Space-charge particle resonances and mode parametric resonances

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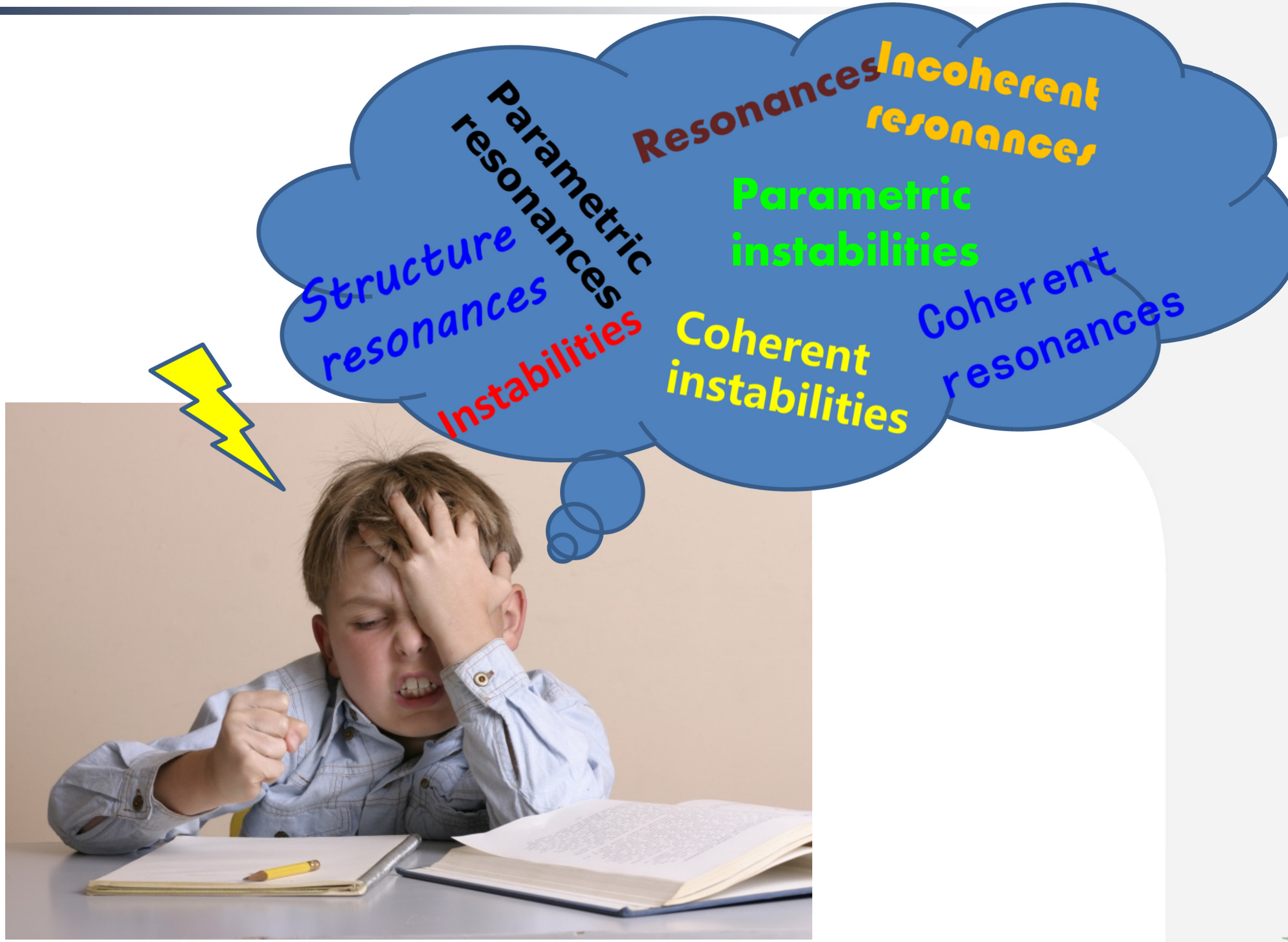
Drowned in a swamp of terms…?



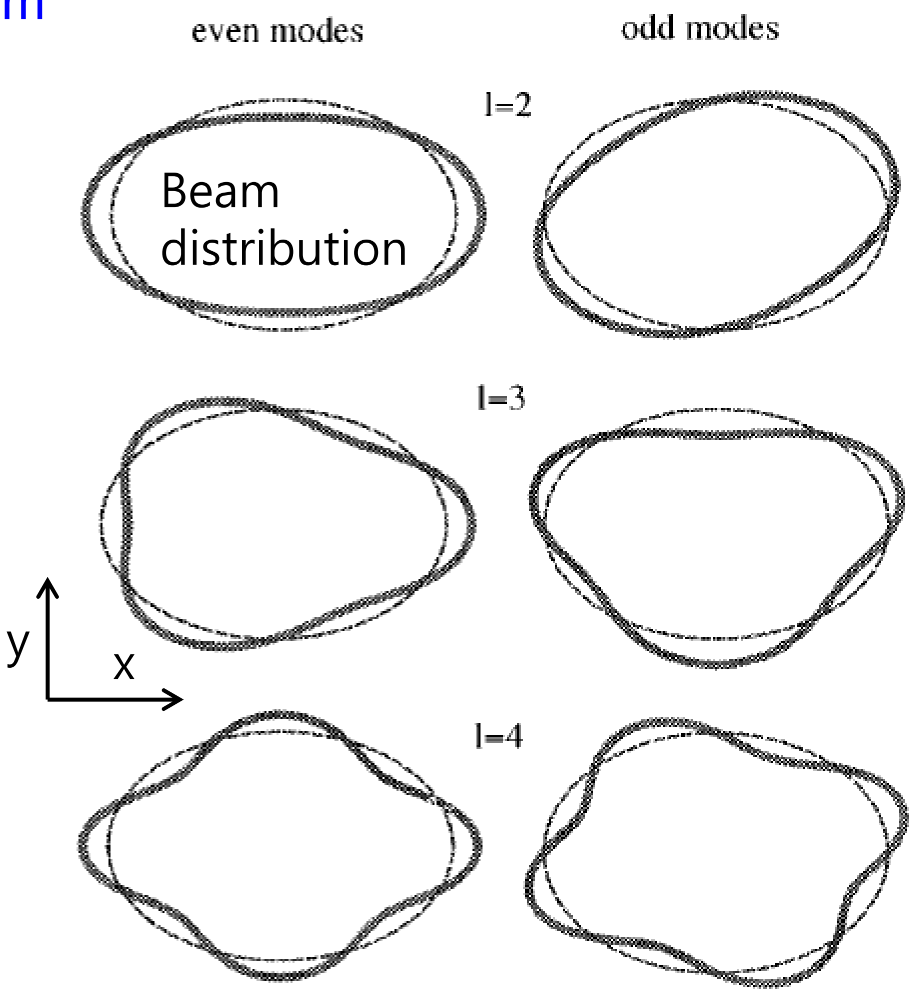
Space-charge Halo Mechanisms

- The community has put a lot of efforts to better understand space charge mechanisms.
- There are two major families of space-charge mechanisms, and yet they need to be differentiated: (mode) instabilities and (particle) resonances.
- (Particle) Resonances: a.k.a. single particle resonances, incoherent resonances …
- (Mode) Instabilities: a.k.a. parametric resonances, structure resonances, coherent resonances, coherent instabilities, parametric instabilities …
- Both families are loosely called “resonances”.
- Many names for the same thing … → confusing even to experts.

