

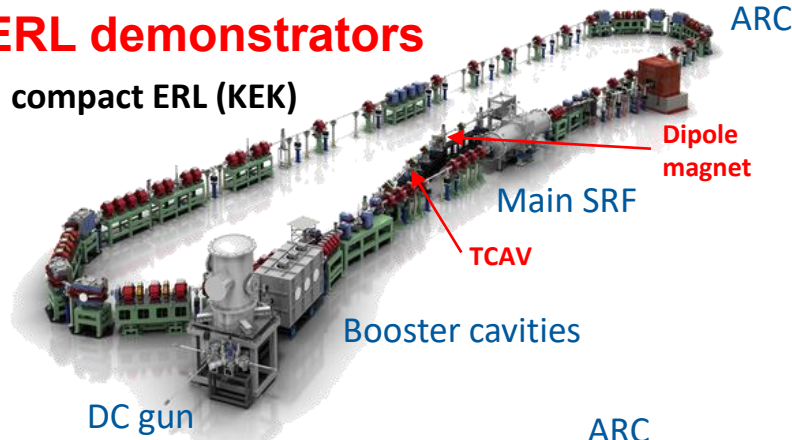


Measurement of Bunch Length and Temporal Distribution using Accelerating Radio Frequency Cavity in Low Emittance Injector

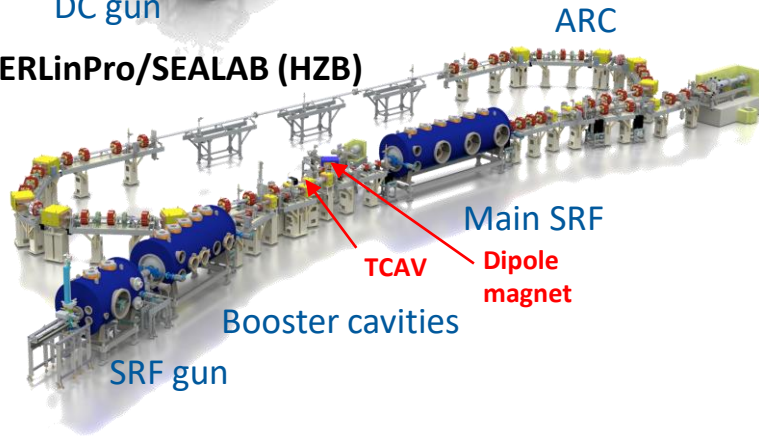
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Tsukasa Miyajima, Yosuke Honda | High Energy Accelerator Research Organization (KEK)
Eun-San Kim | Korea University Sejong Campus

ERL demonstrators

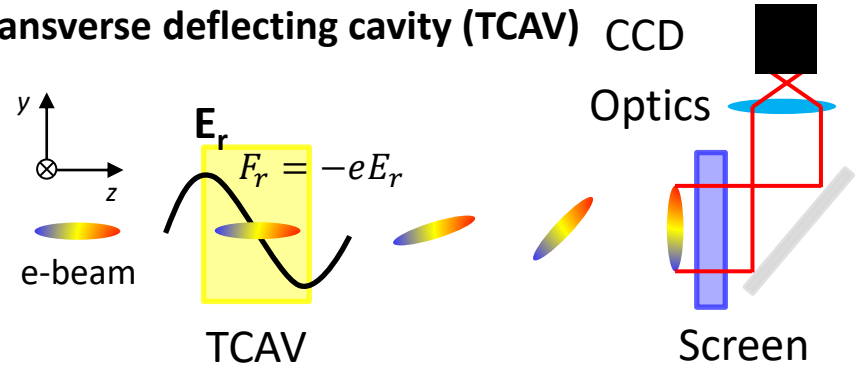
compact ERL (KEK)



bERLinPro/SEALAB (HZB)



Transverse deflecting cavity (TCAV)



