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EXPLORING QUASI-INTEGRABLE OPTICS WITH THE IBEX PAUL TRAP

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Abstract

An ideal accelerator built from linear components will exhibit bounded and stable particle motion. However, in reality, any imperfections in the magnetic field strength or slight misalignments of components can introduce chaotic and unstable particle motion. All accelerators are prone to these non-linearities but the effects are amplified when studying high intensity particle beams with the presence of space charge effects. This work aims to explore the non-linearities which arise in high intensity particle beams using a scaled experiment called IBEX. The IBEX experiment is a linear Paul trap which allows the transverse dynamics of a collection of trapped particles to be studied. It does this by mimicking the propagation through multiple quadrupole lattice periods whilst remaining stationary in the laboratory frame. IBEX is currently undergoing a nonlinear upgrade with the goal of investigating Quasi-Integrable Optics (QIO), a form of Nonlinear Integrable Optics (NIO), in order to improve our understanding and utilisation of high intensity particle beams.

INTRODUCTION

When designing new particle accelerators, constructing test accelerators can be costly in terms of both financial expense and energy consumption. Once an accelerator is built, it is often difficult to change the lattice structure and thus it is impractical to investigate beam properties over large parameter spaces. Therefore, research and development of accelerator lattices are often performed using simulations. Simulations are a vital part of the accelerator design process, however they struggle to reproduce the intricate physics of the many-body Couloumb interactions (space charge forces) over long timescales (tens of thousands of turns) and can never be a replacement for experimental verification.

These challenges led to the design and construction of linear Paul traps to investigate transverse beam dynamics more efficiently at Hiroshima University, Japan [1], Princeton University, US [2] and, most recently, the Intense Beams Experiment (IBEX), at the Rutherford Appleton Laboratories (RAL), UK [3]. IBEX is a table-top sized experiment that can replicate the transverse betatron motion in alternating gradient accelerators in a dispersion- and chromaticity-free environment.

This paper aims to test a lattice proposed by the theory of Nonlinear Integrable Optics (NIO) by exciting a 4th order resonance and measuring particle loss and phase space evolution. Two different octupole potentials are compared; a normal square wave octupole and an octupole with a strength that scales with the beta function to make the Hamiltonian time-independent. The time-independent Hamiltonian becomes an invariant of motion and therefore creates a quasi-integrable lattice that should be robust to small perturbations.

THE IBEX PAUL TRAP

The current IBEX trap consists of four stainless steel cylindrical rods and two sets of end caps, each made from four shorter cylinders as seen in the ionisation region of Fig. 1. Argon gas is introduced into the trap via a VAT Series 59 variable leak valve and is ionised with an electron gun. Typically, a sinusoidal RF voltage is applied to the central rods with a maximum peak-to-peak of 300 V and frequency of 1 MHz. Voltages of the same form but opposite polarity are applied to the blue and red outlined rods in Fig. 1 to provide transverse confinement of the ions. A sinusoidal voltage replicates a simple FODO lattice, although it is possible to create more complex lattices in IBEX. Longitudinal confinement of the ions is achieved by applying a DC offset to the end caps.

Adjusting the peak voltage applied to the rods is analogous to changing the quadrupole strength in an accelerator, which in turn changes the betatron tune in both the horizontal and vertical planes. Thus, scanning the rod voltage allows for a wide range of tunes to be scanned over in a short period of time. This is impractical in conventional accelerators and when studying space charge dominated beams over long timescales, it is also impractical to use simulations. Ions can be stored for around 1 s in IBEX, corresponding to 10^6 RF periods. The DC voltage on the end cap is then dropped and the ions are directed onto a Micro-Channel Plate (MCP) or Faraday Cup (FC) detector. The number of ions stored in the trap is controlled by adjusting the length of time that the electron gun is on. This allows for a wide range of intensities to be studied within the trap. Due to the low energy of the ions (< 1 eV), high intensity beam loss studies can be carried out in the trap without damaging or activating components. IBEX has already been used to study coherent and incoherent resonances at high intensities [4].

