Electrostatic finite-element code to study geometrical nonlinear effects of BPMs in 2D

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We have developed a 2D finite FEM-based software for Matlab to study non-resonant effects in BPMs of arbitrary geometry, in particular the geometric nonlinearities. The developed code called ***BpmLab*** utilizes an open-source tetrahedral mesh generator *DistMesh*, combined with a short implementation of FEM with linear basis functions to find the electrostatic field distribution for boundary electric potential excitation.

The BPM response as a function of beam position is calculated in a single simulation for all beam positions using the potential ratios, according to the Green's reciprocity theorem. The code offers ways to correct the geometrical nonlinear distortion, either by polynomials or by direct inversion of the electrode signals through numerical optimization.
The results are tested and benchmarked on the showcase pickup labeled ***pilot-BPM***.

Geometry

done by *signed distance functions*: they give the shortest distance from every node to the boundary of the domain: the sign of the metric is negative inside the region and positive outside.

Fig. 1 shows the initial node disribution of sample shapes based on their distance metrcs.

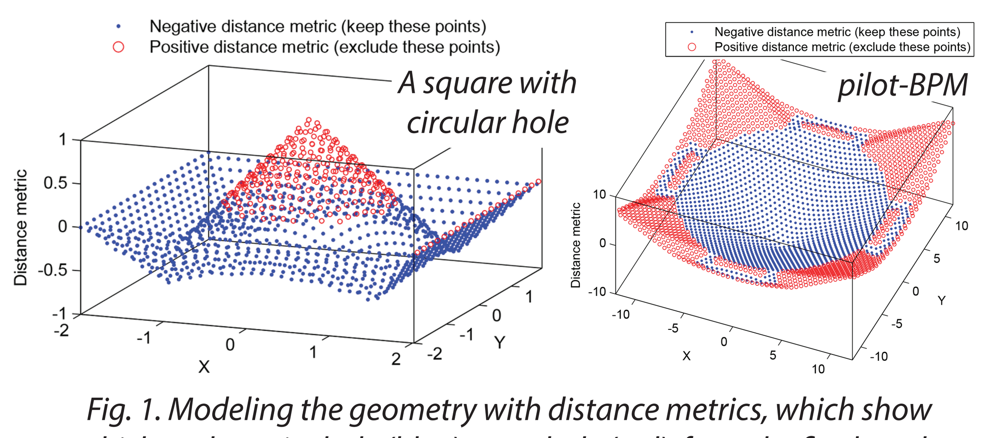


Fig. 1. Modeling the geometry with distance metrics, which show which nodes to include (blue) or exclude (red) from the final mesh.

FEM Solver

A short Laplace's equation solver in 2D with mixed (Dirichlet + Neumann) boundary conditions for unstructured grids with linear elements using the standard Galerkin discretization [2].

The FEM solver calculates the electrostatic potential in each mesh node based on the boundary conditions. Its convergence is tested on analytic shapes with exact solutions, Fig 2.

