IBS near transition crossing in NICA collider

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IBS - Coulomb scattering of charged particles in a beam results in an exchange of energy between different degrees of freedom

➔ Causes the beam size to grow up ➔ limits luminosity lifetime

$$\begin{cases} \dfrac{1}{\tau_x} = \dfrac{1}{\varepsilon_x} \dfrac{d\varepsilon_x}{dt} \\[2ex] \dfrac{1}{\tau_y} = \dfrac{1}{\varepsilon_y} \dfrac{d\varepsilon_y}{dt} \\[2ex] \dfrac{1}{\tau_p} = \dfrac{1}{\sigma_p^2} \dfrac{d(\sigma_p^2)}{dt} \end{cases}$$

IBS is important constraint for circular machines

1974 --- A. Piwinski derived original theory of IBS applicable for weak focusing only [1]

1983--- J. Bjorken & S. Mtingwa [2] derived formalism for X-Y uncoupled case in absence of vertical dispersion (applicable to majority of accelerators)

1. A. Piwinski, Proc. 9th Int. Conf. on High Energy Accelerators, Stanford (1974) p.405
2. J. D. Bjorken, S. K. Mtingwa, Part. Accel. **13**, p. 115 (1983)

Well known

$\gamma < \gamma_{tr}$ - quasi-equilibrium between three “temperatures” (of each degree of freedom) may exists

IBS leads to

- relaxation (equation) between 3 “temperatures” in the beam faster than 6D-emmitance growth rate

NICA

$\gamma = 5.8$

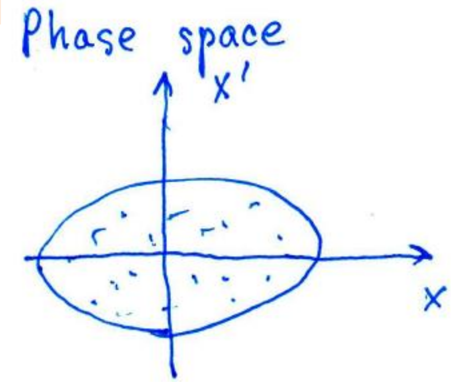
$\gamma_{tr} = 7.1$

$\gamma \approx \gamma_{tr}$?

$\gamma > \gamma_{tr}$ - quasi-equilibrium between local temperatures in the beam **does not** exists

IBS leads to

- infinite beam 6D-phase space volume growth in circular machines



distr. function.

$$f(x,x') = \frac{1}{2\pi\sigma_x\sigma_{x'}} \exp\left\{-\frac{x^2}{2\sigma_x^2} - \frac{x'^2}{2\sigma_{x'}^2}\right\}$$

Beam temp.

$$T \stackrel{def}{=} \frac{m\sigma_{x'}^2}{2k_B} = \frac{m\sigma_{v_x}^2}{2k_B}$$

$$\sigma_{v_x} \equiv \sqrt{\overline{v_x^2}}$$

Evolution of the velocity distribution function $f(v_x, v_y, v_z)$ is described by Landau collision integral

$$\frac{df}{dt} = -2\pi n r_0^2 c^4 L_c \frac{\partial}{\partial v_i} \int \left(f \frac{\partial f'}{\partial v_j'} - f' \frac{\partial f}{\partial v_j} \right) \omega_{ij} d^3 v'$$

$$\omega_{ij} = \frac{\left(\mathbf{v} - \mathbf{v}'\right)^2 \delta_{ij} - \left(v_i - v_i'\right)\left(v_j - v_j'\right)}{|\mathbf{v} - \mathbf{v}'|^3}$$

$$\int f(\mathbf{v}) d^3 v = 1,$$

Plasma perturbation theory

Works only when ***Lc>>1*** **!**

(logarithmic approximation)

When the particles kinetic energy much higher than their interaction potential

Landau collisions integral

General time-dependent solution does not exist

$L_c = \ln(\rho_{max}/\rho_{min})$ is the Coulomb logarithm

$$\rho_{min} = r_0 c^2 / \overline{v^2}, \qquad \rho_{max} = \sqrt{\overline{v^2} / 4\pi n r_0 c^2},$$

$$\overline{v^2} = \sigma_{vx}^2 + \sigma_{vy}^2 + \sigma_{vz}^2$$

$\sigma_{vi} \equiv \sqrt{\overline{v_i^2}}, i = (x, y, z)$ are the rms velocity spreads

If $f(v_x, v_y, v_z)$ - Gaussian function, it can be reduced to 3-temperature distribution function $\Rightarrow$ $$f = \frac{1}{(2\pi)^{3/2}\sigma_{vx}\sigma_{vy}\sigma_{vz}}\exp\left(-\frac{1}{2}\left(\frac{v_x^2}{\sigma_{vx}^2}+\frac{v_y^2}{\sigma_{vy}^2}+\frac{v_z^2}{\sigma_{vz}^2}\right)\right)$$

Growth rate for the distribution function $$\Sigma_{ij} = \int f\, v_i v_j d^3 v.$$ - second order moments (only diagonal elements non zero)

Rate of change of these second order moments due to Coulomb scattering in plasma $$\frac{d}{dt}\Sigma_{ij} = \int \frac{\partial f}{\partial t} v_i v_j d^3 v.$$ $\Rightarrow$ put in the Landau collisions integral