(Mode) Instabilities

- (Mode) Instabilities (or parametric resonances) are instabilities of beam eigenmodes.
- Beam eigenmodes of Vlasov and Poisson equations.
- Analytical solution is available only for the KV distribution.

Examples of eigenmodes of Vlasov-Poisson model



Courtesy of Hofmann, PRE **57**, 4713 (1998)

(Mode) Instabilities

- Instabilities of KV distribution were reported in the early literatures, and the 2nd order mode instability is widely known as "the envelope instability".

- These instabilities of beam eigenmodes are also known as parametric resonances.

- They are parametric resonances of the modes of Vlasov-Poisson equations.
- Are they resonances of the beam particle? No.

(Mode) Instabilities of real beams?

- (Mode) Instabilities of Vlasov-Poisson equations have been observed in multiparticle PIC simulations for KV and waterbag distributions.
- But, not observed for Gaussian distributions.

multiparticle PIC simulations

Instability	KV distribution	Waterbag distribution	Gaussian distribution
2^{nd} order	O	O	O
3^{rd} order, 4^{th} order	O	O	X
higher order	O	X	X

- This suggests that the 3^{rd} or higher order instabilities may not be observable for real beams.

Jeon, J. Korean Phys. Soc. **72**, 1523 (2018)
Space Charge 2019

(Particle) Resonances in Linacs

- (Particle) Resonances in circular accelerators are well known.

- 2009, Jeon et al discovered the 4th order particle resonance in high-intensity linacs [PRST-AB (2009)].
- 2009, Groening et al confirmed experimentally the 4th order resonance using the GSI UNILAC [PRL (2009)].
- 2016, Jeon confirmed the 4th order resonance using the SNS linac [PRAB (2016)].

- 2015, Jeon et al found the 6th order resonance in high-intensity linacs [PRL (2015)].
- 2016, Hofmann found the 8th and 10th order resonances [HB2016].

What is the difference?

Mode Instabilities
(or parametric resonances)

→ No FFT resonance frequency component

Mode instabilities (or mode parametric resonances) :
Fixed points do not exist in the phase space.

Particle Resonances

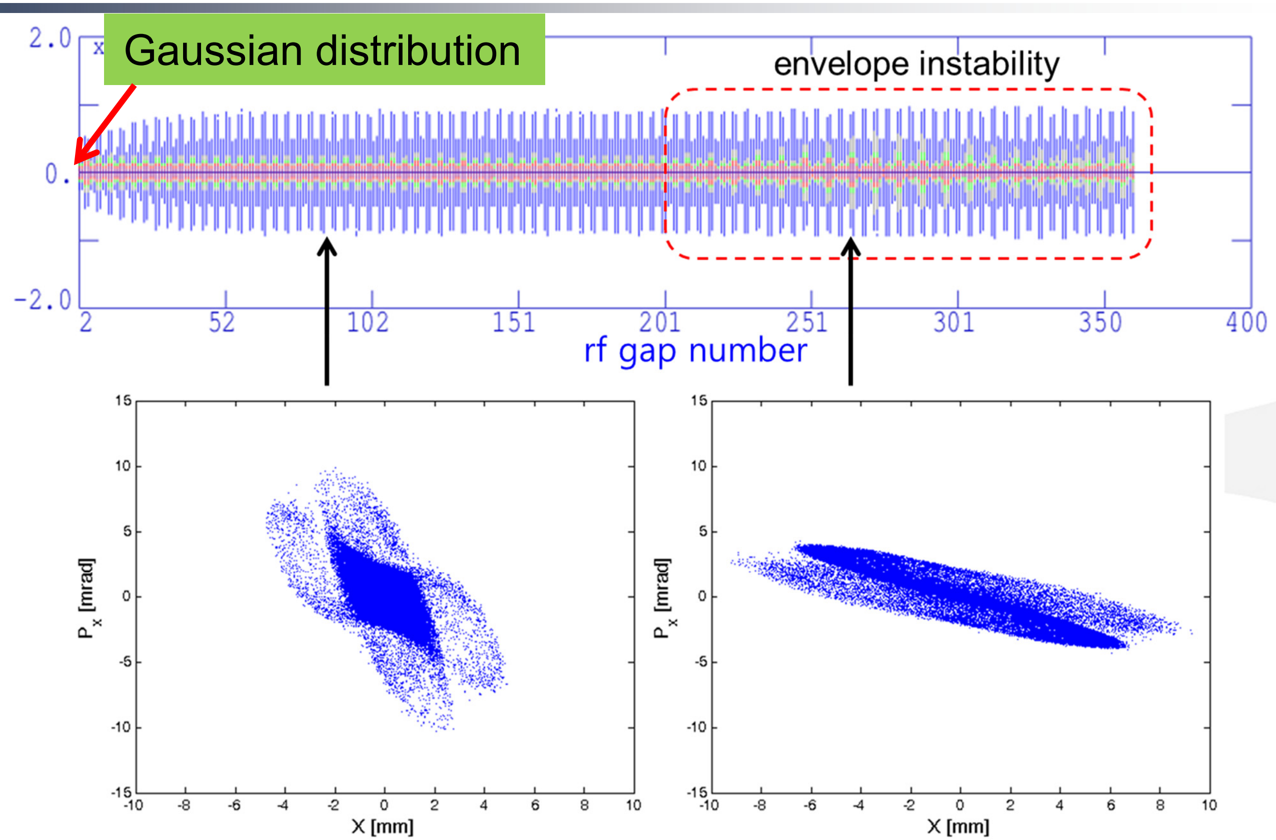
→ Yes FFT resonance frequency component

Particle resonances :
Fixed points exist in the phase space.

(Mode) Instabilities

a.k.a. **parametric resonances** or **structure resonances**, **coherent resonances**, parametric instabilities, etc

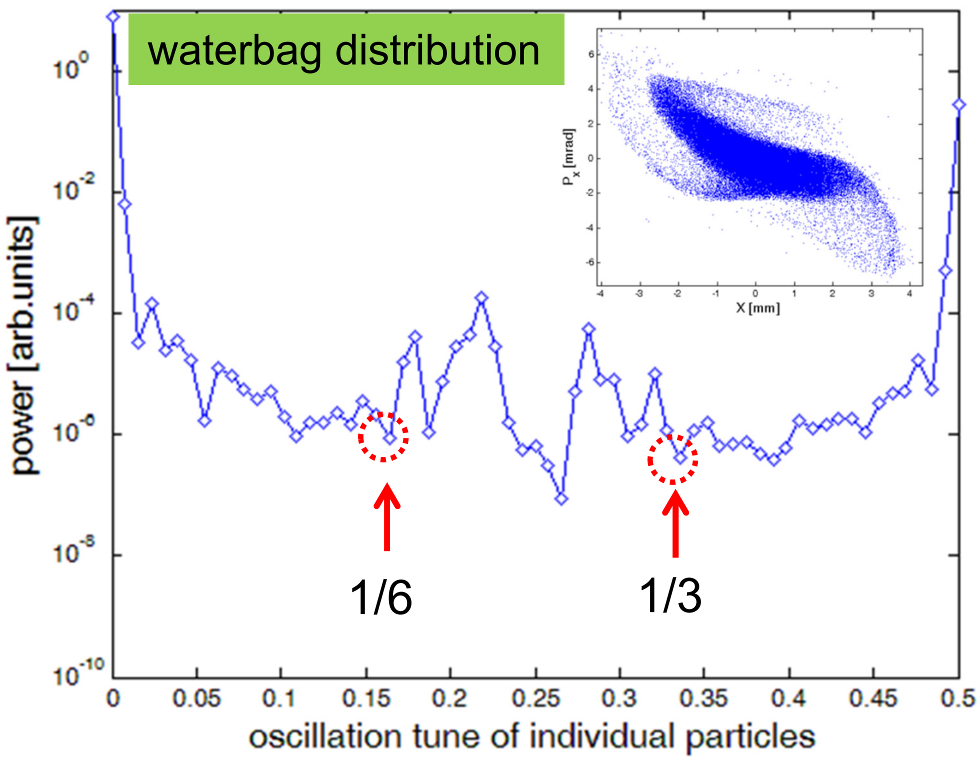
(2nd order) envelope instability
in high intensity linear accelerators