NONLINEAR UPGRADE TO IBEX

In IBEX's current quadrupolar rod configuration it is only possible to study linear lattice designs. Thus, in order to investigate QIO and NIO, a nonlinear upgrade to IBEX is required. The upgrade will duplicate the linear trap but will also include the addition of four plate electrodes positioned between the rods (shown in Fig. 1 side view). Grounding the four cylindrical rods whilst applying an equal voltage to all four plate electrodes will allow for octupole fields to be realised in the trap. Grounding the four plates while applying voltages of opposite polarity to the red and blue outlined rods will continue to create a quadrupole field. The additional

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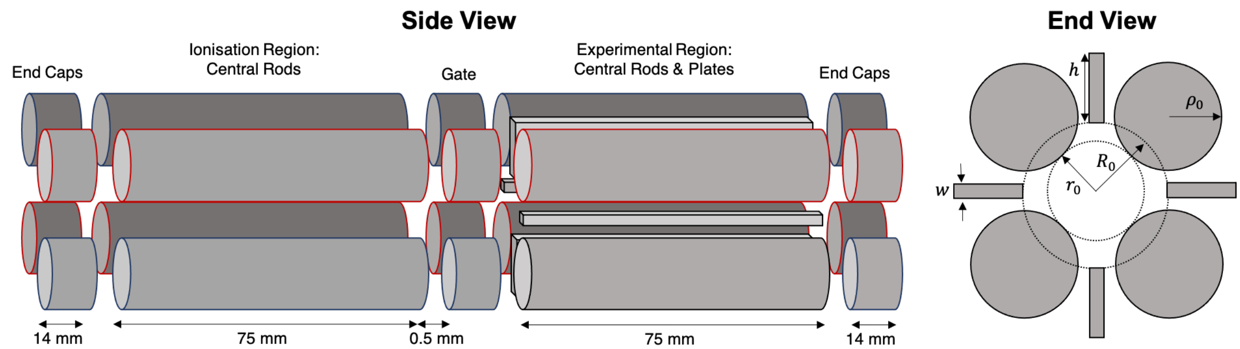

Figure 1: Schematic of the IBEX trap (ionisation region) and nonlinear upgrade (addition of experimental region). Opposing RF voltages are applied to the red and blue outlined rods for transverse confinement of ions. A DC voltage is applied to the end caps and gate electrodes to provide longitudinal trapping. In IBEX $r_0 = 5$ mm and $\rho_0 = 5.75$ mm. Four additional plates between the rods are present in the nonlinear trap at an inscribed radius of R_0 to enable the creation of octupole fields.

electrodes will affect the quadrupole field quality within the trap, thus it is important to minimise these effects to ensure the upgrade has minimal impact on ion confinement.

The scalar potential ϕ of the trapping field can be expressed as a sum of multipoles

$$\phi(r, \theta) = \sum_{n=1}^{\infty} c_n \left(\frac{r}{r_0}\right)^n \cos(n\theta). \tag{1}$$

The multipole coefficients c_n are calculated by evaluating ϕ along the circumference of a circle within the trap aperture and taking the Fast Fourier Transform (FFT) of the solutions. The quadrupolar focusing potential in Eq. (1) corresponds to the c_2 coefficient which will be the dominant term in the multipole expansion of the quadrupole field. Due to the symmetry of the trap, in the ideal case, only every fourth harmonic will be excited; the next two terms will thus be c_6 (12-pole) and c_{10} (20-pole).

The width w and inscribed radius of the plates, R_0 were optimised to reduce the c_6 (12-pole) component of the quadrupole field in the experimental region. The results agree well with a previous study at Hiroshima University which found the optimum values to be $R_0 = 8.5$ mm and $w = 1$ mm [5]. With these values and perfect electrode alignment, the c_6 component was found to be completely eliminated. The alignment tolerances of the new trap will be within 10 µm to further reduce unwanted nonlinearities.

Argon gas will now be ionised and trapped in the linear ionisation region before being transported to the nonlinear experimental region of the trap (Fig. 1). Ions will then be stored in this region of the trap for the desired length of time before the end cap voltage is removed and the ions are detected on an MCP and phosphor screen. The current incident on the phosphor screen can be read out to get an ion count, or it can also be operated in a mode where the phosphor screen florescence is captured by a CCD camera to create an integrated image of the ion distribution. With this upgrade to the trap we hope to be able to test the proposed nonlinear, integrable lattices [6, 7].