Result - rate of energy exchange between degrees of freedom in plasma:

$$\frac{d\mathbf{\Sigma}}{dt} = \frac{d}{dt}\begin{pmatrix}\sigma_{vx}^{\ 2}\\ \sigma_{vy}^{\ 2}\\ \sigma_{vz}^{\ 2}\end{pmatrix} = \frac{(2\pi)^{3/2} n r_0^{\ 2} c^4 L_c}{\sqrt{\sigma_{vx}^2+\sigma_{vy}^2+\sigma_{vz}^2}}\begin{pmatrix}\Psi\left(\sigma_{vx},\sigma_{vy},\sigma_{vz}\right)\\ \Psi\left(\sigma_{vy},\sigma_{vz},\sigma_{vx}\right)\\ \Psi\left(\sigma_{vz},\sigma_{vx},\sigma_{vy}\right)\end{pmatrix}$$

Assumptions:

- ❑ Initial particles' distribution – Gaussian ➔ does not stay Gaussian-like in evolution process (but stay similar)
- ❑ Integral does not take into account single collisions (responsible for non-Gaussian tails)

$$\frac{d}{dt}\begin{pmatrix}\sigma_{vx}{}^2\\ \sigma_{vy}{}^2\\ \sigma_{vz}{}^2\end{pmatrix}=\frac{(2\pi)^{3/2}\,nr_0{}^2c^4L_c}{\sqrt{\sigma_{vx}^2+\sigma_{vy}^2+\sigma_{vz}^2}}\begin{pmatrix}\Psi(\sigma_{vx},\sigma_{vy},\sigma_{vz})\\ \Psi(\sigma_{vy},\sigma_{vz},\sigma_{vx})\\ \Psi(\sigma_{vz},\sigma_{vx},\sigma_{vy})\end{pmatrix}$$

$\Psi(x,y,z)$

expressed through the symmetric elliptic integral of the second kind

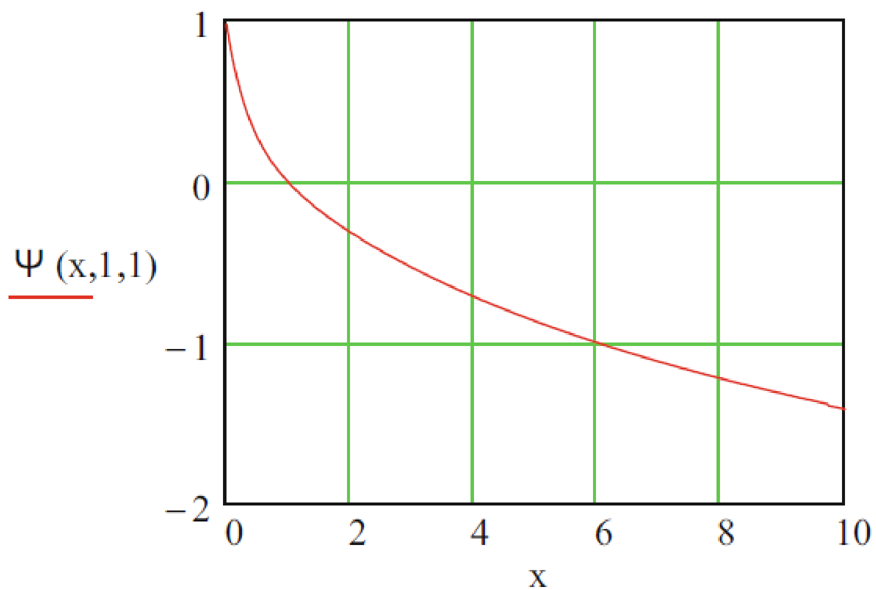
$$\Psi(x,y,z)=\frac{\sqrt{2}r}{3\pi}\left(y^2R_D(z^2,x^2,y^2)+z^2R_D(x^2,y^2,z^2)-2x^2R_D(y^2,z^2,x^2)\right)$$

$$R_D(u,v,w)=\frac{3}{2}\int_0^\infty\frac{dt}{\sqrt{(t+u)(t+v)(t+w)^3}}$$

can be evaluated numerically

$$r=\sqrt{x^2+y^2+z^2}\qquad x,y,z\geq 0$$

- ***Ψ(1,1,1)*** - depends on ratios of its variables (not on r)
- normalized that ***Ψ(0,1,1)=1***
- ***Ψ(1,1,1)=0*** – no energy transfer between degrees of freedom
- ***Ψ(x,y,z) + Ψ(y,z,x) + Ψ(z,x,y) = 0*** energy conservation



Function for two equal temperatures

In the ring accelerator (collider)

In difference to plasma where the energy is conserved, in a storage ring the binary collisions do not conserve energy in the beam frame (BF) .
It results in unlimited 3D-emittance growth supported by energy transfer from the longitudinal beam motion to the internal particle motion in BF.

How to calculate ??

