Recent progress on nonlinear beam manipulations in circular accelerators

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Shaping transverse beam distributions

- Exploit nonlinear effects in tranverse motion:
  - Change phase space topology (new separatrices, islands)
- Slow variation of parameters
  - Change surfaces of phase-space regions
  - Perform particle trapping & transport in phase-space regions
- Manipulate transverse beam distribution:
  - Beam splitting
  - Sharing of transverse emittances
  - Cooling of annular beams

Theoretical frameworks

Hénon-like maps

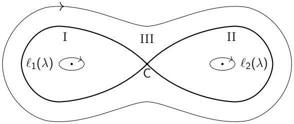
$$\begin{pmatrix} q_{n+1} \\ p_{n+1} \end{pmatrix} = \mathrm{R}(\omega_0) \begin{pmatrix} q_n \\ p_n + f(x_n) \end{pmatrix}$$

$\rightarrow$ interpolating Hamiltonians

Normal Forms

$$\begin{array}{ccc} \mathrm{z} & \xrightarrow{\mathrm{F}} & \mathrm{z}' \\ \Phi \downarrow & & \uparrow \Psi \\ \zeta & \xrightarrow[\mathrm{U}]{} & \zeta' \end{array}$$

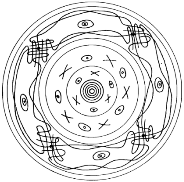
Separatrix crossing theory



$$P_{\mathrm{III}\rightarrow i} = \frac{\mathrm{d}A_i/\mathrm{d}t}{\mathrm{d}A_{\mathrm{III}}/\mathrm{d}t}$$

$$J_f = A_i/2\pi$$

Poincaré-Birkhoff theorem



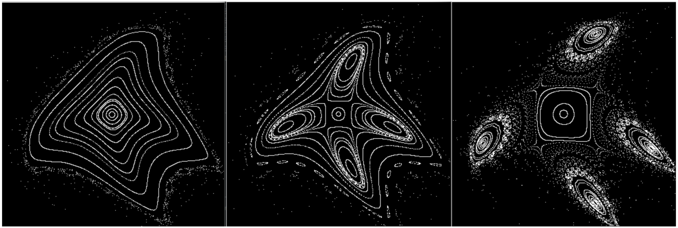
The starting point: Multi-Turn Extraction (MTE)

Hénon map:

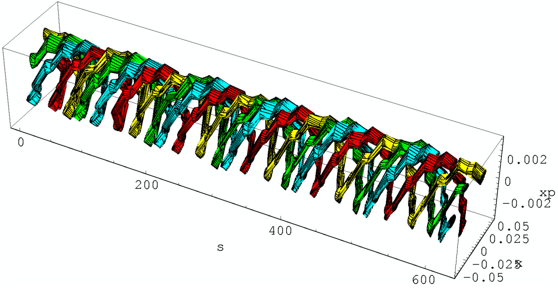
$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_0) \begin{pmatrix} x_n \\ p_n + x_n^2 \end{pmatrix}$$

$\omega_0 \approx 2\pi r/s$: s islands.

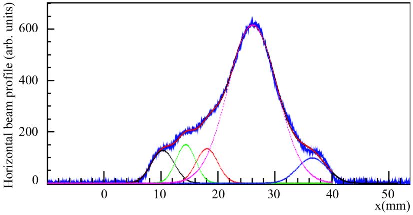
- split beam in $s+1$ beamlets
- used for beam transfer from PS to SPS



The starting point: MTE



Split beam structure along the ring

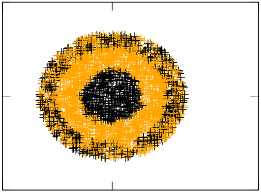


Example of a measured beam profile of a split beam

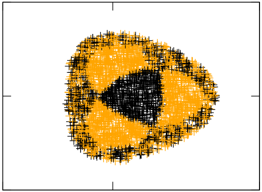
Extending MTE: an external exciter

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_0) \begin{pmatrix} x_n \\ p_n + x_n^2 + \varepsilon x_n^{\ell-1} \cos(\omega t) \end{pmatrix}$$

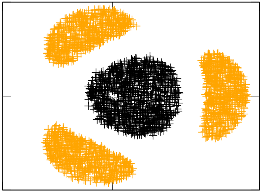
ω_0 fixed, $\omega \approx m\omega_0$: m islands.