Pros: Space-charge free

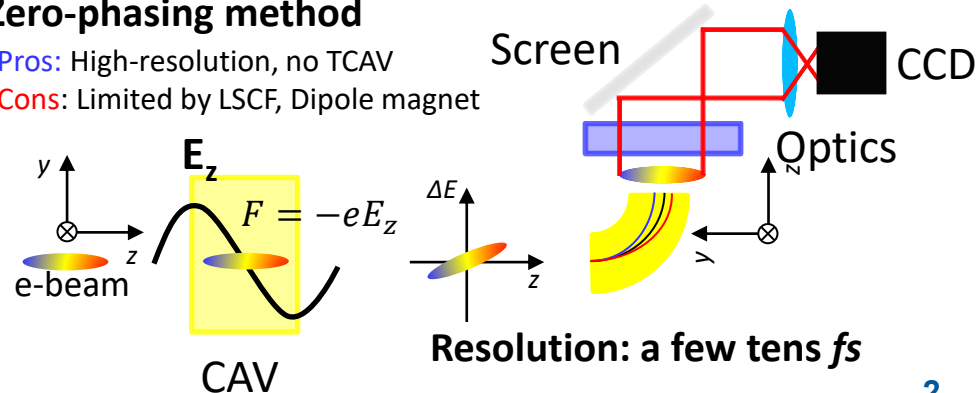
Cons: Difficult to install TCAV near e-gun

Resolution: a few hundred fs

Zero-phasing method

Pros: High-resolution, no TCAV

Cons: Limited by LSCF, Dipole magnet



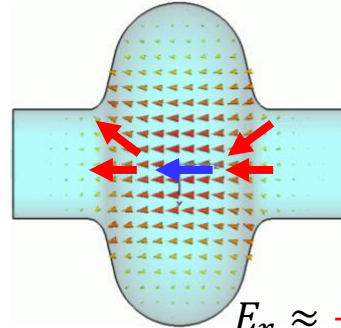
Resolution: a few tens fs

2-cell booster cavity
in compact-ERL

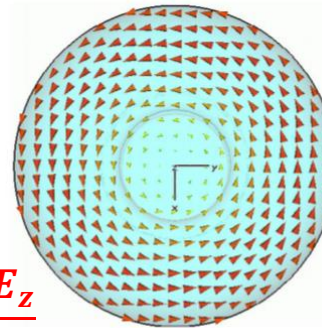


EM field map of TM01 mode

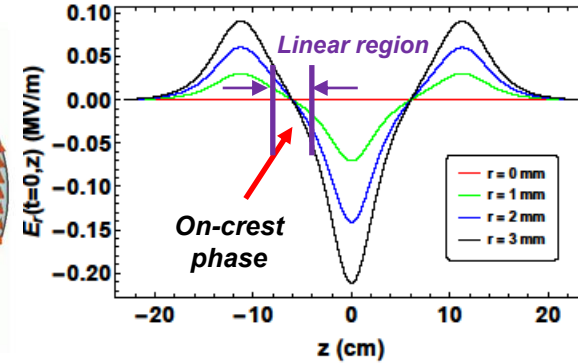
E-field



H-field



$$E_r \approx -\frac{r}{2} \frac{\partial E_z}{\partial z}$$



An **accelerating RF cavity** can provide a **phase-dependent transverse kick** to the electrons, resulting in the linear coupling of the trajectory angle with the longitudinal position.

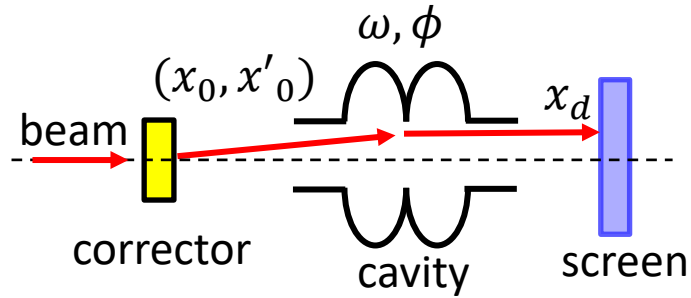
The equation of motion with the maximum acceleration phase (on-crest phase)

$$\frac{dp_r}{dt} = e(E_r - \beta c B_\theta) = -\frac{er}{2} \left[\frac{\partial E_z}{\partial z} - \frac{\beta}{c} \frac{\partial E_z}{\partial t} \right]$$

For high energy beams ($\beta \simeq 1$), the transfer matrix can be calculated analytically (in Rosenzweig's paper) which is well known as a **SRF focusing effect**. [J. Rosenzweig and L. Serafini, Phys. Rev. E 49, 1599 \(1994\)](#)

For low energy beams, the transverse motion is complicated by the significant changes in the velocity of an electron inside the cavity owing to its low initial kinetic energy.

