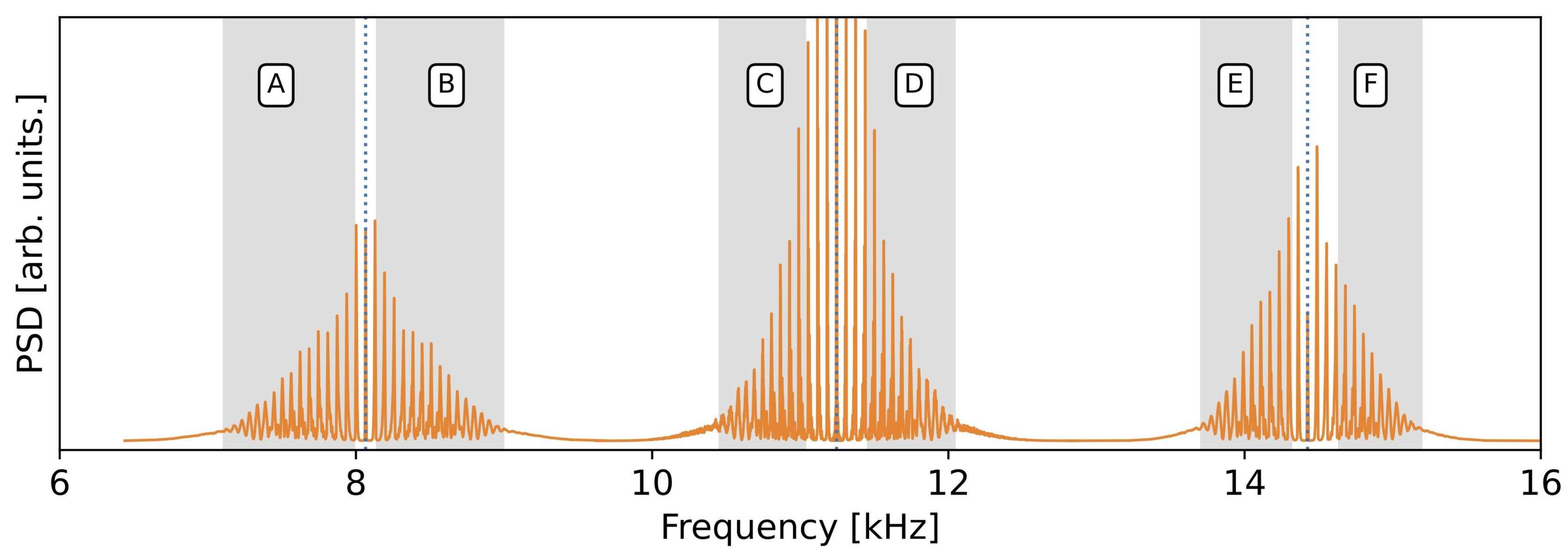
III. Results

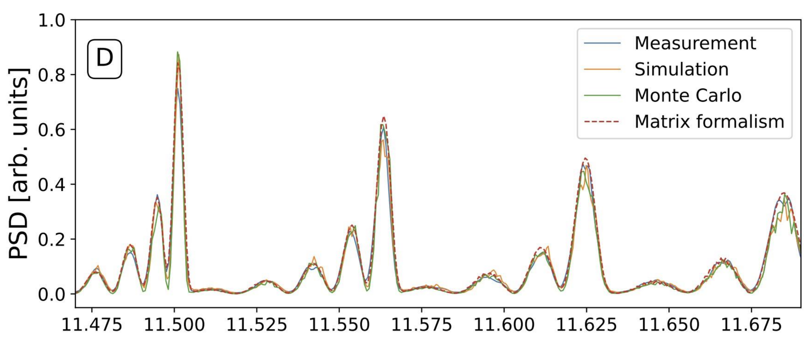
- The simulation method (analytical Fourier transform) is **benchmarked against experimental Schottky spectra** obtained with lead ion beams during LHC Run 2.
- The experimental Schottky data consists of horizontal measurement of beam 2 at injection energy for the fill 7443.
- Among the PyHEADTAIL input parameters, some are based on the fitting of the experimental Schottky spectra as done in Ref. [4] and allows to determine precise machine and beam parameters for the specific fill we want to reproduce.
- Other parameters come from direct measurements or machine design.

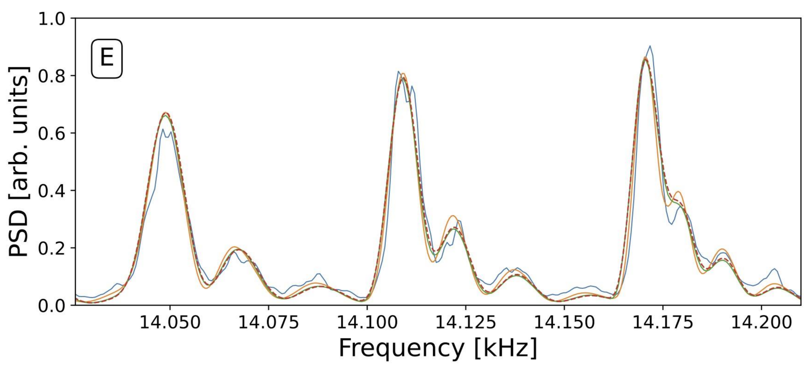
PyHEADTAIL simulation parameters for fill 7443