$\varepsilon = 0$, $\omega = \omega_\mathrm{i} < \omega_0$



$\varepsilon > 0$, $\omega = \omega_\mathrm{i}$



$\varepsilon > 0$, $\omega = \omega_\mathrm{f} < \omega_\mathrm{i}$

Extending MTE: an external exciter

- Trapping explained via time variation of islands' surface for maps and Hamiltonians
- Scaling laws, parameter dependence established
- Possibility of beam splitting without varying tune

Sharing transverse emittances

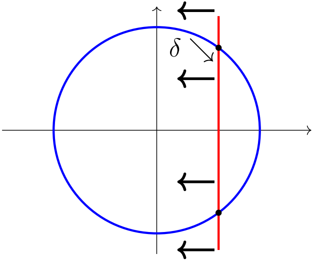
4D Hénon-like map

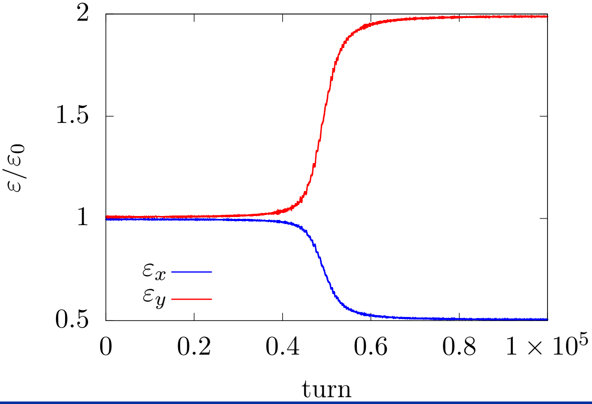
$$\begin{pmatrix} x' \\ p_x' \\ y' \\ p_y' \end{pmatrix} = R(\omega_x, \omega_y) \begin{pmatrix} x & \\ p_x & + \operatorname{Re} f(x,y) \\ y & \\ p_y & - \operatorname{Im} f(x,y) \end{pmatrix}$$

- Resonance: $\delta = m\,\omega_x - n\,\omega_y \approx 0$
- 2D Hamiltonian, $I_2 = mJ_x + nJ_y$ approximately conserved
- Vary ω_x, ω_y $\rightarrow$ separatrix crossing
- For each particle, can we make $J_{y,f} = (m/n)J_{x,i} \implies \varepsilon_{x,f} = (m/n)\varepsilon_{y,i}$?

Sharing transverse emittances

$m = 1, n = 2$:



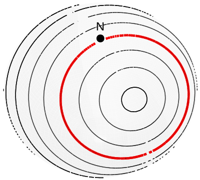


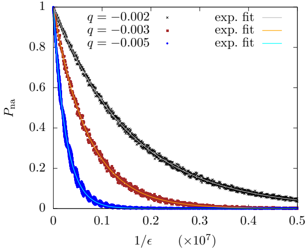
Sharing transverse emittances

- Studied 2D Hamiltonian and 4D map models, exchange mechanism explained via separatrix crossing theory
- Resonances higher than 3: presence of additional fixed points $\rightarrow$ more phase-space regions
- Improved adiabatic theory: error on final J depends on adiabaticity
  - Resonance $(1, 2)$ and higher: power-law
  - Resonance $(1, 1)$ (coupling resonance): exponential

Sharing transverse emittances: linear coupling

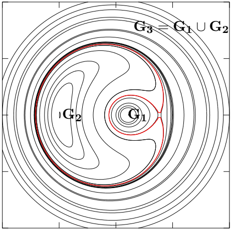
- Exponential behaviour of coupling exchange already observed: now explained with adiabatic theory
- Relationship between coupling strength and adiabaticity established

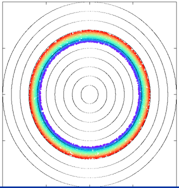


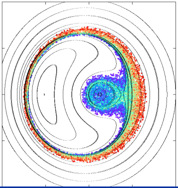


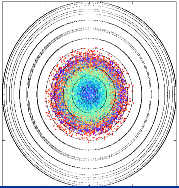
Cooling an annular beam

- AC dipole & nonlinearity: $H = \omega_0 J + \Omega_2 J^2/2 + \varepsilon\sqrt{2J}\cos\phi\cos\omega t$
- vary ω, ε; engineer areas to optimize trapping & transport: up to 90% cooling

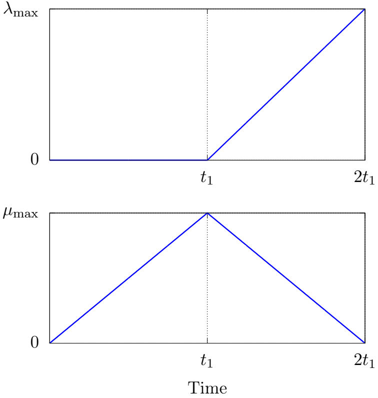


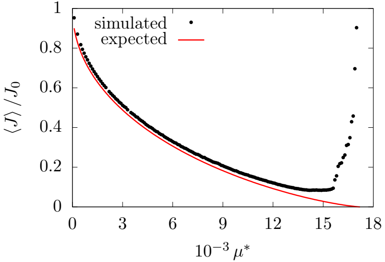






Cooling an annular beam





An alternative cooling protocol has been devised

Next steps

- Design experimental configurations to perform beam tests of some of these techniques (mainly on PS).
- Study the double-resonance case, like in the MTE case, in which the resonance crossing is improved by means of an AC dipole.

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