- $2\sigma_o - \Delta\sigma_{2,coh}$ = 180° instability for a constant- σ_o lattice with σ_o = 100° and σ = 70°.
- Observed for KV, Gaussian, waterbag distributions.
- The envelope instability is excited following the 4th order resonance for a constant-σ_o lattice.

3rd order instability
in high intensity linear accelerators

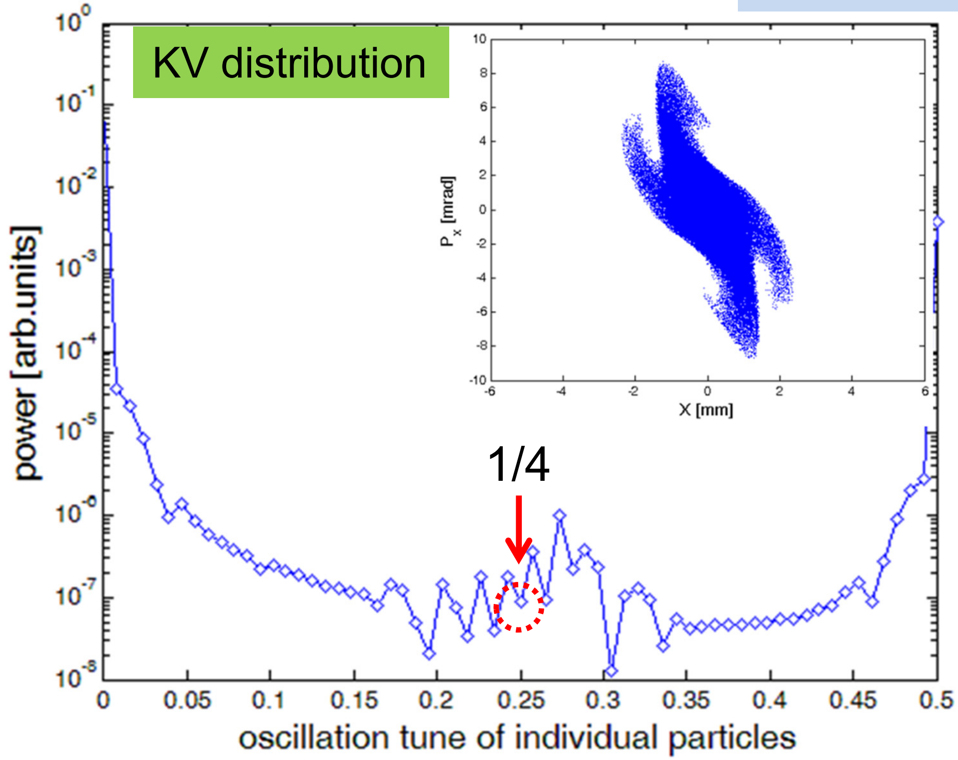
Jeon et al., NIM A **832** (2016) 43



- $3\sigma_o - \Delta\sigma_{3,coh} = 180°$ instability for a lattice with $\sigma_o = 92°$ and $\sigma = 40°$ (90 mA beam).
- Observed for KV and waterbag distributions, but not for Gaussian distribution.
- Not a particle resonance: no resonance peak around 1/3 or 1/6 in the FFT spectrum.

4^{th} order instability
in high intensity linear accelerators

Jeon, J Korean Phys Soc **72**, 1523 (2018)



- $4\sigma_o - \Delta\sigma_{4,coh} = 360°$ instability for a lattice with $\sigma_o = 112°$ and $\sigma = 85°$.
- Observed only for a KV distribution when σ is close to 90°.
- Not a particle resonance: no resonance peak around 1/4 in the FFT spectrum.

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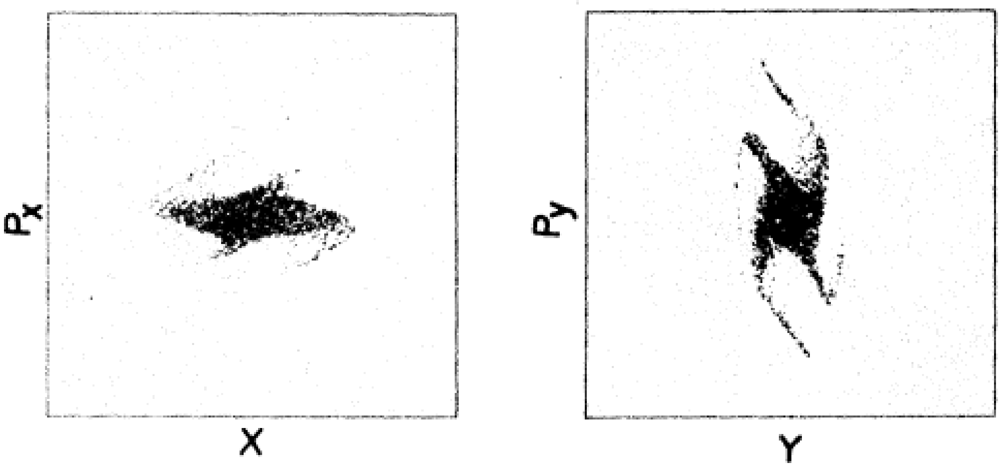
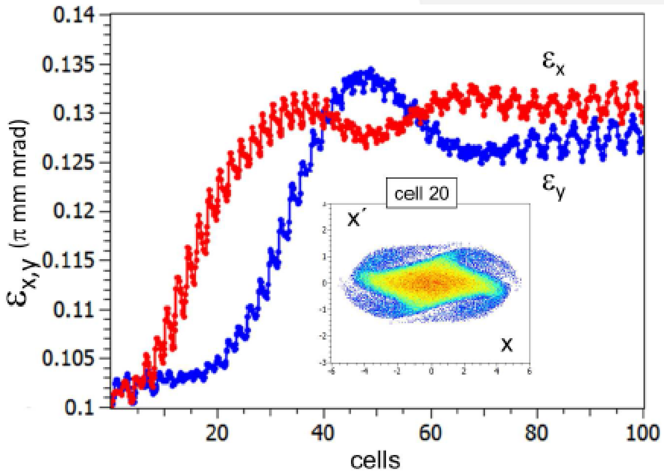
4^{th} order instability
in high intensity linear accelerators





Courtesy of Haber and Maschke, PRL **42**, 1479 (1979)
KV distribution
σ_o = 90° and σ = 30°

Courtesy of Hofmann (HB2016)
waterbag distribution
σ_o = 70° and σ = 35°

- These are $4\sigma_o - \Delta\sigma_{4,coh}$ = 180° instability.
- Observed for KV and waterbag distributions.
- Observed when σ is far away from 90°.

