A PROCEDURE FOR THE CHARACTERIZATION OF CORRECTOR MAGNETS

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Abstract

At Diamond Light source, the main assumption for the Fast Orbit Feedback (FOFB) controller design is that the corrector magnets all have the same dynamic response. In this paper, a procedure to measure the frequency responses of the corrector magnets on the Diamond Storage Ring is presented and the magnet responses are measured and compared in order to assess whether this assumption is valid. The measurements are made by exciting a single corrector magnet with a sinusoidal input and measuring the resulting sinusoidal movement on the electron beam using electron Beam Position Monitors (eBPMs). The input excitation is varied from 10 Hz to 5 kHz using a 10 mA sine wave. The amplitude ratio and the phase difference between the input excitation and the beam position excitation are determined for each input frequency and the procedure is repeated for several magnets. Variations in both gain and phase across magnets are discussed in this paper and the effect of such variations on the performance of the FOFB controller performance is determined.

INTRODUCTION

The Fast Orbit Feedback (FOFB) Controller at Diamond performs global orbit correction to 172 horizontal and vertical correctors respectively using the position from 171 horizontal and 171 vertical electron Beam Position Monitors (eBPMs). The main assumption of the FOFB design is that all corrector magnets in the Storage Ring have the same dynamic effect on beam position. This assumption allows the FOFB controller to be decoupled into a static part (implemented as the inverse of the Response Matrix) and a dynamic part (implemented as IIR filters on the outputs of the inverse Response Matrix). If the dynamics of the corrector magnets are dissimilar, then the decoupled control approach may no longer be valid and significant differences in dynamics may limit the ability of the FOFB controller to attenuate disturbances.

Two straights in the Diamond Storage Ring (I13 and I09) were modified with vertical mini-beta and horizontally focusing optics [1], resulting in the need for two extra correctors in each modified straight in both planes. The additional correctors are different in design to the standard correctors used around the rest of the Storage Ring. Moreover the mini-beta correctors are fitted around a different vacuum chamber cross section. A method to measure the dynamic response of the Storage Ring correctors was developed so that the dynamics of the mini-beta correctors can be compared to the standard corrector magnets and the impact on the FOFB performance can be determined. The procedure to obtain the required measurements for such characterisation and the analysis of the measurements are presented in this paper.

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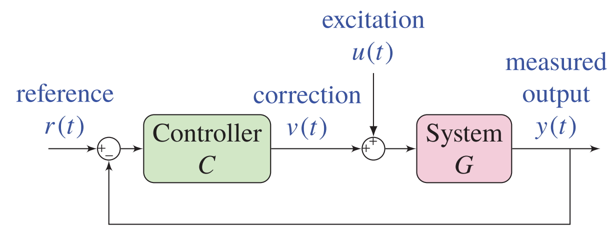

Figure 1: System representation.

FREQUENCY RESPONSE OF A SYSTEM

A general representation of the system to be characterised is shown in Fig. 1 where the system to be characterised is represented by G (referred to as the open loop system) which includes dynamics contributed by the magnet power supplies, the magnet itself and the vacuum vessel. Also included in G are external disturbances acting on the electron beam. The FOFB controller is represented by C, which takes the difference of the beam position at all eBPMs, $y(t)$ and the golden orbit, $r(t)$ as an input. An external excitation, $u(t)$ can be added to the calculated output $v(t)$, which then becomes the correction applied to the corrector magnets.

A common way of modelling the system G, is to find the frequency response, or response to a sinusoid. An input signal $u(t)$ that is a harmonic signal with angular frequency ω, can be expressed as

$$u(t) = u_0 \sin(\omega t) \quad (1)$$

If the system is properly damped, then after some time the transient behaviour of the system will damp and the output $y(t)$ is also harmonic with the same frequency and its amplitude and phase with respect $u(t)$ are determined by the complex value of $G(j\omega)$ i.e. the complex number that is obtained when $s = j\omega$ is substituted in the expression of the transfer function $G(s)$ [2]. Specifically, the gain $|G(j\omega)|$ equals the ratio of the amplitudes of the output and input signals and the phase angle $\angle G(j\omega)$ is equal to the phase shift. The gain and phase shift are shown as functions of the angular frequency in a Bode plot. The information in such a plot is used as a model of the linear, time-invariant system $G(s)$ and can be used to compute the output of the system for a given input.

To measure the frequency response, the system is excited at a user defined set of M excitation frequencies $\{\omega_i\}_{i=1,\ldots,M}$ and associated amplitudes $\{u_{0_i}\}_{i=1,\ldots,M}$. When the system is excited, information is only obtained at the chosen excitation frequencies, so that the frequency grid should normally

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be chosen where required dynamic behaviour is expected. Additionally, the variance of the estimated model decreases with increasing number of excitation frequencies. The amplitude of the excitation is set to obtain optimal signal to noise performance. However, the amplitude should be chosen such that any amplitude or rate limiters in the system are not activated, as such nonlinearities would affect the measured response.