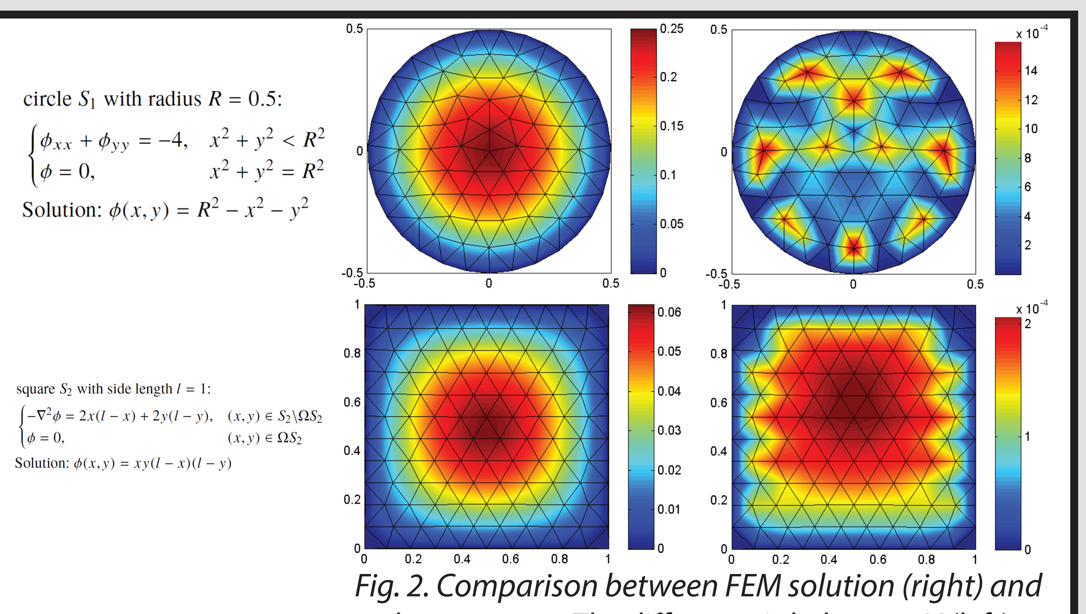
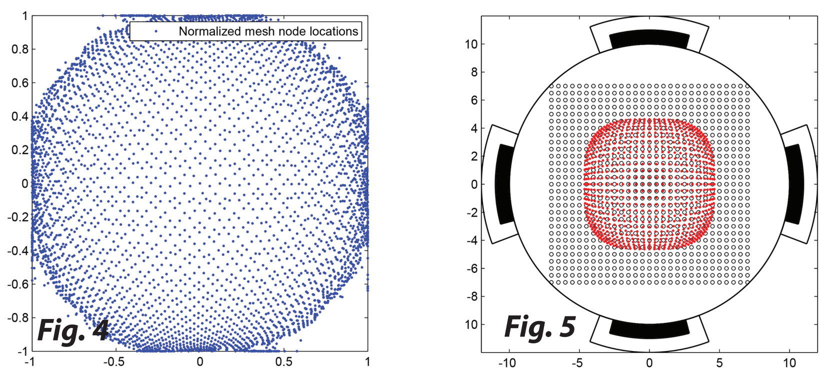


Fig. 2. Comparison between FEM solution (right) and the exact one. The difference is below 1 mV (left).

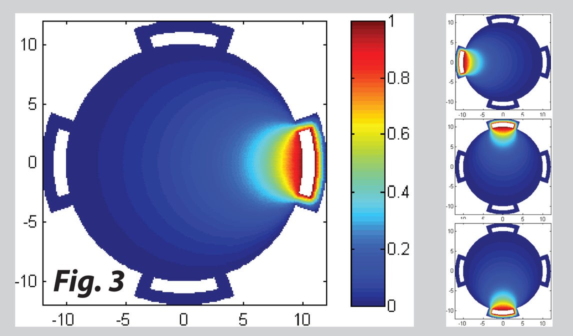
Reciprocity

According to Green's reciprocity theorem (GRT), all beam positions are found in a single calculation [3]:

One electrode is excited to 1 volt, the results are mirrored 3x times (Fig. 3). The DOS expressions combine 4x results and normalized H and V response characteristic in the mesh nodes is obtained (Fig. 4). Figure 5 shows a calibtared response map.