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Instability Condition

- Mode instability condition is basically Mathieu-type instability condition.
  - Instability condition is $m\sigma_o - \Delta\sigma_{m,coh} = n180°$.
  - Mathieu-type instability condition.
- Called "half integer resonance" by some, denoting $m\sigma_o - \Delta\sigma_{m,coh} = n360°/2$.

- But in circular accelerators, half integer resonances are $2\sigma = n360^o$.
- Particle resonance condition $m\sigma = n360^o$ comes from the Fourier expansion of the Hamiltonian.

Particle Resonances

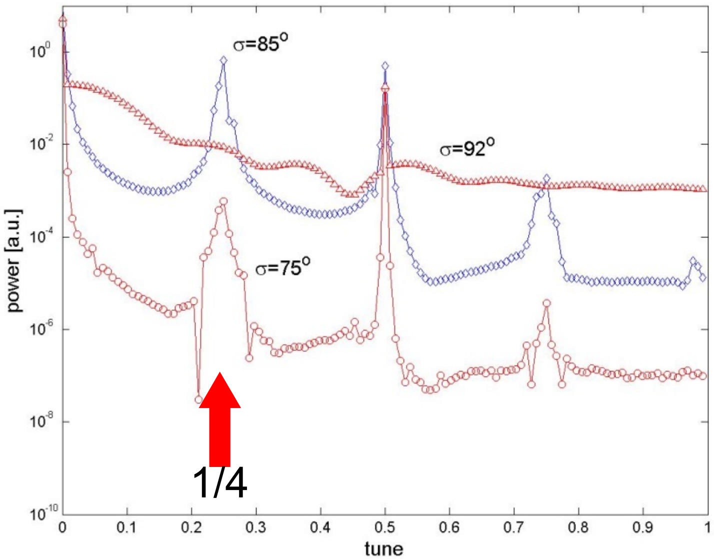
a.k.a. **(single) particle resonances, incoherent resonances**

4^{th} order (particle) resonance

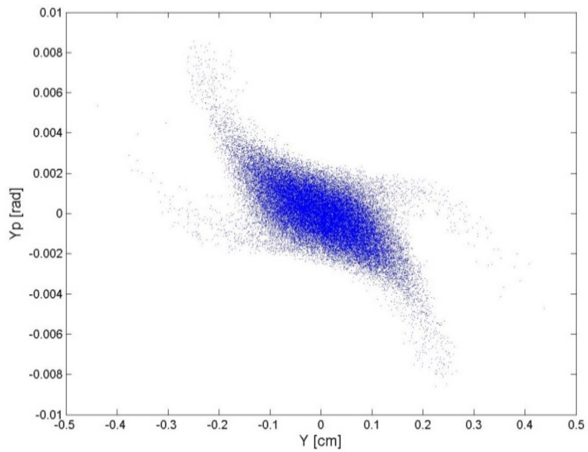
Discovery of the particle resonance in linacs

Jeon et al, PRST-AB **12**, 054204 (2009)

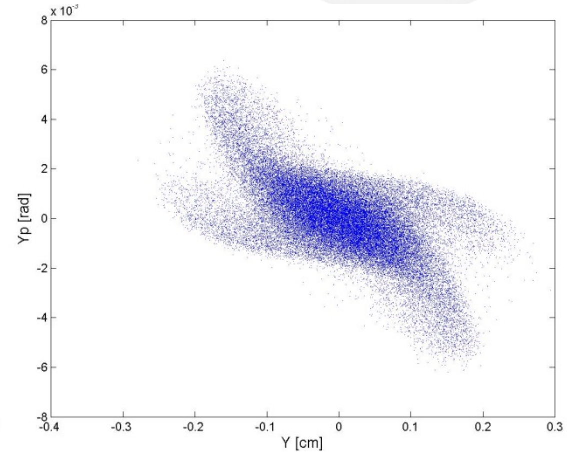
FFT frequency spectrum



Cross from below



Cross from above



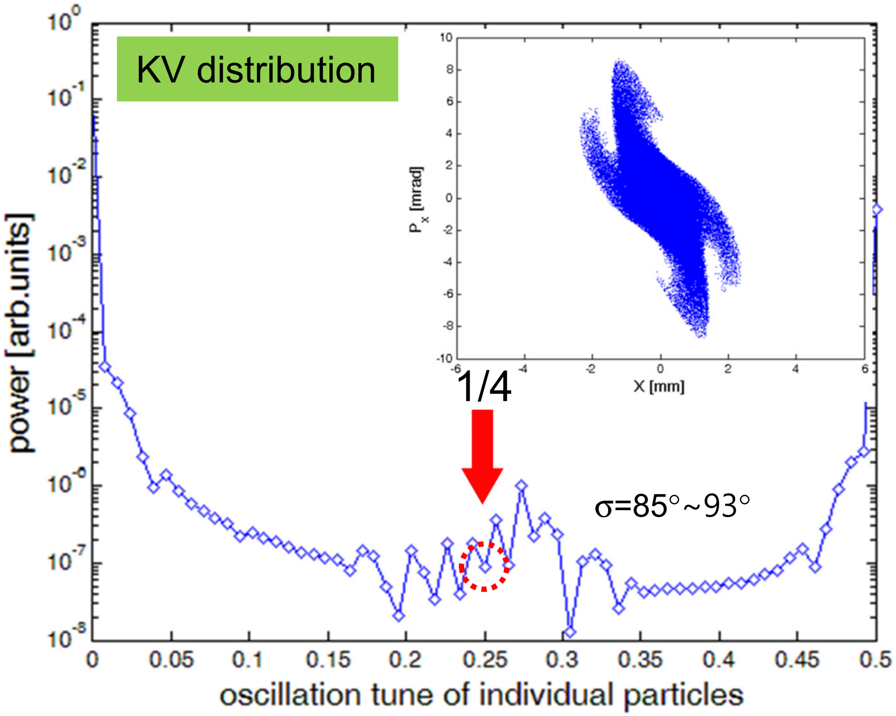
- The 4σ = 360° 4^{th} order resonance was discovered in high-intensity linear accelerators in 2009.
- This mechanism was explicitly identified as a (particle) resonance.
- FFT resonance frequency component observed
- Behavior difference between crossing the resonance "from above" and "from below".

