Identification of the Inter-Bunch and Intra-Bunch Beam Dynamics Based on Dynamic Modal Decomposition (DMD)

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Introduction

Model-based Beam Dynamics

- In general, the model estimation of the beam dynamics (Longitudinal / Transversal), based on measurement, assumes a defined model for the beam.
- The beam parameters of this *model-based* beam dynamics are important for both diagnostics and designs of control systems to stabilize the beam or improve its performance.
- *Data-based* modeling of dynamical systems is becoming an important technique allowing a free initial representation of the dynamics.
- There exists a synergy between the two options: *Physics/Model-based* and *Data-based* estimations, improving the future tools for diagnostics, identification and control.
- *Data-based* modeling is presented to describe intra-bunch and inter-bunch dynamics of beams.

Introduction

Data-based Beam Dynamics

The main objective is to characterize the intra/inter-bunch dynamics based on measurements. The beam dynamics can be represented by

$$\frac{dx(t)}{dt} = f(x(t)) \qquad \text{with } x \in R^n(C^n) \text{ (Inter-bunch dynamics)}$$

$$\frac{\partial x(z,t)}{\partial t} = g\left(\frac{\partial x(z,t)}{\partial z}, x(z,t)\right) \text{ with } x \in R^n(C^n) \text{ and } z \in R \text{ (Intra-bunch dyn.)}$$

if $x(z)$ is not evaluated continuously in z but at some discrete $z \in R^m$, then the partial differential equation (PDE) can be approximated by an ordinary differential equation (ODE).
The ODE can be approximated by a discrete equation, $x_{k+1} \approx F(x_k)$; where $x_k = x(t)\Big|_{t=k\Delta T}$ to represent the beam dynamics. In particular, around the operation point, it can be linearized by $x_{k+1} \approx A\,x_k$

Dynamic Modal Decomposition (DMD)

- Thus, A can be estimated from measurements x_{k+1}, x_k
- If $x_{k+1} \approx A\, x_k$, it is posible to define for N equally separated samples at $t = k\Delta T$

$$X = [x_1\, x_2 \ldots x_{N-1}] \in C^{n\times N-1} \text{ and } X' = [x_2\; x_3 \ldots x_N] \in C^{n\times N-1}, \text{ then}$$

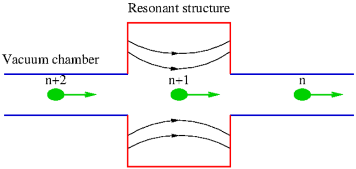
$$X' \approx A\, X \text{ with } A \in C^{n\times n}$$

- If X admits a singular value decomposition (SVD), then $X = U\Sigma V^H$ and then, $A = X'X^{\dagger} = X'V\Sigma^{-1}U^H$.
- A can be reduced to its dominant modes and gives the spatio-temporal behavior of $x(t)$ (eigenvalues Λ, eigenvectors Ψ)
- In particular, future motion can be estimated by $\tilde{x}(t)\Big|_{t=k\Delta T} \approx \sum_{i=1}^{r} \psi_i\; e^{\lambda_i k\Delta T} b_i = \Psi e^{\Lambda k\Delta T} b$

Example - Inter-bunch Dynamics

Longitudinal Dynamics in a Circular Accelerator

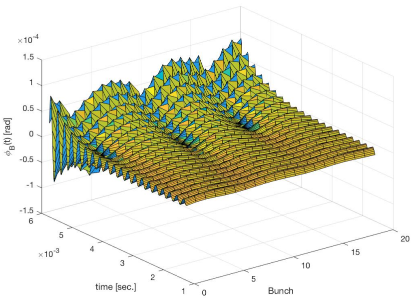
- The longitudinal inter-bunch dynamics is defined by the coupling between the individual bunches through the machine impedance distributed along the ring.
- In particular, for the low-order mode longitudinal beam dynamics the main source of coupling among bunches is the RF station impedance.

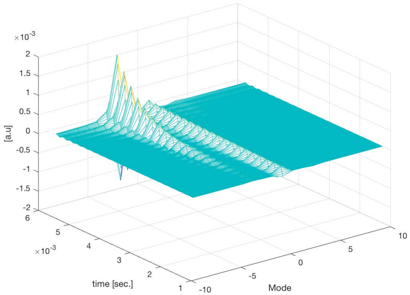


- Example: Simulations of the e^--ring of the EIC machine (Brookhaven Natl. Lab.), $E = 10$ GeV, $I_{b_{DC}} = 2.5$A, $V_{RF} = 17 \times 1.27 = 21.59$MV - Configuration: RF system with LLRF feedback, no comb filters, no longitudinal feedback. Beam unstable

Example - Inter-bunch Dynamics

Longitudinal Dynamics in a Circular Accelerator





Longitudinal motion of 20 macrobunches - Simulation

Modal decomposition using the 'N even-filled bunch' base $\varphi_m(t) = \frac{1}{n}\sum_{\ell=1}^{n} \phi_{B_\ell}(t)e^{-j2\pi\frac{m\ell}{n}}$ (Model-based analysis)