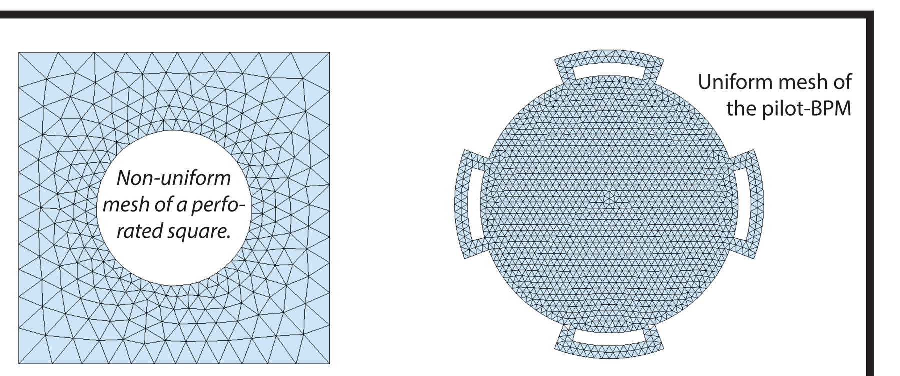


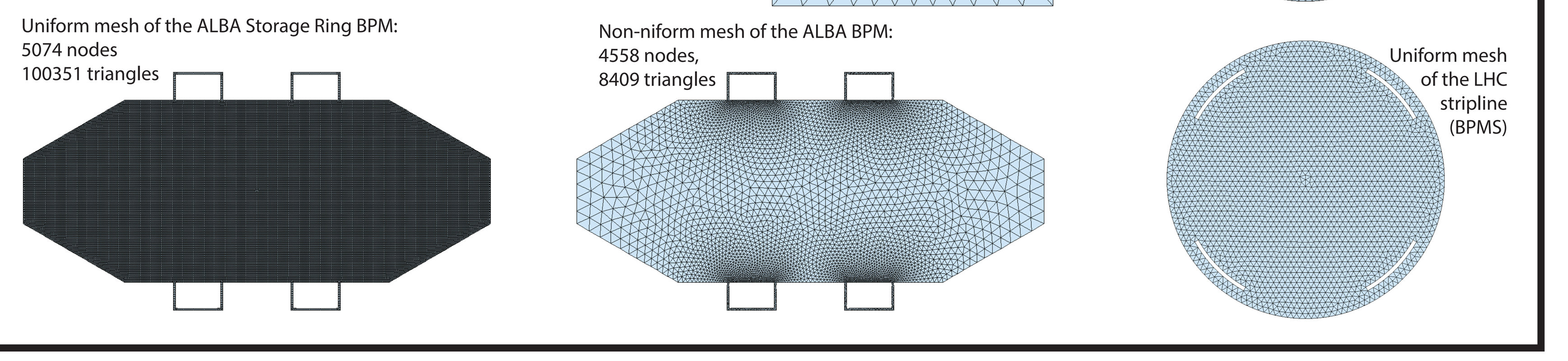
GRT: the charge induced on an electrode surface *qb* due to a test charge *q* at *(x0,y0)* is proportional to the potential *phi* at that same position when the test charge is absent and the electrode is set to a potential *V0*: $q_b V_0 = -q\phi(x_0, y_0)$



Meshing

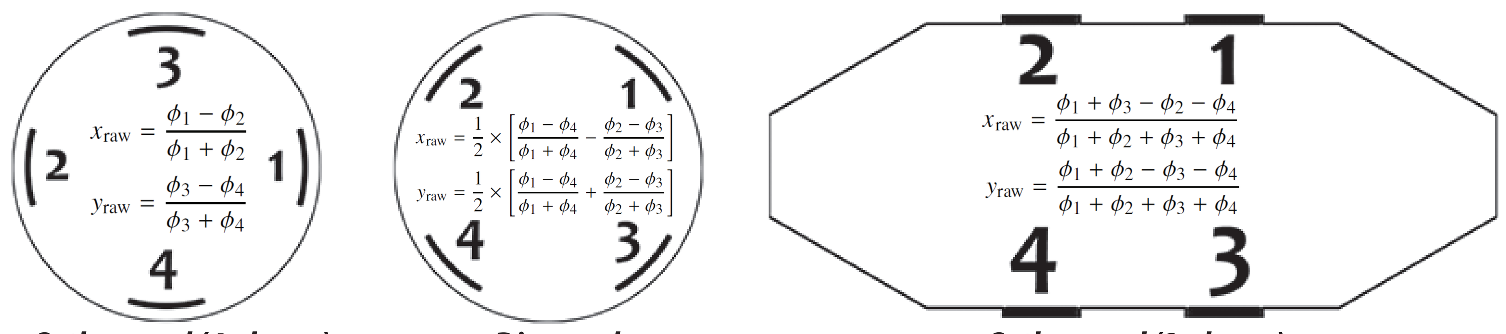
by *DistMesh*: a free Matlab code for generating constrained Delaunay (= equilateral) triangular meshes with node optimizaton and manual density control [1].

