ImpactX Modeling of Benchmark Tests for Space Charge Validation

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Background

ImpactX [1] is a GPU-capable C++ successor to the code IMPACT-Z [2] built on the AMReX software framework [3] for modeling relativistic charged particle beams in linacs or rings. The code is currently under active development. Similar to IMPACT-Z, tracking is performed with respect to the path length variable s, and space charge is included using a second order operator splitting [2]. All tracking methods are symplectic by design, and maps are used where possible for efficient particle pushing. The 3D space-charge fields are computed with an iterative Multi-Level Multi-Grid (MLMG) Poisson solver [3], providing new support for adaptive mesh refinement. The code is continuously benchmarked (after every code change) against a suite of >20 test problems, designed to validate each feature of the code. The space charge benchmarks, to be described, are valid for 3D bunched beams in the presence of open boundary conditions.

Space Charge Benchmark Problems

ImpactX was used to reproduce the standard benchmark problems described in both [4] and [5]. The suite of tests includes:

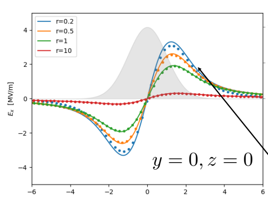
- static tests of the Poisson solve for space charge fields
- dynamical tests involving coasting or stationary beams
- beams matched to periodic focusing channels

Testing is fully open and details are archived online [7].

Space charge fields in a Gaussian bunch

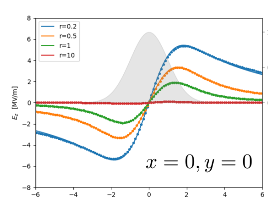
1 nC charge, 1 M particles, grid [128,128,256]

Variable aspect ratio $r = \sigma_z/\sigma_\perp, \quad \sigma_\perp = 1 \text{ mm}$

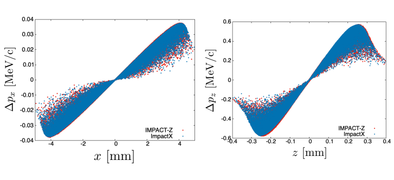


solid: exact
dots: ImpactX

large aspect ratio (blue)



Phase space of a 10 MeV, 1 nC electron bunch, initially Gaussian, $(\sigma_x, \sigma_y, \sigma_z) = (1, 1, 0.1)$ mm after a 1 m drift . ImpactX *vs*. IMPACT-Z shown.



Waterbag beam in a CF channel

Proton bunch matched to a 3D constant focusing section (using the rms envelope equations)
bunch: Q = 10 nC, KE = 2 GeV, ε= 1 μm (in each plane)
focusing: k = 1/m, L = 2 m
We verify the rms beam sizes remain stationary to within a specified tolerance.

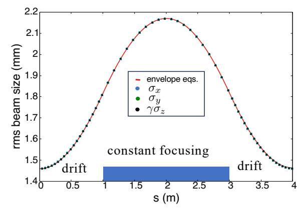
Kurth beam in a periodic focusing channel

Analogous to a K-V beam in a FODO channel, but appropriate for 3D bunched beams [6].

Described by 3D envelope equations (SC is linear).

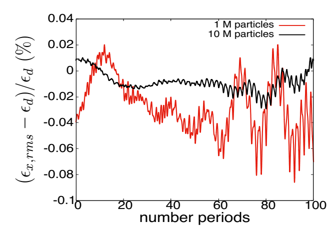
Proton bunch: Q = 10 nC, KE = 2 GeV, ε = 1 μm
Focusing: alternating drifts and CF ($k = 0.7 \text{ m}^{-1}$)

Matched beam envelopes over a single period



Phase advance: depressed from 121° ⟶ 74°

Emittance evolution over 100 periods



(Red) 1 M particles, [72,72,72] and (Black) 10 M particles, [128,128,128]. Fluctuation about the design value $\epsilon_d = 1\ \mu\text{m}$ is purely numerical.

Free expansion of a cold uniform bunch

Cold, uniform ellipsoidal bunch increasing in size (in a drift) due to its own space charge fields.

