Sum resonances with space charge

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Outline

1. Incoherent vs Coherent
2. Coupled sum resonances without SC
3. Coupled sum resonances with SC
4. Application example
5. Summary/Outlook

Incoherent vs. Coherent

Debye length

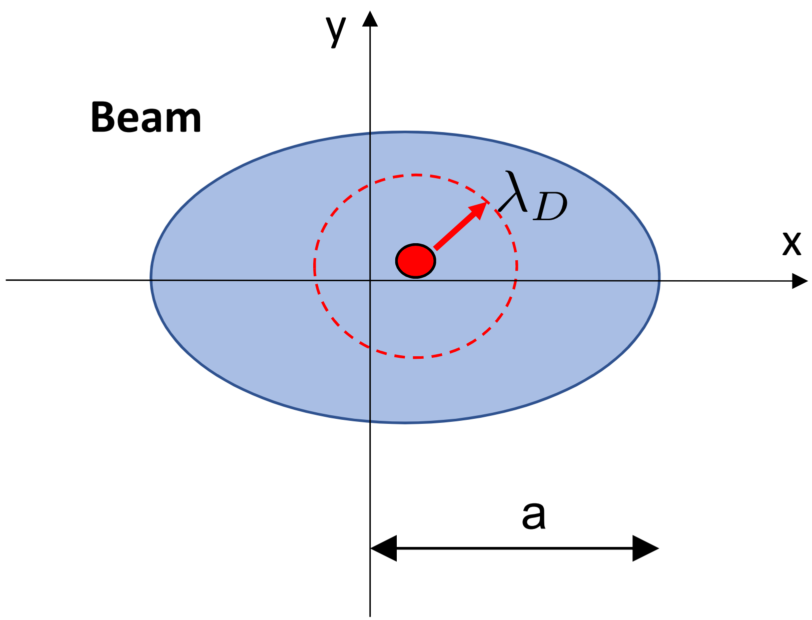
If a test charge is placed into a neutral plasma having a temperature T and equal positive ion and electron densities ***n,*** the excess electric potential set up by this charge is effectively screened off in a distance $\boldsymbol{\lambda_D}$ by charge redistribution in the plasma. This effect is known as ***Debye shielding.***

$$\lambda_D = \frac{\tilde{v}_x}{\omega_p} \qquad \omega_p = \sqrt{\frac{q^2 n}{\epsilon_0 \gamma^3 m}}$$

M. Reiser book

(Wikipedia)

Debye length in a beam



M. Reiser book

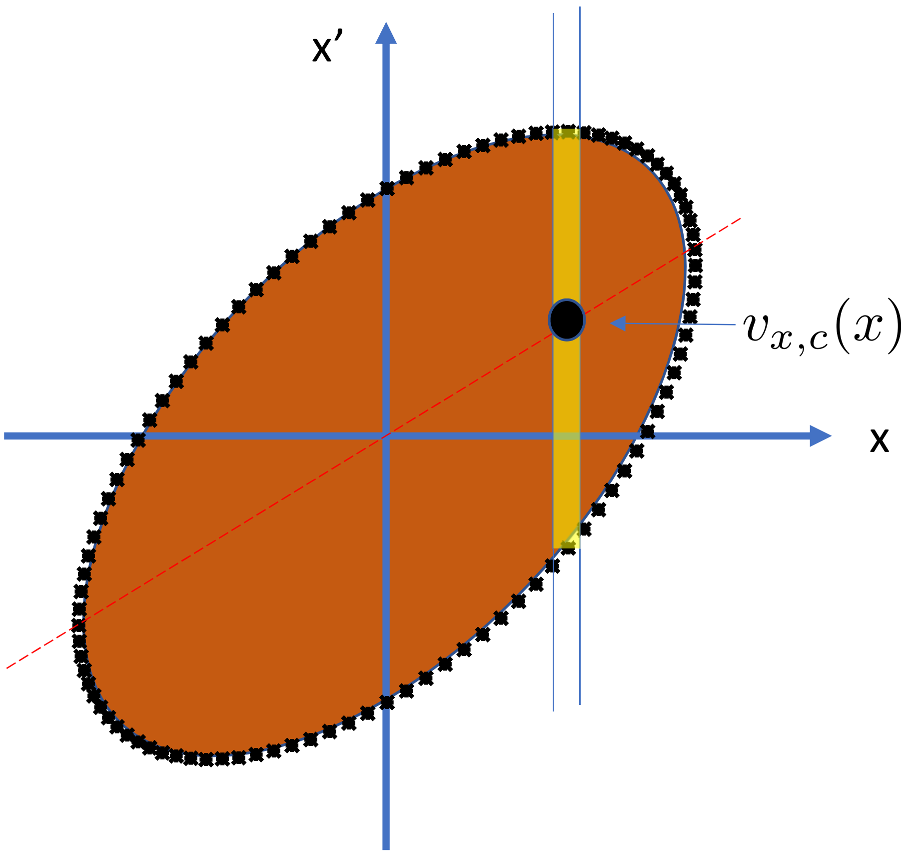
If λ_D >> a the screening will be ineffective and single-particle behavior will dominate.

If λ_D << a collective effects due to the self fields of the beam will play an important role.

1) the Debye length increases with energy

2) at sufficiently high energy the space-charge forces become insignificant in comparison to the external forces acting on a beam.

Thermal velocity of a matched beam



$$v_x = v_{x,th} + v_{x,c}(x)$$

For a "linear correlation"

$$\langle v_{x,th}^2 \rangle = \frac{v_0^2}{\langle x^2 \rangle} \tilde{\epsilon}_x^2$$

In terms of high intensities jargon

The beam density can be re-expressed using the perveance

$$n = K\frac{\epsilon_0 m\gamma^3 v_0^2}{q^2\tilde{a}^2}e^{-\frac{1}{2}\frac{x^2+y^2}{\tilde{a}^2}}$$

Thermal velocity for a matched beam

$$\frac{\tilde{v}_{x,th}^2}{v_0^2} = \frac{\tilde{\epsilon}_x}{\beta_x}$$

For an axi-symmetric Gaussian beam

$$\frac{\Delta Q_x}{Q_x} = -\frac{R^2}{Q_x^2}\frac{K}{4\tilde{a}^2}$$

(incoherent tune-shift)

Debye length

$$\lambda_D^2(r) = \frac{Q_{x0}}{4\Delta Q_x}\tilde{a}^2 e^{\frac{1}{2}\frac{r^2}{\tilde{a}^2}}$$