- Be sure that particle collision time ρ_{max}/v in BF is much smaller than period of betatron oscillations
- Assume that in each location of the accelerator the distribution function in the BF is Gaussian in 6D phase space
- Use results for plasma in each location of the ring => calculate the growth rate in BF
- Convert these rates into the Laboratory frame (LF) emittance growth rates
- Average the this results over whole accelerator length to obtain overall IBS rates:

$$\mathrm{Rate} := \sum_i \frac{\mathrm{rate}_i\, ds}{C_{ring}}$$

rate_i -local rate of the emittance growth at the lattice element of small length ds with fixed Twiss parameters

Smooth focusing, unbunched beam (variation of beta- and dispersion- functions ~0)

$$\Sigma_v = \gamma\cdot\beta\cdot c\cdot\begin{pmatrix}\theta_x^2 & 0 & 0\\ 0 & \theta_y^2 & 0\\ 0 & 0 & \theta_p^2\end{pmatrix}$$

- matrix of second moments of local velocity distribution in BF

$$\theta_x^2 = \frac{\varepsilon_x}{\beta_x} \qquad \theta_y^2 = \frac{\varepsilon_y}{\beta_y} \qquad \theta_p^2 = \sigma_p^2\cdot\frac{\beta_x\cdot\varepsilon_x}{\gamma^2\cdot\sigma_x^2}$$

$$\sigma_x = \sqrt{\varepsilon_x\cdot\beta_x + \sigma_p^2\cdot D^2} \qquad \sigma_y = \sqrt{\varepsilon_y\cdot\beta_y}$$

$$\frac{d}{dt}\varepsilon_k = \frac{\sqrt{\pi}}{2\cdot\sqrt{2}}\cdot\frac{e^4\cdot N\cdot L_c}{M^2\cdot c^3\cdot\sigma_x\cdot\sigma_y\cdot L\cdot\beta^3\cdot\gamma^5\cdot\sqrt{\theta_x^2+\theta_y^2+\theta_p^2}}\cdot E_k\cdot\begin{pmatrix}\Psi_{IBS}(\theta_x,\theta_y,\theta_p) & 0 & 0\\ 0 & \Psi_{IBS}(\theta_y,\theta_p,\theta_x) & 0\\ 0 & 0 & \gamma^2\cdot\Psi_{IBS}(\theta_p,\theta_x,\theta_y)\end{pmatrix}$$

where

$$E_x = \begin{pmatrix}\beta_x & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \frac{D^2}{\beta_x}\end{pmatrix} \qquad E_y = \begin{pmatrix}0 & 0 & 0\\ 0 & \beta_y & 0\\ 0 & 0 & 0\end{pmatrix} \qquad E_s = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 2\end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p^2\end{pmatrix} = \frac{\sqrt{\pi}}{2\cdot\sqrt{2}}\cdot\frac{e^4\cdot N\cdot L_c}{M^2\cdot c^3\cdot\sigma_x\cdot\sigma_y\cdot L\cdot\beta^3\cdot\gamma^5\cdot\sqrt{\theta_x^2+\theta_y^2+\theta_p^2}}\cdot\begin{pmatrix}\beta_x\cdot\Psi_{IBS}(\theta_x,\theta_y,\theta_p)+\gamma^2\cdot\frac{D^2}{\beta_x}\cdot\Psi_{IBS}(\theta_p,\theta_x,\theta_y)\\ \beta_y\cdot\Psi_{IBS}(\theta_y,\theta_p,\theta_x)\\ 2\cdot\gamma^2\cdot\Psi_{IBS}(\theta_p,\theta_x,\theta_y)\end{pmatrix}$$

R. Carrigan, V. Lebedev, N. Mokhov, S. Nagaitsev, V. Shiltsev, G. Stancari, D. Still, and A. Valishev, chapt. 6. Accelerator Physics at the Tevatron Collider

Smooth focusing, unbunched beam

at quasi-equilibrium state of the coasting beam:

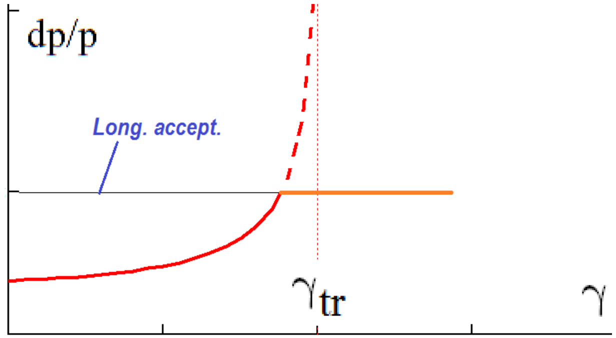
$$\theta_x = \theta_y = \theta_p \qquad \Psi_{IBS}(\theta_x, \theta_y, \theta_p) = \Psi_{IBS}(\theta_y, \theta_p, \theta_x) = \Psi_{IBS}(\theta_p, \theta_x, \theta_y) = 0$$

