Fermilab U.S. DEPARTMENT OF ENERGY Office of Science

Nonlinear integrable optics to facilitate high intensity operation

Alexander Valishev

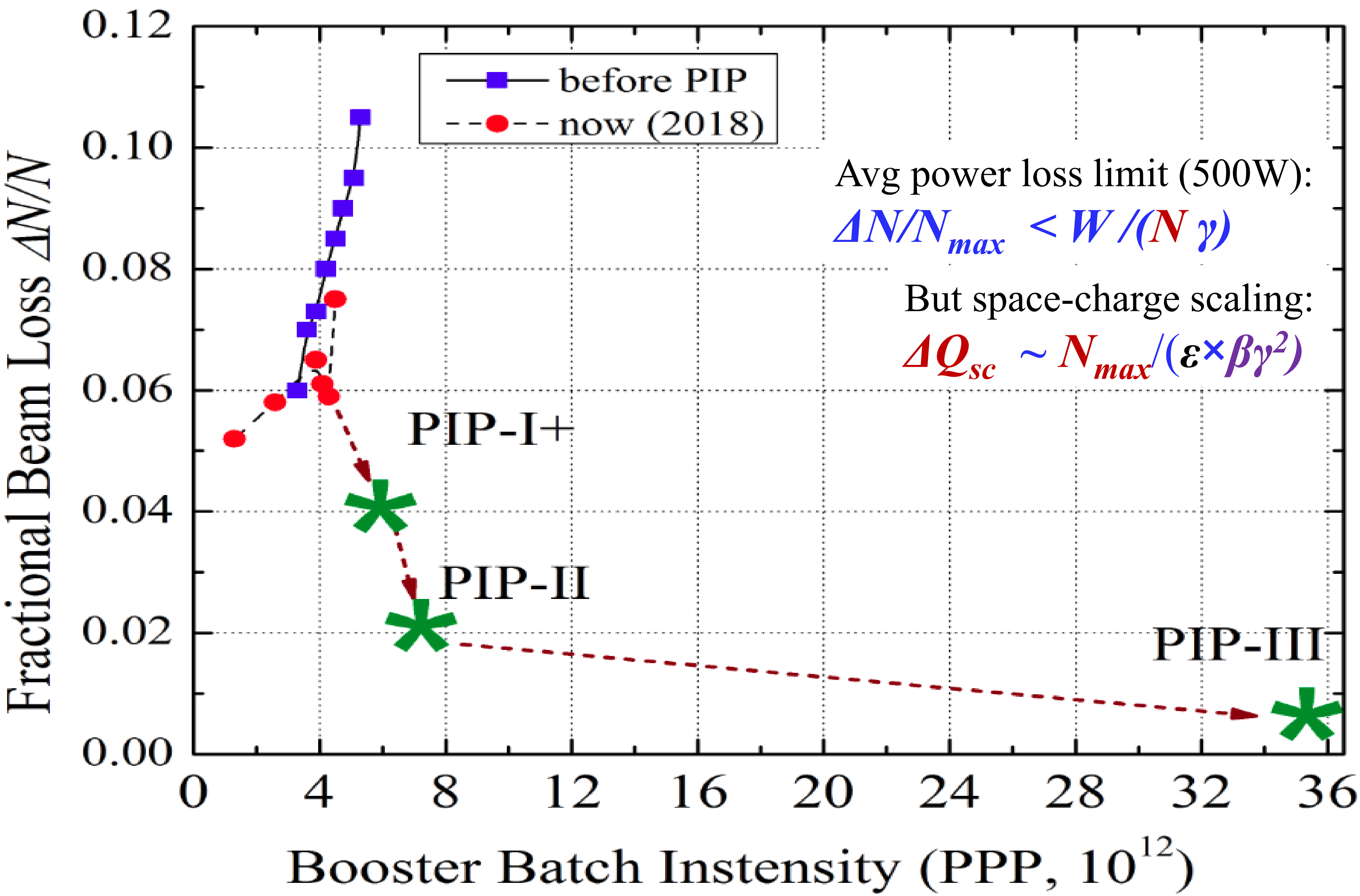
HB2018, Daejeon, Korea

20 June 2018

Acknowledgments

- R.Andrews, D.Barak, V.Bocean, D.Broemmelsiek, A.Burov, K.Carlson, D.Crawford, A.Didenko, N.Eddy, D.Edstrom, J.Eldred, R.Espinoza, D.Franck, J.Jarvis, V.Kashikhin, R.Kellett, M.Kucera, V.Lebedev, J.Leibfritz, C.Jensen, W.Johnson, P.Li, M.McGee, S.Nagaitsev, L.Nobrega, M.Obrycki, H.Piekarz, E.Prebys, I.Rakhno, A.Romanov, J.Ruan, J.Santucci, V.Shiltsev, G.Stancari, A.Valishev, J.VanBogaert, A.Warner, S.Wesseln(FNAL)
- M.Andorff, S.Chattopadhyay, P.Piot, S.Szustkowski (NIU)
- D.Shatilov (BINP)
- S.Antipov, N.Kuklev (U of Chicago)
- D.Abell, D.Bruhwiler, N.Cook, C.Hall, S.Webb (RadiaSoft)
- C.Mitchell (LBNL)
- R.Agustsson, A.Murokh, F.O'Shea (RadiaBeam)
- H.Baumgartner, R.Kishek, K.Ruisard (UMD)

FNAL Booster Protons Per Pulse Challenge: PIP →PIP-I+ →PIP-II →PIP-III



Linear Focusing - Limitations

- The key principle of all accelerator lattices – linear focusing

$$H' \approx \frac{p_x^2 + p_y^2}{2} + \frac{K_x(s)x^2}{2} + \frac{K_y(s)y^2}{2}$$

- What about higher order terms?
  - Imperfections in magnet construction
  - Chromatic aberrations
  - IBS
  - Wakes
  - Beam-beam
  - Intentionally introduced multipole magnets (e.g. sextupoles to correct chromaticity)
- All are aberrations to the initially decoupled system of two linear oscillators

Aberrations of Linear Focusing

$$x'' + K_x(s)x = S(s)x^2 + O(s)x^3 + \cdots$$

- Nonlinearities result in dependence of oscillation frequency on amplitude
- Explicit time-dependence of multipole coefficients results in resonances
- Coupling between x and y further complicates the dynamics
- Ultimately, chaos and loss of stability
  - Beam blow-up
  - Particle loss from accelerator
- We call this single particle stability or Dynamical Aperture

Collective Instabilities

- In addition to the single-particle chaos, the beam can become unstable as a whole if resonantly excited by external field or via self-interaction through environment
- These instabilities can be suppressed by
  1. External damping system – presently the most commonly used mechanism to keep the beam stable.
  2. Landau damping – the beam's own "immune system" related to the spread of betatron oscillation frequencies. The larger the spread, the more stable the beam is against collective instabilities.

Landau Damping of Collective Instabilities

COLLIDING BEAMS: PRESENT STATUS; AND THE SLAC PROJECT*

B. Richter

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

The discovery in the early '60's at the Princeton-Stanford ring of what was thought to be the resistive wall instability brought the realization that circular accelerators are fundamentally unstable devices because of the interaction of the beam with its environment. Stability is achieved only through Landau damping and/or some external damping system.