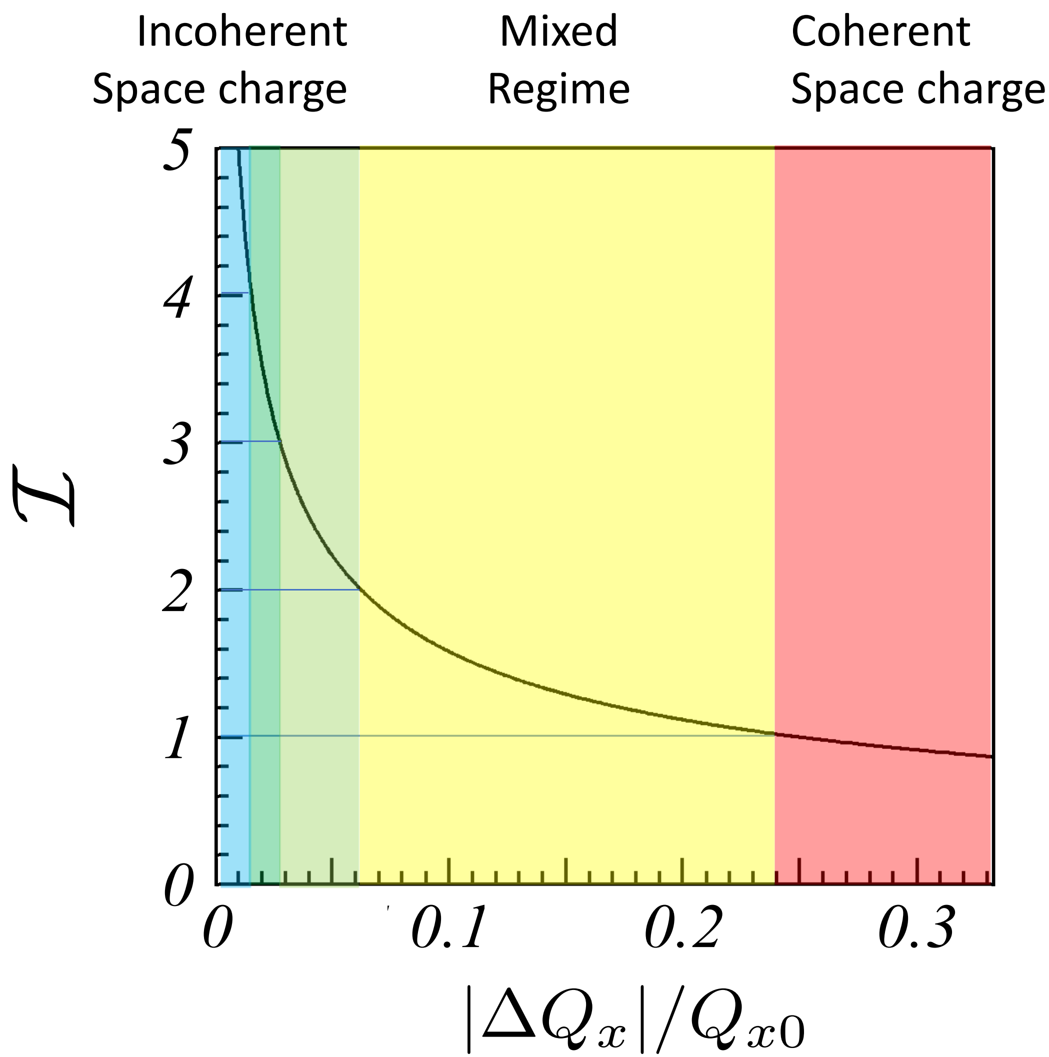
Incoherent vs Coherent

Lets define a "parameter of incoherence" $\mathcal{I}$ as the minimum $\dfrac{\lambda_D}{\tilde{a}_0}$

$$\mathcal{I} = \sqrt{\frac{1}{4}\frac{Q_{x0}}{\Delta Q_x}}$$

The bigger $\mathcal{I}$ the more space charge is incoherent

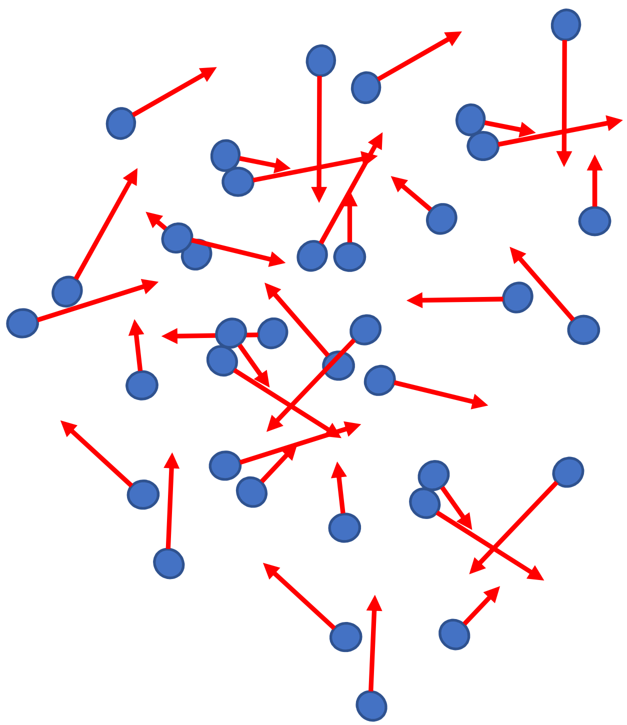
Incoherent vs Coherent



	Q_{x0}	ΔQ_{x0}	$\mathcal{I}$
PS-Exp.	6.1	0.05	5.5
GSI-Exp	4.3	0.05	4.6
SIS100	18.7	0.5	3.05
SIS18	4.3	0.5	1.46

Average interparticle distance

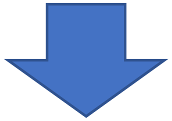
$$l_p = n^{-1/3}$$



M. Reiser Book

When $\boldsymbol{\lambda_D} \gg \mathbf{l_p}$, *smooth functions for the charge and field distributions can be used*

When $\boldsymbol{\lambda_D} \sim \mathbf{l_p}$, a particle will be affected by its nearest neighbors more than by the collective field of the beam distribution as a whole.



1) Intrabeam scattering in high-energy storage rings;
→ play a fundamental role in driving a beam toward a Maxwell-Boltzmann distribution;

2) At extremely **low temperature or very large density**, the mutual interaction of single particles leads to crystal-like configurations in the particle distribution

Brightness, IBS ?

Coulomb Coupling Constant

$$\Gamma = \frac{q^2}{4\pi\epsilon_0 l_p}\frac{1}{k_B T}$$

$\Gamma \sim 1$ Liquid

$\Gamma < 170$ Crystalline

Beam temperature

$$\gamma m \tilde{v}_x^2 = k_B T$$

$$\frac{\lambda_D}{l_p} = \Gamma^{-1}\frac{\gamma^2}{4\pi}$$

$$\frac{\lambda_D}{l_p} = 1 \quad \Rightarrow \quad \Gamma = \frac{\gamma^2}{4\pi}$$

Can we use $\mathbf{\Gamma}$ to define a brightness regime?

Coupled sum resonances without SC

The framework: perturbative

$$H = H_0 + H_1$$ ← Nonlinear errors

↑ Quadratic

Solution of system with H_0 → $x = \sqrt{\beta_x \epsilon_x} \cos(\psi_x(s) + \phi_0)$

Solution of the perturbed system → $x = \sqrt{\beta_x a_x} \cos(\psi_x(s) + \varphi_x)$

New dynamical variables

Canonical equation of the slow variables

$$\frac{da_x}{ds} = -2\frac{\partial H_1}{\partial \varphi_x}, \quad \frac{d\varphi_x}{ds} = 2\frac{\partial H_1}{\partial a_x}$$

$$\frac{da_y}{ds} = -2\frac{\partial H_1}{\partial \varphi_y}, \quad \frac{d\varphi_y}{ds} = 2\frac{\partial H_1}{\partial a_y}$$

Instead of discussing x,x',y,y' we discuss $a_x, \varphi_x, a_y, \varphi_y$

Example: third order

$$-\tilde{a}_x' = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{\varphi}_x} \quad = -2\sqrt{\tilde{a}_x}\tilde{a}_y \Lambda \sin(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha)$$

$$\tilde{\varphi}_x' = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{a}_x} \quad = \quad 2\frac{1}{2\sqrt{\tilde{a}_x}}\tilde{a}_y \Lambda \cos(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha) + t_x \frac{2\pi\Delta_r}{L}$$

$$-\tilde{a}_y' = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{\varphi}_y} \quad = -4\sqrt{\tilde{a}_x}\tilde{a}_y \Lambda \sin(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha)$$

$$\tilde{\varphi}_y' = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{a}_y} \quad = \quad 2\sqrt{\tilde{a}_x}\Lambda \cos(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha) + t_y \frac{2\pi\Delta_r}{L}$$

$$\Lambda_c = -\sum_j \frac{1}{8L} K_{2j}\sqrt{\beta_{xj}}\beta_{yj} \times \\ \times \cos\left[2\pi\frac{s_j}{L}N + \mathscr{D}_x(s_j) + 2\mathscr{D}_y(s_j)\right],$$

$$\Lambda_s = -\sum_j \frac{1}{8L} K_{2j}\sqrt{\beta_{xj}}\beta_{yj} \times \\ \times \sin\left[2\pi\frac{s_j}{L}N + \mathscr{D}_x(s_j) + 2\mathscr{D}_y(s_j)\right].$$

Λ Is the amplitude driving term

α Is the phase of the driving term

→ $t_x + 2t_y = 1$

Quantitative prediction

Resonance or fixed line → $\tilde{a}'_x = 0, \tilde{\varphi}'_x = 0, \tilde{a}'_y = 0, \tilde{\varphi}'_x = 0$

$$0 = \Delta_r \Lambda (-1)^M \left[\frac{1}{2\sqrt{\tilde{a}_x}} \tilde{a}_y + 2\sqrt{\tilde{a}_x} \right] + \frac{\Delta_r^2}{2}$$