NONLINEAR INTEGRABLE OPTICS

Accelerators are mainly built from linear elements such as dipole and quadrupole magnets. Assuming no coupling between the horizontal and vertical particle motion, an ideal linear lattice is fully integrable. In other words, the Hamiltonian associated with a single particle is time-independent and can be separated into two invariants of motion. However, in reality these linear lattices are susceptible to perturbations, and so components such as sextupoles and octupoles are often used to apply higher order corrections. In general, the addition of these nonlinear components creates a non-integrable system, which limits the available area of phase-space where the particle motion is non-chaotic. Therefore, it is desirable to design a lattice which includes nonlinear components in such a way that it remains integrable. If an integrable, nonlinear lattice exists, then the system will always be close to an integrable solution, even when effects such as magnet misalignment and space charge are included.

The theory of NIO was laid out in 2010 by Danilov and Nagaitsev [6] in which they proposed an integrable, nonlinear lattice consisting of a linear T-insert and a drift region for a nonlinear magnet insert. The three conditions of the T-insert lattice are 1. $n\pi$ (where n is an integer) phase advance over the linear section to provide quasi-periodic motion through the nonlinear region. 2. Equal beta functions in the nonlinear insert drift region ($\beta_x = \beta_y$). 3. For Quasi-Integrable optics, the octupole strength will scale with $1/\beta^3(s)$.

Along with the conditions for integrability, the lattice is also constrained by the experimental set up of IBEX. The maximum beta function needs to be kept below around 1500 m to reduce scraping on the rods. The maximum voltage that can be applied to the rods is 150 V and the maximum voltage that can be applied to the plates is 500 V. The shortest voltage pulse duration is limited by the slew rate of the custom made high voltage amplifiers which is expected to be around 1750 V/µs. The beta functions are plotted for a T-insert lattice designed for IBEX in Fig. 2. The lattice length of $s = 890$ m corresponds to a lattice time period of

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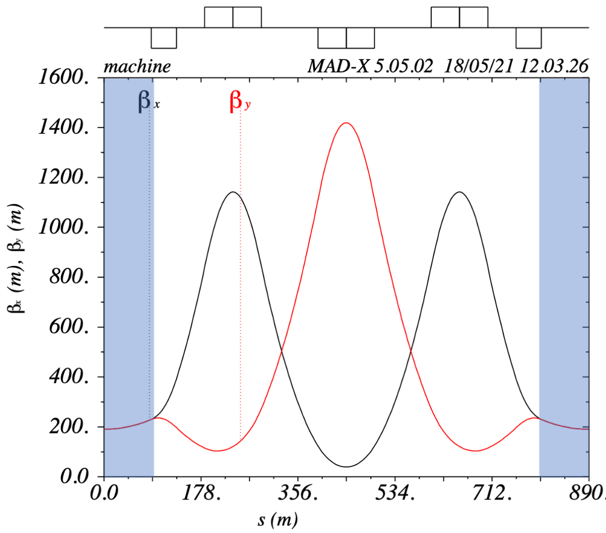

Figure 2: Beta functions of the T-insert lattice designed for the IBEX trap. Blue shaded region is the drift region for the nonlinear insert.

$T = s/c = 2.966\,\mu s$ in IBEX, where we assume a particle bunch traveling at the speed of light, c.

The Integrable Optics Test Accelerator (IOTA), Fermilab [8] will test the fully integrable solution, which requires a complex elliptical potential in the nonlinear insert region. However, the elliptical potential is a challenge to create experimentally. Therefore, IBEX will first test the quasi-integrable case, which uses an octupole potential of the form

$$V(x, y, s) = \frac{\kappa}{\beta^3(s)}\left(\frac{x^4}{4} + \frac{y^4}{4} - \frac{3x^2y^2}{2}\right), \qquad (2)$$

where κ is a constant and $\kappa/\beta^3(s)$ defines the strength of the octupole field. Assuming equal beta functions in the horizontal and vertical direction ($\beta_x(s) = \beta_y(s) = \beta(s)$), the octupole potential creates the time-independent Hamiltonian

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N) \qquad (3)$$

where $U(x_N, y_N) = \beta(s)V(x, y, s)$ and the coordinate transform, $z_N = z/\sqrt{\beta(s)}$ is used. The time-independent Hamiltonian becomes an integrable of motion, creating a quasi-integrable lattice which should be robust to small perturbations. In this work, the T-insert lattice was tested by applying an octupole perturbation, with and without applying an octupole field in the nonlinear insert, and measuring the particle loss and transverse phase space distribution.

