# THE EVALUATION OF BEAM INCLINATION ANGLE ON THE CAVITY BPM POSITION MEASUREMENT* 

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## Abstract

Cavity beam position monitor (CBPM) is widely used to measure the transverse position in free-electron laser (FEL) and international linear collider (ILC) facilities due to the characteristic of high sensitive. In order to study the limiting factors of the position resolution of cavity BPM, the influence of beam inclination angle on the measurement of CBPM position and the direction of beam deflection was analyzed. The simulation results show that the beam inclination angle is an important factor limiting the superiority of CBPM with extremely high position resolution. The relative beam experiments to change the relative inclination angle between the cavity and the electron beam based on the kicker were performed in Shanghai Soft X-ray FEL (SXFEL) facility, the experiment results will also be mentioned as well.

## INTRODUCTION

The free electron laser (FEL) is a fourth-generation light source based on the interaction between electromagnetic fields and ultra-relativistic electron bunches which travel along the axis of a vacuum beam-pipe. In order to achieve high efficiency operation of FEL, the electron beam and the generated photo beam need to be overlapped strictly and that both can pass through the entire undulator section. Therefore, requirements on the BPM system for the FEL are very stringent, especially on position stability.

Cavity BPM systems [1] adopt a resonant cavity structure and through the use of anti-symmetric characteristic mode, coupled from the cavity, to measure the beam position which can meet that requirements, so it is widely used in FEL facilities. In order to maximize the advantages of cavity BPM with extremely high-resolution, a detailed analysis of the limiting factors affecting CBPM performances is needed, especially the effect of beam inclination angle on CBPM performance. Based on this purpose, the influence of beam trajectory angle on the amplitude of position signal and the direction judgement of beam position was simulated in this paper.

Shanghai soft X-ray FEL facility is a user experiment facility with an expected capacity to generate 9 -nm X-ray laser by adopting an FEL frequency doubling of ultraviolet band seeded laser of 265 nm . A total of 20 CBPM systems are installed in the undulator section for the beam position measurement. Therefore, SXFEL is also an excellent test platform for experiment verification of beam trajectory angle simulation which mentioned above. De-
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tailed calculation principles, simulation results and online beam verification results will be given in the following sections.

## CALCULATION PRINCIPLE

In order to calculate the signal intensity generated when the beam trajectory has an angle through the cavity, a method of dividing a large cavity into a plurality of small cavities is adopted. As shown in Fig. 1. The position of the bunch in each small cavity can be considered to be parallel to the Z axis. Then, all the small cavities can be integrated to obtain the signal intensity when the beam has an angle and pass through the entire cavity.


Figure 1: Model of cavity segmentation.
When the beam is parallel to the Z axis and the distance from the electrical center is $x$, the cavity excitation signal can be expressed by Eq.(1):

$$
\begin{equation*}
V_{p}=\frac{\omega q}{2} \sqrt{\frac{Z}{Q_{e x t}}\left[\frac{R}{Q}\right]_{0}} * e^{-\frac{\omega^{2} \sigma_{2}^{2}}{c^{2}}} * \frac{x}{x_{0}} * e^{-\frac{t}{2 \tau} * \sin (\omega \mathrm{t})} \tag{1}
\end{equation*}
$$

Assume that the length of the bunch is constant during this process, the cavity parameters are also fixed, and the constant term can be separated, it can expressed by Eq. (2) briefly:

$$
\begin{equation*}
V_{p}=A x * e^{-\frac{t}{2 \tau}} * \sin (\omega \mathrm{t}) \tag{2}
\end{equation*}
$$

When the beam passes through the center of the cavity but with an inclination angle $\theta$, the excitation voltage of the small cavity whose cavity length is dz can be written by Eq. (3):

$$
\begin{equation*}
d v=A^{*} z * \tan (\theta) * e^{-\frac{t}{2 \tau}} * \sin (\omega \mathrm{t}) * \frac{d z}{L} \tag{3}
\end{equation*}
$$