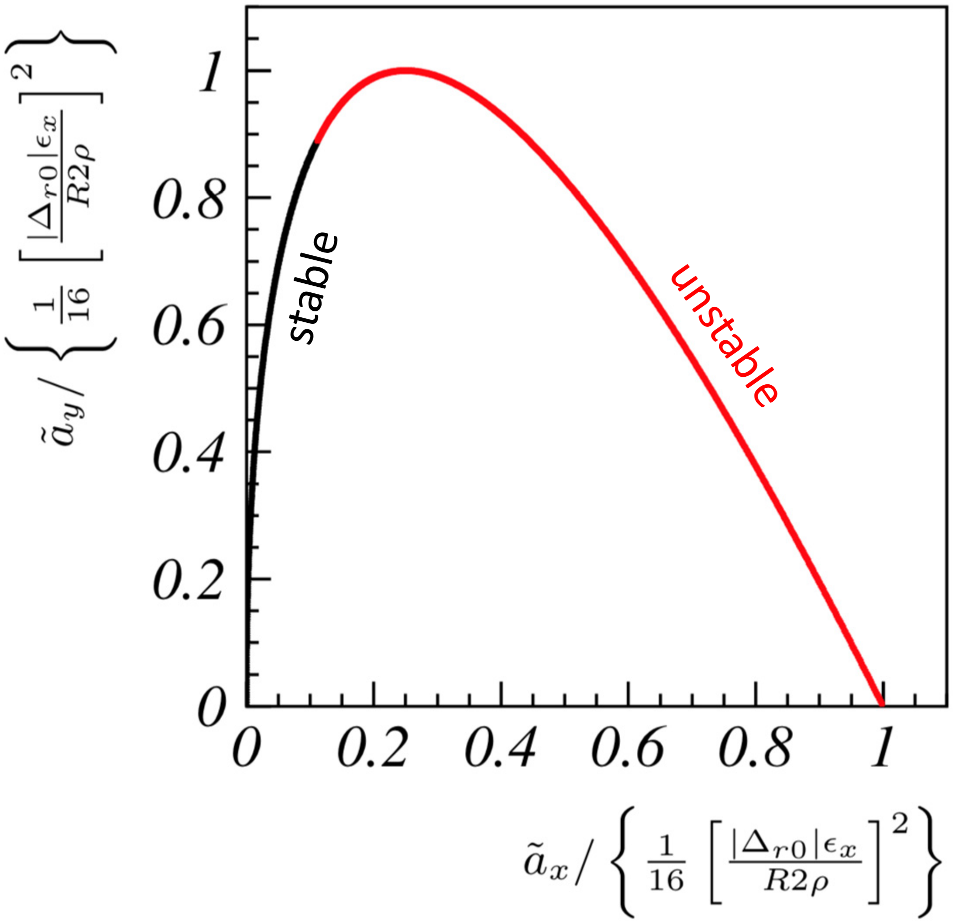
This are all the fixed lines!

$$\Delta_r = Q_x + 2Q_y - m$$

Distance from the resonance defined from the dynamics

Infinite fixed lines

$Q_x + 2Q_y = m$



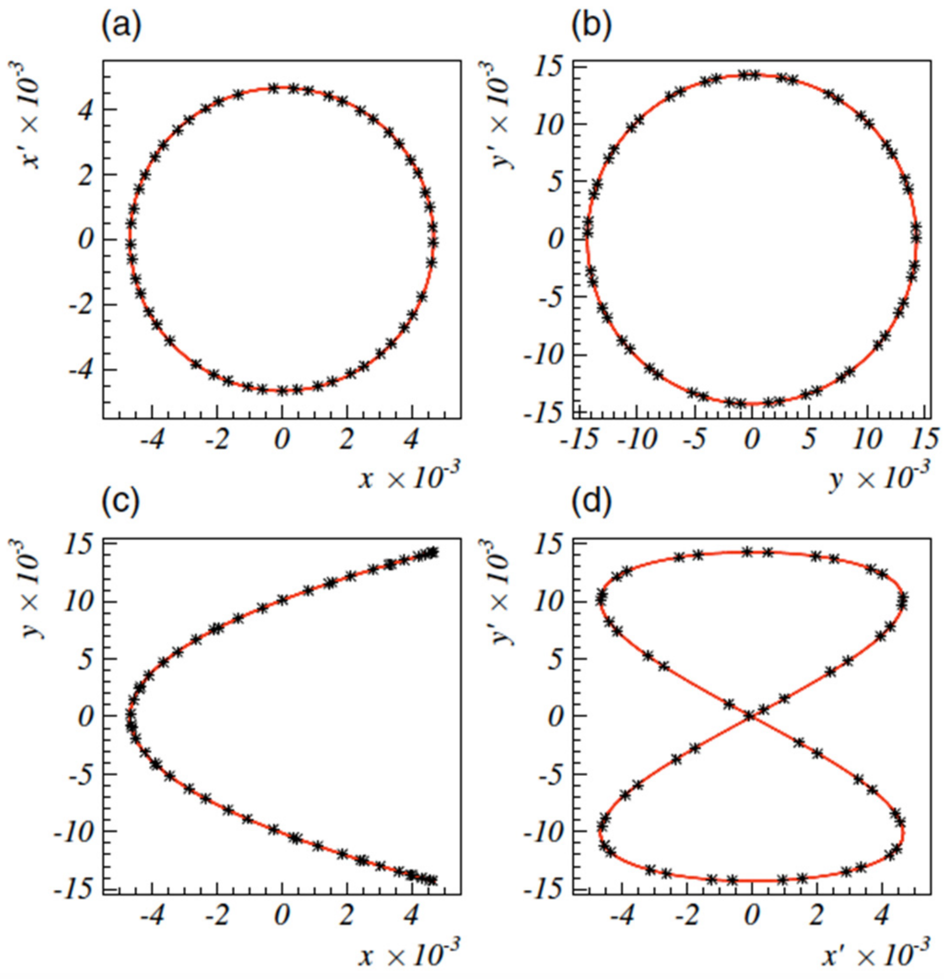
$$\tilde{a}_y = \frac{(2\pi\Delta_r)^2}{4\Lambda^2 L^2}\tau(1-\tau)$$

$$\tilde{a}_x = \frac{(2\pi\Delta_r)^2}{16\Lambda^2 L^2}(1-\tau)^2$$

$$0 \le \tau \le 1$$

On the physical space

$$x(t) = \sqrt{\beta_x a_x}\cos(-2t + \pi M),$$
$$y(t) = \sqrt{\beta_y a_y}\cos(t),$$



G.Franchetti & F. Schmidt
Phys. Rev. Lett. **114**, 234801 (2015)

Coupled sum resonances with SC

Direct Space Charge

	Coherent Effects $\mathcal{I} \lesssim 1$	Incoherent Effects $\mathcal{I} \gtrsim 3$
Feature	Space charge forces create a collective beam response:	Space charge forces acts only on particles like "external forces"
Example	• Envelope oscillations • Envelope instabilities • Coherent tunes • ..	• Amplitude dependent detuning • Tune-spread • Modification of optics • Structure resonances
Time scale	Fast	Fast

Resonance theory with space charge

To discuss convergence properties one has to consider scaled quantities

$$\rho = \frac{\lambda}{\pi a_0 b_0} F'\left(\frac{x^2}{a_0^2} + \frac{y^2}{b_0^2}\right) \qquad a_0 = \sqrt{\beta_x \epsilon_x}, \quad b_0 = \sqrt{\beta_y \epsilon_y} \qquad \hat{a}_x = a_x/\epsilon_x$$

Beam sizes

$$V_{sc} = -\frac{K}{2}\int_0^\infty \frac{F(T(t)) - F(0)}{(a_0^2+t)^{1/2}(b_0^2+t)^{1/2}} dt \qquad T(t) = \frac{x^2}{a_0^2+t} + \frac{y^2}{b_0^2+t}$$

$$C = N_y \hat{a}_x - N_x \hat{a}_y$$

This is an invariant of motion
(in the slow harmonics approximation)

Dynamics of slow variables

$$\mathsf{a}' = \frac{4\rho_s}{R} N_x \mathsf{a}^{n_x/2} \mathsf{a}_y^{n_y/2} \sin(\Phi)$$

$$\Phi' = \frac{2\rho_s}{R} \mathsf{a}^{n_x/2} \mathsf{a}_y^{n_y/2} \left(N_x \frac{n_x}{\mathsf{a}} + N_y \frac{n_y}{\mathsf{a}_y} \right) \cos(\Phi) + \frac{\Delta_{r0}}{R} + \frac{\mathscr{D}_{r,sc}}{R} N_x \frac{d\mathcal{V}_{sc}}{d\mathsf{a}} + \sum_n \frac{\mathscr{D}_{r,m}^n}{R} N_x \frac{d\mathcal{V}_m^n}{d\mathsf{a}}$$

$$\mathscr{D}_{r,sc} = N_x \Delta Q_x + N_y \Delta Q_y$$

"Resonance tune-spread" is naturally obtained from the theory.