To characterise the open loop response, the FOFB controller is turned off (i.e. $v(t) = 0$ in Fig. 1) and the excitation signal, $u(t)$ is turned on, so that the output signal is a direct measurement of the system dynamics which can be written in terms of transfer functions described by s i.e.

$$Y(s) = G(s)U(s) \tag{2}$$

By substituting $s = j\omega$, the magnitude of the system $G(j\omega)$ is expressed as the ratio of the amplitudes of the excitation and the output signals and the phase of the system $G(j\omega)$ is expressed as the difference between the phases of the output signal and the excitation signal i.e.

$$\begin{aligned} |G(j\omega)| &= \frac{|Y(j\omega)|}{|U(j\omega)|} \\ \angle G(j\omega) &= \angle Y(j\omega) - \angle U(j\omega) \end{aligned} \tag{3}$$

When the FOFB controller is switched on and the excitation is active, the output signal, $y(t)$ includes the effect of the FOFB controller dynamics as well as the system dynamics and is given by

$$\begin{aligned} Y(s) &= G(s)(U(s) + V(s)) \\ Y(s) &= \frac{G(s)}{1 + G(s)C(s)} \end{aligned} \tag{4}$$

The sensitivity function [2] is defined as transfer function between the disturbances acting on the beam and the beam position which is described by

$$S(s) = \frac{1}{1 + G(s)C(s)} \tag{5}$$

The magnitude of the sensitivity function at a particular frequency indicates the level of attenuation the closed loop achieves. From (2) and (4), by dividing the measured frequency response when the FOFB controller is on by the response with the FOFB turned off, the sensitivity function can be determined.

MEASUREMENT PROCEDURE

The FOFB system calculates the orbit correction on distributed VxWorks PowerPC processors at s sample rate $f_s = 10072$ Hz. The processors can be programmed via EPICS to provide individual excitation signals for each corrector which is added to the calculated output from the FOFB controller, or DC set-point when open loop. Almost arbitrary excitation frequencies are possible as the controller advances the sine wave excitation by a programmed phase advance each tick. The resulting orbit data is collected by the eBPMs and is also archived at f_s [4], which is later read back to provide data for the analysis.

The algorithm for measurement of the frequency response of a corrector is described in the following steps:

1. The required corrector is excited with a sinusoidal signal of amplitude u_0 and frequency $f = \frac{\omega}{2\pi}$ Hz at time t_{start}, ending at time $t_{\text{end}} = t_{\text{start}} + N/f$ where N is the number of cycles of the excitation and is chosen by the user.

2. Beam position data from all enabled BPMs is collected from time t_{start} to t_{end}. To ensure synchronization of the input and output signals, a large window of data is taken and then sliced to the exact duration of the excitation using a global timestamp.

3. The output sinusoid from each BPM can be expressed as
$$\begin{aligned} y(t) &= y_0 \sin(\omega t + \phi) \\ &= I \sin(\omega t) + Q \cos(\omega t) \end{aligned} \tag{6}$$
The IQ data is extracted by multiplying the output by $\sin(\omega t)$ and $\cos(\omega t)$ respectively and filtering the high frequency component by taking the mean over N.

The above process is repeated to generate measurements for any required excitation frequency (up to Nyquist ($f_s/2$). Therefore for each excitation frequency, a complex number is obtained which has an amplitude (normalised by the input amplitude) equal to the gain of the system and a phase corresponding to the phase of the system i.e.

$$\begin{aligned} y &= \sqrt{I^2 + Q^2} \\ \phi &= \arctan \frac{Q}{I} \end{aligned} \tag{7}$$

where the gain of the system k is give by $k = y_0/u_0$.

CHARACTERISATION PROCEDURE

Fig. 2a and Fig. 2b show the measured frequency response observed by a single eBPM between 10 Hz to 3 kHz for the two mini-beta corrector magnets in straight I13 and a standard corrector magnet. The mini-beta correctors for both straights were found to have the same response, therefore only the responses of those in straight I13 are presented in this paper. Typically, an excitation amplitude of 10 mA was used however, the second corrector in each straight required a larger excitation amplitude (40 mA) which was determined by preliminary excitation tests. The magnitude of the responses shown in Fig. 2a and Fig. 2b are normalised to unity gain for comparison of the dynamic behaviour. It should be noted that the measured gain (in mm/A) is equivalent to the response matrix element for the corresponding corrector magnet and eBPM.