So the signal intensity when the beam has an angle $\theta$ and pass through the entire cavity can be calculated by
integrating Eq. (3) over the entire length $L$ of the cavity, which can be expressed by Eq. (4):

$$
\begin{align*}
& v_{\theta} \approx \frac{A \tan (\theta)}{L} * e^{-\frac{t}{2 \tau}} * \int_{-\frac{L}{2}}^{\frac{L}{2}} z^{*}\left[\cos (\omega \mathrm{t}) * \sin \left(\frac{\omega z}{c \cos (\theta)}\right)\right] d z  \tag{4}\\
& =\frac{A \tan (\theta)}{L} * e^{-\frac{t}{2 \tau}} * \cos (\omega \mathrm{t})\left[\frac{2 c^{2} \cos ^{2} \theta}{\omega^{2}} \sin \frac{\omega L}{2 c \cos \theta}-\frac{L c \cos \theta}{\omega} \cos \frac{\omega L}{2 c \cos \theta}\right]
\end{align*}
$$

From the Eq. (4), when the beam pass through the cavity center with an angle $\theta$, the phase of the excitation signal is $90^{\circ}$ out of phase with the positional offset signal. Therefore, the signal excited by the angle cannot be eliminated by the movement of the beam.

## SIMULATION RESULTS

Based on the Eq. (4) and the cavity parameters designed by SINAP [2-3], the simulation results on the relationship between the beam angle $\theta$ and the equivalent beam offset is shown in Fig. 2.


Figure 2: Relationship between the beam angle and the equivalent beam offset.

The results show if the beam trajectory has an angle about 1 mrad , the excited signal equivalent to the beam offset about 0.6 um and this excited signal cannot be eliminated by the movement of the beam. It means that CBPM cannot be resolved if the beam offset is less than 0.6um.

When the beam trajectory has an angle, the effect of cavity length $L$ on the equivalent beam offset is also evaluated. The results can be seen in Fig. 3.


Figure 3: The effects of cavity length $L$ on the equivalent beam offset.

The simulation results show that when the angle of beam trajectory is fixed, the equivalent beam offset increases with the increase of the cavity length, which is consistent with the physical theory.

For another case, when the beam trajectory with an angle of $\theta$ but not pass through the electrical center of the cavity, as shown in Fig. 4.


Figure 4: Diagram when the beam with an angle but not pass through the electrical center.

Assume the beam trajectory with an angle of $\theta$ and the cross point with the Z axis is the m , then the excited signal about the can be expressed by Eq. (5):

$$
\begin{align*}
& v_{\theta}=A \int_{-\frac{L}{2}}^{\frac{L}{2}}(z-m) * \tan (\theta) * e^{-\frac{t+\frac{z}{c \cos \theta}}{2 \tau}} * \sin \left[\omega\left(\mathrm{t}+\frac{z}{c \cos (\theta)}\right)\right] * \frac{d z}{L}  \tag{5}\\
& =\frac{A \tan (\theta)}{L} * e^{-\frac{t}{2 \tau}} * \cos (\omega \mathrm{t})\left[\frac{2 c^{2} \cos ^{2} \theta}{\omega^{2}} \sin \frac{\omega L}{2 c \cos \theta}-\frac{L c \cos \theta}{\omega} \cos \frac{\omega L}{2 c \cos \theta}\right] \\
& -\frac{A m \tan (\theta)}{L} * \frac{2 c \cos (\theta)}{\omega} * \sin \left(\frac{\omega L}{2 c \cos (\theta)}\right) * e^{-\frac{t}{2 \tau} * \sin (\omega \mathrm{t})}
\end{align*}
$$

It can be divided into two parts to explain, the first part is the same with the Eq. (4) which can be equivalent to the case where the bunch has an angle but passes through the center of the cavity. The second part which can be equivalent to the case that the beam parallel to the Z axis with a beam offset. So the relationship between beam offset and equivalent beam offset with different beam trajectory angle can be illustrated in Fig. 5.


Figure 5: Relationship between beam offset and equivalent beam offset with different beam trajectory angle.