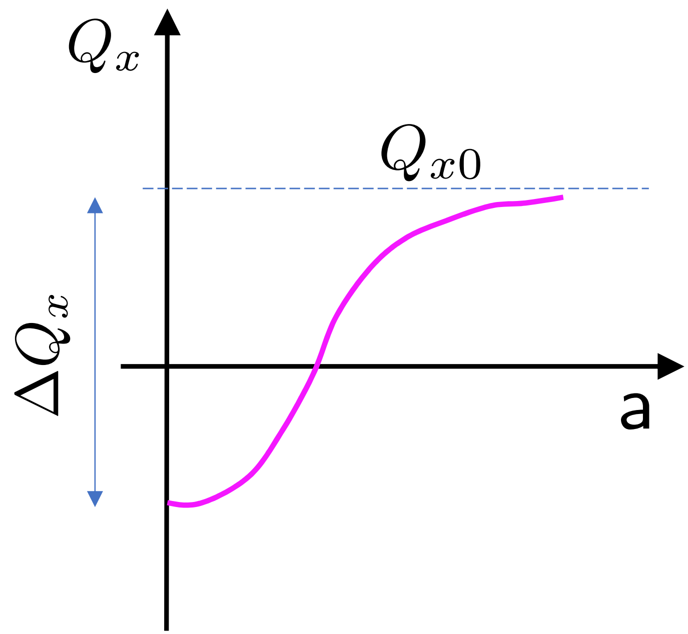
ρ_s is a normalized driving term from **lattice nonlinear error** or **space charge** (no dimension)

Similarity with 1D resonances

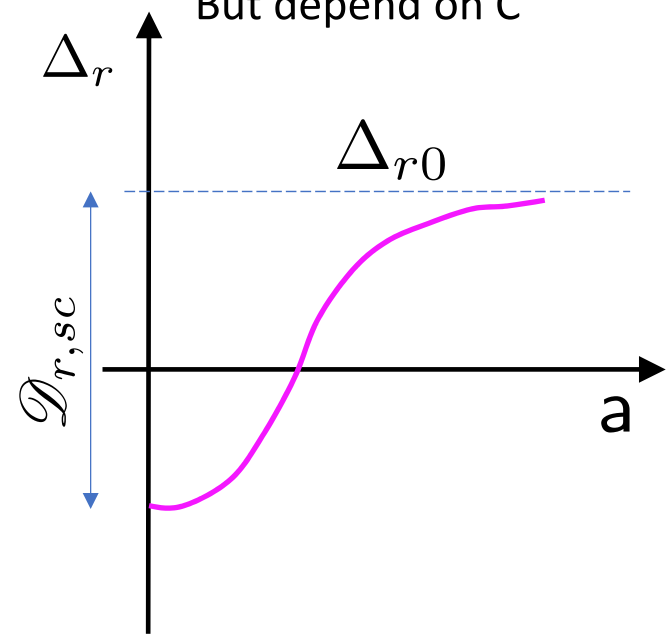
This term is an "amplitude dependent *resonance detuning*"

$$\frac{\Delta_{r_0}}{R} + \frac{\mathscr{D}_{r,sc}}{R} N_x \frac{d\mathcal{V}_{sc}}{da} \quad \Rightarrow \quad \Delta_r = \Delta_{r0} + \mathscr{D}_{r,sc} N_x \frac{d\mathcal{V}_{sc}}{da}$$