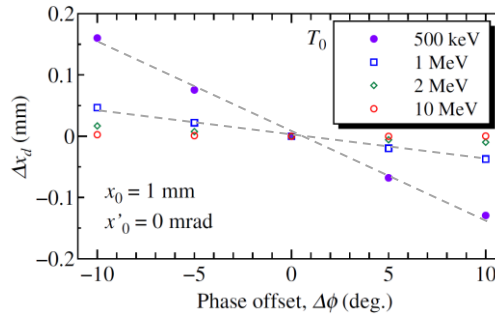
→ Numerical simulation using **General Particle Tracer**



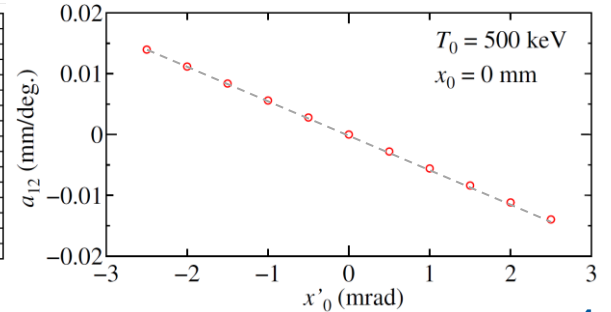
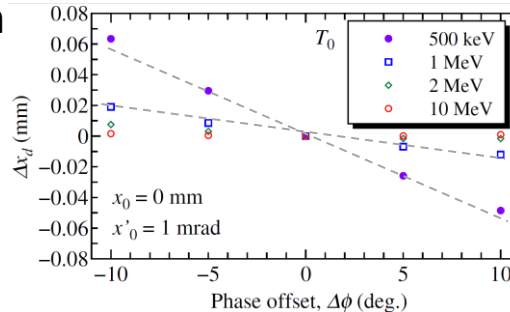
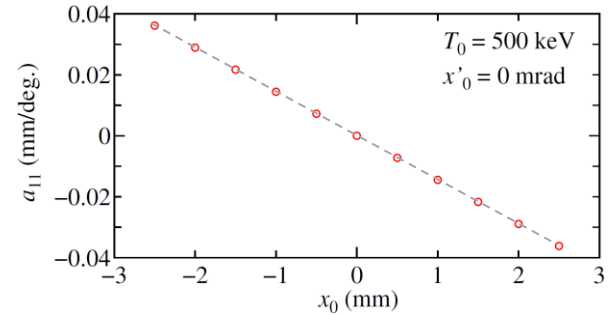
With a paraxial approximation, we can describe the transverse motion using the transfer-matrix as

$$\begin{aligned}
 x_d &= a_{11}x_0 + a_{12}x'_0 \\
 &= x_c + (d_{11}\phi x_0 + d_{12}\phi x'_0) \\
 &= x_c + (\mathbf{d}_{11}x_0 + \mathbf{d}_{12}x'_0) \phi
 \end{aligned}$$

Is it linear on the phase ϕ ?



Is it linear on the x_0 and x'_0 ?



Since the phase can be expressed as $\phi = \omega t$, the horizontal distribution convoluted by the initial temporal distribution is given by

$$G(x) = \int f(\tau)g(x - (d_{11}x_0 + d_{12}x'_0)\omega\tau) d\tau$$

, where $f(x)$ and $g(x)$ are a temporal and horizontal distribution, respectively.