Due to the existence of beam trajectory angle, the effects on resolution is greater when working near the electricity center, while the effects is relatively small when the beam is off center ( $>50 \mathrm{um}$ ).

What is discussed above is the case when the beam trajectory has an angle, so it is also necessary to discuss the effects when the bunch itself has an angle. Assuming that the bunch distributed in a Gaussian, the length of the
bunch is $\sigma$, the charge of the bunch is q , and the inclination of the bunch is $\alpha$. The model can be seen in Fig. 6.


Figure 6: Diagram when the bunch itself has an angle.
When the bunch passes through the cavity along the $Z$ axis, the equivalent result is considered to divide the bunch into a group of electrons with a charge dq distributed along the z-axis, can be written as Eq. (6):

$$
\begin{equation*}
d q=\frac{q}{\sqrt{2 \pi} \sigma_{z}} * e^{-\frac{z^{2}}{2 \sigma_{z}^{2}}} d z \tag{6}
\end{equation*}
$$

Integrating the entire space, the excited signal can be expressed by Eq. (7):

$$
\begin{align*}
& v_{\alpha}=A * \frac{\tan (\alpha)}{\sqrt{2 \pi} \sigma_{z}} \int_{-\infty}^{+\infty} z * e^{-\frac{z^{2}}{2 \sigma_{z}^{2}}} * e^{-\frac{\left(t-\frac{z}{c}\right)}{2 \tau}} * \sin \left[\omega\left(\mathrm{t}-\frac{z}{c}\right)\right] \mathrm{d} \mathrm{~d} \\
& =A \frac{\tan (\alpha)}{\sqrt{2 \pi} \sigma_{z}} * e^{-\frac{t}{2 \tau}} \int_{-\sigma_{z}}^{+\sigma_{z}} z^{*} e^{-\frac{z^{2}}{2 \sigma_{z}^{2}}} * e^{\frac{z}{2 c \tau}} * \sin \left[\omega\left(\mathrm{t}-\frac{z}{c}\right)\right] \mathrm{d} \mathrm{z}  \tag{7}\\
& \approx A \frac{\omega \sigma_{z}^{2} \tan (\alpha)}{c} * e^{-\frac{t}{2 \tau}} * \cos (\omega \mathrm{t})
\end{align*}
$$

Therefore, when the bunch itself has an angle, the phase of the excitation signal and the position offset signal are also $90^{\circ}$ out of phase. The simulation results on bunch angle and equivalent beam offset is shown in Fig. 7.


Figure 7: Simulation results on bunch angle and equivalent beam offset.

Compared with the beam trajectory angle, at the same angle of inclination, beam itself angle differs by about 18 orders of magnitude in the equivalent beam offset. Therefore, it can be considered that in the cosine term introduced by this case, the influence of the angle of the bunch itself on the beam position measurement is negligible.

## BEAM EXPERIMENTS

Some experiments based on the cavity BPM and corrective magnet which can change the angle of the beam
trajectory to verify the simulation qualitatively. Schematic of the experiment is shown in Fig. 8.


Figure 8: Experiment schematic to verify the simulation results.

Through the corrective magnet located in front of the CBPM to change the angle $\theta$ and cross point with the $Z$ axis and calculate current position.


Figure 9: Experiment results when bunch with an angle.
As shown in Fig. 9, we adjusted the beam with an angle and closed to the cavity center, but we cannot find the position of electrical center. From the Fig. 9, we can find there has an indistinguishable area about 8 um . Verified the simulation qualitatively but a more accurate verification test still needs to be built on the four-dimensional mobile platform for testing.

## CONCLUSION

In this paper, the influence of the beam trajectory angle and beam's angle on the CBPM for beam position measurement was considered, related calculation and simulation has been done. Simulation results shown that the beam trajectory angle will have effects for the resolution of CBPM when electron beam closed to the electric center. Preliminary validation experiment was performed in SXFEL, but more quantitative experiments need to build a 4D mobile platform in SXFEL. And we are looking forward to getting better results in the future.

## REFERENCES

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