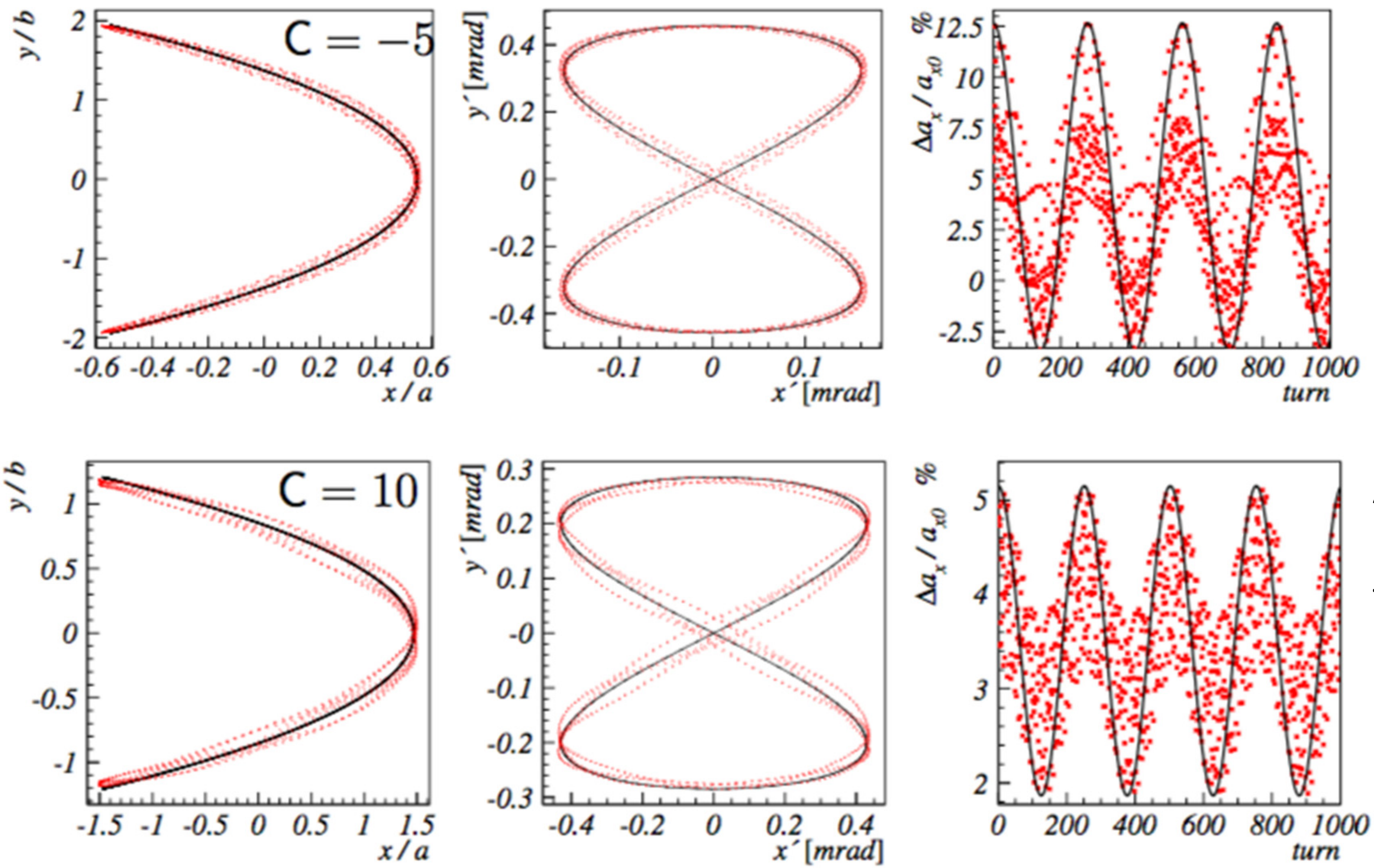
1D resonance



Coupled resonance
But depend on C



Fixed lines with space charge



PS-Exp. parameters

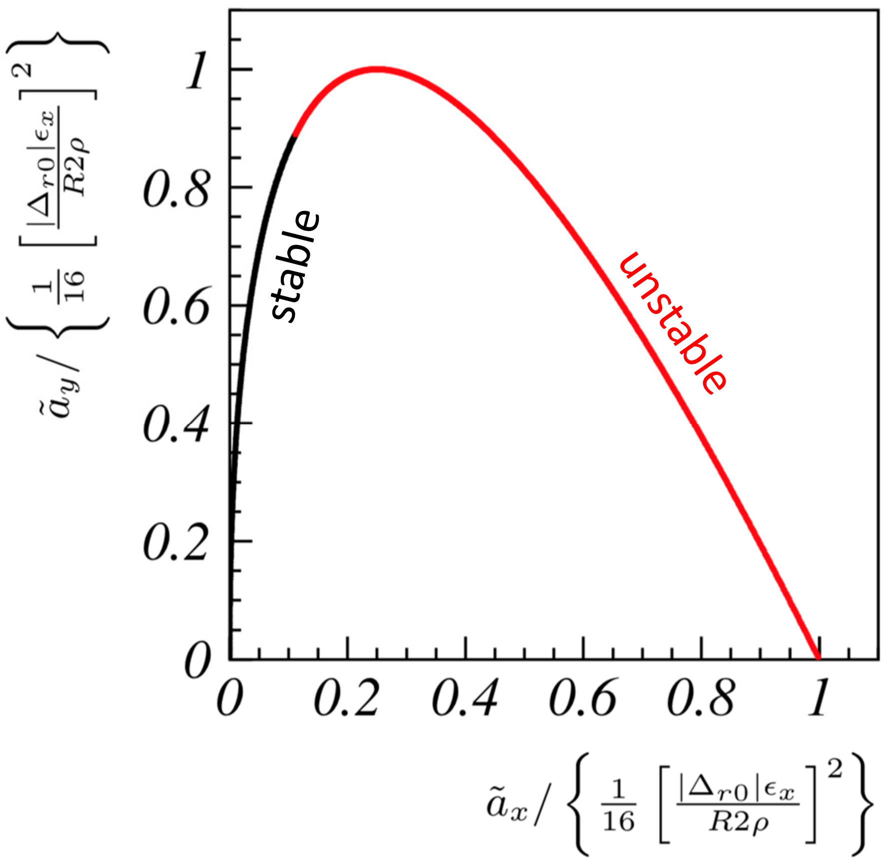
Qx + 2Qy = 19

DQx = -0.05

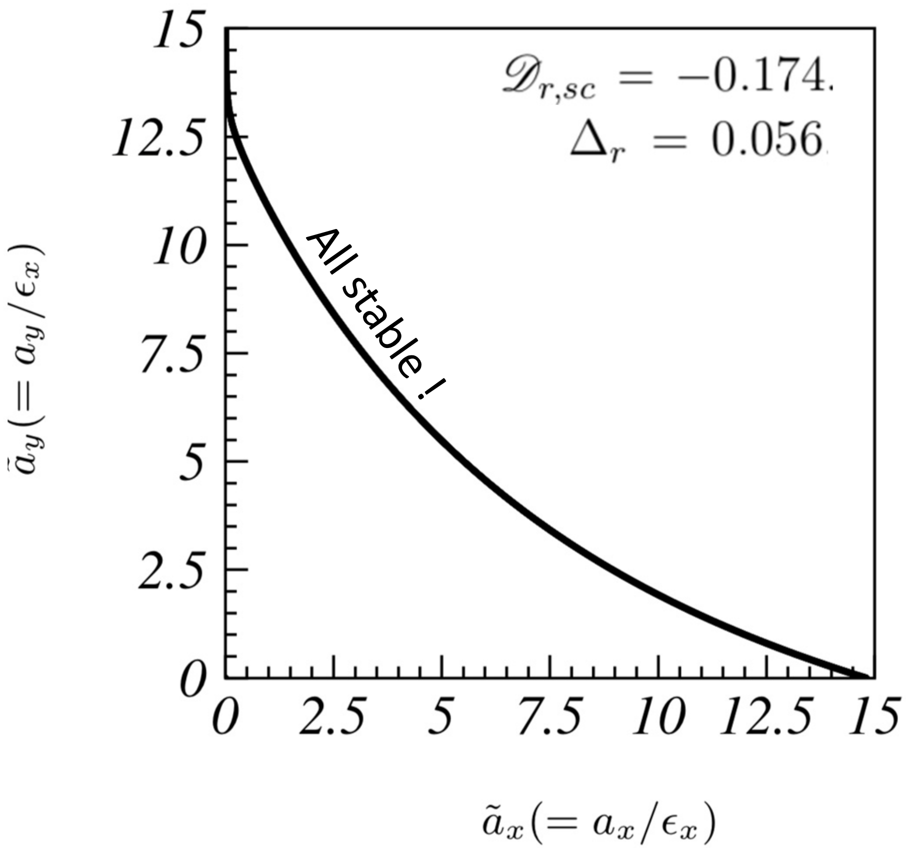
Analytic prediction of the secondary tunes for resonances of any orders … (hence the stability)