If the beam has a Gaussian distribution and is transversally small, the transverse beam size is determined by the bunch length, initial offset and angle, and accelerating RF cavity parameters as

$$\sigma_x = \sqrt{\sigma_{x0}^2 + ((d_{11}x_0 + d_{12}x'_0)\omega\sigma_t)^2}$$

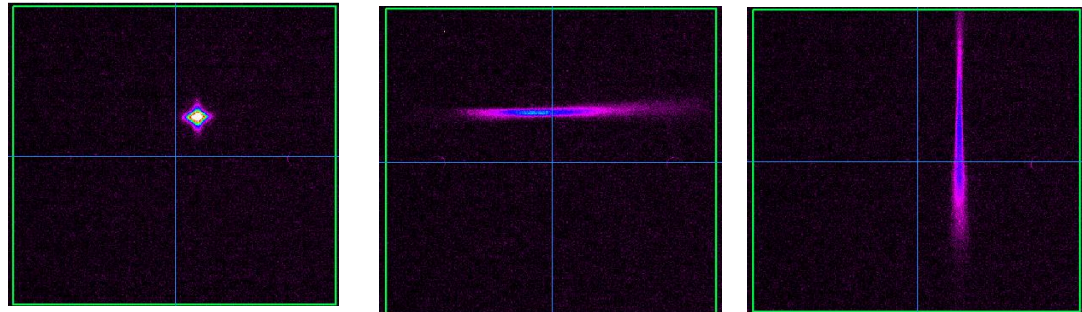
, where σ_{x0} is the initial beam size at the screen without offset and σ_t is the bunch length.

Remark:

1. Small σ_{x0} (focusing on the screen downstream) improves the resolution of the method.
2. Since the σ_{x0} is determined by the integration of complex physics processes including optics, energy variation as well as low-energy beam dynamics, **it is not necessary to deconvolute the emittance dilution by space charge effects.**



First proof-of-principle test with a long pulse generation by a laser stacking of 8 pulses with 2 ps separation



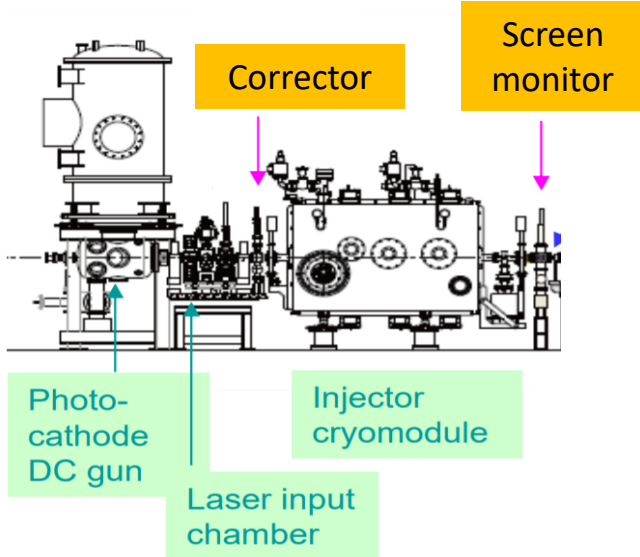
Beam on cavity center

TM01 cavity with $\Delta x' = 23$ mrad

TM01 cavity with $\Delta y' = 23$ mrad

Condition for the PoP studies

Parameters	Values
K.E.	0.39 keV (@ ent. cavity) ~ 2 MeV (@ after cavity)
Bunch charge	10 fC
$\epsilon_{nx,ny}$	~ 0.1 mm-mrad
E_{acc}	7.95 MV/m @ 1 st booster cavity



For determining the absolute bunch length, we must know: ω , d_{11} , d_{12} , x_0 , x'_0 .