This equivalent to:

$$\frac{\varepsilon_x}{\beta_x} = \frac{\varepsilon_y}{\beta_y} = \sigma_p^{\,2} \cdot \frac{\beta_x \cdot \varepsilon_x}{\gamma^2 \cdot \left(\varepsilon_x \cdot \beta_x + \sigma_p^{\,2} \cdot D^2 \right)} \Rightarrow \frac{\sigma_p^{\,2}}{\gamma^2} = \frac{\varepsilon_x}{\beta_x} \cdot \frac{1}{1 - \dfrac{\gamma^2}{\gamma_{tr}^{\,2}}}$$

can be fulfilled only below critical energy

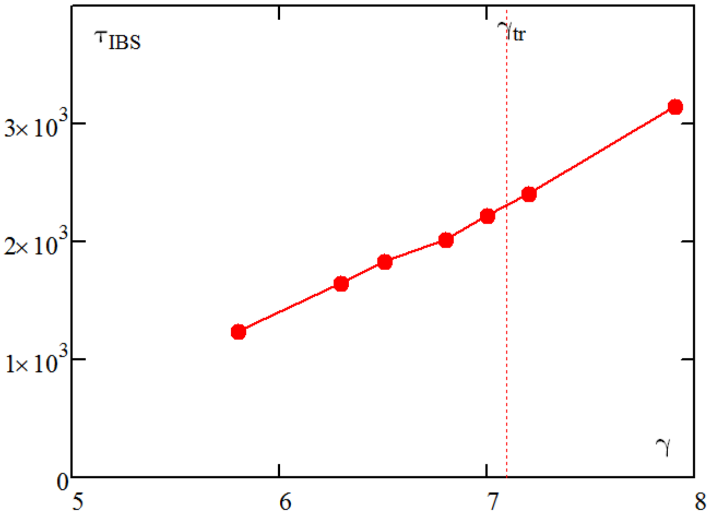
$$1 - \frac{\gamma^2}{\gamma_{tr}^{\,2}} > 0$$



For fixed transverse emittance the equilibrium momentum spread grows to infinity when the beam energy approaches transition

Equilibrium does not exist above transition in smooth approximation

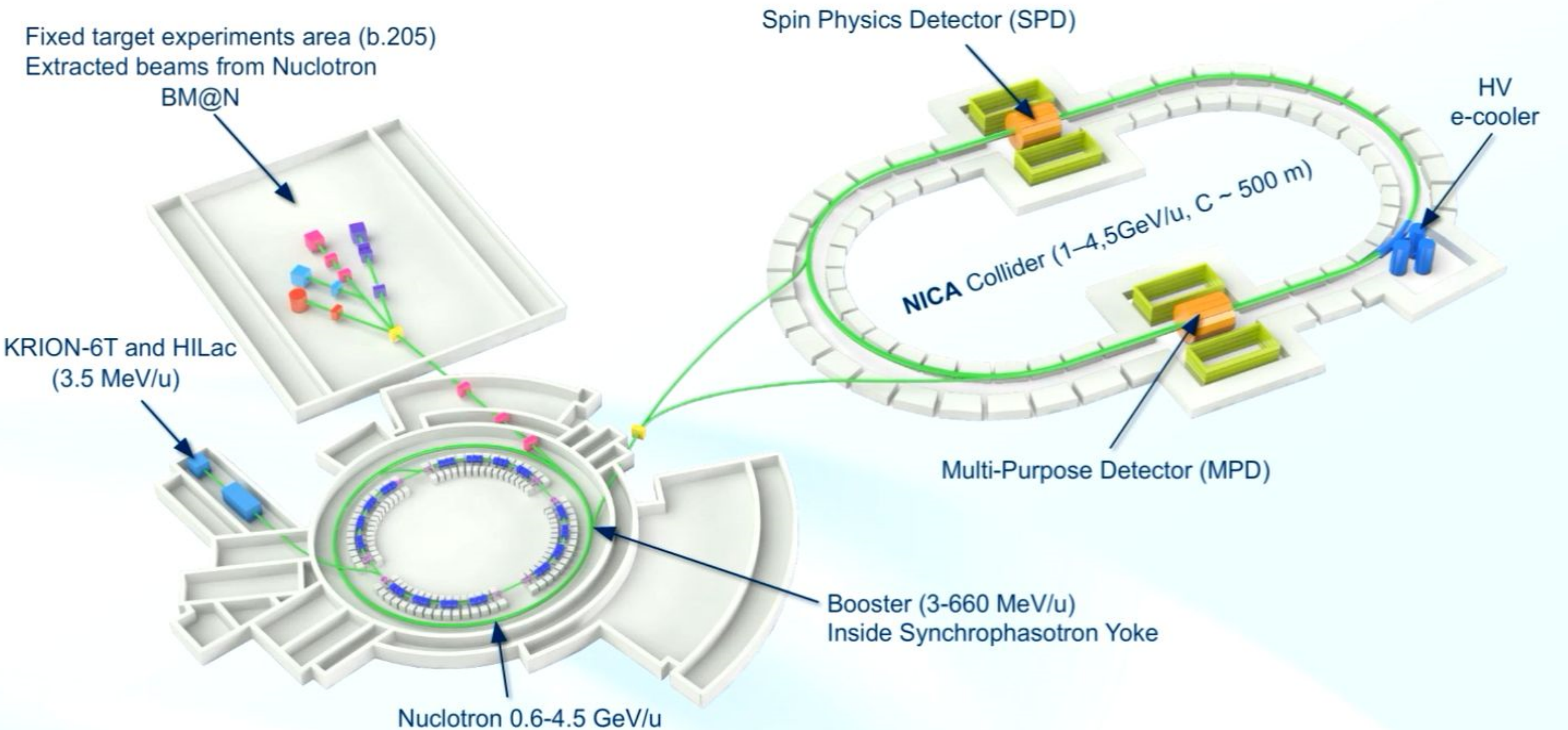
For FODO equilibrium does not exist. 6D emittance grows before and after,
and there is no jump for emittance growth at transition