- **1965 Priceton-Stanford CBX: First mention of an 8-pole magnet**
  - Observed vertical resistive wall instability
  - With octupoles, increased beam current from ~5 to 500 mA
- **CERN PS: In 1959 had 10 octupoles; not used until 1968**
  - At 10^{12} protons/pulse observed (1st time) head-tail instability. Octupoles helped.
  - Once understood, chromaticity jump at transition was developed using sextupoles.
  - More instabilities were discovered; helped by octupoles, fb
- **LHC has 336 octupoles that run close to 500A to create 0.001 tune spread**
- **FCC will require ~ 20,000 octupoles to retain stability**

Intermediate Summary

- All present machines are built around concept of linear focusing
  – Nonlinear aberrations ruin beam quality and particle stability
  – Nonlinear aberrations are intrinsic to charged particle beams and scale with beam brightness
  – Nonlinearities must be introduced to maintain beam's own immunity to coherent instabilities through Landau damping
- Can we attempt a major paradigm change to leave the linear focusing?
  – Let us build machines that are nonlinear by design but stable (like Solar system)
  – Easier said than done…

History of Search for Stable Nonlinear Solutions

- Orlov (1963)
- McMillan (1967) – **1D solution**
- Perevedentsev, Danilov (1990) – generalization of McMillan case to 2D, round colliding beams. **Require non-Laplacian potentials to realize**
  - Implemented at VEPP-2000 collider at BINP (Novosibirsk, Russia) commissioned in 2006. Record-high beam-beam tune spread 0.25 attained in 2013
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- Danilov, Nagaitsev (2010) – Solution for nonlinear lattice with 2 invariants of motion that **can be implemented with Laplacian potential**, i.e. with special magnets – *Phys. Rev. ST Accel. Beams 13, 084002 (2010)*

Invariants of Motion

Let us return to the linear Hamiltonian (note x and y separate)

$$H' \approx \frac{p_x^2 + p_y^2}{2} + \frac{K_x(s)x^2}{2} + \frac{K_y(s)y^2}{2}$$

One can transform to normalized variables

$$\psi = \int_0^s \frac{dz}{\beta(z)}$$ – new time

$$x_n = \frac{x}{\sqrt{\beta(s)}} \qquad p_n = p\sqrt{\beta(s)} - \frac{\beta'(s)x}{2\sqrt{\beta(s)}}$$

Equations of motion become that of linear oscillator

$$\frac{d^2 x_n}{d\psi^2} + Q^2 x_n = 0 \qquad I = \frac{1}{2\pi}\int p_n dx_n$$

And Hamiltonian $H = Q_x I_x + Q_y I_y$

has two invariants (Courant-Snyder invariants, or actions)

Motion is 2D integrable

Stability of Hamiltonian System

- Nekhoroshev (1971) studied which systems are more stable to perturbations

Russian Math. Surveys 32:6 (1977), 1–65
From Uspekhi Mat. Nauk 32:6 (1977), 5–66

AN EXPONENTIAL ESTIMATE OF THE TIME OF STABILITY OF NEARLY-INTEGRABLE HAMILTONIAN SYSTEMS

N. N. Nekhoroshev

1.1 Nearly-integrable Hamiltonian systems. Perpetual stability and stability during finite intervals of time. In this article we investigate the behaviour of the variables I in the Hamiltonian system of canonical equations

$$\dot{I} = -\frac{\partial H}{\partial \varphi}, \quad \dot{\varphi} = \frac{\partial H}{\partial I}$$

with the Hamiltonian

$$H = H_0(I) + \varepsilon H_1(I, \varphi), \tag{1.1}$$

where $\varepsilon \ll 1$ is a small parameter, the perturbation $\varepsilon H_1(I, \varphi)$ is 2π-periodic in $\varphi = \varphi_1, \ldots, \varphi_s$, and I is an s-dimensional vector, $I = I_1, \ldots, I_s$.