To make the problem simple, we performed our experiments with $x_0 = 0$.

$$\sigma_x = \sqrt{\sigma_{x0}^2 + (d_{12}x'_0\omega\sigma_t)^2}$$

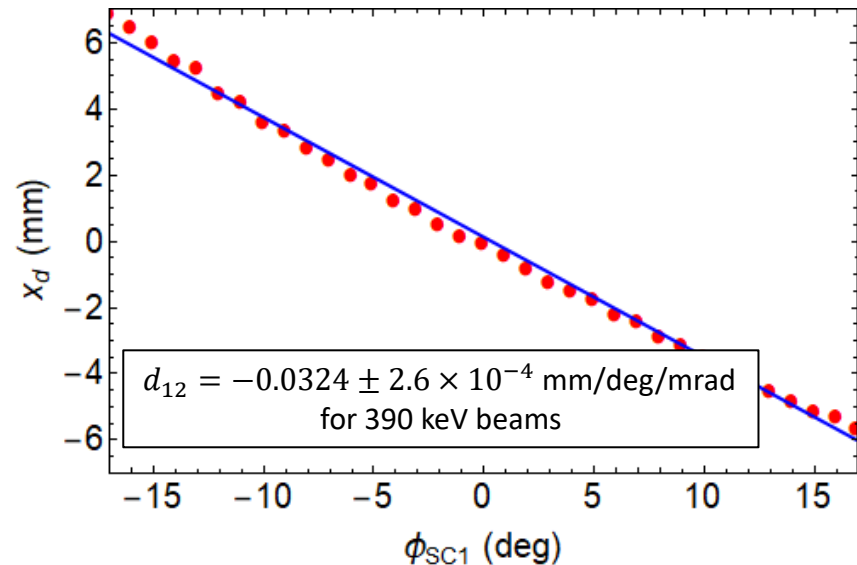
The ω , x'_0 can be estimated by pre-calibrations, and then, unknown parameter is only d_{12} .

Can we measure d_{12} in the experiment?

Yes! From the definition, $\Delta x = d_{12}x'_0\phi$,
the $d_{12} = \Delta x / (x'_0\phi)$.

Then it is necessary to measure the position variation as a function of the phase with a fixed initial angle.

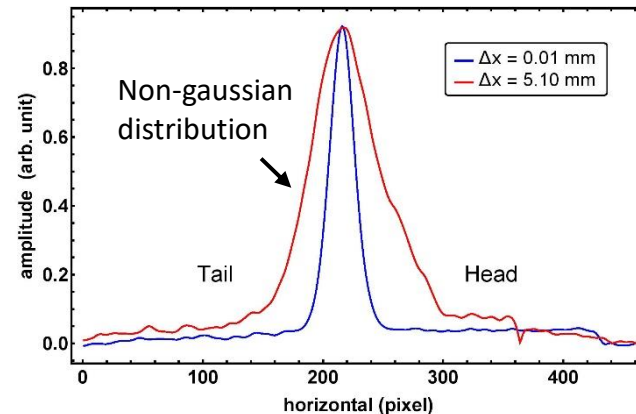
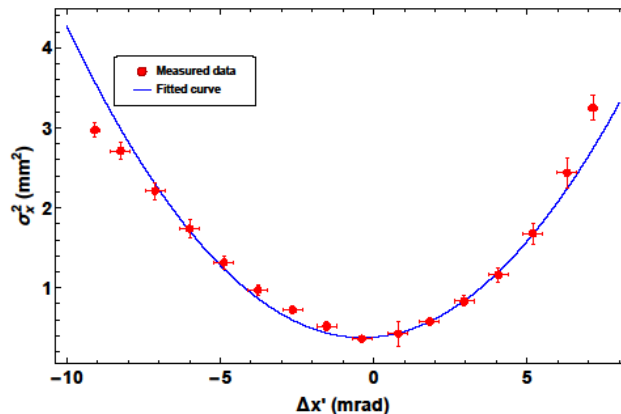
Resolution: about **0.98 ps**. (It can be improved by increasing the field gradient of the cavity or lowering the initial beam energy.)