SIMULATION RESULTS

Previous work has shown that resonances driven by a small quadrupole perturbation are damped in a T-insert lattice with octupole insert [9]. In this paper, VSim 11 was used to simulate an octupole perturbation applied to the T-insert lattice in order to excite the 4th order resonance. Figure 3 plots the voltage waveform applied to the rods in IBEX to create the beta functions of the T-insert shown in Fig. 2. The blue waveform corresponds to the voltage applied to one pair of opposing rods and the red waveform corresponds to a voltage, which is equal and opposite in magnitude, applied to the other pair of rods. The octupole perturbation is plotted as the black dashed line and corresponds to a sinusoidal voltage of $V = A\sin(4 \cdot 2\pi Q/T)$ applied to the plates where $A = 5.88\,V$ is the amplitude of the perturbation, $Q = 0.6356$ is the horizontal and vertical tune of the lattice and $T = 2.966\,\mu s$ is the lattice time period. In the left plot of Fig. 3, no octupole is applied in the nonlinear insert region, only the octupole perturbation is present (black dashed). In the center plot of Fig. 3, a constant octupole field is applied in the nonlinear insert (green) in the presence of the octupole perturbation. As the octupole strength does not vary with the beta function, this lattice is no longer time independent and is non-integrable. The right plot of Fig. 3 shows the quasi-integrable lattice, where an octupole scaling as $1/\beta^3(s)$ is applied in the nonlinear region of the perturbed T-insert lattice (black, solid).

A 2D Gaussian distribution of 25,865 particles was tracked in a VSim 11 model of IBEX for 200 T-insert lattice periods. A low number of particles was chosen to avoid space charge effects in these simulations. Figure 4 shows the initial phase space (x, x') of the particle distribution as well as the phase space after 50, 100 and 200 turns for the T-insert lattice with octupole perturbation but no octupoles turned on in the nonlinear insert (Fig. 3, left). The four tails forming in the phase space distribution show that the 4th order resonance is being excited by the octupole perturbation. Over the 200 lattice periods, 25.6 % of particles were lost in the simulation.

Figure 5 shows the evolution of the phase space distribution for the perturbed, quasi-integrable lattice. An octupole with a strength that scales with the beta function is applied in the nonlinear insert (Fig. 3, right). The phase space distribution has been restored to one that resembles the initial distribution. The particle loss over the 200 lattice periods has been reduced from 25.6 % to 5.1 %. These results show that the T-insert lattice can successfully dampen the instability excited by the octupole perturbation with voltages that can be realistically achieved in the nonlinear IBEX trap.

In order to prove the true benefits of quasi-integrable optics, we must compare the quasi-integrable lattice to a lattice that creates an equal tune spread but is non-integrable. The simplest way to do this is with a lattice where a constant octupole is applied in the nonlinear insert, with an integrated octupole strength equal to the quasi-integrable lattice (Fig. 3, center). Tracking the particle distribution over 200 turns through this non-integrable lattice also gives a particle loss of 5.1 % and has a similar phase space evolution to Fig. 5.

These results do not show a difference between the non-integrable and quasi-integrable lattices over the limited number of periods they were studied. The simulations will need to be run for many more lattice periods (>1000 lattice periods) to potentially show a significant difference. This will take days of computing time as it currently takes around 20

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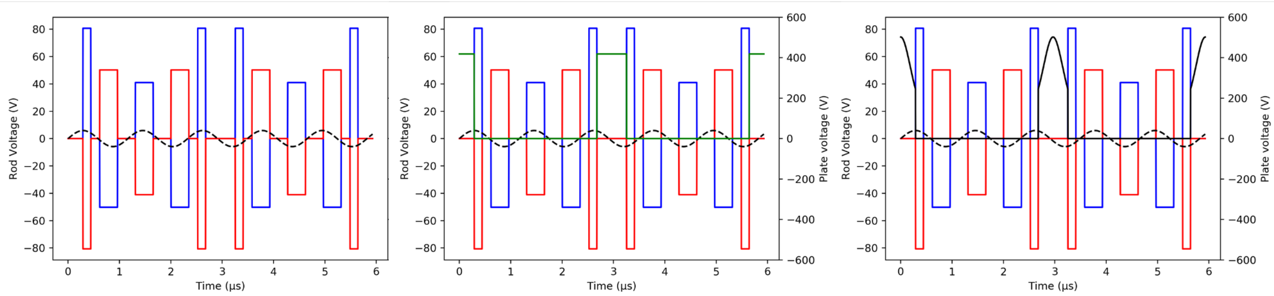