The procedure for identifying a transfer function model that is appropriate of the design of the FOFB controller is described in the following steps:

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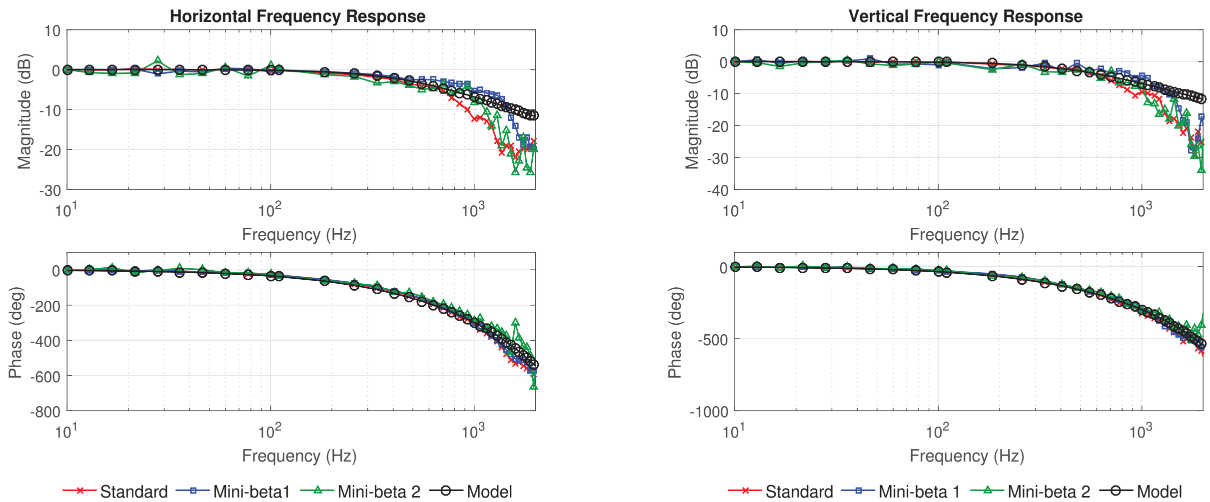

(a) Horizontal frequency response measurements.

(b) Vertical frequency response measurements.

Figure 2: Frequency response measurements for a standard corrector (red '×') and mini-beta correctors (blue '□' and green '△') compared to the modelled frequency response (black '○') for horizontal and vertical planes. The model is a first order model with bandwidth of 500 Hz and delay of 600 μs.

1. The approximate order of the model is determined. The high frequency roll-off is determined by the order of the model i.e. for a first order model, the high frequency roll-off is −20 dB/decade. The measured responses exhibit an approximate first order response which takes the form

$$g(s) = k\frac{a}{s+a} \tag{8}$$

where k is the steady state gain. The user should decide whether a first order model is accurate enough or that a higher order model is required for the purpose of the model.

2. The open loop bandwidth is determined. The bandwidth, a of the open loop system is defined as the frequency (in rad.s^{-1}) at which the magnitude of the frequency response drops by 3 dB. Table 1 shows the measured bandwidths for the different corrector magnets.

3. The open loop delay is determined. The measured phase responses of the corrector magnets are greater than that expected for a first order system, indicating that there is a delay element in the transfer function. The delay can be extracted from the phase information of the frequency response and fitted by a linear regression. The model with a delay term included is written as

$$g(s) = k\frac{a}{s+a}e^{-s\tau_d} \tag{9}$$

where τ_d is the delay in the system. To determine the delay, τ_d the measured phase of the system dynamics $\angle G(j\omega)$ is expressed as a first order system plus a delay, taking the form

$$\begin{aligned}\angle G(j\omega) &= \angle\left(\frac{a}{j\omega+a}e^{-j\omega\tau_d}\right)\\ \angle G(j\omega) &= \angle\left(\frac{a}{j\omega+a}\right) + \angle\left(e^{-j\omega\tau_d}\right)\end{aligned} \tag{10}$$

By using the relationship in (10) the delay element can be extracted and expressed as

$$-\tau_d\omega = \angle G(j\omega) - \angle\left(\frac{a}{j\omega+a}\right) \tag{11}$$

Fig. 3 shows the measured delays for each magnet for horizontal and vertical planes.

Table 1: Measured Bandwidth for a Standard Corrector and the Mini-beta Correctors for Vertical and Horizontal Planes

Magnet	Horizontal	Vertical
Standard	500 Hz	500 Hz
Mini-beta 1	700 Hz	700 Hz
Mini-beta 2	500 Hz	500 Hz

The measured responses show that the first mini-beta corrector has a bandwidth of 700 Hz and the second, a bandwidth of 500 Hz which matches the bandwidth of the standard corrector magnet. The measurements for the second mini-beta corrector are noisier than that taken for the standard corrector and first mini-beta corrector. This indicates that the excitation amplitude was too small for satisfactory signal to noise performance, however using larger amplitudes for the excitation activated a rate limiter which limited the bandwidth of the open loop measurement. A delay of 600 μs was determined to be the best fit for all magnets.