Long pulse

: laser stacking of 8 pulses
with 2 ps separation

13.51 +/- 0.03 ps
for $E_{acc} = 7.21$ MV/m

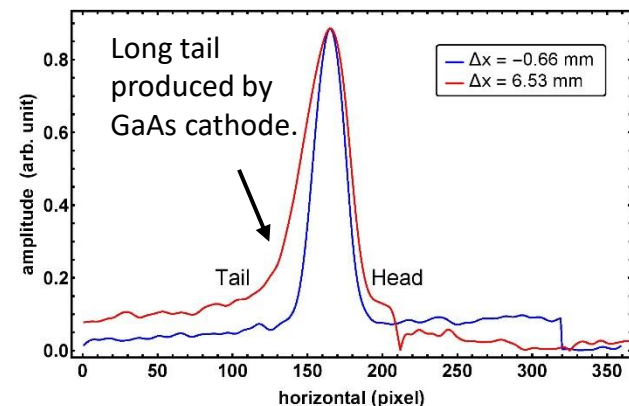
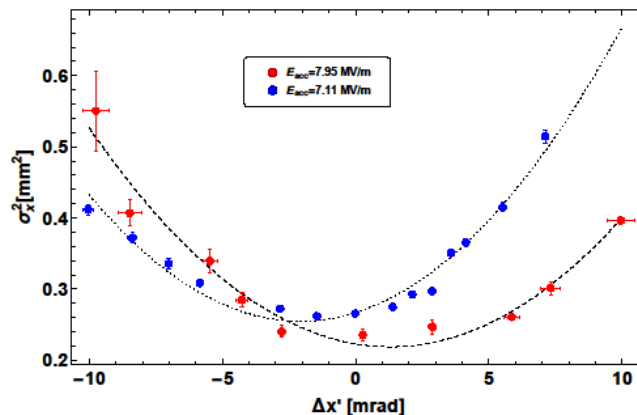


Short pulse

: a single laser with a
duration of 3.3 ps rms

3.39 +/- 0.12 ps (rms)
for $E_{acc} = 7.11$ MV/m

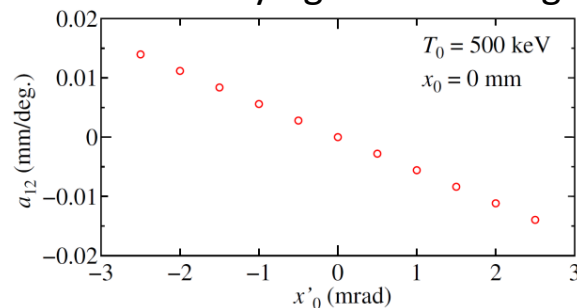
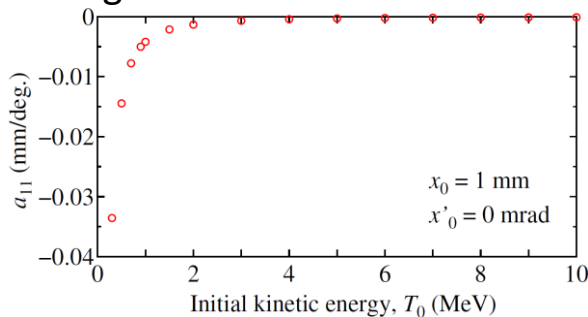
2.94 +/- 0.15 ps (rms)
for $E_{acc} = 7.95$ MV/m



Radial electric-field always **exists** in a TM01-mode accelerating cavity.

It was considered **harmful and interventive** (dilutes emittance) for high-quality low-energy beams.

Here, we propose to use it for measuring a **temporal profile/bunch length** of the high-quality beam with only existing instruments in the beamline. (below 3 MeV relying on the field gradient)



The center of the cavity should be defined to use this method.

We have proven that this can be done without special instruments

1. Measure $dY_C/d\phi_{SC1}$ with various offset (at least three point)
2. Estimate the electromagnetic center by $dY_C/d\phi_{SC1} = 0$
3. Confirm it by varying beam offset inside a cavity

See J.-G. Hwang et.al., Nucl. Instrum. Methods Phys. Res. Sect. A, 753, 97 (2014)

THANK YOU FOR YOUR ATTENTION



**Special thanks to compact-ERL collaborators
for their efforts in constructing and performing the machine commissioning.**