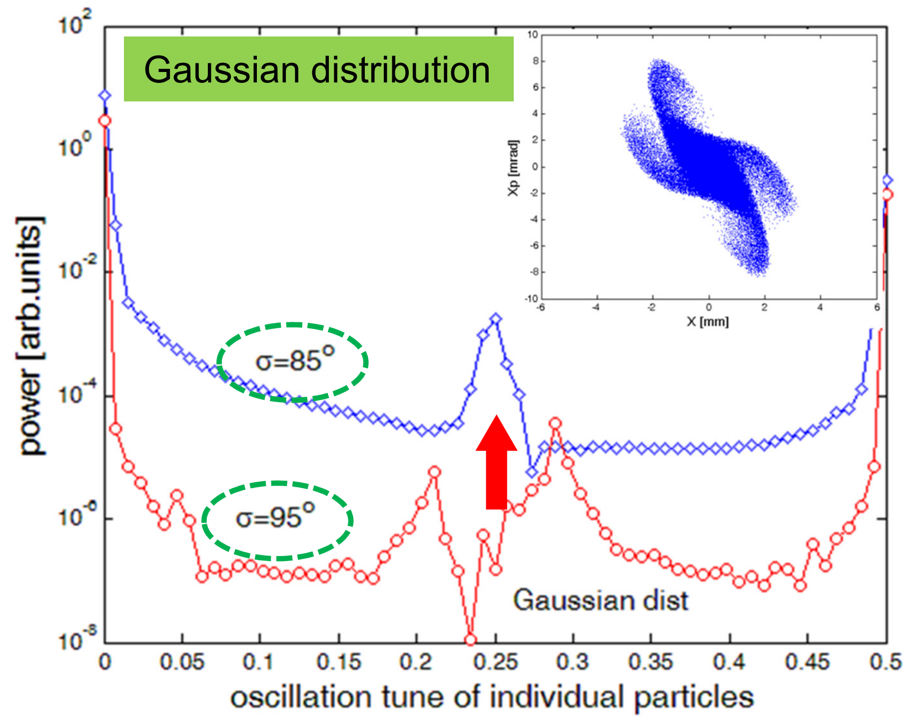
There are look-alike people!!

- There are look-alike people but not related at all.
- Their parents (origin) are different.

Appearance can be deceiving!

Jeon, J Korean Phys Soc **72**, 1523 (2018)





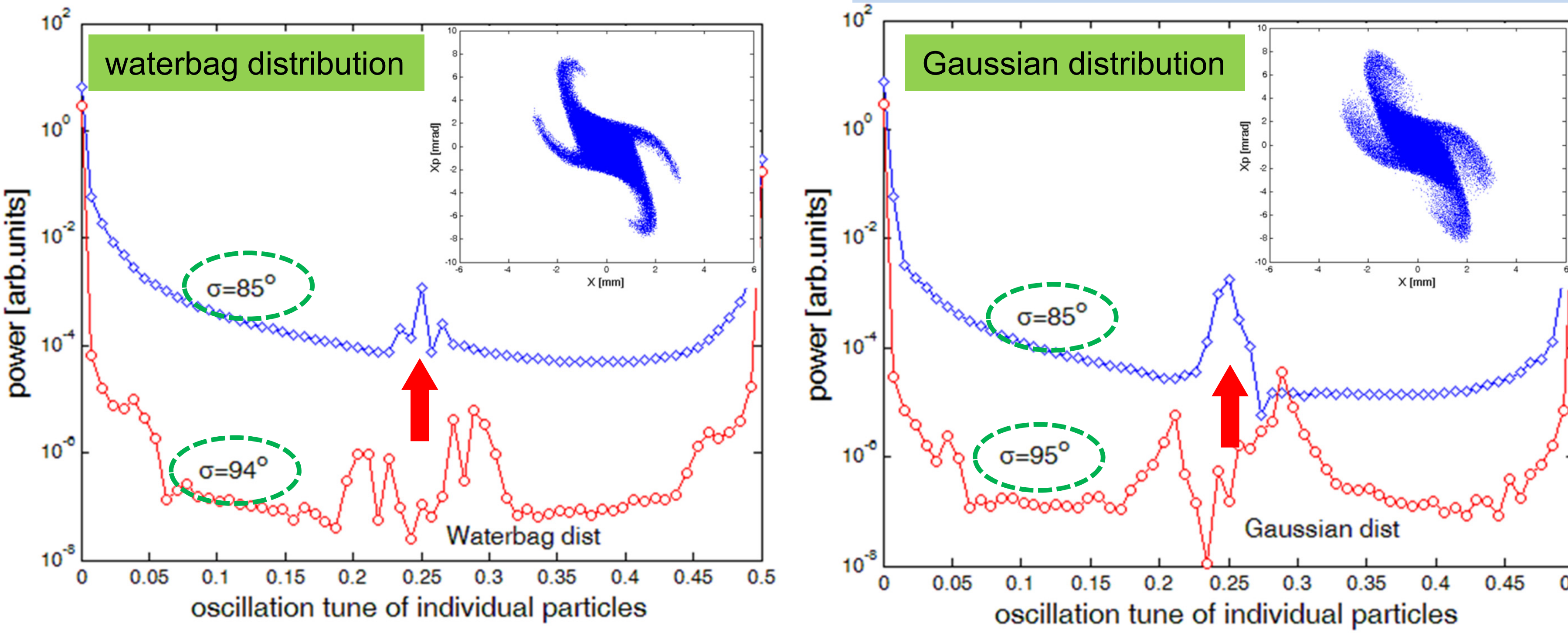
4th order instability
KV distribution

4th order resonance
Gaussian distribution

- Instabilities and resonances, their appearances may look alike in the phase space. But they are completely different mechanisms.
- No resonance frequency component is observed for the 4th order mode instability of a KV distribution.

FFT resonance frequency component

Jeon, J Korean Phys Soc **72**, 1523 (2018)



4th order resonance
Waterbag distribution

4th order resonance
Gaussian distribution

- Clear resonance frequency peak at 1/4 = 90°/360° is observed for non-KV beam distributions.

