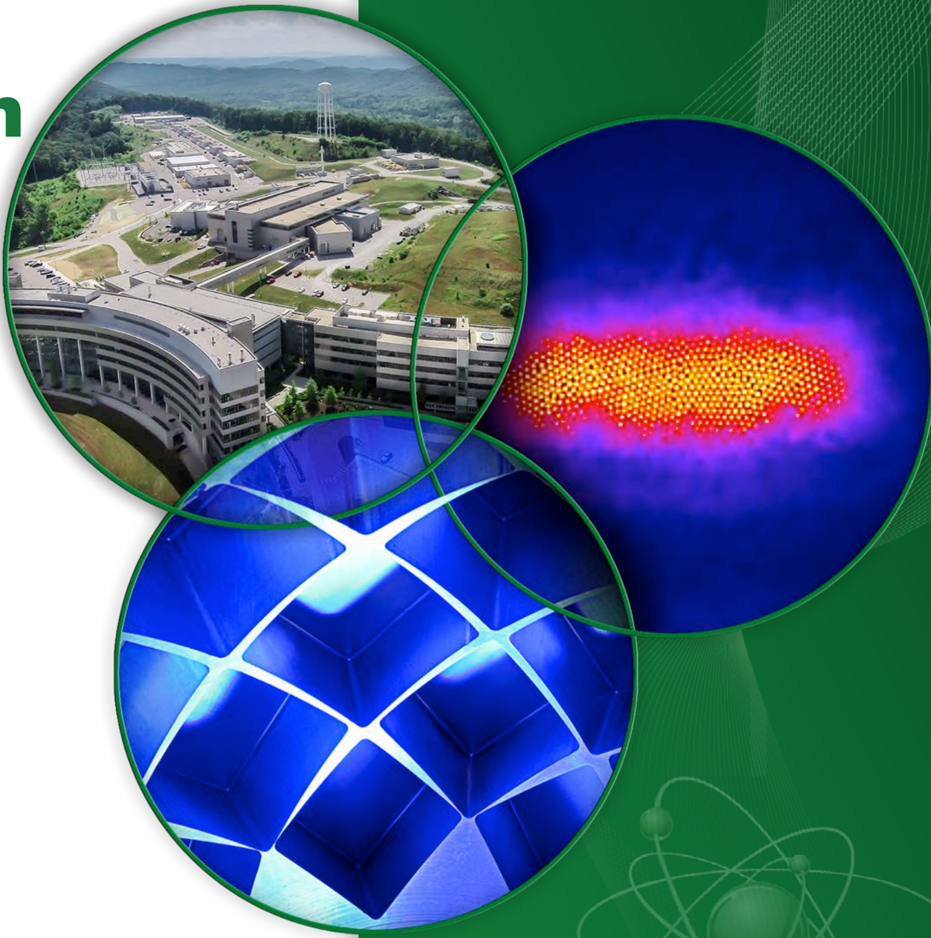
Longitudinal Diagnostics Methods and Limits for Hadron LINACs



Andrei Shishlo,

SNS, ORNL, USA

September 14, 2016

IBIC 2016, Barcelona, Spain

ORNL is managed by UT-Battelle
for the US Department of Energy

Outline

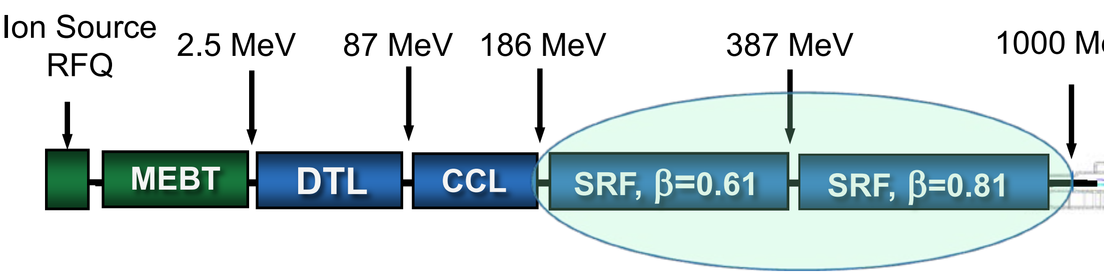
- SNS Accelerator
- Longitudinal Diagnostics in SNS
- BPM for Bunch Length Measurements
  - How to
  - Limitations
- Longitudinal Twiss Parameters Measurements
  - Method = RF + Drift + BPM
  - Conditions of Applicability and Errors
- Results for SNS Superconducting Linac
- Conclusions

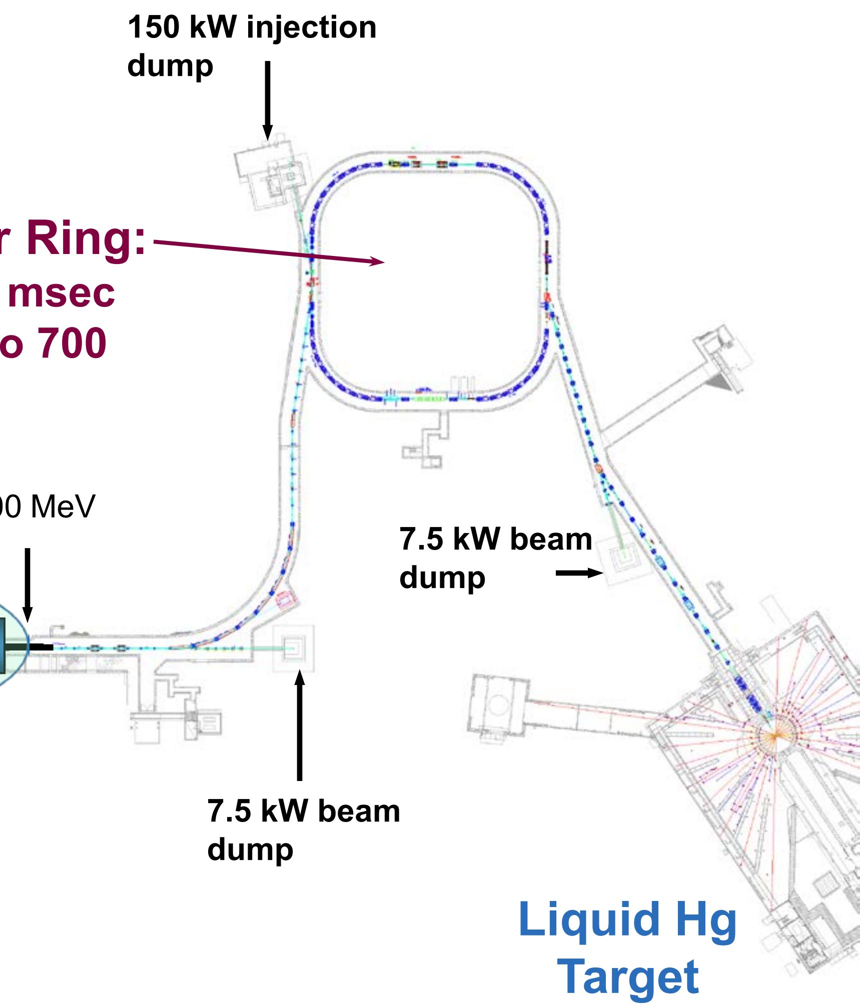
Spallation Neutron Source Accelerator (SNS)