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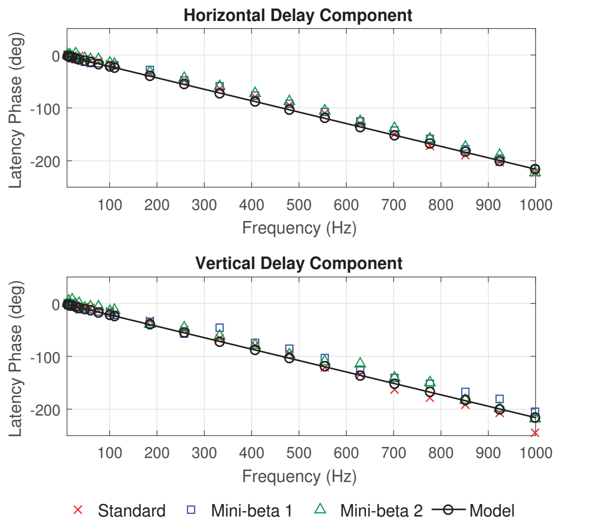

Figure 3: Phase introduced by the system latency in horizontal and vertical planes for a standard corrector (red '×') and mini-beta correctors (blue '□' and green '△') compared with a linear fit (black '○') of a 600 μs delay.

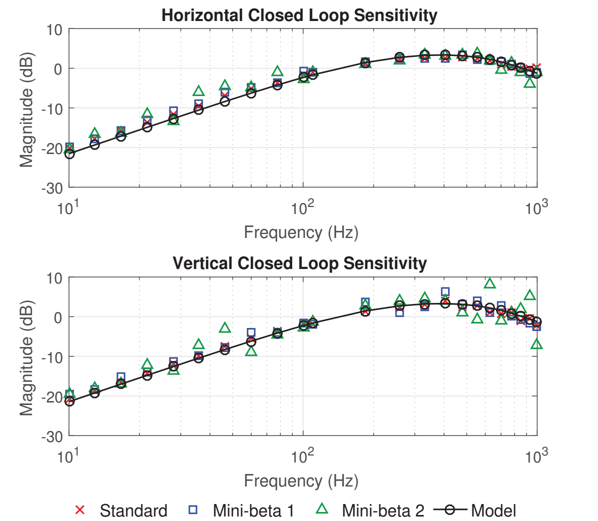

Figure 4: Measured sensitivity in horizontal and vertical planes for a standard corrector (red '×') and mini-beta correctors (blue '□' and green '△') compared to the theoretical response (black '○').

The derived models are also included in Fig. 2a and Fig. 2b. The model captures the dynamic behaviour of the system up to the bandwidth and fits the phase roll-off well. The model with the structure given in (9) using the bandwidth of the standard corrector magnet for each plane was discretized [3] and used for the FOFB design. Therefore it is important to assess the impact of the dynamic differences between the magnets on the performance of the FOFB controller.

The sensitivity function given in (5) is used to determine how well the corrector performs in terms of disturbance rejection and is shown in Fig. 4 for each corrector magnet in the horizontal and vertical planes. Also included in Fig. 4 is the theoretical sensitivity for the standard corrector which predicts that at 10 Hz, the closed loop provides around 20 dB attenuation of disturbances but at the cost of amplifying disturbances above 150 Hz by a maximum of 3 dB at 400 Hz. The sensitivity of the standard and mini-beta correctors provide similar levels of attenuation at most frequencies. The data shows that there is no significant difference in sensitivity measurements at low frequencies, which is expected because the frequency response of the corrector magnets do not differ below 500 Hz in each plane.

CONCLUSIONS

The advantage of the frequency response approach to modelling the open loop response is that it can be measured directly and gives the frequency response immediately. Furthermore, the user experiment parameters such as the relevant dynamic frequencies, the duration of each excitation, the sample frequency and the type of input signal can be easily modified by the user. Also, as the input and output signals are only analysed at specific frequencies, the amount of data is reduced significantly from the number of time domain samples to the number of considered frequencies. Finally, frequency domain identification can deal equivalently with time continuous models as with time discrete models, which are useful for the design of the FOFB controller. The main advantage of the procedure outlined in this paper is the ease with which the experimental data can be used for design purposes. No significant processing is required to obtain a Bode plot which can then be used to derive a simple model of the open loop to inform the design of an appropriate compensator.

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