Emittance Measurement Algorithm and Application to HIMM Cyclotron*

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Introduction

HIMM, a Heavy Ion Medical Machine, developed by Institute of Modern Physics, has been in operation since April 2020. The beam emittance of the cyclotron exit is measured with the most often used techniques, i.e. slit-grid, Q-scan and 3-grid at a dedicated beam line which is not the actual HIMM optical line. The high speed data acquisition architecture is based on FPGA, and motion control system is constructed based on the NI module.

The data post processing and emittance calculation is based on Python code with self-developed algorithm, including Levenberg–Marquardt optimization algorithm, thick lens model, dispersion effect correction, error bar fit, mismatch check, image denoise and "Zero-thresholding" calculation. .

Algorithm

- **optimization algorithm for Q-scan and 3-grid**

Assuming the transfer matrix can be written as

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

After the implementation of the matrix, the squared beam distribution, with the dispersion considered and chromaticity ignored, in the real space at the exit of the element is

$$\Sigma_{11}^{f} =$$

$$a^2\left(\langle x_\beta^2\rangle + D^2\langle\delta^2\rangle\right) + 2ab\left(\langle x_\beta x'_\beta\rangle + DD'\langle\delta^2\rangle\right) + b^2\left(\langle x'^2_\beta\rangle + D'^2\langle\delta^2\rangle\right)$$

For a configuration of focus lens followed by a drift section,

$$a(k) = cos\sqrt{k}l - \sqrt{k}\mathrm{L}\sin\sqrt{k}l$$

$$b(k) = \frac{1}{\sqrt{k}}\sin\sqrt{k}l + \mathrm{L}\ cos\sqrt{k}l$$

with $k = \frac{B'}{B\rho}, B'l = C_1 + C_2 I$. C_1 and C_2 for calibrated coefficient. Therefore, for Q-scan schedule, the desired values, i.e. $\langle x_\beta^2\rangle$, $\langle x_\beta x'_\beta\rangle$, $\langle x'^2_\beta\rangle$,can be obtained via solving a non-linear least squares problem with the objective function given as follows

$$\Sigma_{11}^{f}(I, \boldsymbol{\alpha}; D, D', \delta), with\ \boldsymbol{\alpha} = \left(\langle x_\beta^2\rangle, \langle x_\beta x'_\beta\rangle, \langle x'^2_\beta\rangle\right)$$

To find a solution, the following minimization procedure would be implemented,

$$\hat{\boldsymbol{\alpha}} = argmin_{\boldsymbol{\alpha}} S(\boldsymbol{\alpha}) = argmin_{\boldsymbol{\alpha}} \sum_{i=1}^{n}\left[y_i - \sqrt{\Sigma_{11}^{f}(I_i, \boldsymbol{\alpha}; D, D', \delta)}\right]^2$$