Front-End:
Produce a 1-msec long, chopped, H^- beam

1 GeV LINAC

Accumulator Ring:
Compress 1 msec long pulse to 700 nsec



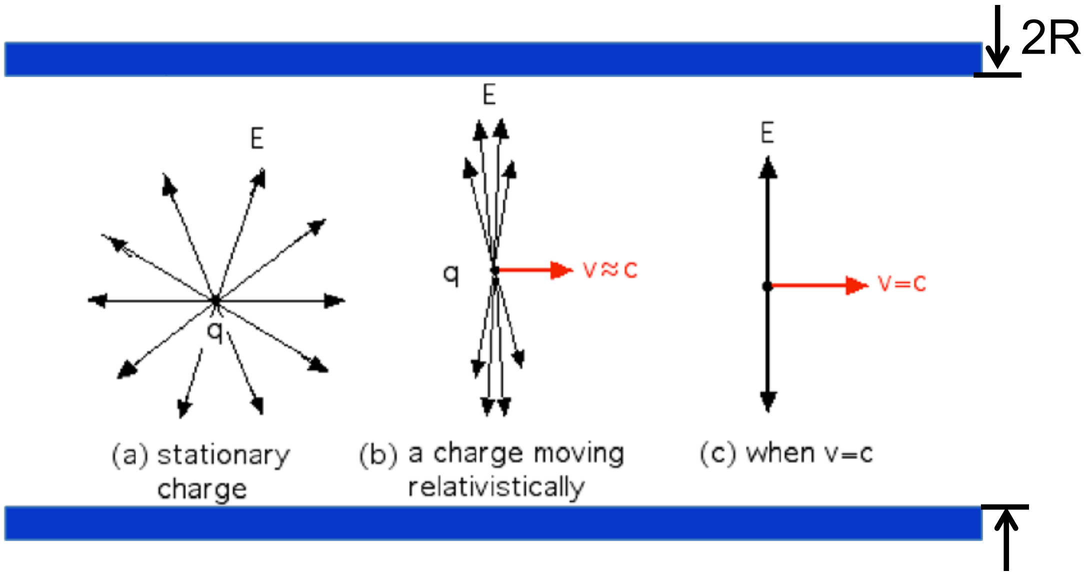


Design parameters	
Kinetic Energy [GeV]	1.0
Beam Power [MW]	1.4
Repetition Rate [Hz]	60
Peak Linac Current [mA]	38
Linac pulse length [msec]	1.0
SRF Cavities	81

Longitudinal Diagnostics in SNS Linac

- Acceptance phase scan
  - was used at the DTL entrance to measure the bunch length
  - was used in SCL to estimate the longitudinal emittance
- Bunch Shape Monitors (BSMs) in CCL
  - Were used to measure bunch length in CCL
  - Were used to confirm the new method
- **SCL RF + Drift + Beam Position Monitors with the amplitude signals (non-intercepting method)**

Bunch Shape from Stripline Signals



Picture from Alex Chao's book "Collective Instabilities in Accelerators"

Hadron accelerators:
v<c
Case (b)

There is no direct way to interpret the stripline BPM signal as the bunch density

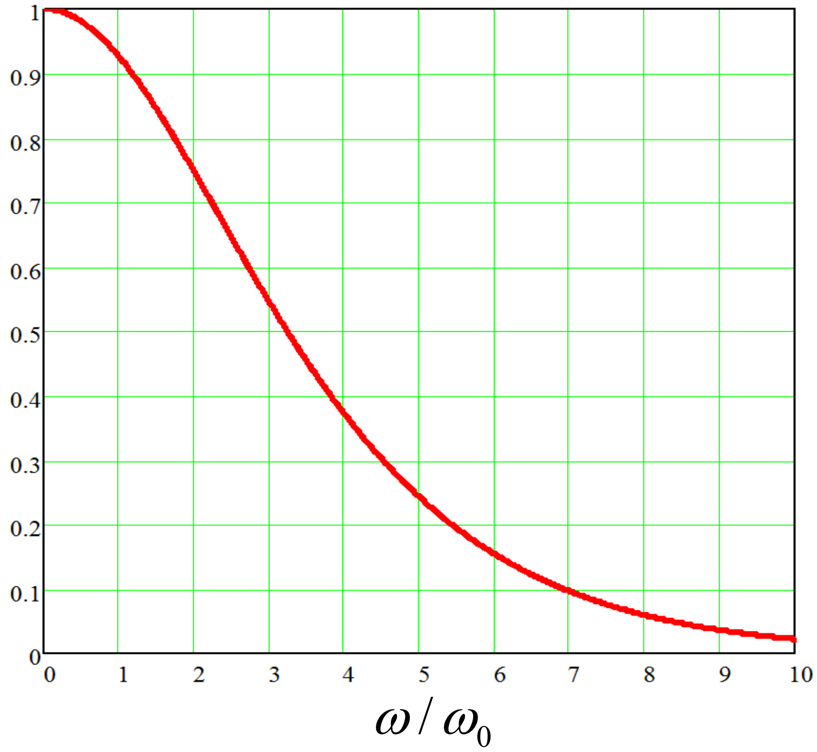
$$u_\omega \propto \frac{A_\omega}{I_0\left(2\pi \frac{R}{\gamma \cdot \lambda_\omega}\right)}$$

J. H. Cuperus, NIM 145, 219 (1977)
MONITORING OF PARTICLE BEAMS AT HIGH FREQUENCIES

A_ω - Fourier harmonics of the longitudinal density distribution

BPM Amplitude vs. Frequency

$$\frac{1}{I_0\left(2\pi\frac{R}{\gamma\cdot\lambda_\omega}\right)}$$



SNS SCL Case:

R= 40 mm

β= 0.55

γ= 1.2

f_0 = 402.5 MHz

- **The frequency of the BPM response is limited even by the simple geometry**
- **There is a dependency on the energy of the beam. It should be taken into account during the calibration**

Harmonic of Gaussian Distribution

$$\lambda(z) = q \cdot N \cdot \frac{1}{\sqrt{2\pi\sigma_z^2}} \cdot \exp\left(-\frac{z^2}{2\cdot\sigma_z^2}\right)$$

Gaussian Longitudinal Distribution

BPM harmonic after Fourier transformation

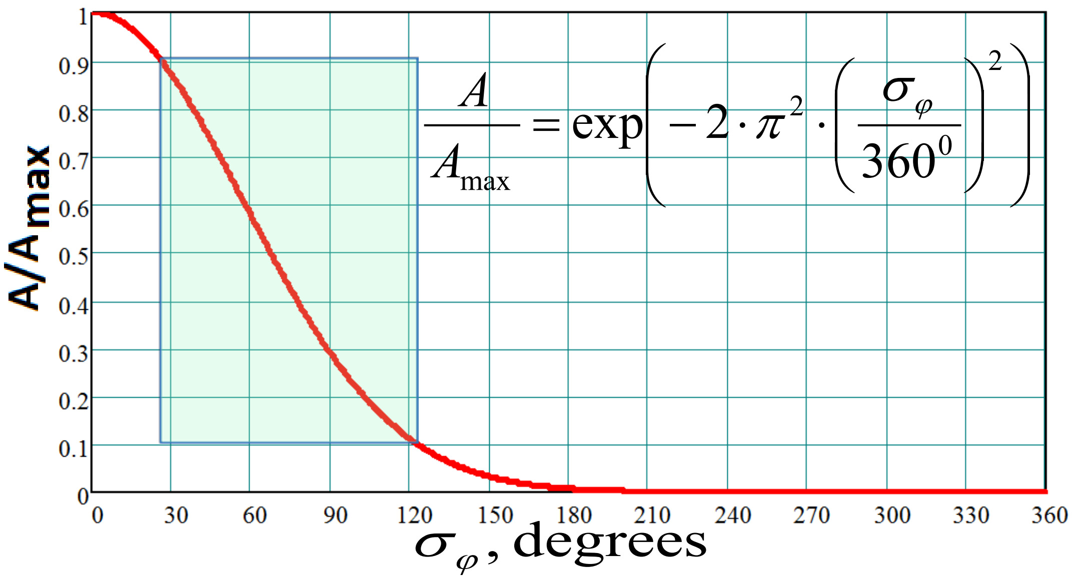
$$A_\omega(\sigma_\varphi) = A_{\max} \cdot \exp\left(-2\cdot\pi^2\cdot\left(\frac{\sigma_\varphi}{360^0}\right)^2\right)$$

σ_φ **- Longitudinal RMS bunch size in degrees**

$$\sigma_\varphi = \frac{360^0}{\sqrt{2}\cdot\pi}\sqrt{\ln\left(\frac{A_{\max}}{A_\omega}\right)}$$

$A_{\max}$ - should be found during the calibration

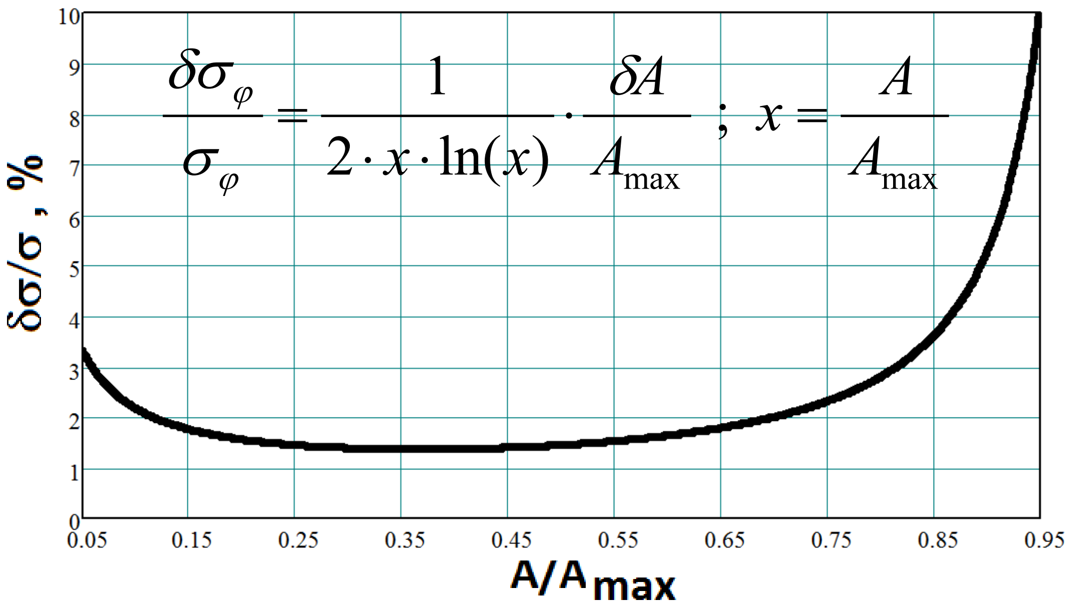
Acceptable Bunch Length Range



Reasonable bunch length for this method is between 30^0 and 120^0

Bunches in linacs are much shorter!

In a normal situation this method is useless!