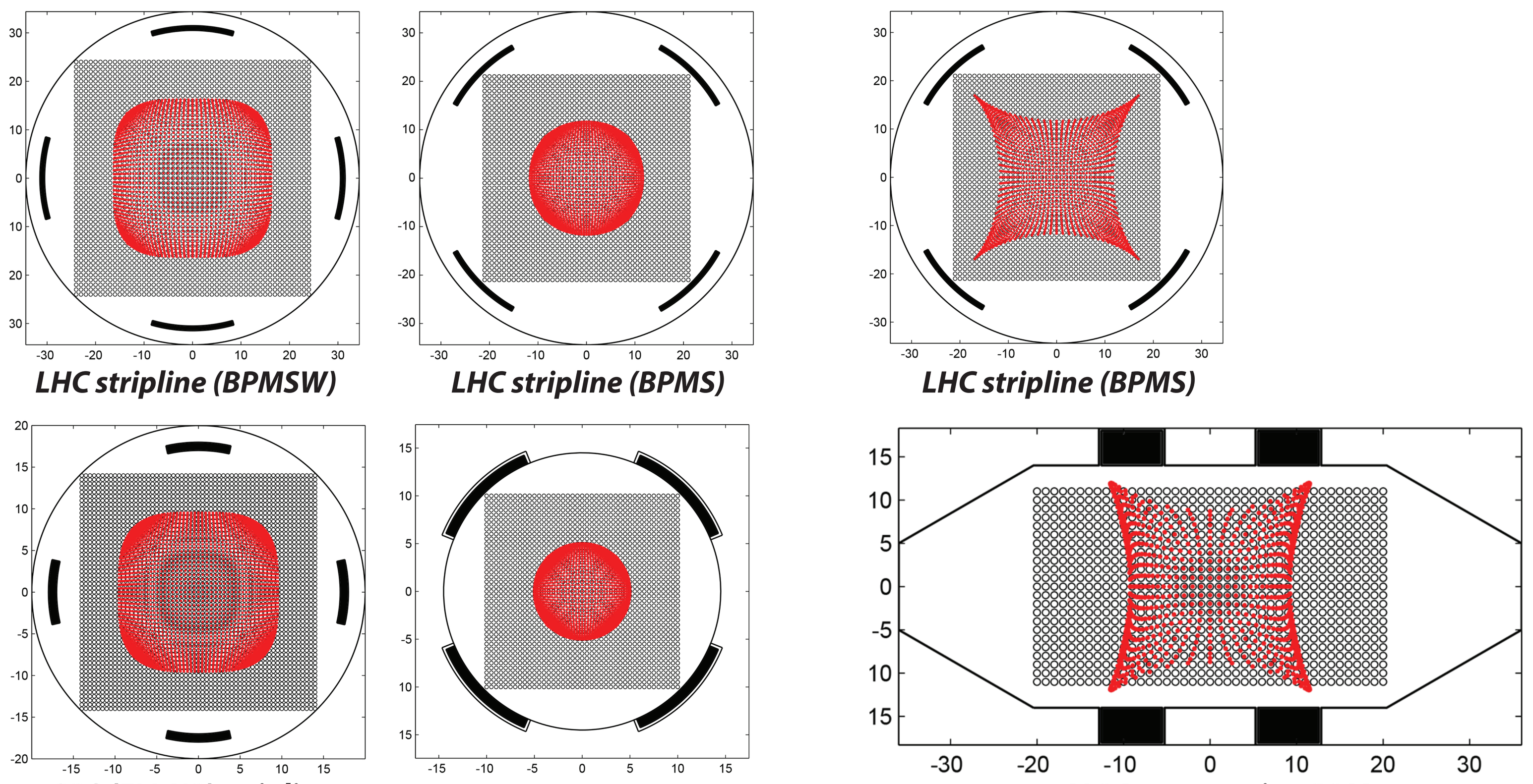




DOS Treatments

The normalized beam position characteristic in the XY plane, based on 4 pickup electrodes, is described in general by **orthogonal** and **diagonal** Difference-Over-Sum (DOS) treatments:



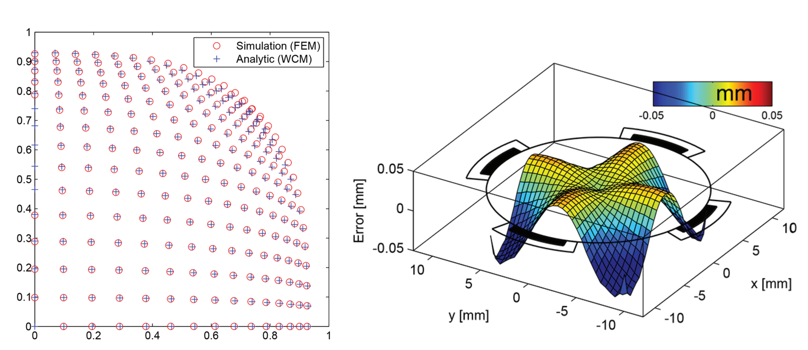


Benchmarking vs. other methods

The pilot-BPM geometry was simulated by each of the following methods. Differences between position maps, treated by DOS (4x planes), are shown with respect to *BpmLab* simulation in Fig 5.