Effect of space charge on the "infinite" fixed lines: third order resonance

Pure Q_x+2Q_y=19. No space charge, no additional detuning

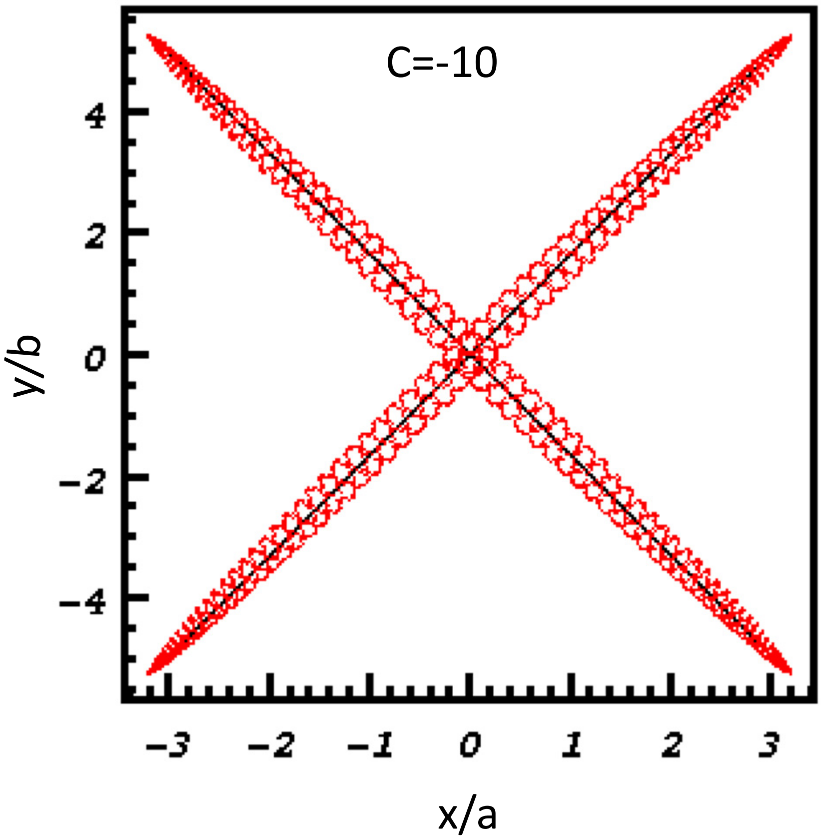


Collection of fixed-lines Q_x+2Q_y=19 in presence of space charge

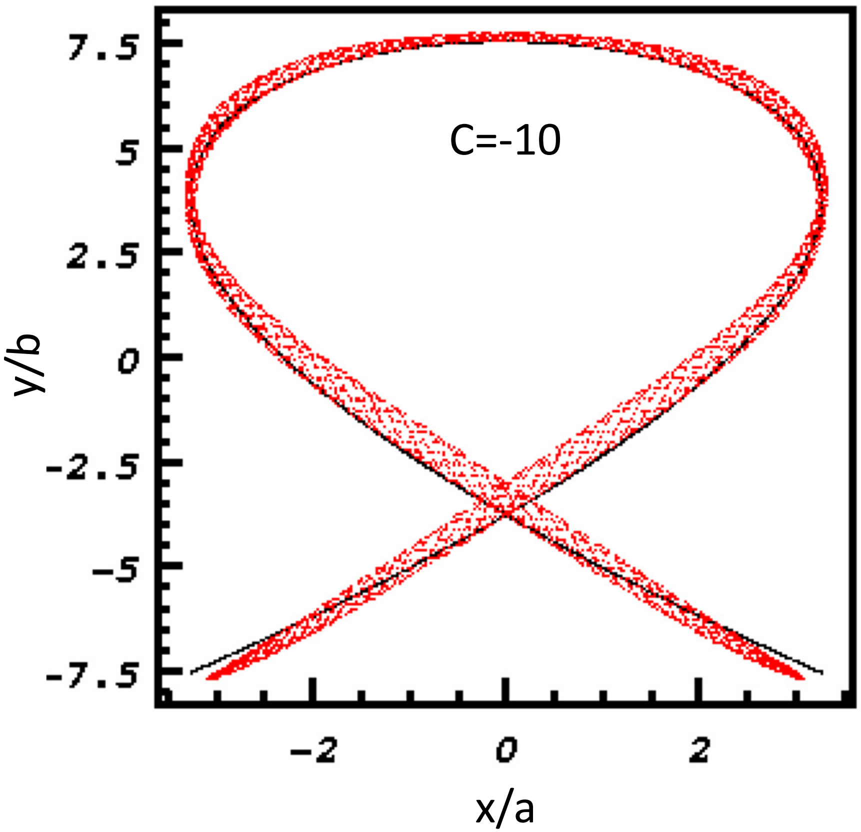


With higher order resonances, fixed lines and space charge

2 Qx + 2 Qy = 19 (normal)

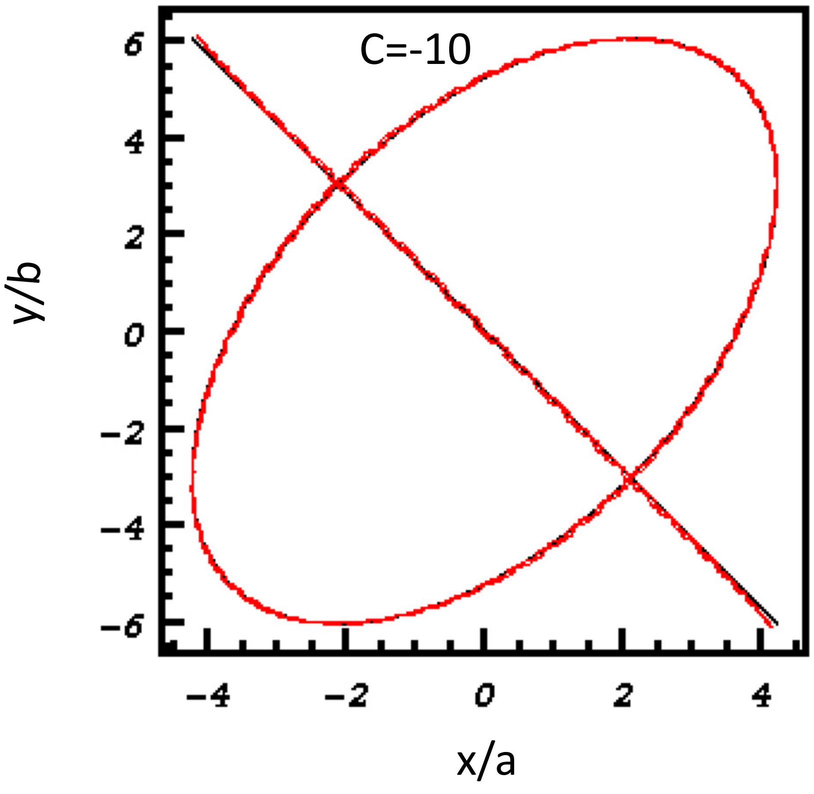


3 Qx + 2 Qy = 19 (skew)

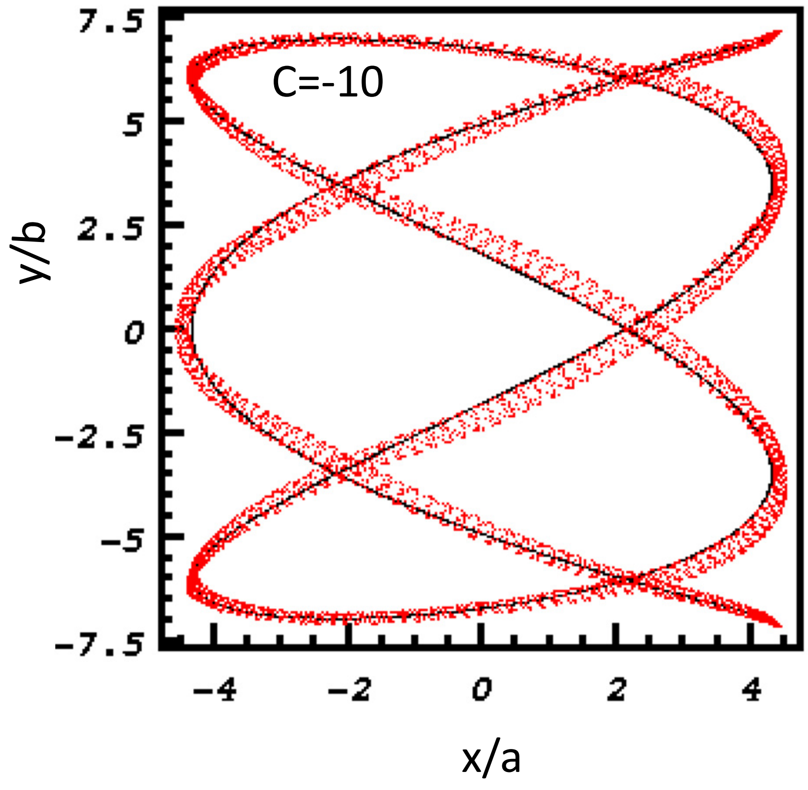


With higher order resonances, fixed lines and space charge

3 Qx + 3 Qy = 29 (skew)

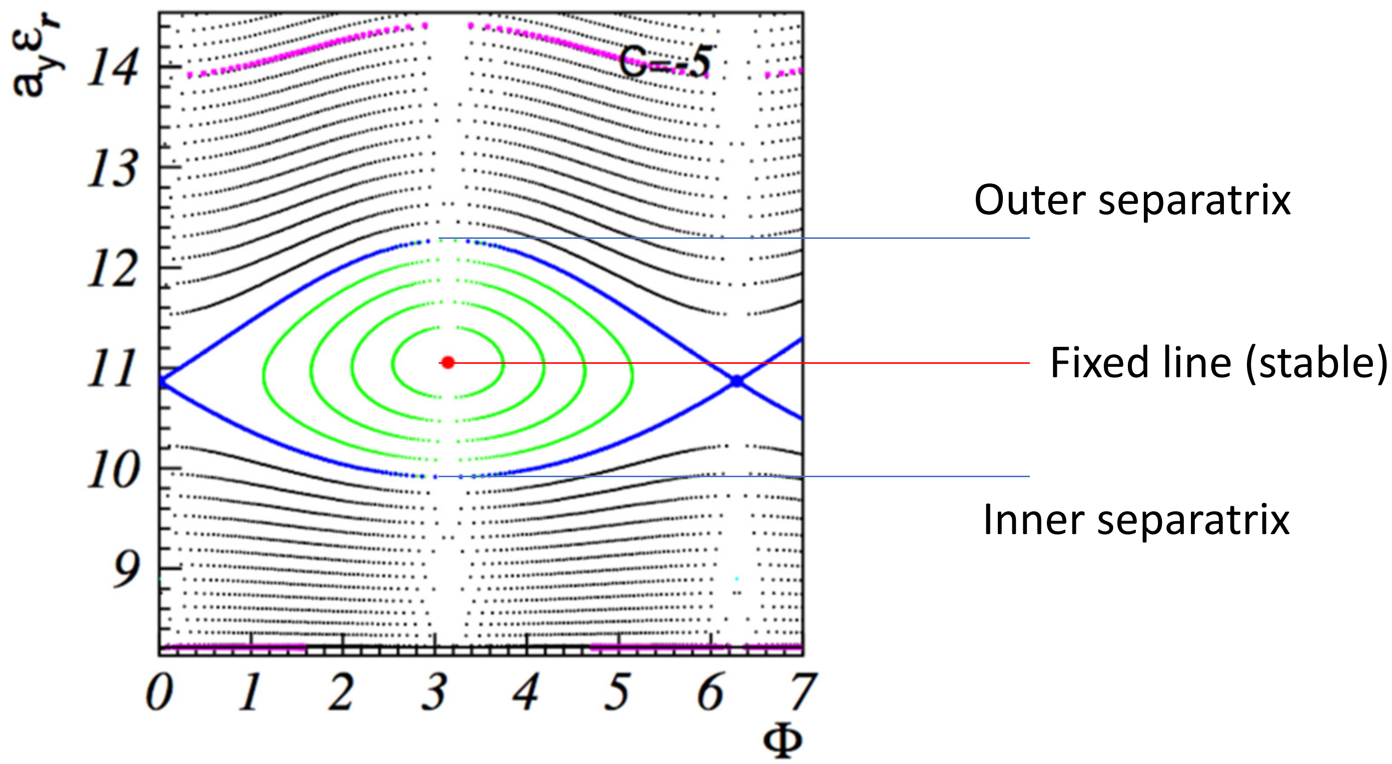


3 Qx + 6 Qy = 49 (normal)



The shape of resonances

Qx + 2Qy = 19



Particle In Cell tests

Comparison with PIC: coasting beam

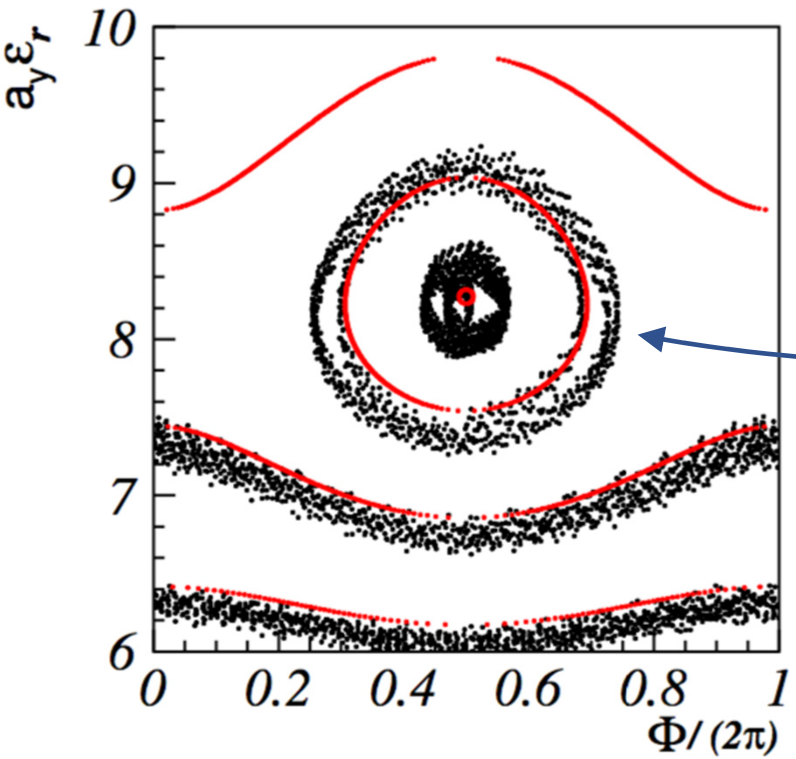
10^6 macro-particles

512×512 grids