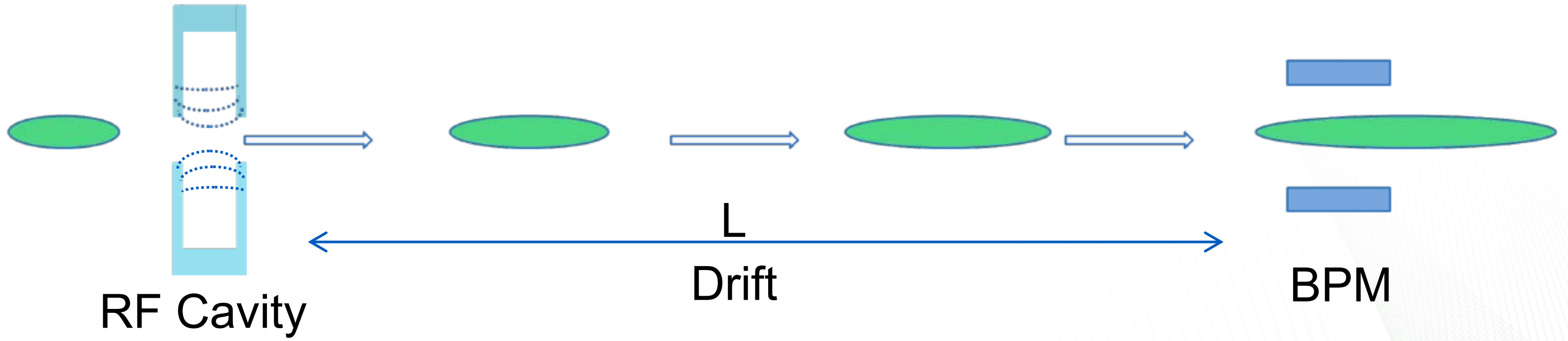
Errors for Bunch Length assuming
$\delta A/A_{max}$ = 1%

A between 10% and 90% **A_{max}** will give us the acceptable errors

OAK RIDGE National Laboratory | HIGH FLUX ISOTOPE REACTOR | SPALLATION NEUTRON SOURCE

The New Method: RF + Drift + BPM

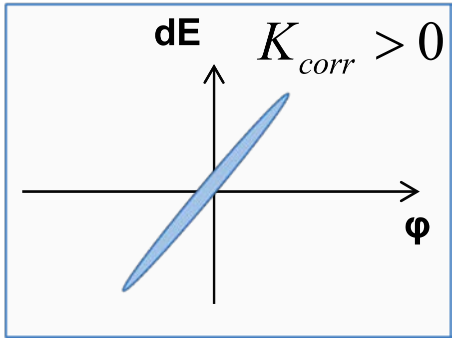
- We want to know the bunch length at places were it is not possible to measure with BPMs
- We want to know not only the bunch length but also Twiss parameters which can be used in the tracking models
- Solution is a combination of three components:
  - RF cavity to manipulate the longitudinal phase space. It is a point of Twiss parameters measurements
  - Drift – long enough for de-bunching of the beam to 30^0-120^0
  - BPM for the bunch length measurements after the drift

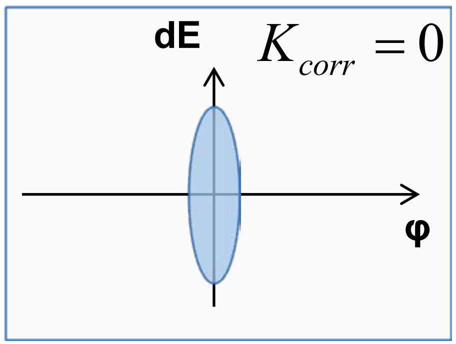


$$E_{out} = E_{in} + qV_0 \cdot \cos(\phi_{RF})$$

Definitions of Variables and Parameters

Longitudinal phase space





(φ, dE) **Longitudinal coordinates – phase and energy deviation from the synchronous particle**

Statistics:

$$\langle \varphi^2 \rangle = \sigma_\varphi^2$$ square of RMS bunch length

$$\langle \varphi \cdot dE \rangle = K_{corr} \cdot \sigma_\varphi \cdot \Delta E$$ phase-energy correlation

$$\langle dE^2 \rangle = \Delta E^2$$ square of RMS energy spread

Initial Twiss Parameters !

$$\varepsilon_{rms} = \sqrt{\langle \varphi^2 \rangle \cdot \langle dE^2 \rangle - \langle \varphi \cdot dE \rangle^2}$$

$$\alpha_{Twiss} = -\frac{\langle \varphi \cdot dE \rangle}{\varepsilon_{rms}}$$

$$\beta_{Twiss} = \frac{\langle \varphi^2 \rangle}{\varepsilon_{rms}}$$

Transformation of Parameters

--- Coordinate transformation by transport matrices ---

$$M_{Drift} = \begin{pmatrix} 1 & 2\pi \dfrac{L}{\lambda_{RF}} \cdot \dfrac{1}{m\gamma^3\beta^3} \\ 0 & 1 \end{pmatrix} \quad M_{RF} = \begin{pmatrix} 1 & 0 \\ -qV_0 \cdot \sin(\phi_{RF}) & 1 \end{pmatrix} \quad \lambda_{RF} = 2\pi \frac{c\beta}{\omega_{RF}}$$

$$M = M_{Drift} \times M_{RF} = \begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix}$$

Total transport matrix

$$\begin{pmatrix} \varphi_{BPM} \\ dE_{BPM} \end{pmatrix} = M \times \begin{pmatrix} \varphi_0 \\ dE_0 \end{pmatrix}$$

No space charge effects included!

$$\varphi_{BPM} = m_{1,1} \cdot \varphi_0 + m_{1,2} \cdot dE_0$$

$$\langle \varphi_{BPM}^2 \rangle = m_{1,1}^2 \langle \varphi_0^2 \rangle + 2m_{1,1}m_{1,2} \langle \varphi_0 dE_0 \rangle + m_{1,2}^2 \langle dE_0^2 \rangle$$

$$\sigma_{BPM}^2(\phi_{RF}) = \left(1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3}\right)^2 \sigma_0^2 + \{Corr\} + \left(2\pi \frac{L}{\lambda_{RF}} \frac{\Delta E}{m\gamma^3\beta^3}\right)^2$$

$$\{Corr\} = 2\left(1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3}\right)\left(2\pi \frac{L}{\lambda_{RF}} \frac{1}{m\gamma^3\beta^3}\right) \cdot K_{corr} \cdot \sigma_0 \Delta E_0$$

RF Cavity Effect

$$\sigma_{BPM}(\phi_{RF}) = \left|1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3}\right| \sigma_0$$

$$>> 1$$

No correlation, no energy spread.

