ISBN: 978-3-95450-201-1 doi:10.18429/JACoW-IBIC2018-MOPB10

A STUDY ON THE INFLUENCE OF BUNCH LONGITUDINAL DISTRIBUTION ON THE CAVITY BUNCH LENGTH MEASUREMENT*

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Abstract

Cavity bunch length measurement is used to obtain the bunch length depending on the eigenmodes exciting inside the cavity. For today's FELs, the longitudinal distribution of particles in electron bunch (bunch shape) may be non-Gaussian, sometimes very novel. In this paper, the influence of bunch shape on the cavity bunch length measurement is analyzed, and some examples are given to verify the theoretical results. The analysis shows that the longitudinal distribution of particles in electron bunch has little influence on the cavity bunch length measurement when the bunch length is less than 1 ps and the eigenmodes used in measurement are below 10GHz.

INTRODUCTION

Bunch length is one of the important characteristics of charged particle beam in accelerators. Compared with the traditional methods, bunch length monitor based on resonant cavities has great potential especially for high quality beam sources, for it has superiority, such as simple structure, wide application rage, and high signal to noise ratio [1]. What's more, the eigenmodes of cavities are used in combined measurement of bunch length, beam intensity, position and quadrupole moment. For example, the monopole modes can be used to measure the bunch length and the beam intensity [2]. At the same time, the dipole modes are always utilized to obtain the beam position offset [3]. What's more, we could decide the quadrupole moment by analyzing the TM220 modes of the square resonators [4]. Therefore, the measurement device shows the characteristic of terseness and compaction.

The present FELs show characteristic of very short bunch. For example, the bunch length of Shanghai soft X-ray free Electron laser (SXFEL) is several hundred femtoseconds. At the same time, the longitudinal distribution of particles in electron bunch (bunch shape) may be non-Gaussian, sometimes very novel. While analyzing the beam-cavity interaction, we assume the longitudinal distribution of particles in electron bunch is Gaussian in a general way. Then what is the impact of the non-Gaussian bunch on the cavity bunch length monitor? In this paper, the influences of the different bunch shapes on the cavity bunch length measurement are analyzed under different circumstances, and the results provide theoretical support for the future FEL bunch length measurements using resonant cavities.

MESUREMENT OF GAUSSIAN BUNCH

While passing through a cavity, an electron whose charge is q can excite a series of eigenmodes. The voltage amplitude of an eigenmode can be expressed as

$$V_q = 2k_n q \qquad (1)$$

Where k_n is the loss factor which is related to R/Q of the eigenmode [5]. It can be seen that the voltage is related to the charge q. As for a bunch whose total charge is Q_b, the voltage amplitude of an eigenmode excited inside the cavity is the superposition of the electrons in the bunch which arrive at the cavity at different times. The moments when the electrons arrive at the cavity depend on the longitudinal distribution of particles in electron bunch. Assume an electron bunch whose longitudinal normalized distribution function of electrons is f(t). The moment when the center of the bunch arrives at the center of the cavity is defined as zero time. So the voltage amplitude of an eigenmode when the charge arriving at the cavity at time t can be expressed as

$$dV = 2k_n dq \qquad (2)$$

Considering the voltage phases excited by electrons in different places is disparate, the voltage amplitude of an eigenmode can be written as

$$\begin{aligned} V_b &= \int_{-t_m}^{t_m} e^{i\omega_n t}\, dV \\ &= 2k_n \int_{-t_m}^{t_m} e^{i\omega_n t}\, dq \\ &= 2k_n \int_{-t_m}^{t_m} I(t)\times e^{i\omega_n t}\, dt \\ &= 2k_n Q_b \int_{-t_m}^{t_m} f(t)\times e^{i\omega_n t}\, dt \end{aligned} \qquad (3)$$

Where ω_n is the frequency of the eigenmode, t_m is the range of the bunch longitudinal distribution, and I(t) represents beam current. In a general way, the longitudinal distribution of the bunch is regarded as Gaussian distribution, so