1000 turns

$Q_x + 2Q_y = 19$

Constant focusing

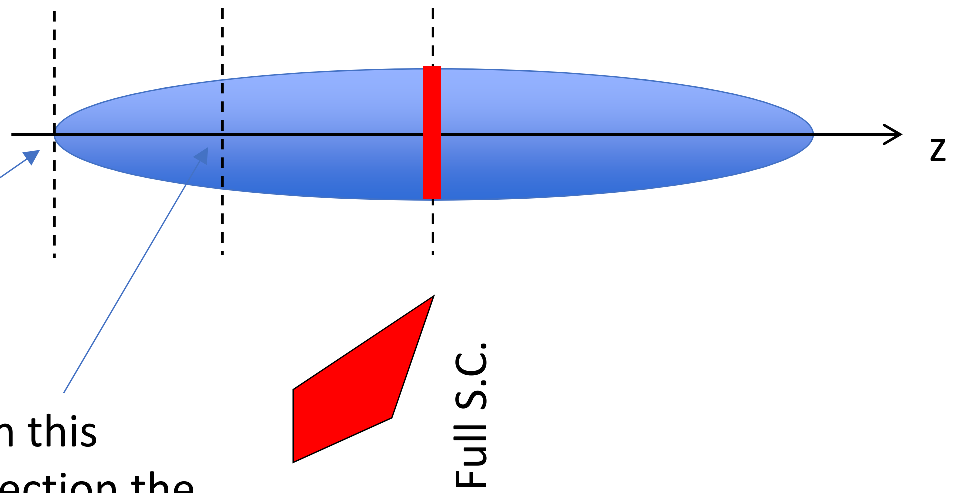


topological stability

Application example

The effect of space charge on the adiabatic crossing

Set an artificially very slow synchrotron motion



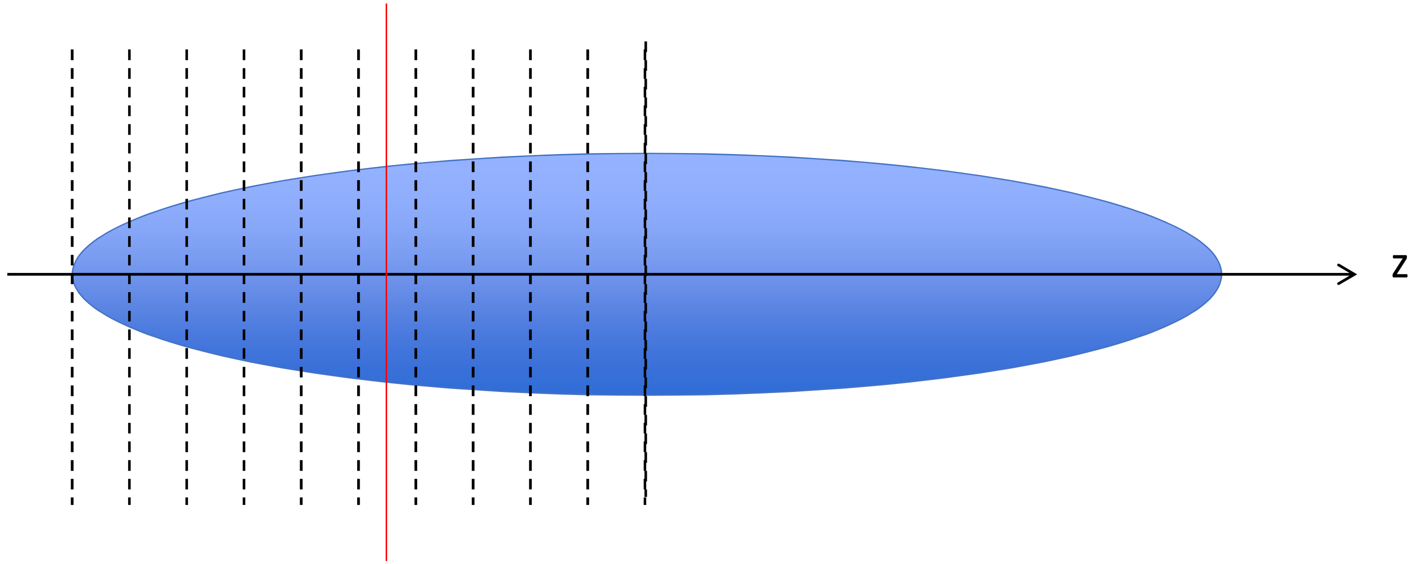
Initial position of a test particle

Very small transverse amplitude

In this section the tune-spread starts to overlap the resonance

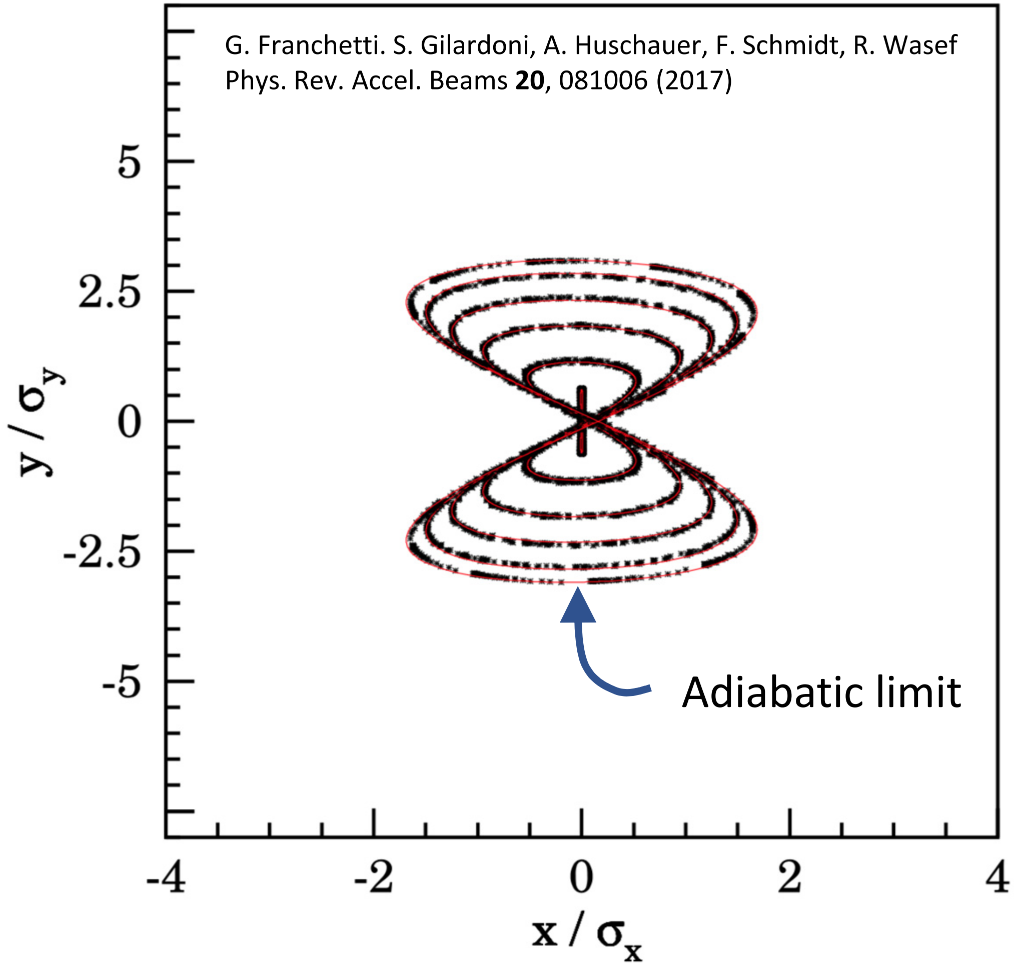
The effect of space charge on the adiabatic crossing

Take 10 snapshots along ¼ synchrotron oscillation



In each of these "section" 1000 turns are taken

The adiabatic limit

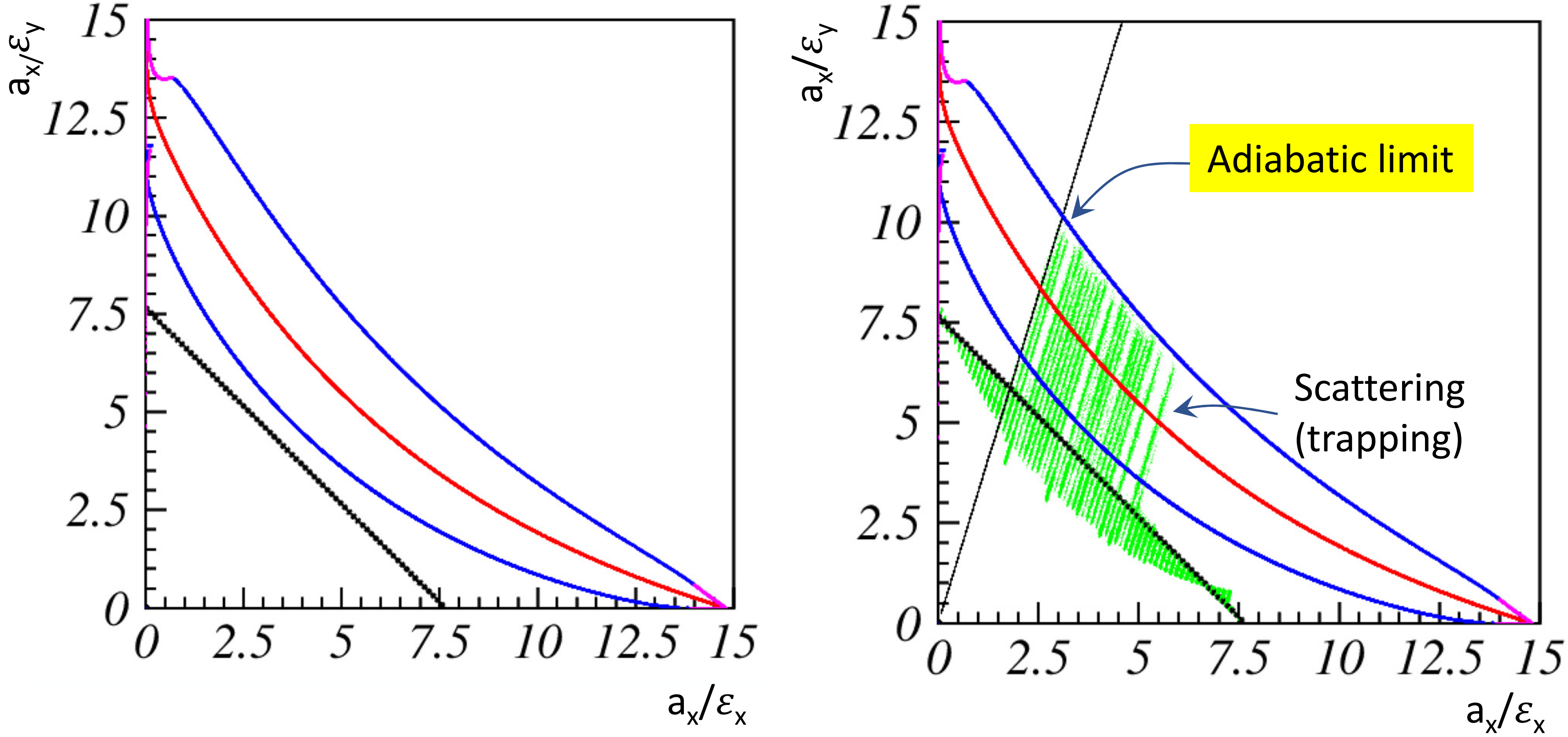


It seems that the "adiabatic limit" predicts the maximum amplitudes of the diffusing process due to periodic resonance crossing

Why the adiabatic limit works ?

Periodic “fixed lines” crossing, alias periodic resonance crossing

$Q_x + 2Q_y = 19$ $\quad Q_x = 6.104 \quad \Delta_r = 0.056 \quad \Delta Q_x = -0.05 \quad \mathscr{D}_{r,sc} = -0.174$



Summary / Outlook

It is proposed an incoherence parameter $\mathcal{I}$

Theory of resonances with frozen space charge “almost” complete (4D)

From theory → secondary tunes for all orders

SC stabilizes all resonances, which otherwise would be unstable

Prediction of amplitudes of all fixed lines

Periodic resonance crossing in a bunch: diffusion bounded by outer separatrix,
under verification

“Some” consistency with PIC is verified… more tests underway

Necessary further tests for broad range of parameters

To be checked if $\mathcal{I}$ really allows to distinguish incoherent from coherent regimes