Intensity	1.76×10^8 ions per bunch
Energy per ion	36.9 TeV
Ion charge	82 e
Ion mass	193.687 GeV/c^2
ϵ_x, ϵ_y	1.5 µm
Tunes	$Q_x = 64.2827$, $Q_y = 59.2985$
Chromaticities	$Q'_x = 18.56$, $Q'_y = 11.64$
α	3.479×10^{-4}
h_{rf}	35640
RF voltage	8.22 MV
LHC circumference	26.659 km
Rice parameters of $\hat{\tau}$ distribution [6]	$\sigma = 1.306$ ns, $b = 0.216$

Simulated longitudinal (C, D) and transverse horizontal (A, B, E, F) Schottky spectra for the machine and beam configuration of LHC fill 7443. The dotted lines indicate respectively (from left to right): $(1-Q_x)\,f_0$, f_0 and $(1+Q_x)\,f_0$.



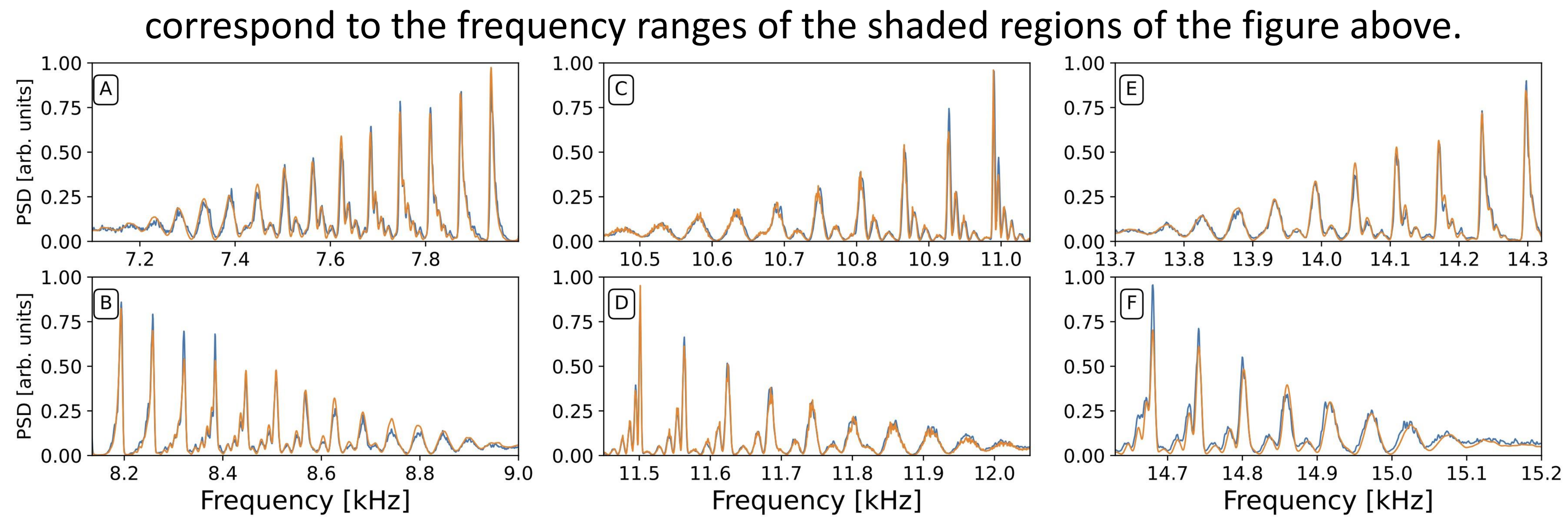




- Comparison made with two theoretical formalisms:
  - Matrix formalism [4, 6].
  - Monte Carlo [7].
- The different methods are in very good agreement with each other and reproduce the overall shape of the spectrum as well as the detailed internal structure of the synchrotron satellites.
- Effect of chromaticity is well reproduced by the simulation. The upper betatron sideband is higher and thinner than the lower one.

Frequencies on all the plots have been shifted from the LHC Schottky harmonic, $h = 427725$, to the first harmonic.

Measured (blue lines) longitudinal and transverse horizontal Schottky spectra compared with the simulated spectra (orange lines) for LHC fill 7443. The A–F region labels correspond to the frequency ranges of the shaded regions of the figure above.



IV. Conclusion

- The aim of this work was to validate the development of a new method for calculating Schottky spectra from macroparticle simulations.
- This method will enable future studies on how effects such as beam-coupling impedances impact the measured spectra.
- This method allows to closely reproduce the Schottky spectrum of a given LHC fill. The obtained results were shown to be in good agreement with reference measurements as well as with other theory-based methods, and reproduce the overall shape of the spectrum together with the detailed internal structure of the synchrotron satellites.

References

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