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{t^2}{2\sigma^2}) \qquad (4)$$

* Supported by National Key R&D Program of China (Grant No. 2016YFA0401900, 2016YFA0401903), the National Science Foundation of China (Grant No. 11375178, 11575181) and the Fundamental Research Funds for the Central Universities (WK2310000046).
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ISBN: 978-3-95450-201-1 doi:10.18429/JACoW-IBIC2018-MOPB10

Where σ represents the bunch length. The voltage amplitude of an eigenmode excited by the bunch whose charge is Qb can be described as

$$V_b = 2k_n Q_b \int_{-t_m}^{t_m} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} e^{i\omega_n t} dt \\ = 2k_n Q_b \, e^{-\frac{\omega_n^2\sigma^2}{2}} \tag{5}$$

Both Q_b and σ are unknown quantity, so we need two eigenmodes at least and the bunch length can be calculated by solving these equations.

$$\begin{cases} V_{b1} = 2k_{n1} Q_b \, e^{-\frac{\omega_{n1}^2\sigma^2}{2}} \\ V_{b2} = 2k_{n2} Q_b \, e^{-\frac{\omega_{n2}^2\sigma^2}{2}} \end{cases} \tag{6}$$

That is the conventional method used to obtain the bunch length.

THE INFLUENCE OF BUNCH SHAPE

In this section, the influence of the longitudinal distribution of particles in electron bunch on the bunch length measurement mentioned above will be analyzed.

The root-mean-square value of the bunch longitudinal distribution is defined as bunch length σ. When the centre of the bunch is the origin of the normalized distribution function, the root-mean-square value is the same size as the standard deviation of the longitudinal distribution,

$$\sigma = \sqrt{\int_{-\infty}^{+\infty} f(t) \times t^2 dt} \tag{7}$$

It means a centralized region of most particles, not the entire range of the bunch longitudinal distribution. Define the integral in Eq. (3) is the bunch shape factor V_{shape},

$$V_{shape} = \int_{-t_m}^{t_m} f(t) \times e^{i\omega_n t} dt \tag{8}$$

Comparing Eq. (8) and Eq. (3), it can be seen that the voltage amplitude of an eigenmode excited by a particle and by a bunch are similar. The bunch shape factor V_{shape} is considered in Eq. (8). The following is the analysis of V_{shape}. The n-order Maclaurin series with Lagrange remainder term can be expanded as

$$e^{i\omega_n t} = 1 + i\omega_n t - \frac{\omega_n^{\,2} t^2}{2!} - i\frac{\omega_n^{\,3} t^3}{3!} + \frac{\omega_n^{\,4} t^4}{4!} \\ + \ldots\ldots + \frac{(\omega_n t)^n}{n!} + \frac{e^{i\theta\omega_n t}(\omega_n t)^{n+1}}{(n+1)!} \tag{9}$$

Where 0 < θ < 1. We extract the real part of Eq. (9) and take the sum of first two terms,

$$V_{shape} = \int_{-t_m}^{t_m} f(t) \times [1 - \frac{\omega_n^{\,2} t^2}{2} + \frac{e^{i\theta\omega_n t}}{4!}(\omega_n t)^4] dt \\ = \int_{-t_m}^{t_m} f(t) dt - \int_{-t_m}^{t_m} f(t) \times \frac{\omega_n^{\,2} t^2}{2} dt \\ + \int_{-t_m}^{t_m} f(t) \times \frac{e^{i\theta\omega_n t}}{4!} (\omega_n t)^4 dt \tag{10}$$

Considering that f(t) is the normalized distribution function, according to the definition of bunch length, Equation (10) can be described as

$$V_{shape} \approx 1 - \frac{\omega_n^{\,2}\sigma^2}{2} \tag{11}$$

The approximation error can be expressed by the Lagrange remainder term,

$$\Delta = \int_{-t_m}^{t_m} [f(t) \times \frac{\exp(i\theta\omega_n t)}{4!} (\omega_n t)^4] dt \tag{12}$$