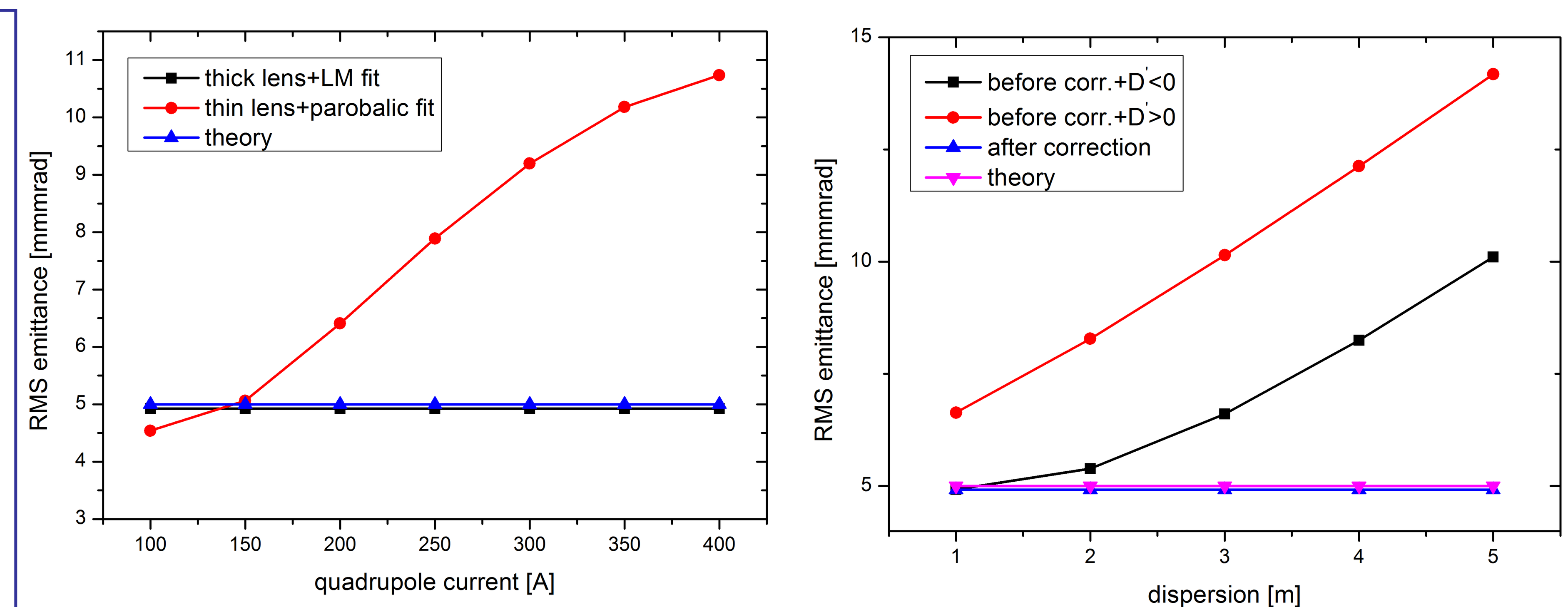
If data uncertainties involved, the minimization may be scaled with a weighting factor ε_i that refers to the uncertainties of the profile y_i,

$$\hat{\boldsymbol{\alpha}} = argmin_{\boldsymbol{\alpha}} \sum_{i=1}^{n}\left[\frac{y_i - \sqrt{\Sigma_{11}^{f}(I_i, \boldsymbol{\alpha}; D, D', \delta)}}{\varepsilon_i}\right]^2$$

The most widely used optimization algorithm for this is the so called Levenberg–Marquardt algorithm (LMA or just LM), which can be thought of a combination of the method of gradient descent and the Gauss–Newton algorithm (GNA).

For 3-grid schedule, the variable would be drift length L, hence, the objective function will be

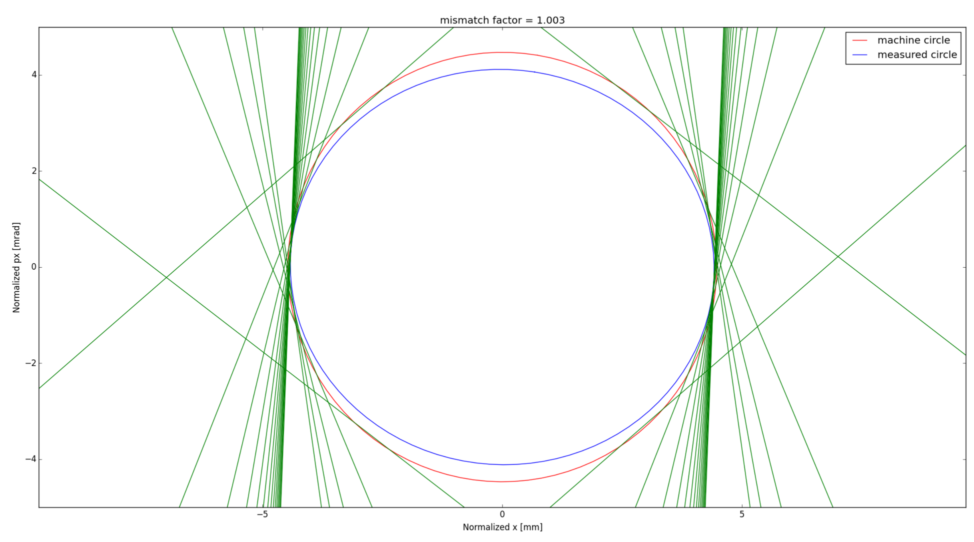
$$\Sigma_{11}^{f}(L_i, \boldsymbol{\alpha}; D, D', \delta)$$



Algorithm performance simulation. Left: Levenberg–Marquardt algorithm based thick lens model reflects the theory emittance better. Right: after the dispersion correction, with the Levenberg–Marquardt algorithm employed, the theory emittance is reconstructed well.