Bunch length should grow from few to 30^0-60^0

Maximal size at $\sin(\phi_{RF}) = \pm 1$

$$L \ge \frac{\lambda_{RF}}{2\pi} \cdot \frac{m\gamma^3\beta^3}{qV_0} \cdot \frac{\sigma_{BPM}}{\sigma_0}$$

Condition on drift space

For SNS:

At SCL entrance E_{kin}=186 MeV

$$qV_0 \approx 10 \ \text{MeV} \qquad m\gamma^3\beta^3 \approx 939 \cdot (1.2)^3 \cdot (0.55)^3 \approx 270 \ \text{MeV}$$

$$\lambda_{RF} = 0.37 \ \text{m} \qquad \sigma_{BPM} = 60^0 \qquad \sigma_0 = 3^0$$

$$L \ge \frac{0.37}{2\pi} \cdot \frac{270}{10} \cdot \frac{60}{3} = 32 \ \text{m}$$

- **We have to transport the beam on significant distance**
- **Weaker the cavity – more drift space we need**
- **Higher the energy – more drift space we need**
- **At the end of SCL_Med we need more than 200 m drift**

Energy Spread Effect

$$\sigma_{BPM}^2 = \left(2\pi \frac{L}{\lambda_{RF}} \frac{qV_0}{m\gamma^3\beta^3}\right)^2 \sigma_0^2 + \left(2\pi \frac{L}{\lambda_{RF}} \frac{\Delta E}{m\gamma^3\beta^3}\right)^2$$

Cavity — Energy spread

No correlation.

Cavity at max focusing/defocusing

$$\sin(\phi_{RF}) = \pm 1$$

Effects are equal when (bunch length in radians)

$$qV_0 \cdot \sigma_0 \approx \Delta E$$

$$\sigma_0 \approx \frac{\Delta E}{qV_0} \approx \frac{0.1 - 0.3\ \mathrm{MeV}}{10\ \mathrm{MeV}} \approx 0.6^0 - 2^0$$

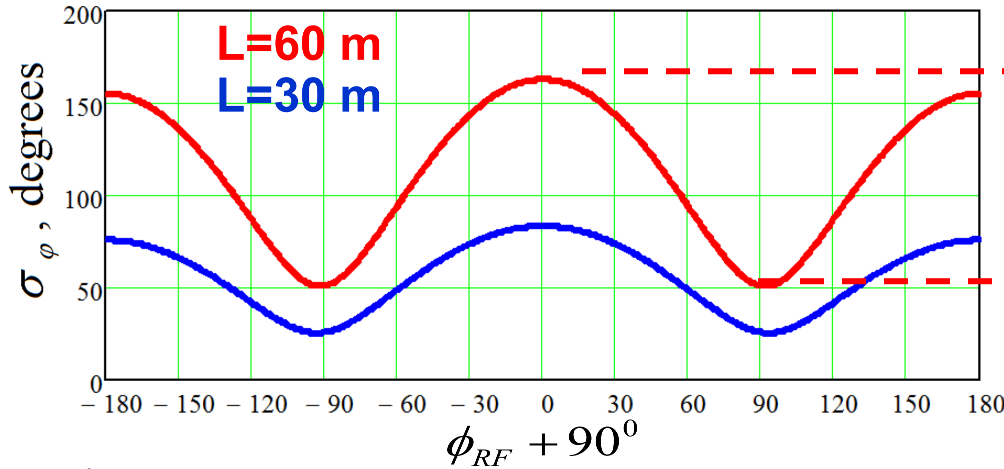
Border value for the bunch length

- **Energy spread should be small** $\Delta E << qV_0$
- **We will see only energy spread if** $\sigma_0 << \dfrac{\Delta E}{qV_0}$
- **Space charge may significantly increase the energy spread, so the cavity should be strong enough**

Phase-Energy Correlation Effect - No

$$\sigma^2_{BPM}(\phi_{RF}) = \left(m_{1,1}(\phi_{RF})\right)^2 \sigma_0^2 + 2m_{1,1}(\phi_{RF}) \cdot m_{1,2} \cdot K_{corr} \cdot \sigma_0 \cdot \Delta E + m_{1,2}^2 \cdot \Delta E^2$$

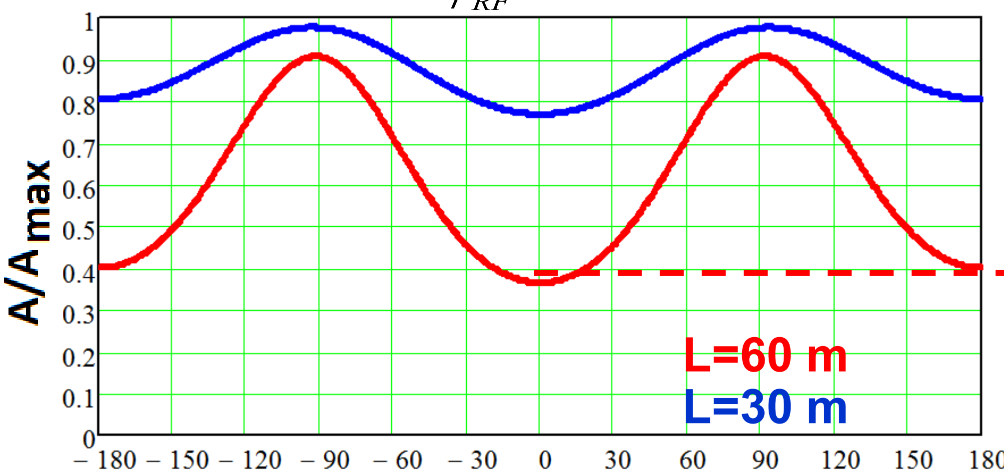
$K_{corr} = 0 \quad qV_0 = 10\,\text{MeV} \quad \sigma_0 = 4^0 \quad \Delta E = 235\,\text{keV}$



Bunch length in degrees

Almost $\sin^2$ from cavity

Const. from energy spread



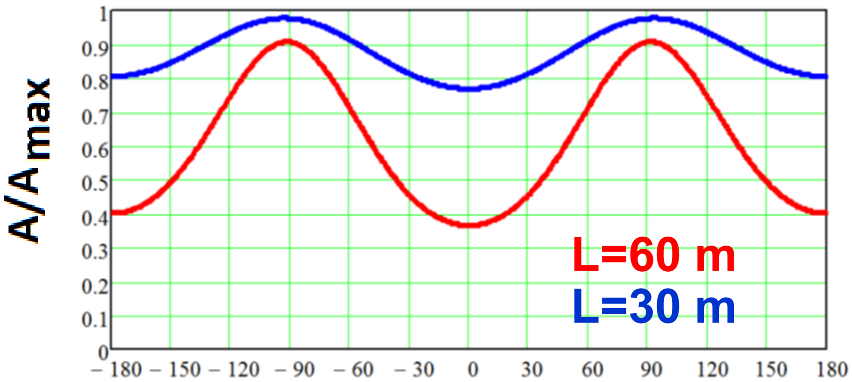
BPM's amplitudes

Almost the same level

SNS SCL:

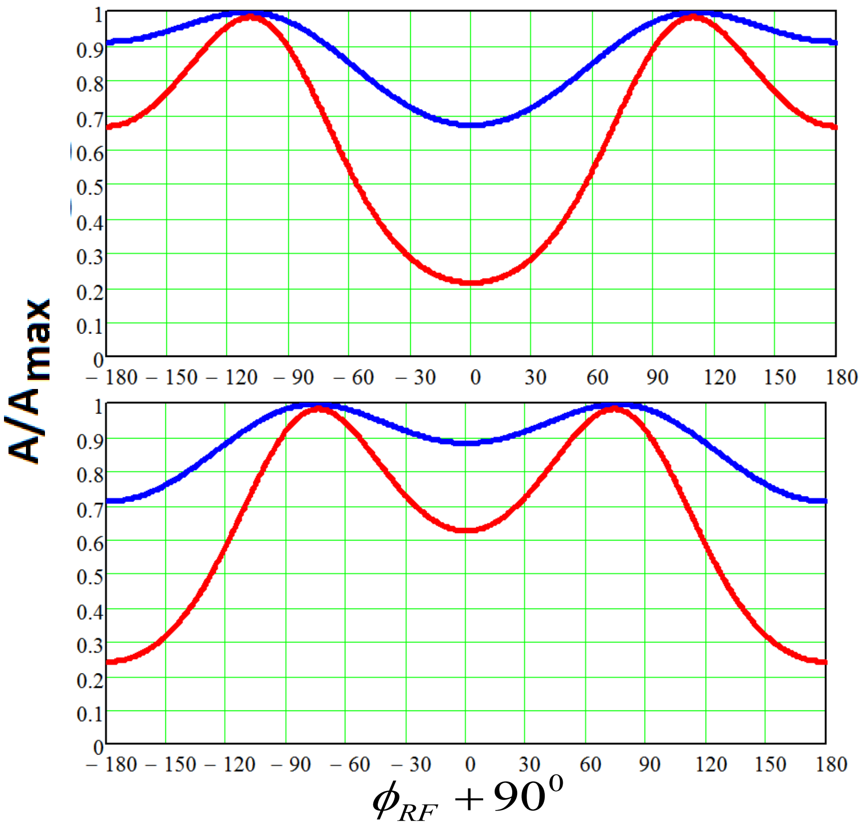
f_{RF}=805 MHz f_{BPM}=402.5 MHz

Phase-Energy Correlation Effect - Yes



$$K_{corr} = 0$$

No correlation



$$K_{corr} = 0.9 > 0$$

The shape of the BPM's amplitude vs. RF phase curve is sensitive to Phase – Energy Correlation

$$K_{corr} = -0.9 < 0$$

How to Get Parameters from Curves

We scan RF phase

These values are from the model (like Trace3D)

$$\sigma_{BPM}^2(\phi_{RF}) = \left(m_{1,1}(\phi_{RF})\right)^2\sigma_0^2 + 2m_{1,1}(\phi_{RF})\cdot m_{1,2}\cdot K_{corr}\cdot\sigma_0\cdot\Delta E + m_{1,2}^2\cdot\Delta E^2$$

That we measure with BPM amplitude

These 3 we want to know!

$$\begin{cases} \sigma_{BPM}^2(\phi_{RF}^{(1)}) = \left(m_{1,1}(\phi_{RF}^{(1)})\right)^2\sigma_0^2 + 2m_{1,1}(\phi_{RF}^{(1)})\cdot m_{1,2}\cdot K_{corr}\cdot\sigma_0\cdot\Delta E + m_{1,2}^2\cdot\Delta E^2 \\ \sigma_{BPM}^2(\phi_{RF}^{(2)}) = \left(m_{1,1}(\phi_{RF}^{(2)})\right)^2\sigma_0^2 + 2m_{1,1}(\phi_{RF}^{(2)})\cdot m_{1,2}\cdot K_{corr}\cdot\sigma_0\cdot\Delta E + m_{1,2}^2\cdot\Delta E^2 \\ \sigma_{BPM}^2(\phi_{RF}^{(3)}) = \left(m_{1,1}(\phi_{RF}^{(3)})\right)^2\sigma_0^2 + 2m_{1,1}(\phi_{RF}^{(3)})\cdot m_{1,2}\cdot K_{corr}\cdot\sigma_0\cdot\Delta E + m_{1,2}^2\cdot\Delta E^2 \end{cases}$$

- **We have 3 unknown variables** $\sigma_0,$ $\Delta E,$ $(K_{corr}\cdot\sigma_0\cdot\Delta E)$
- **3 equations - it is enough (but more is better!)**
- **If there are more than 3 eq. we have a linear system for the least square method**

Errors of Parameters

$$\begin{pmatrix} \sigma^2_{BPM}(\phi^{(1)}_{RF}) \\ \dots \\ \sigma^2_{BPM}(\phi^{(N)}_{RF}) \end{pmatrix} = M_{N\times 3} \times \begin{pmatrix} \sigma_0^2 \\ K_{corr}\cdot\sigma_0\cdot\Delta E \\ \Delta E^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \sigma_0^2 \\ K_{corr}\cdot\sigma_0\cdot\Delta E \\ \Delta E^2 \end{pmatrix} = \left(M^T\cdot W\cdot M\right)^{-1} M^T W \begin{pmatrix} \sigma^2_{BPM}(\phi^{(1)}_{RF}) \\ \dots \\ \sigma^2_{BPM}(\phi^{(N)}_{RF}) \end{pmatrix}$$

$$W_{N\times N} = \begin{pmatrix} \frac{1}{\delta^2(\sigma^2_{BPM}(\phi^{(1)}_{RF}))} & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & \frac{1}{\delta^2(\sigma^2_{BPM}(\phi^{(N)}_{RF}))} \end{pmatrix}$$

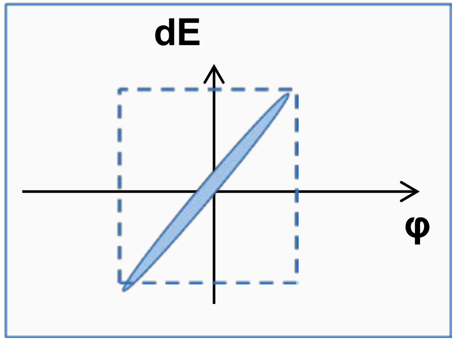
$$\begin{pmatrix} \delta^2(\sigma_0^2) \\ \delta^2(K_{corr}\cdot\sigma_0\cdot\Delta E) \\ \delta^2(\Delta E^2) \end{pmatrix} = \left[\left(M^T\cdot W\cdot M\right)^{-1}\right]_{Diagonal}$$