Given f(t) > 0, divide Eq. (12) by V_{shape},

$$\frac{\Delta}{V_{shape}} = \frac{\int_{t_{\min}}^{t_{\max}} [f(t) \times \frac{\exp(i\theta\omega_n t)}{4!} (\omega_n t)^4] dt}{\int_{t_{\min}}^{t_{\max}} f(t) \times \exp(i\omega_n t) dt} \\ < \frac{\int_{-t_m}^{t_m} f(t) \times \frac{(\omega_n t)^4}{4!} dt}{\int_{-t_m}^{t_m} f(t) \times \exp(i\omega_n t_m) dt} \\ < \frac{\int_{-t_m}^{t_m} [f(t) \times \frac{(\omega_n t_m)^4}{4!}] dt}{\exp(i\omega_n t_m) \times \int_{-t_m}^{t_m} f(t) dt} \\ = \frac{(\omega_n t_m)^4}{24} \cos(\omega_n t_m) \tag{13}$$

The maximum of the approximation relative error can be obtained by this equation. The relationship between $\omega_n t_m$ and the maximum of the approximation relative error is shown in Fig. 1.

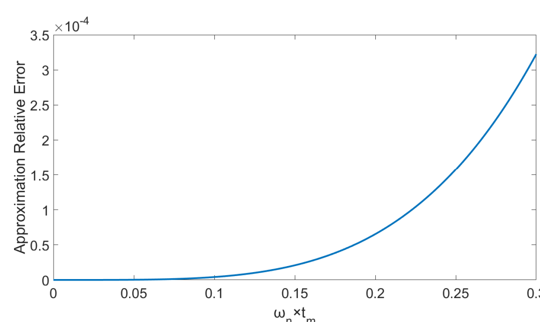

Figure 1: $\omega_n t_m$ versus the maximum of the approximation relative error.

It can be seen that we preferred smaller $\omega_n t_m$ which means the approximate value in Eq. (11) approaches to

ISBN: 978-3-95450-201-1 doi:10.18429/JACoW-IBIC2018-MOPB10

the value of V_{shape}. The bunch length of today's FELs is about hundreds of femtoseconds, and the entire range of the bunch longitudinal distribution is approximately from -2 ps to 2 ps. The frequency of the eigenmode used for measurement can reach to about 10GHz. In this case,

$$\frac{\Delta}{V_{shape}} < 1.03\times10^{-5} \tag{14}$$

It can be seen from the results that we can take the approximation like Eq. (11) in spite of bunch shape and the approximation relative error is less than 1.03×10^{-5}. The voltage amplitude of an eigenmode excited by a bunch can be written as

$$V = 2k_nQ_bV_{shape} = 2k_nQ_b(1-\frac{\omega_n{}^2\sigma^2}{2}) \tag{15}$$

The voltage is irrelevant to the longitudinal distribution of particles in electron bunch. Therefore, the bunch is able to be regard as Gaussian bunch in this case.

VERIFICATION

Many different kinds of bunch longitudinal distributions are shown in Fig. 2 to Fig. 8. There are the normalized probability density function analytic expressions of uniformly distributed bunch, flat-topped bunch, parabolic bunch, Gaussian bunch, triangular bunch and two kinds of double-humped bunches, respectively.

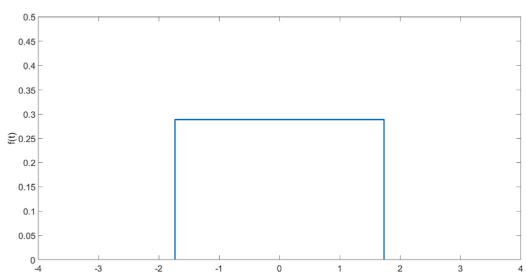

Figure 2: The normalized probability density function of uniformly distributed bunch.