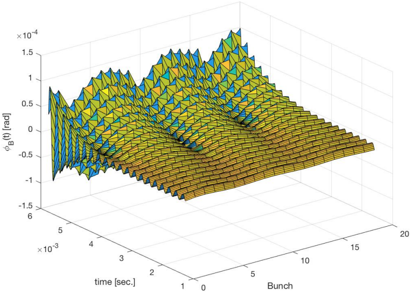
Example - Inter-bunch Dynamics

Longitudinal Dynamics in a Circular Accelerator

- Dominant modes (instables) eigenvalues of $\tilde{A}$:
  $\lambda_{(-3)} = 1198 + j2\pi 4.674e^3 sec^{-1}$
  $\lambda_{(-4)} = 818 + j2\pi 4.389e^3 sec^{-1}$
  $\lambda_{(-2)} = 106 + j2\pi 4.611e^3 sec^{-1}$
- Now if $\tilde{A}$ is known, it is possible to evaluate the response of the model

$$\tilde{\Phi}_B(t)\Big|_{t=k\Delta T} \approx \sum_{i=1}^{r} \psi_i \, a_i(k) b_i,$$

with $a_i(k) = e^{\lambda_i \Delta T \, k}$ and b_i the initial condition



Estimated longitudinal motion of 20 macrobunches

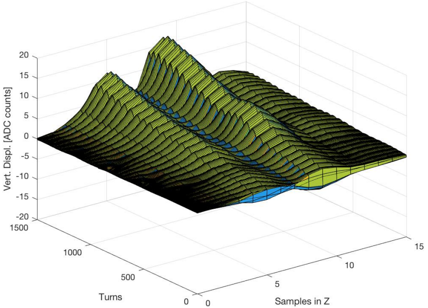
Example - Intra-bunch Dynamics

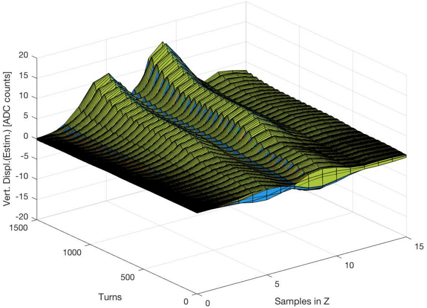
Transverse Bunch Dynamics in a Circular Accelerator

- The estimation of the intra-bunch dynamics is based on the transverse motion measurements of a single bunch circulating in the SPS ring at CERN.
- To measure the vertical motion along the bunch, a 3.2 GSamp/sec processing system is synchronized with the bunch and able to take 16 samples along the 5 ns bucket.
- The acquired magnitude corresponds to the dipole motion, that is the product of the vertical displacement $y(z)$ and the charge Q(z) at the longitudinal coordinate of the bunch z.
- The bunch dynamics is estimated based on measurements of the natural motion of an unstable bunch.

Example - Intra-bunch Dynamics

Transverse Bunch Dynamics in a Circular Accelerator



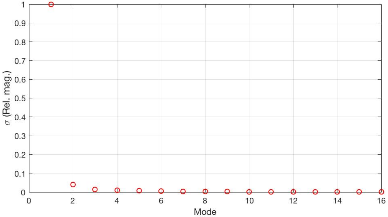


Vertical dipole displacement along the bunch - (Measured)

Vertical dipole displacement along the bunch - (Estimated)

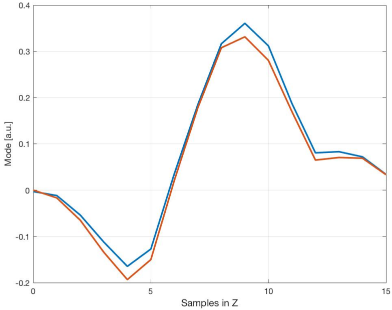
Example - Intra-bunch Dynamics

Transverse Bunch Dynamics in a Circular Accelerator



Singular Values of matrix X, define the dominant modes of the motion

- Eigenvalues of dominant modes
  $\lambda_a = 0.0013 + j2\pi 0.196$ 1/rev
  $\lambda_b = 0.0001 + j2\pi 0.1965$ 1/rev



Eigenfuncion $\psi_i(z)$ for the dominant modes

Conclusion

- Dynamical Modal Decomposition (DMD) is presented to analyze and identify the intra-bunch /inter-bunch dynamics in accelerator systems
- The method can be applied to create tools for bean diagnostics and could be extended to identify beam models to design feedback systems to stabilize the beam motion.
- The advantage of the method is that the model created does not require the knowledge of the eigenstructure of the system dynamics and based on the measurements or simulation results can estimate the eigenvalues - eigenvectors/eigenfunctions of the dominant modes of the motion.
- DMD has ben applied successfully in other fields dealing with ordinary/partial differential equations to describe the system dynamics.
- Further research it is necessary to improve the method and create tools for diagnostic and beam model identification.