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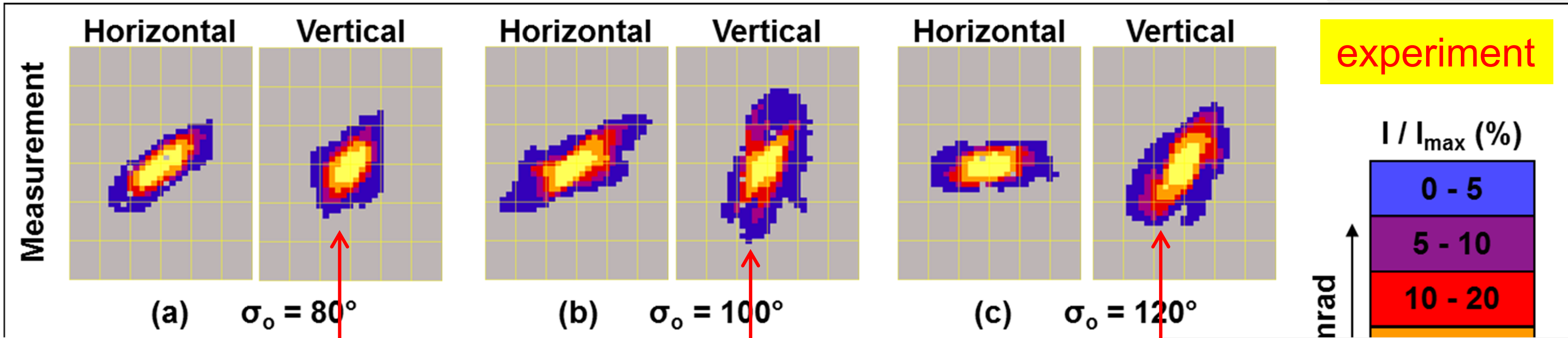
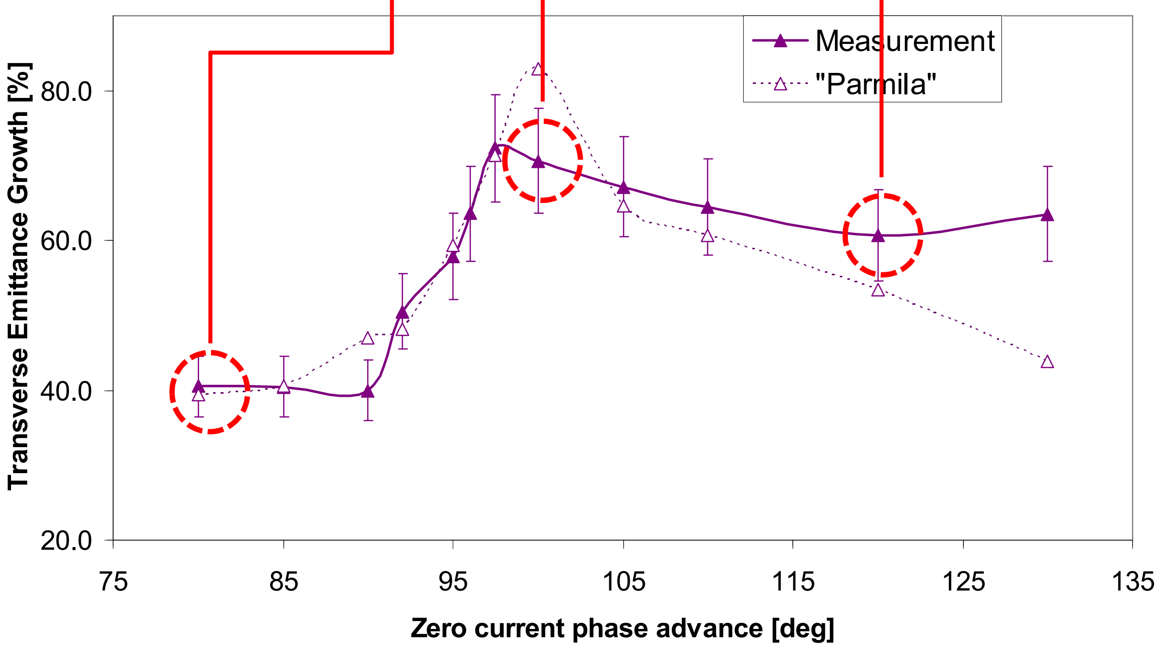
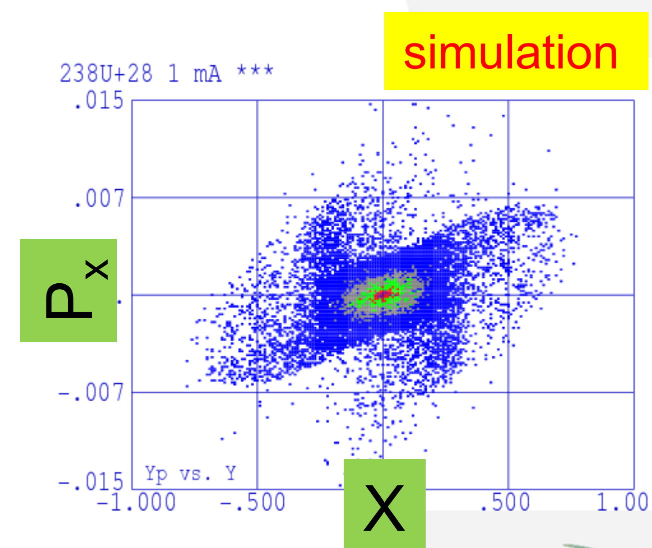
(Experiment #1) the 4th order resonance
using GSI UNILAC

Groening et al., PRL **102**, 234801 (2009)