Figure 3: Two periods of the voltage waveform, applied to the rods in IBEX (red and blue) to create the T-insert lattice. A 5.88 V octupole perturbation is applied to the plates (black, dashed). **Left:** No octupoles applied in drift region. **Center:** Constant square wave octupole pulse applied in drift region (green). **Right:** Octupole pulse with $1/\beta^3$ strength scaling (black, solid).

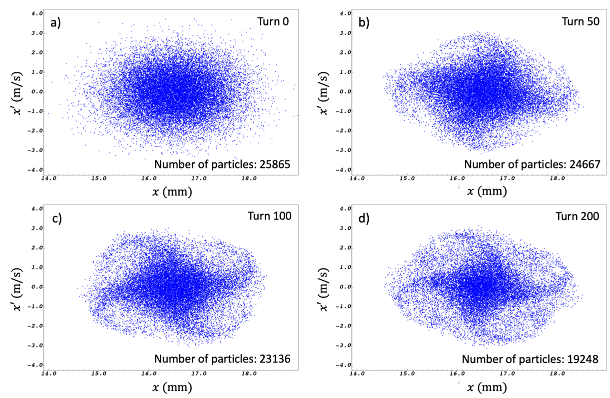

Figure 4: Phase space evolution when an octupole perturbation is applied to the T-insert lattice, at a frequency proportional to $4Q$, with the octupole potential switched off.

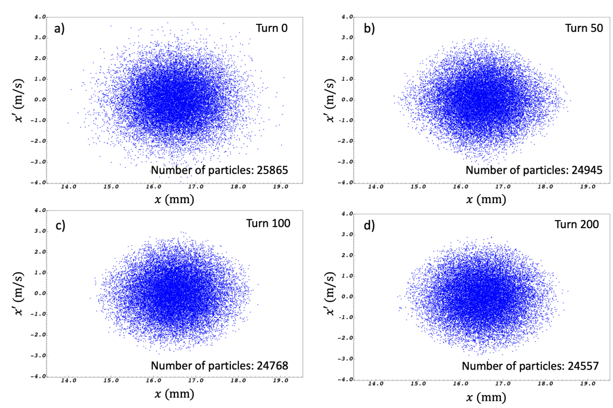

Figure 5: Phase space evolution when an octupole perturbation is applied to the T-insert lattice, at a frequency proportional to $4Q$, with the octupole potential switched on.

hours running on 16 cores to simulate the lattices for 200 periods. Extending these simulations to include space charge will make them even more computationally intensive.

Fortunately, with the nonlinear upgrade to IBEX, particles can be stored for tens of thousands of lattice periods in less than a second to help probe the differences between the quasi-integrable and non-integrable lattices.

CONCLUSION

A nonlinear upgrade to the IBEX Paul trap is currently underway which will allow for the simultaneous excitation of quadrupole and octupole fields in the experimental region of the trap. A T-insert lattice has been designed to create the conditions needed for QIO, while being constrained by realistic voltages applied to the rods (< 150 V) and plates (< 500 V) and the maximum beta functions to avoid scraping on the rods. Simulation results show that when trapping particles in the linear T-insert for 200 periods, whilst applying a small octupole perturbation to the plates, the octupole resonance is excited and 25.6 % particle loss is seen. When the octupoles are applied in the nonlinear region with correct scaling with beta function, the resonance is damped and only 5.1 % particle loss is seen. When compared to a non-integrable lattice with the same integrated octupole strength but constant in the nonlinear insert, the same particle loss of 5.1 % over 200 turns was observed. Although a significant difference between the quasi-integrable and non-integrable lattice was not observed, the simulations showed that the T-insert lattice with an octupole present in the nonlinear region was able to successfully dampen an octupole resonance. We hope that the nonlinear upgrade to IBEX will allow us to examine the differences between the quasi-integrable and non-integrable lattices over longer time scales.

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