For a bunch spherical in its own rest frame, the distance required to double in size is given explicitly by:

$$\Delta s = \beta\gamma\kappa\sqrt{\frac{R_0^3}{r_c N_b}}, \quad \kappa = 1 + \frac{\sqrt{2}}{4}\log(3 + 2\sqrt{2})$$

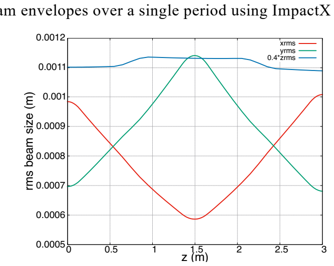
We verify the bunch has the correct second moments at Δs.

Cold beam in a FODO channel with RF cavities

Proton bunch: Q = 0.14 nC, KE = 250 MeV, ε = 0 (cold)
Focusing: FODO channel with RF cavities for long. focusing [5]

On-axis RF electric field: $E_z(z) \propto \exp(-(4z)^4)\cos\left(\frac{5\pi}{2}\tanh(5z)\right)$

Beam envelopes over a single period using ImpactX



Matched initial beam moments were obtained from MaryLie/IMPACT. This case is challenging due to RF-induced energy evolution and absence of symmetry among the *x*/*y*/*z* planes.

Bithermal beam in a CF channel

Self-consistent model of a stationary 3D bunch with a nontrivial core-halo distribution, now supported within ImpactX.

Phase space density (6D):

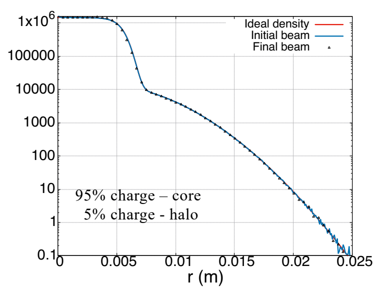
$$f = c_1 \exp(H/kT_1) + c_2 \exp(H/kT_2)$$

H – particle Hamiltonian, including the space charge potential
c_1, c_2 – constants controlling the weights of core & halo populations

A system of 4 ODEs is solved to yield the space charge potential and cumulative density function of each of the two populations.

Proton bunch: Q = 0.14 nC, $kT_1 = 36\times10^{-6}$, $kT_2 = 900\times10^{-6}$
Focusing: $k = 2\pi \text{ m}^{-1}$, L = 10 m

Spatial density of a bithermal distribution as a function of radius, showing that the distribution remains stationary in a CF channel



Using 10 M particles, [128,128,128] grid. A log scale is used to visualize the beam halo. The distribution is stationary over 7 decades.

Conclusion

In this work, we have used the code ImpactX to reproduce documented space charge benchmarks appropriate for 3D particle-in-cell codes in the context of high intensity bunched beams [4,5]. All numerical results shown here are archived in [7], and additional benchmark tests can be found in [1]. Future plans include detailed code performance and scaling studies, detailed exploration of benchmark tests with mesh refinement, the implementation in ImpactX of 2D and/or 2.5D space charge models appropriate for long or unbunched beams, and the implementation of additional collective effects (including resistive wall wakefields and CSR models). In the future, we hope to participate in benchmarks involving 2.5D space charge appropriate to multi-turn tracking of long beams, such as the GSI benchmark on space charge induced trapping.

References

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[3] https://amrex-codes.github.io/amrex/

[4] C. Mayes et al, in Proc. IPAC 2018, pp. 3428-3430, Vancouver, BC, Canada, 2018.

[5] R. D. Ryne et al, in Proc. EPAC 2004, pp. 1942-1944 , Lucerne, Switzerland, 2004.

[6] C. E. Mitchell et al, in Proc. IPAC 2021, pp. 3213-3216, 21, Campinas, SP, Brazil, 2021.

[7] Supplementary materials, doi.org/10.5281/zenodo.8295849

Acknowledgments

This work is supported by the Director, Office of Science, Office of Basic Energy Sciences of the U.S. Department of Energy, and makes use of computer resources at the National Energy Research Scientific Computing Center.