Collider basic parameters:
$\sqrt{s_{NN}}$ = **4 - 11** GeV; ***beams***: *from* **p** to **Au**; **L** ~ **10^{27}** $cm^{-2}\,c^{-1}$ (Au),

The NICA accelerator facility will consist of:

- cryogenic heavy ion source KRION of ESIS type,
- heavy ion linear accelerator (HILac)
- a superconducting Booster synchrotron
- the superconducting heavy ion synchrotron Nuclotron
- collider: two new superconducting storage rings with two interaction points



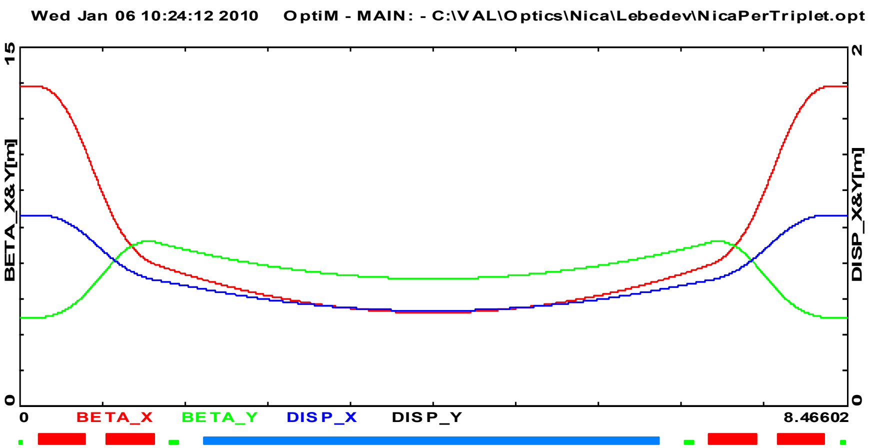
IBS beam emittance growth rate minimization → Smooth focusing → Collider concept

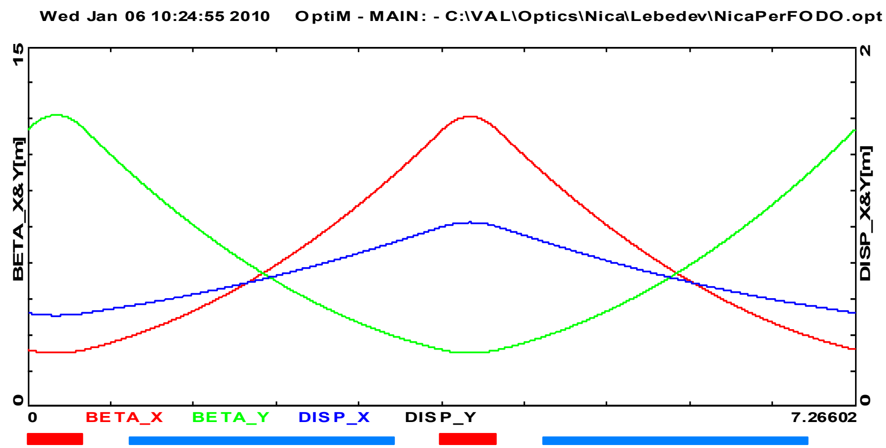
IBS rates calculation → Lattice optimization

Collider concept ↕ Lattice optimization

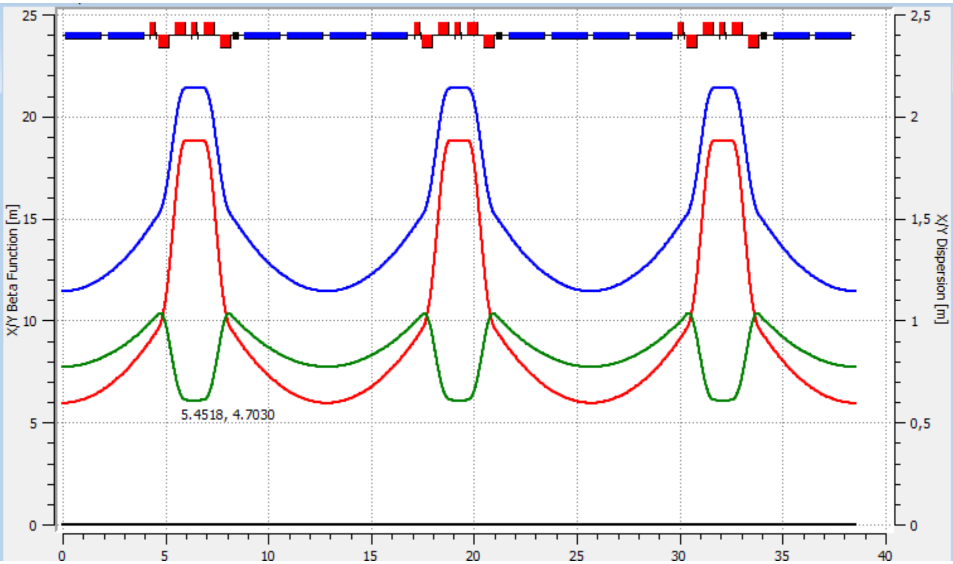
$$
\begin{cases}
\dfrac{1}{\tau_x} = \dfrac{1}{\varepsilon_x}\dfrac{d\varepsilon_x}{dt} \\[2ex]
\dfrac{1}{\tau_y} = \dfrac{1}{\varepsilon_y}\dfrac{d\varepsilon_y}{dt} \\[2ex]
\dfrac{1}{\tau_p} = \dfrac{1}{\sigma_p^2}\dfrac{d(\sigma_p^2)}{dt}
\end{cases}
$$