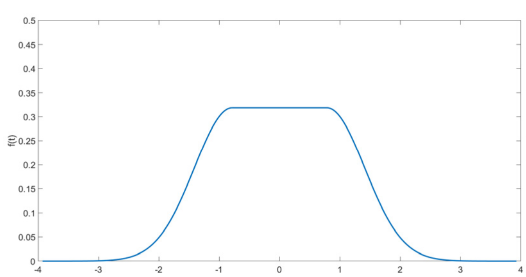

Figure 3: The normalized probability density function of flat-topped bunch.

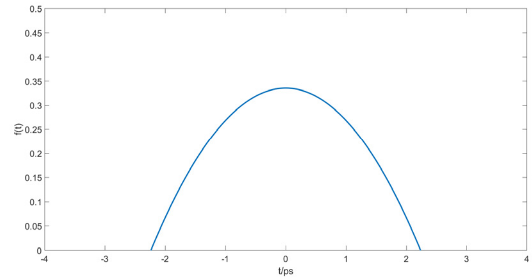

Figure 4: The normalized probability density function of parabolic bunch.

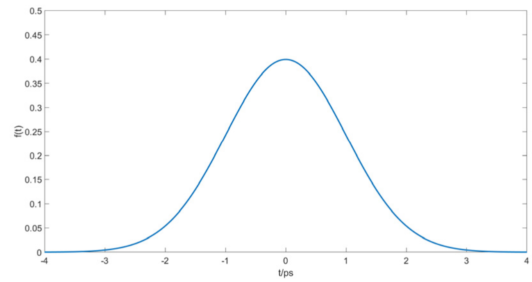

Figure 5: The normalized probability density function of Gaussian bunch.

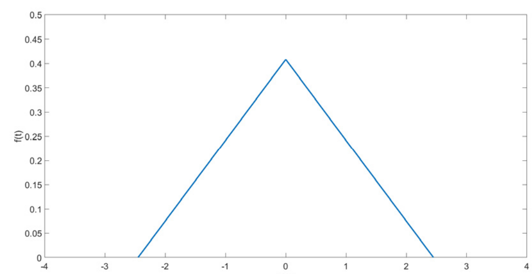

Figure 6: The normalized probability density function of triangular bunch.

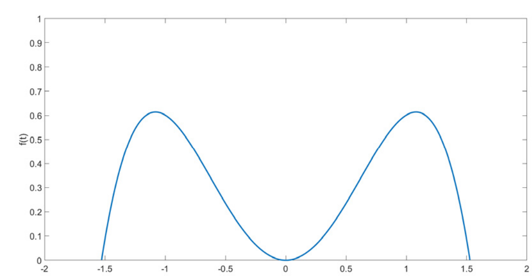

Figure 7: The normalized probability density function of double-humped bunch 1.

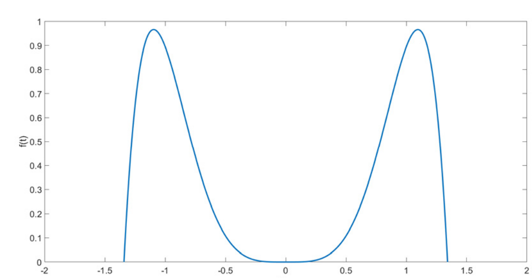

Figure 8: The normalized probability density function of double-humped bunch 2.