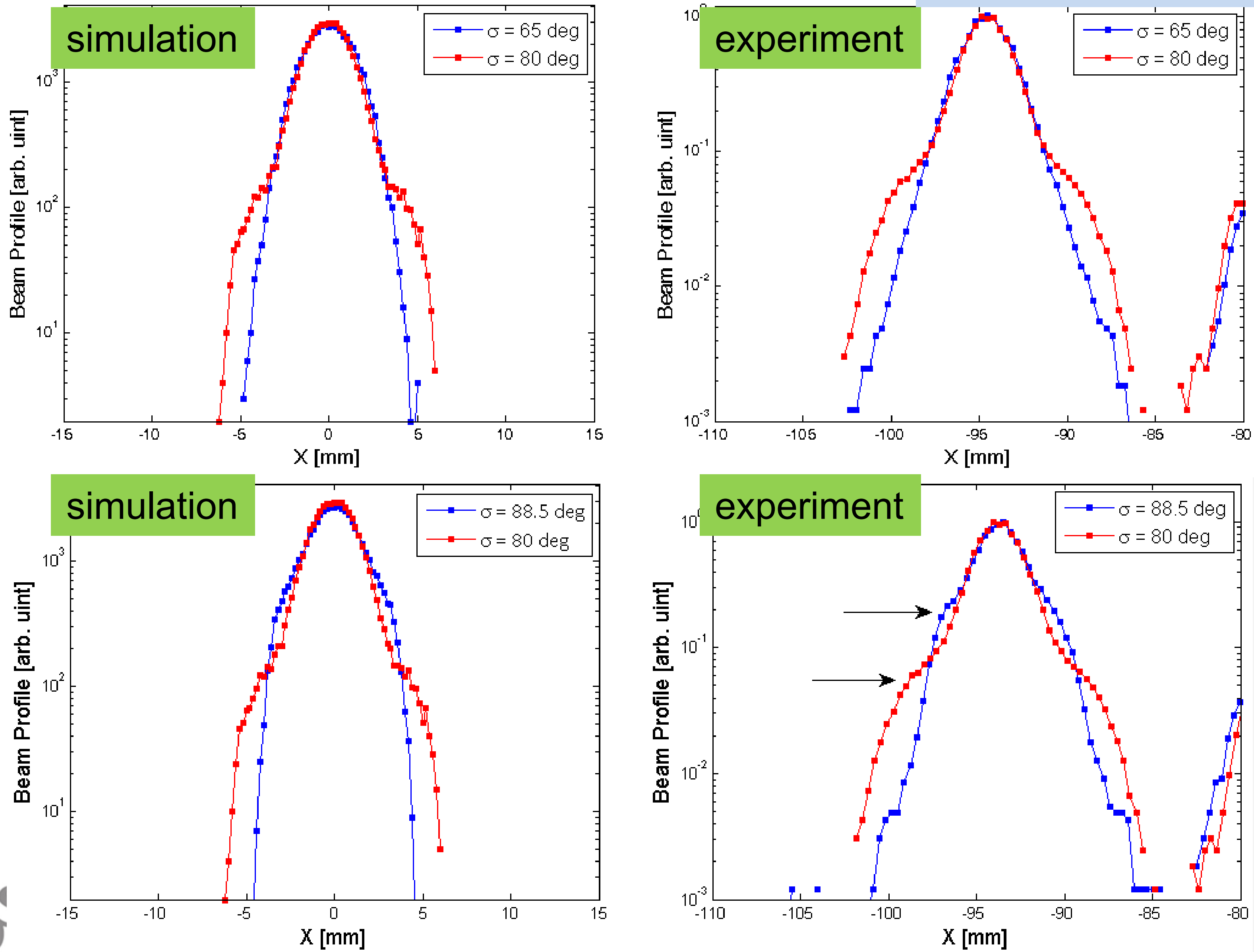






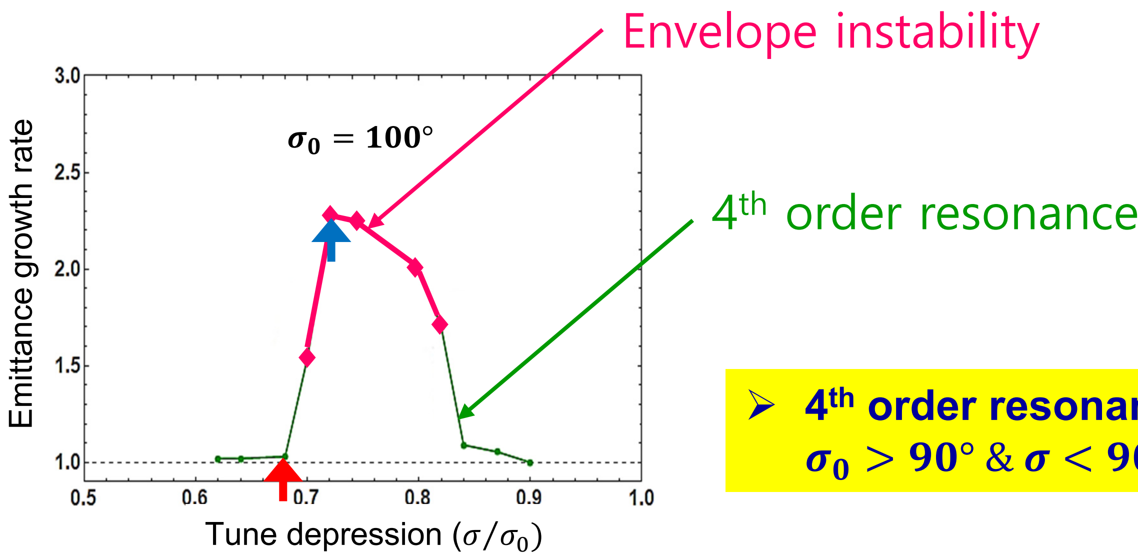
(Experiment #2) the 4th order resonance
using SNS linac

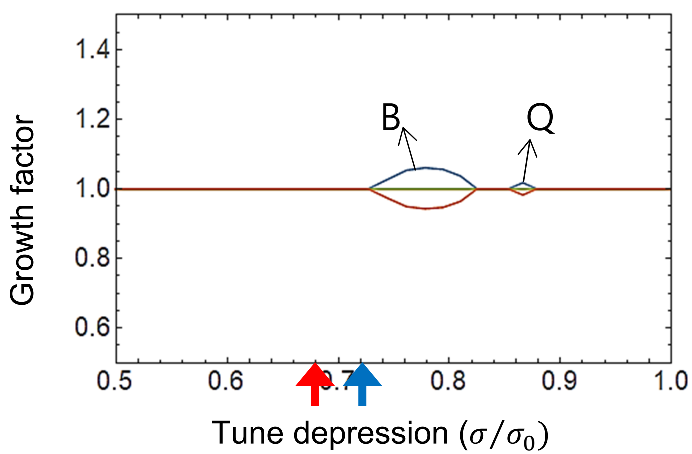
Jeon, PRAB **19**, 010101 (2016)



Stopband of 4th order resonance and envelope instability

Cheon et al., Phys Plasmas **27**, 063105 (2020)





- **4th order resonance stop band :** $\sigma_0 > 90° \;\&\; \sigma < 90°$

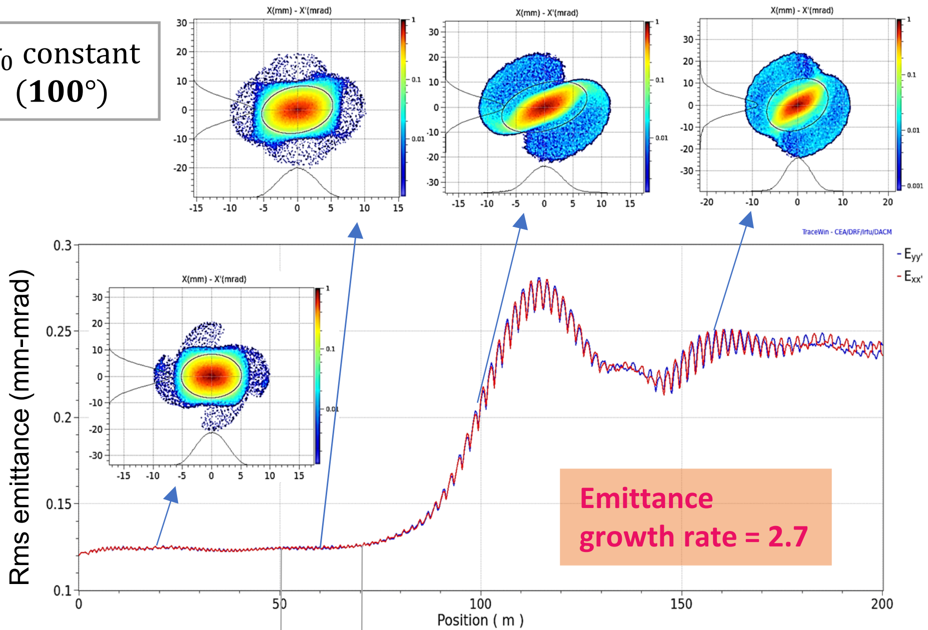
The 4th order particle resonance stop band is wider than the envelope instability stopband.

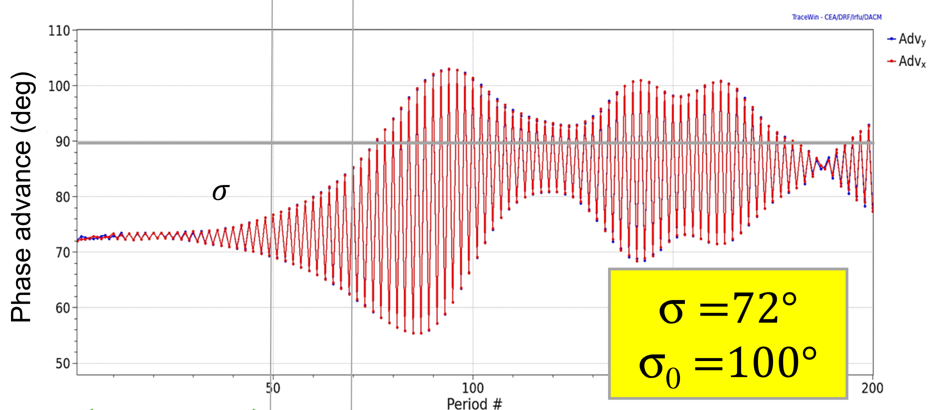
The envelope instability is induced only within the envelope instability stopband.