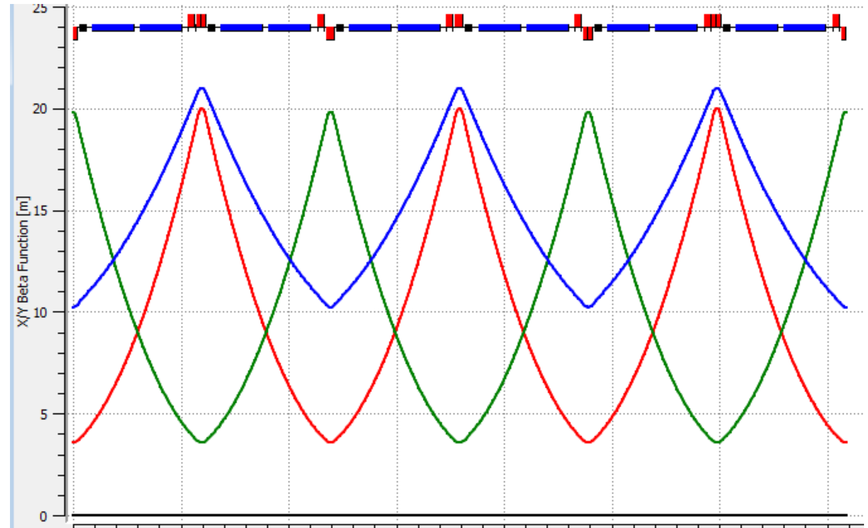
Lattices with FODO- and triplet- focusing were tested





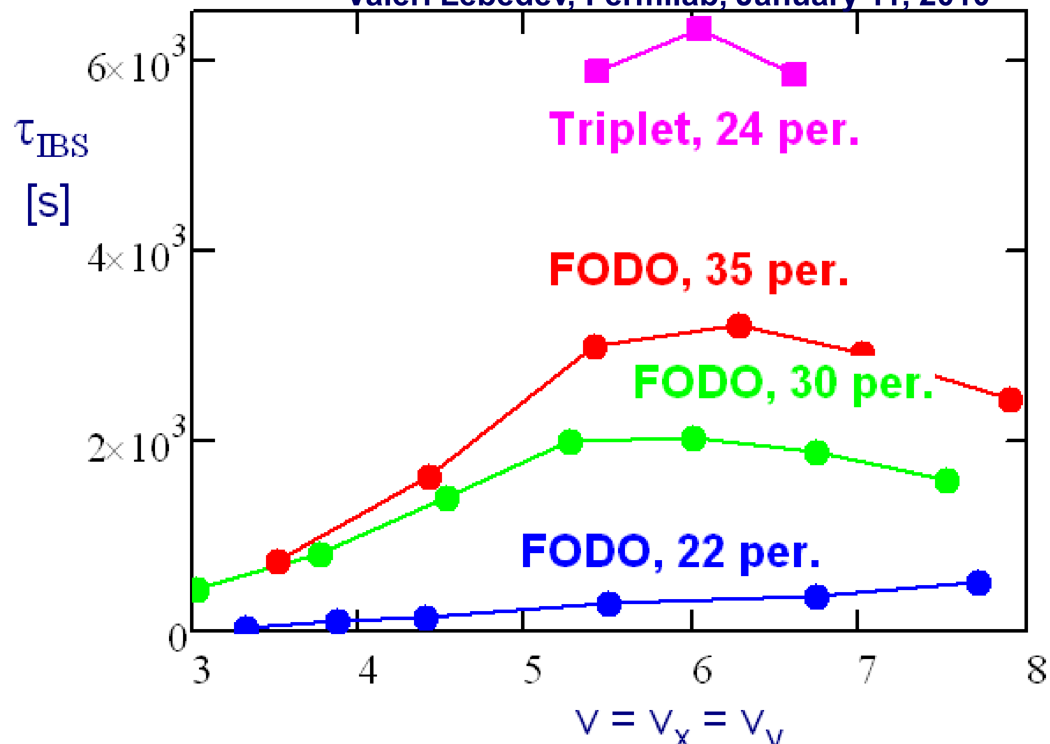
Ideal storage ring - no IPs





NICA: Conceptual Proposal for Collider,
Valeri Lebedev, Fermilab, January 11, 2010

- perimeter of the ring ➔ keep constant
- change focusing strength ➔ adjust total ring tune (number periods per ring is varied)
- fix $\delta\nu_{SC}$ = 0.05 - limited ➔ number of ions in the beam
- adjust ε_x, ε_y and σ_p to keep equal all 3 growth times