- **Errors of the initial parameters are defined by the BPMs' amplitudes errors, cavity's strength, and RF phases**
- **More the RF phase points is better for accuracy**
- **Usually the Twiss parameters errors will be small if the our second order correlations have small errors, but is not always true**

Summary of Method

- We need:
  – several lattice elements: RF+Drift+BPM
  – Data
  – transport matrices from the RF entrance to BPM
- The method will give us beam longitudinal Twiss parameters
- Is it model based?
  – If there is no space charge – all formulas are here!
  – If there is space charge effect – we need only something very simple – envelope model like Trace3D
  – We can use more complicated models (IMPACT,TraceWin etc.), but the error estimation should be done with linear lattice model

Some Special Cases for Twiss



$$\varepsilon_{rms} << \sigma_{\varphi} \cdot \Delta E$$

$$\varepsilon_{rms} = \sqrt{\left(\sigma_{\varphi} \cdot \Delta E\right)^2 - \left(K_{corr} \cdot \sigma_{\varphi} \cdot \Delta E\right)^2} = \sigma_{\varphi} \cdot \Delta E \cdot \sqrt{1 - K_{corr}^2}$$

$$K_{corr} \approx 1$$

$$\frac{\delta\varepsilon_{rms}}{\varepsilon_{rms}} >> 1$$

- **Our results bunch length, energy spread, and phase-energy correlation. We have errors for these values**
- **If the correlation coefficient between phase and energy is close to 1, even the small errors will give us a big relative error for the emittance**
- **RMS emittance is important integral of motion in linear lattices, so it could be measured in convenient places**

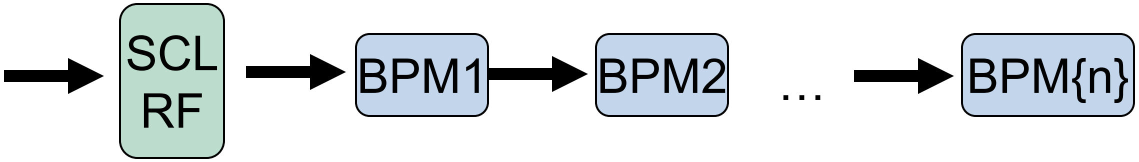
BPM's Amplitude Calibration

$$u_\omega(\sigma_\omega) \propto \frac{A_\omega(\sigma_\omega)}{I_0\left(2\pi \frac{R}{\gamma \cdot \lambda}\right)}$$

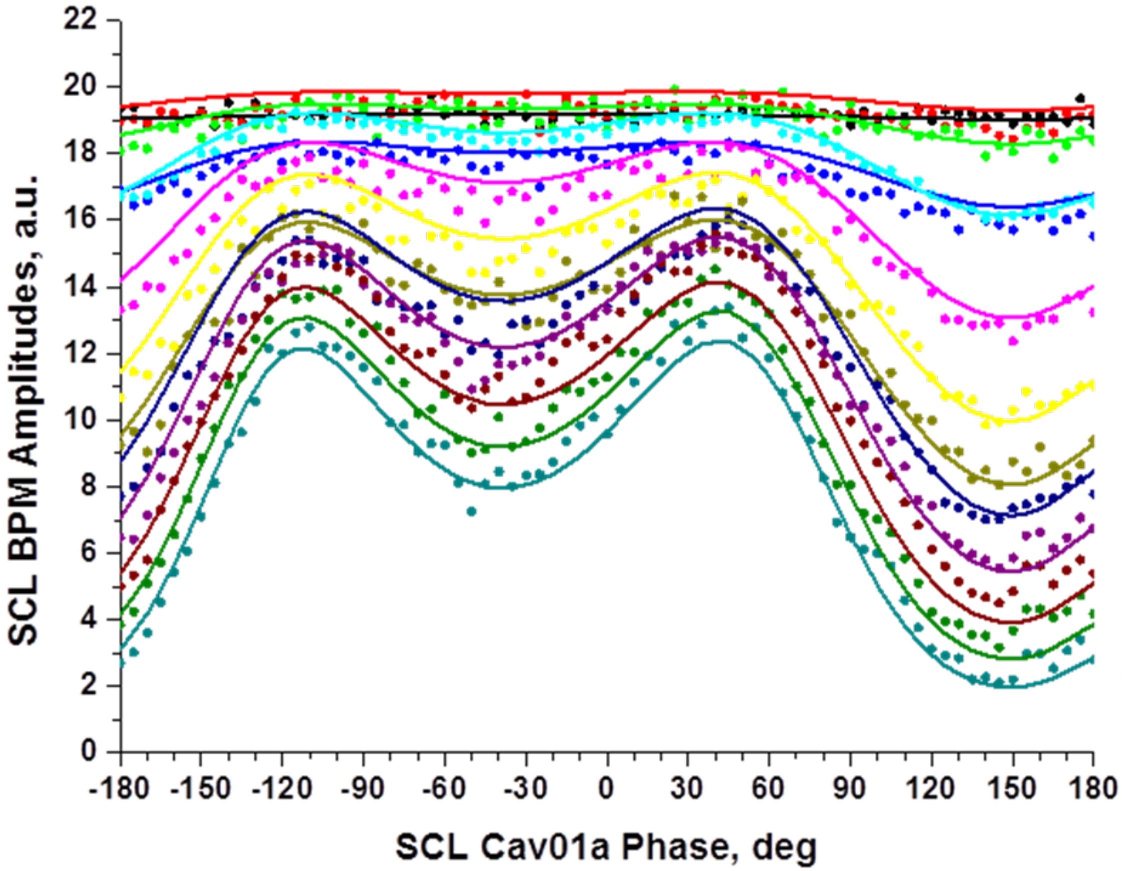
$$A_\omega(\sigma_\varphi) = A_{\max} \cdot \exp\left(-2 \cdot \pi^2 \cdot \left(\frac{\sigma_\varphi}{360^0}\right)^2\right)$$

- We have to know A_{max}
- We can use a very short bunch (few degrees) for calibration
- We have to take into account the beam energy
- **For the case of SNS Superconducting linac we used the production setup when we knew the energy at each BPM and the fact that the bunches are short**

SNS SCL:Cav01a Scan Results



We included all BPMs up to 80 m downstream (BPM1-BPM14).
The phase scan was with 5^0 step.
The system of equations had (72x14) = 1008 equations .