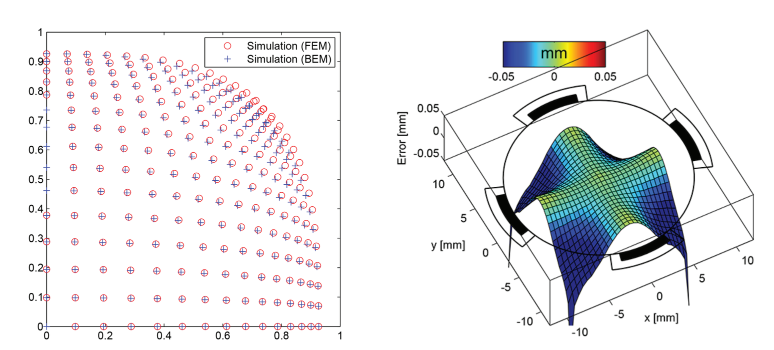
Wall Current Model

Analytic integration of the wall current distribution induced on an electrode due to a line-charge [4]. Difference below 30 um around most of the map:



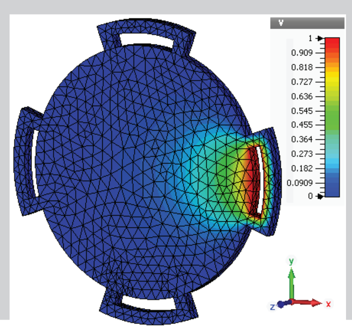
Boundary Element Method

Numerical solution of the 2D electrostatic problem of finding the induced charge on the boundary of the domain containing a line-charge [5]. Difference below 20 um around most of the map:

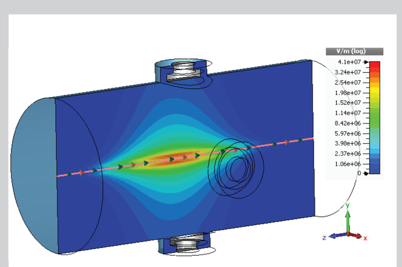


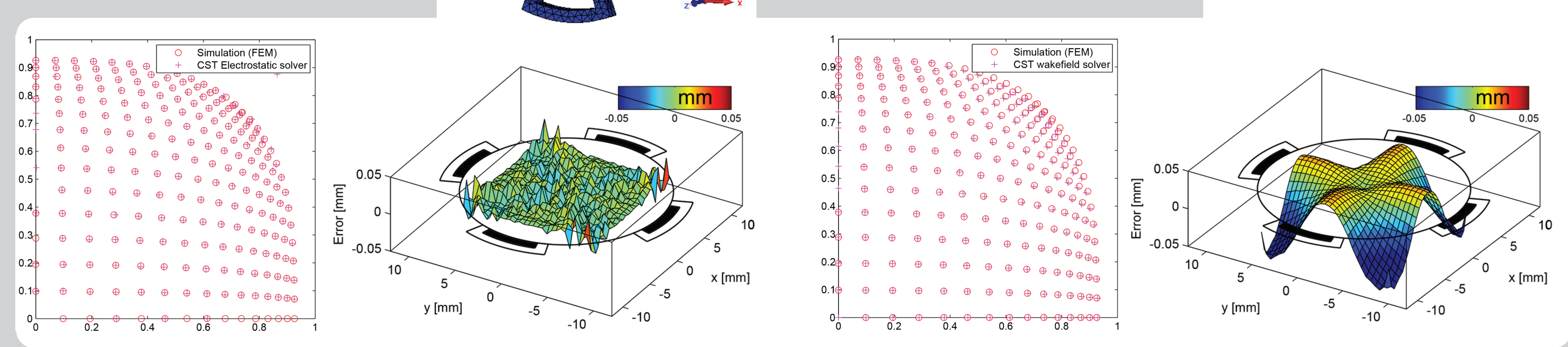
CST: Electrostatic & Wakefield solvers

Potential excitation of the right electrode to 1V is simulated in the 3D Electrostatic solver of CST with under 10 um difference wrt. *BpmLab*:



Time domain wakefield mapping in 3D by a single transient bunch in CST Particle Studio (225 simulations in 5h) with under 30 um difference:

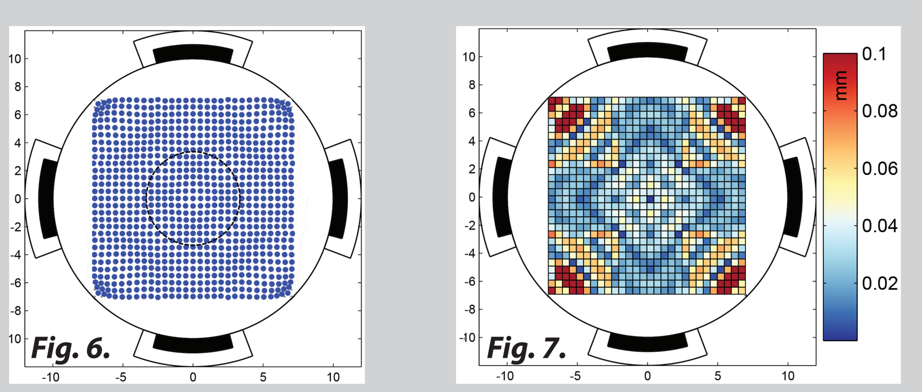




Nonlinearity corrections

With polynomials

Using polynomial coefficients of a nonlinear fit of two normalized quantities (Xraw,Yraw), derived from DOS ratios of potentials. Polynomial correction allows real-time beam position reconstruction taking into account the coupling between electrodes [4]. Accuracy of correction depends mainly on the polynomial's power, Figs. 6, 7.



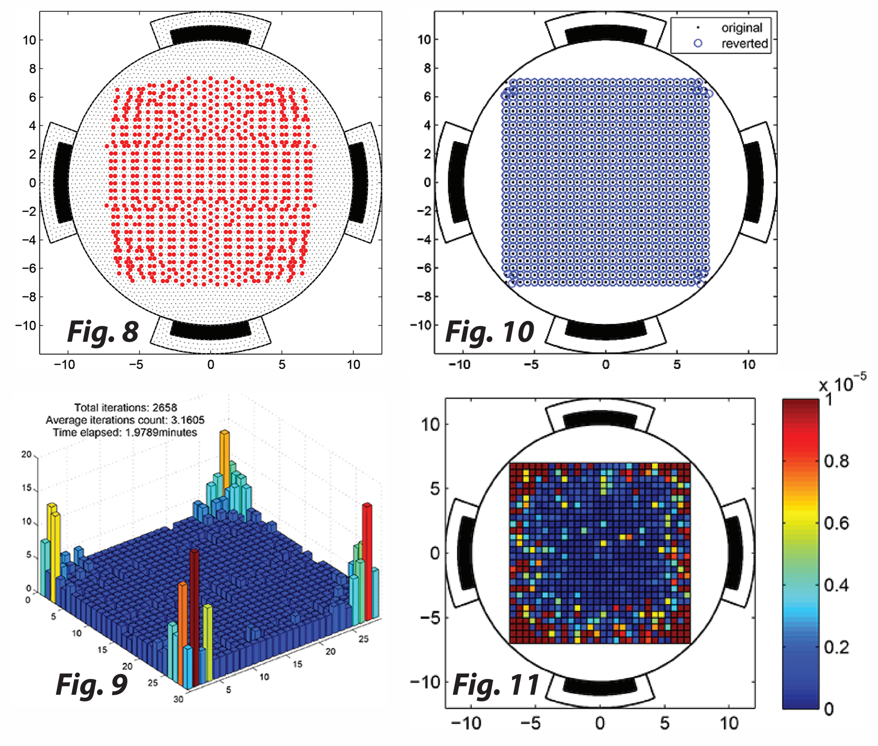
The result of applying a 9-th order 2D polynomial to correct the DOS distortion of the pilot-BPM in Fig. 5. Low-power polynomials are often sufficient for good precision.

By direct voltage inversion

The electrodes' voltages can be inverted back to beam position using the electrostatic model of a BPM through off-line numerical optimization [3,6]. It offers a much more precision over polynomial fits.

To invert a set of voltages, the target function is minimized using its analytic gradient and the iterative *Gradient Descent Method*. The starting point for the search is taken as the *nearest mesh node* (Fig 8) which assures quick minimization in ~3 iterations/point, Fig. 9.

The procedure was tested for backward convergence on the pilot-BPM using self-generated signals on a 29x29 grid (Fig. 10) of beam positions with excellent beam position recovery with machine precision, Fig. 11.



Correction based on voltage inversion through optimization. Here the self-generated BPM signals are used for backward convergence test of the FEM method.

References

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[2] J. Alberty, C. Carstensen, S. A. Fun ken, "Remarks around 50 lines of Matlab: short finite element implementation", Numerical Algorithms 20, 117-137 (1999).

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[5] A. Stella, "Analysis of the Dafne beam position monitor with a boundary element method", Dafne Tech. Note, CD-10 (1997).

[6] R. Bartolini, J. Rowland, "Geometric nonlinearities of a four buttons BPM", DLS Int. Rep. No. AP-SRREP-0174 (2010).