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$$f(t)=\frac{\sqrt{3}}{6},[-\sqrt{3},\sqrt{3}] \quad (16)$$

$$f(t)=\begin{cases}\frac{1}{2}N(-\sqrt{\frac{3\pi}{9+2\pi}},\frac{6}{9+2\pi}),(-\infty,-\sqrt{\frac{3\pi}{9+2\pi}}) \\ \frac{1}{2}\times\sqrt{\frac{12\pi}{9+2\pi}},[-\sqrt{\frac{3\pi}{9+2\pi}},\sqrt{\frac{3\pi}{9+2\pi}}] \\ \frac{1}{2}N(\sqrt{\frac{3\pi}{9+2\pi}},\frac{6}{9+2\pi}),(\sqrt{\frac{3\pi}{9+2\pi}},+\infty)\end{cases} \quad (17)$$

$$f(t)=-0.03\times\sqrt{5}\times t^2+0.15\times\sqrt{5},[-\sqrt{5},\sqrt{5}] \quad (18)$$

$$f(t)=N(0,1),(-\infty,+\infty) \quad (19)$$

$$f(t)=\begin{cases}\frac{t}{6}+\frac{\sqrt{6}}{6},[-\sqrt{6},0) \\ -\frac{t}{6}+\frac{\sqrt{6}}{6},[0,\sqrt{6}]\end{cases} \quad (20)$$

$$f(t)=-\frac{135}{196}\times\sqrt{\frac{3}{7}}\times t^4+\frac{45}{28}\times\sqrt{\frac{3}{7}}t^2,[-\sqrt{\frac{7}{3}},\sqrt{\frac{7}{3}}] \quad (21)$$

$$f(t)=-\frac{4375}{8748}\times\sqrt{5}t^6+\frac{875}{972}\times\sqrt{5}t^4,[-\sqrt{\frac{9}{5}},\sqrt{\frac{9}{5}}] \quad (22)$$

Their charges are all 1 nC and their bunch lengths (root-mean-square value of the distribution) are all 1 ps. The relationship between their corresponding shape factors V_{shape} and the eigenmode working frequencies ω_n is shown in Fig. 9 and Fig. 10. Figure 10 is the larger version of Fig. 9 at the low frequency part.

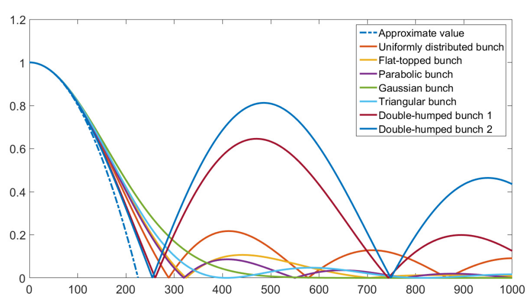

Figure 9: The relationship between V_{shape} and the eigenmode working frequencies ω_n.

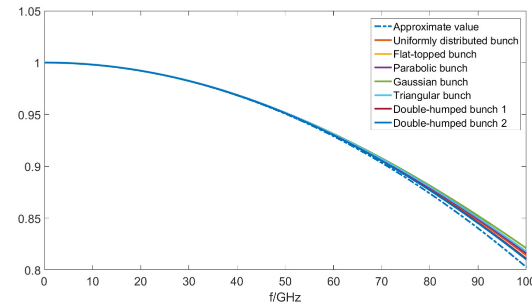

Figure 10: the larger version of Fig. 9 at the low frequency part.

The imaginary lines in the above pictures represent the approximate expression Eq. (11). It can be seen that an eigenmode voltage excited by a bunch in cavity is irrelevant to the longitudinal distribution of particles in electron bunch when the frequencies of the eigenmodes used for measurement are below 50GHz. In that case, the results of the Gaussian bunch measurements are able to take the place of the results of the bunches with other shapes measurements. Even if the frequency of the eigenmode used for measurement is equal to 50GHz, the maximum relative error is merely 5.25×10^{-3}.

CONCLUSION

In this paper, the impact of bunch shape on cavity bunch length measurement is analyzed in theory. The shorter the bunch, the less the influence from the longitudinal distribution of particles in electron bunch is. For present and future FEL whose bunch length is less than 1ps, the longitudinal distribution of particles in electron bunch do not have to be taken into account when we use the cavity to obtain the bunch length. The bunch is able to be regard as Gaussian bunch in that case. This conclusion provides theoretical support for the future FEL bunch length measurements using resonant cavities.

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