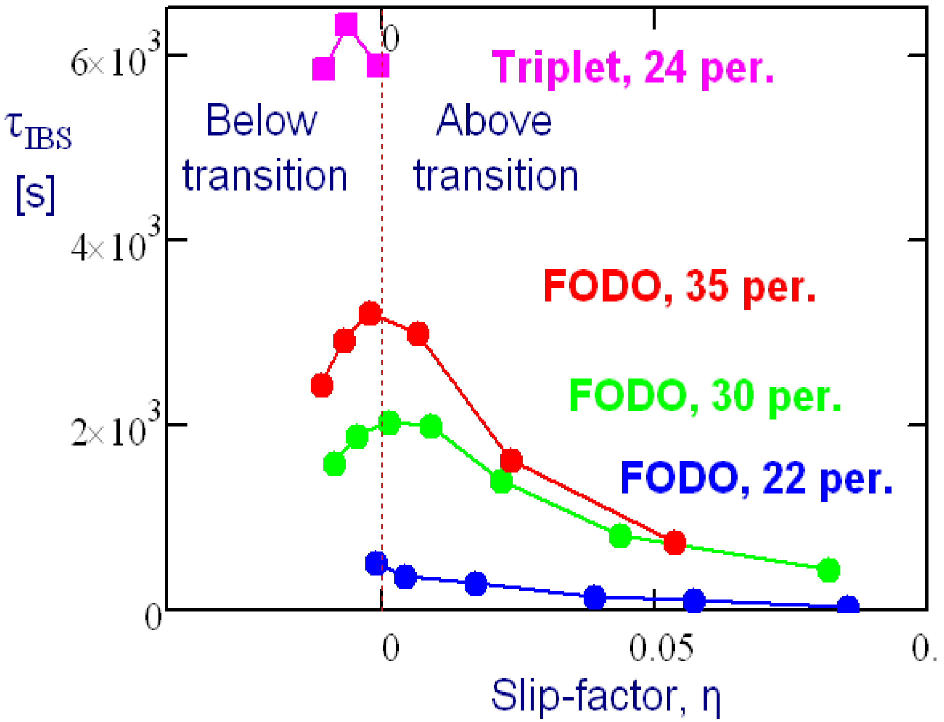
Triplet focusing preferable. It results in doubling IBS growth time