Stopband of 4th order resonance and envelope instability

Cheon et al., Phys Plasmas **27**, 063105 (2020)

σ_0 constant (100°)

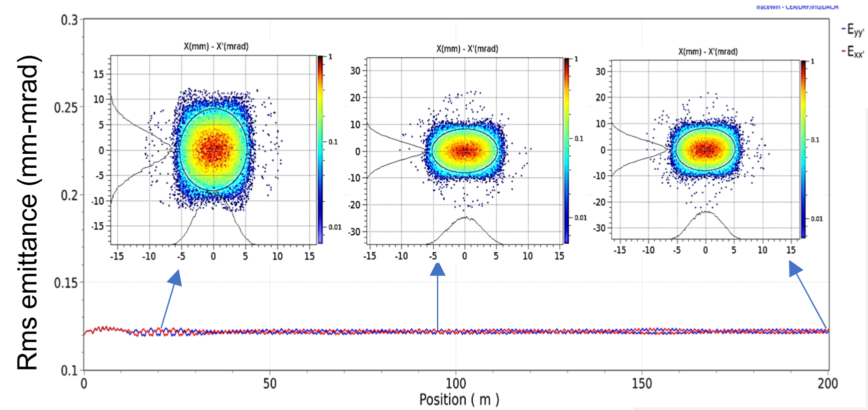


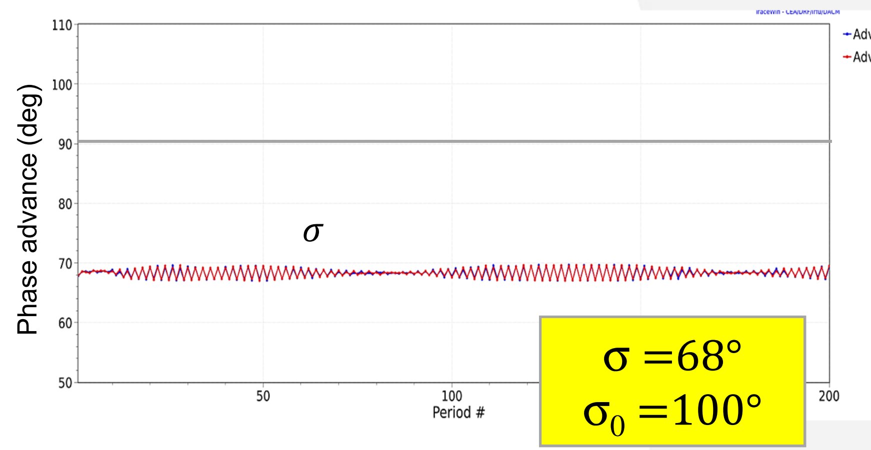


4th order resonance

Envelope instability

Is the envelope instability always excited following the 4th order resonance? - NO.

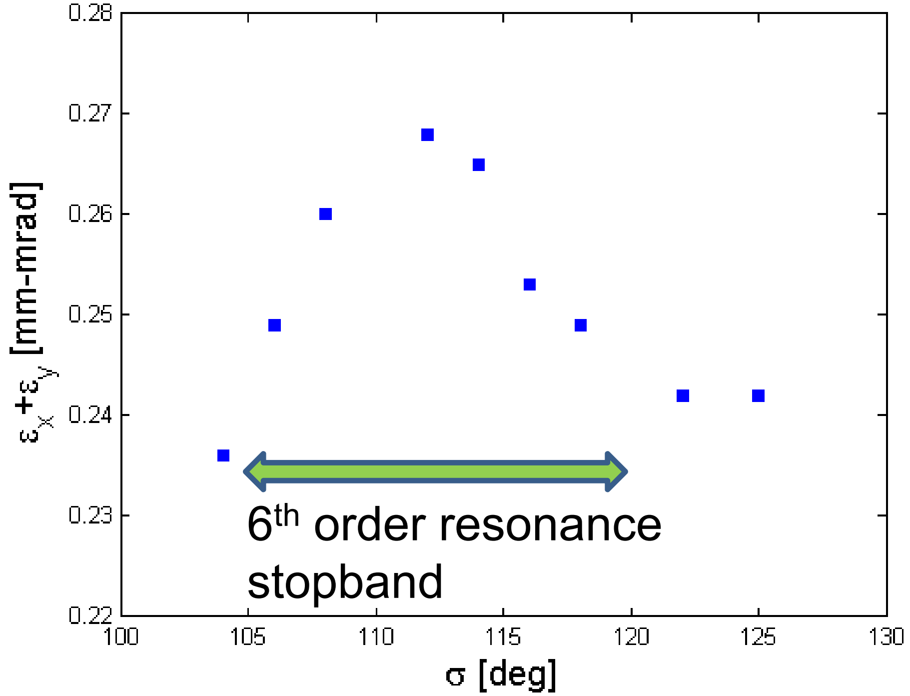


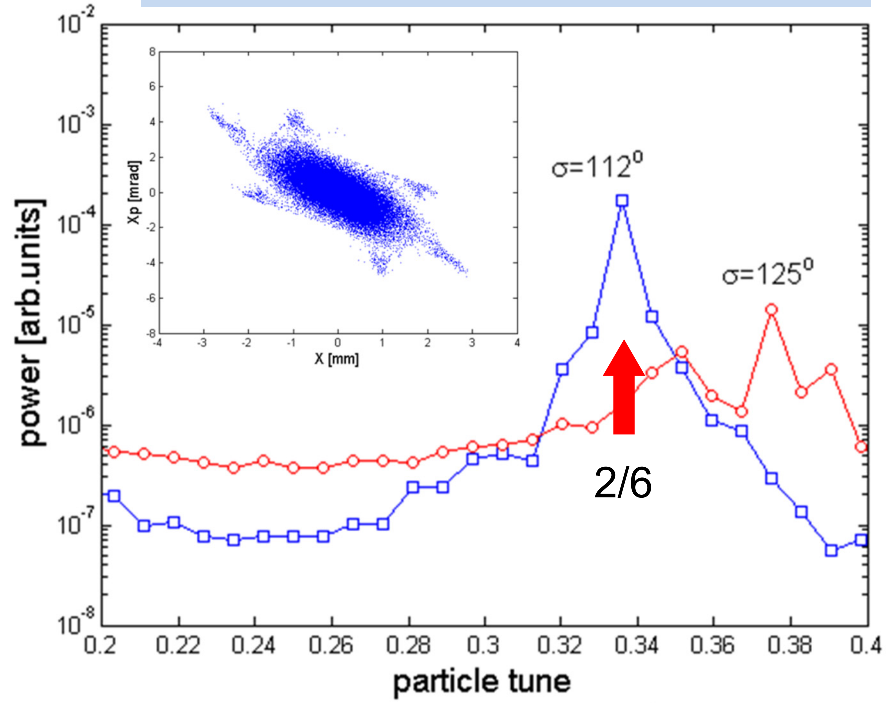


- Envelope instability is **not** induced

6^{th} order resonance
in high intensity linear accelerators

Jeon et al., PRL **114**, 184802 (2015)





- $6\sigma = 720°$ sixth order resonance was discovered when $\sigma < 120°$.
- FFT peak at 1/3.
- No resonance effects when $\sigma > 120°$ (Hamiltonian property).

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Terminology Suggestion

- (Mode) Instabilities are instabilities of modes of Vlasov-Poisson equations:
  - a.k.a. parametric resonances or structure resonances, parametric instabilities, coherent resonances …
  - but would better be called **mode parametric resonances** to distinguish them from particle parametric resonances.

- Resonances are resonances of the beam particle, as well known in circular accelerators:
  - would better be called **particle resonances**,
  - a.k.a. single particle resonances, incoherent resonances …

- Community needs to draw a consensus on the terminology.

Thank you for your attention!
감사합니다

D. Jeon, Classification of Space-Charge Resonances and Instabilities in High-Intensity Linear Accelerators, J. Korean Phys. Soc. **72**, 1523 (2018)

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