After the TWISS parameters are extracted, mismatch will be visualized in the normalized phase space and the mismatch factor is also presented meanwhile.

$$\xi = \frac{1}{2}(\gamma_D\beta - 2\alpha_D\alpha + \beta_D\gamma)$$

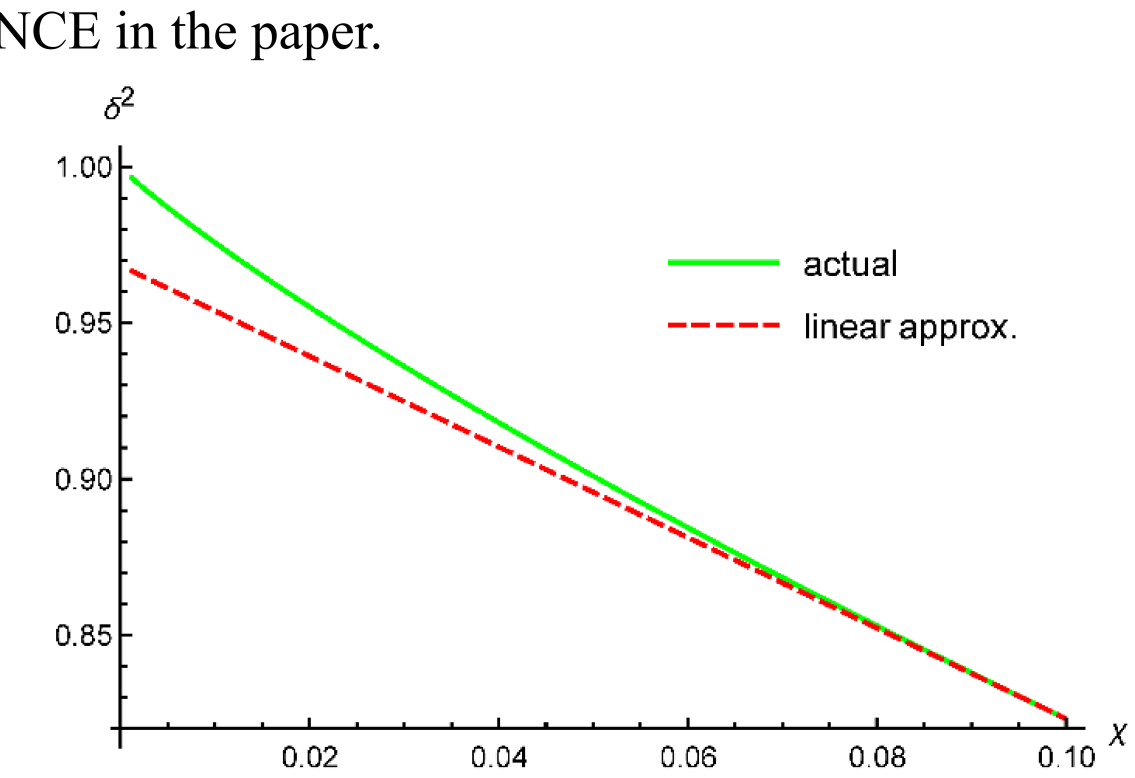


- **'zero-thresholding' approach for slit-grid technique**

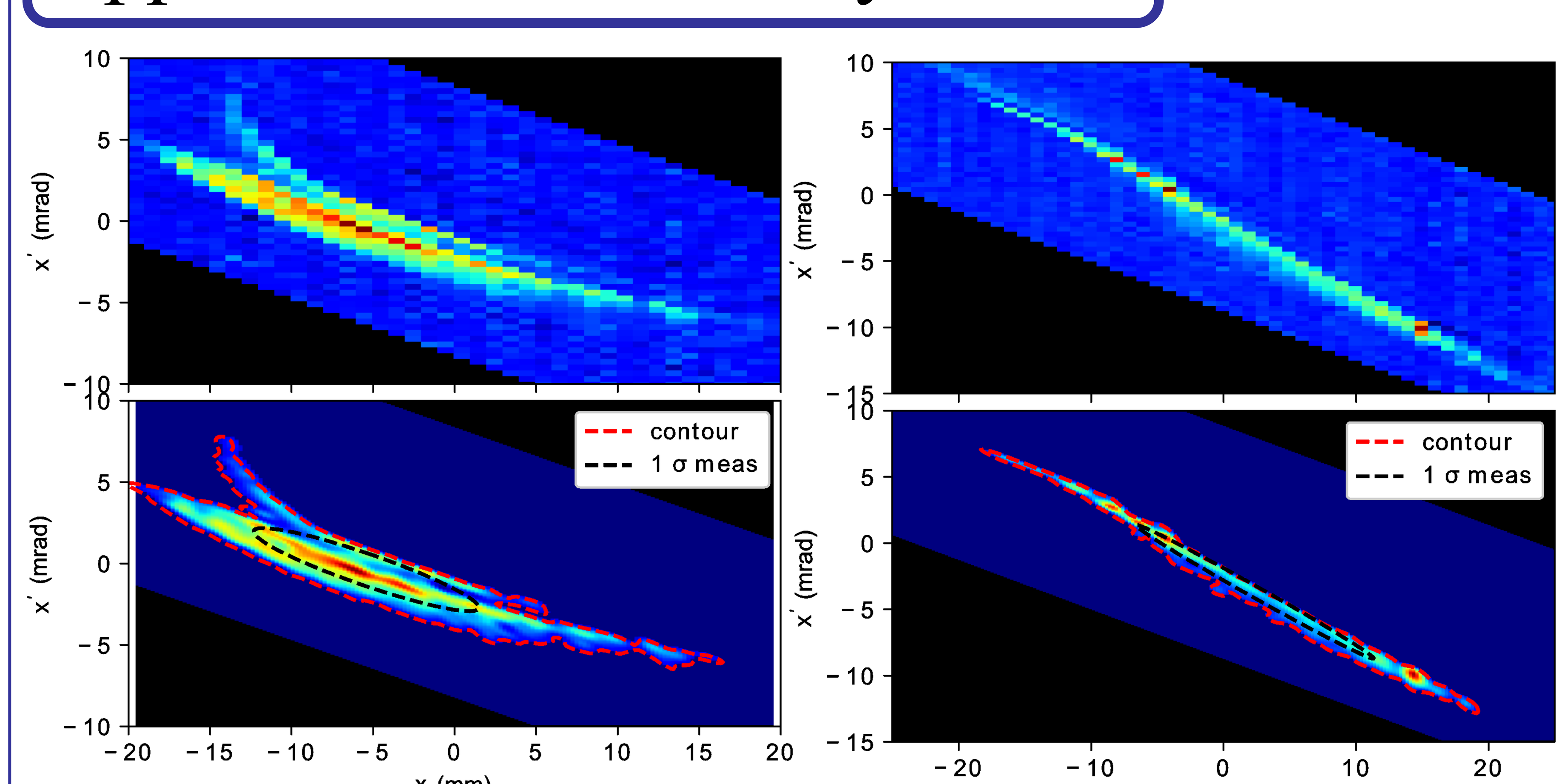
The slit-grid measurement would generate a 2D phase space that is the area the emittance calculation launched. From which, the RMS emittance can be retrieved statistically.

Typically, the data are noisy owing to disturbances from the MW electronics and the environment. It is, therefore, necessary to denoise the data before performing the calculation. A systematic procedure to complete the emittance evaluation is given in [1], in whih, a method called 'zero-thresholding' approach is proposed.

The theory supporting the "zero-thresholding"based emittance extraction is given in APPENDANCE in the paper.



Application to HIMM cyclotron



Phase space distribution obtained from slit-grid technique. Left: the horizontal plane, Right: the vertical plane. Top: the raw phase space, bottom: the denoised phase space, the red dashed line is the contour line with a contour level of 10%, and the black dashed line is the measured 1σ phase ellipse.

References

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