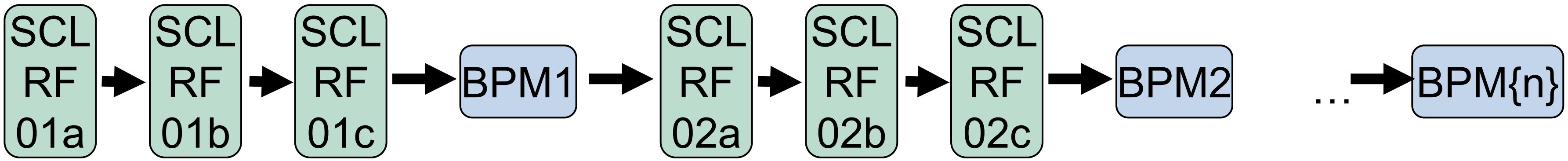


Results (XAL units):
Alpha = 0.56 +- 0.02
Beta = 5.33 +- 0.13
Emitt = (0.928 +- 0.012)*10^{-6}

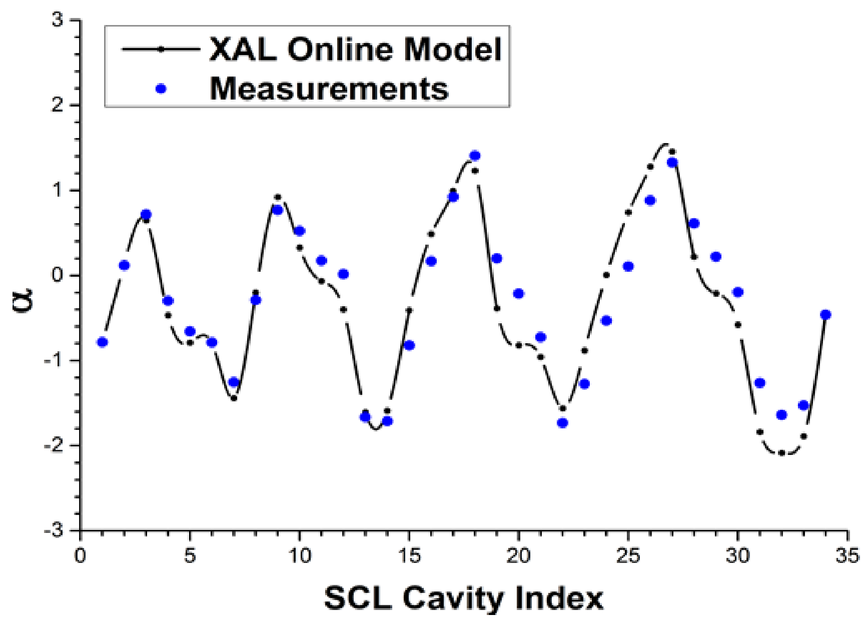
A. Shishlo, A. Aleksandrov, Phys. ST Accel. and Beams 16, 062801 (2013).

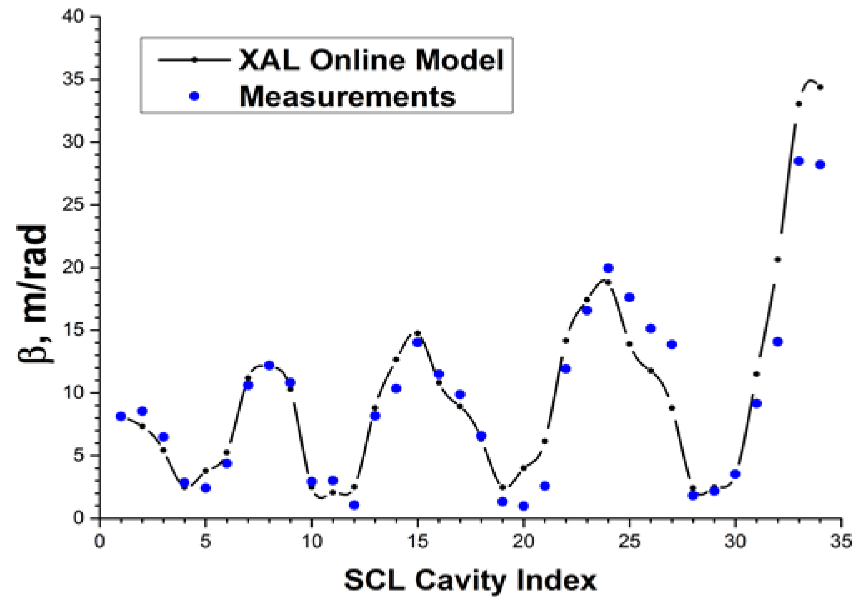
- **The data is a byproduct of tuning procedure**
- **We included all BPMs' data because we have them all anyway**

Each Cavity is A Measuring Device



- We can use each cavity in SCL as the measuring point for the longitudinal Twiss
- We assumed a constant normalized emittance. It means we fitted α, β Twiss parameters only.
- This is a benchmark of the method and the model





Improvements– Some Unchecked Ideas

- **Two-Three cavities simultaneous phase scan**
- **We can reduce the distance**

$$L \geq \frac{\lambda_{RF}}{2\pi} \cdot \frac{m\gamma^3\beta^3}{qV_0} \cdot \frac{\sigma_{BPM}}{\sigma_0} \quad \Rightarrow \quad L \geq \frac{\lambda_{RF}}{2\pi} \cdot \frac{m\gamma^3\beta^3}{3 \cdot qV_0} \cdot \frac{\sigma_{BPM}}{\sigma_0}$$

- **RF1 + (Drift #1) + RF2 + (Drift #2) + BPM**
- **Use the initial energy spread for the beam size increase at the entrance of the 2nd cavity**

$$\sigma_{BPM}^2(\phi_{RF}) = \left(1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3}\right)^2 \sigma_0^2 + \{Corr\} + \left(2\pi \frac{L}{\lambda_{RF}} \frac{\Delta E}{m\gamma^3\beta^3}\right)^2$$

Summary

- The method of the longitudinal Twiss parameters measurements based on the BPMs' signals was developed for the hadron linacs
- The limitations and errors of the method have been analyzed
- The applicability of the new method was demonstrated at the SNS Superconducting Linac

Thanks!