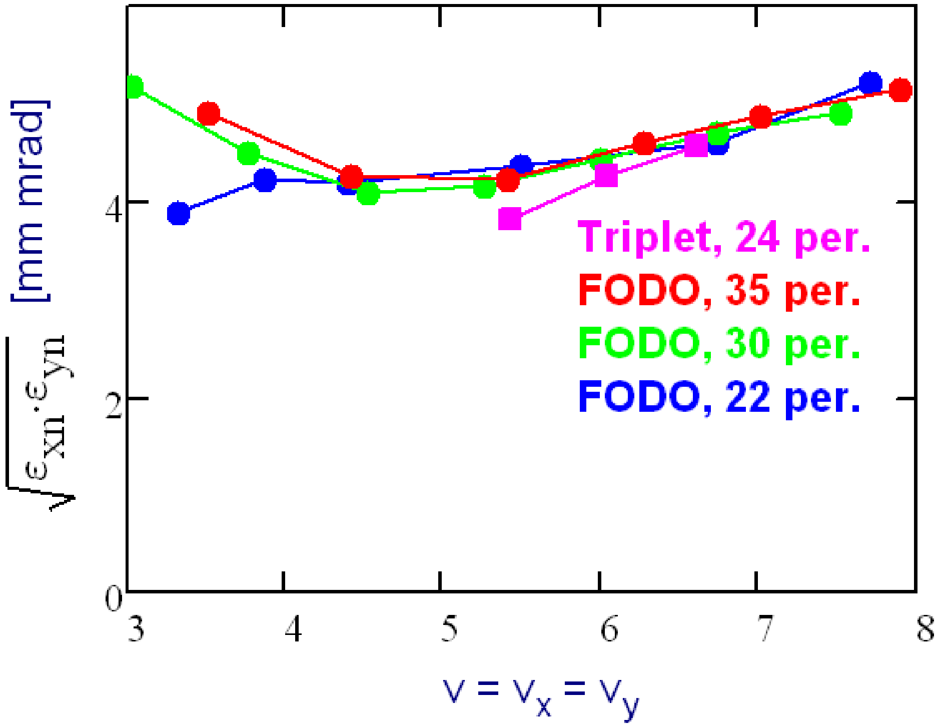
Ideal storage ring - no IPs

- Minimum heating at $\gamma \approx \gamma_{tr}$

❑ below transition (large γ_{tr})
- large $\Delta\nu$ per cell ➔ strong heating

❑ above transition (small γ_{tr})
- heating due to ΔT between H & L planes





Resulting $\varepsilon_x\varepsilon_y$ has weak dependence on other parameters

$\varepsilon_x = \varepsilon_y$ +/– 30% at thermal equilibrium point

NICA: Conceptual Proposal for Collider,
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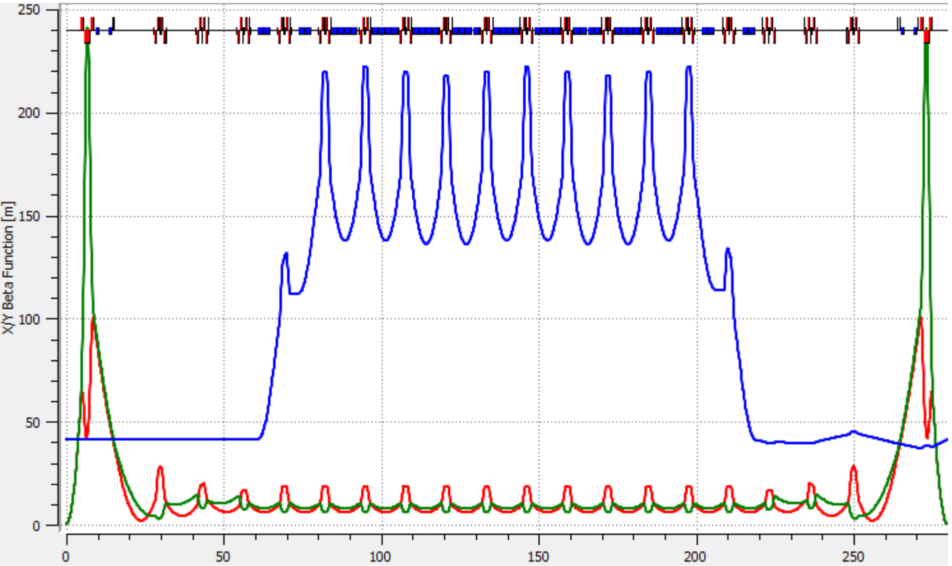
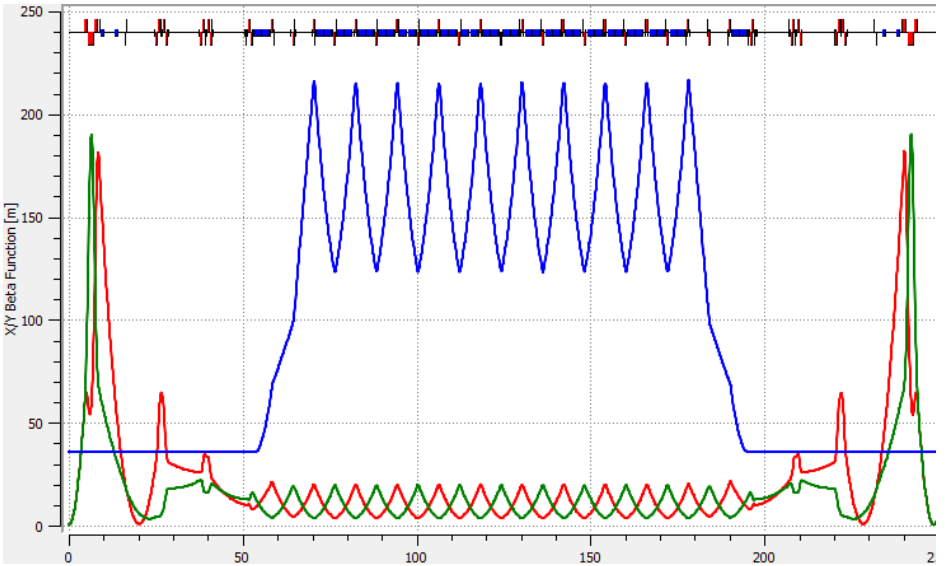
Results of IBS Tests

Add IPs → Ideal storage ring

NICA collider





- Straight lines and IPs increase IBS heating by about 4.5 times
- operation in vicinity of thermal equilibrium still significantly reduces IBS heating

	ε_x [mm mrad]	ε_y [mm mrad]	σ_p %	$\tau_x=\tau_y=\tau_s$ [s]
3.5 GeV/n	1.117	0.692	0.156	1025
4.5 GeV/n	1.291	0.684	0.192	1350

ODFDO- and FODO- give not more than 30% difference in the IBS growth times in "real" rings

- **FODO- was chosen for NICA**

Circumference, m	503.04		
Number of bunches	22		
Rms bunch length, m	0.6		
Beta-function at IP, m	0.35		
Betatron tunes, Qx / Qy	9.44 / 9.44		
Chromaticity, Q'x / Q'y	-33 / -28		
Ring acceptance, $\pi\cdot$mm$\cdot$mrad	40		
Long. acceptance, $\Delta p/p$	± 0.010		
Gamma-transition, $\square_{tr}$	7.088		
Ion energy, GeV/u	1.0	3.0	4.5
Ion number per bunch	$2.0\cdot10^{8}$	$2.4\cdot10^{9}$	$2.3\cdot10^{9}$
Rms $\Delta p/p$, 10^{-3}	0.55	1.15	1.5
Rms beam emittance, hor/vert, (unnormalized), $\pi\cdot$mm$\cdot$mrad	1.1/ 0.95	1.1/ 0.85	1.1/ 0.75
Luminosity, $cm^{-2}s^{-1}$	$0.6\cdot10^{25}$	$1.0\cdot10^{27}$	$1.0\cdot10^{27}$
IBS growth time, sec	160	460	1800