- He proved that for sufficiently small ε $\|I(t) - I(0)\| \le R_* \epsilon^b$ for $|t| \le T_* \exp(\epsilon^{-a})$

provided that $H_0(I)$ meets certain condition known as **steepness**

- Convex and quasi-convex functions $H_0(I)$ are the steepest
- An example of a non-steep function is a linear function

$$H = Q_x I_x + Q_y I_y$$

- A steep (convex) function would be $H = \alpha_x {I_x}^2 + \alpha_y {I_y}^2$

…but to implement in practical accelerator design, fields must satisfy Maxwell equations in vacuum

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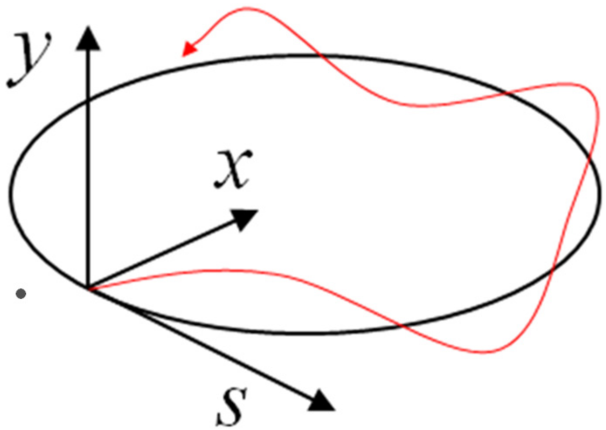
  - IOTA

McMillan Mapping



SOME THOUGHTS ON STABILITY
IN NONLINEAR PERIODIC FOCUSING SYSTEMS

Edwin M. McMillan

September 5, 1967

1D – thin lens kick $f(x) = -\frac{Bx^2 + Dx}{Ax^2 + Bx + C}$

$$x_i = p_{i-1}$$
$$p_i = -x_{i-1} + f(x_i)$$

$$Ax^2p^2 + B\left(x^2p + xp^2\right) + C\left(x^2 + p^2\right) + Dxp = \text{const}$$

- System comprised of two components
  1. Accelerator arc, performing a transformation
     $$x = p_0, p = -x_0$$
  2. Thin nonlinear lens $x = x_0, p = p_0 + f(x_0)$

This system possesses an invariant quadratic in coordinates and momenta

However, no solution for 2D with $\Delta\varphi(x, y) = 0$

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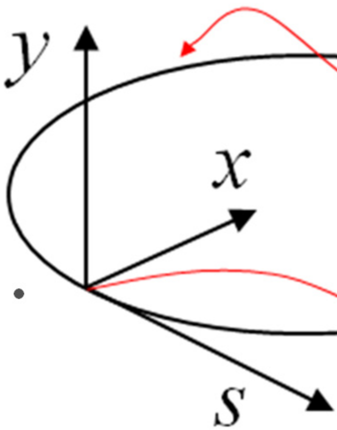
2D Expansion of McMillan Mapping



TWO EXAMPLES OF INTEGRABLE SYSTEMS WITH ROUND COLLIDING BEAMS

V.V. Danilov and E.A. Perevedentsev
Budker Institute of Nuclear Physics, 630090, Novosibirsk, Russia

- 2D – a thin lens solution can be carried over to 2D case in axially symmetric system

$$\begin{pmatrix} cI & sI \\ -sI & cI \end{pmatrix} \begin{pmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix}$$

$$c = \cos(\phi)$$
$$s = \sin(\phi)$$
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\theta(r) = \frac{kr}{ar^2 + 1}$$

can be created with electron lens

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Nonlinear Integrable Optics

- We want to build an optical focusing system that
  a. Is strongly nonlinear = strong dependence of oscillation frequency on amplitude
  b. Is 2D integrable and stable
  c. Can be realized with magnetic fields in vacuum

- Mathematically, that means the system should
  – Possess two integrals of motion
  – Have steep Hamiltonian
  – Field potential satisfies the Laplace equation

- Practical benefits
  – Reduced chaos in single-particle motion
  – Strong immunity to collective instabilities via Landau damping

Danilov-Nagaitsev Solution (2010)

1. Remove time dependence from Hamiltonian thus making it an integral of the motion
   – One integral already gives a better degree of regularity in the motion
      - Demonstrated with round colliding beams (BINP, 2013), 1/2-integer working point in colliders (KEK, 2004)
      - Crab-crossing at DAFNE (INFN/LNF, 2008)

2. Shape the nonlinear potential to find a second integral

Time-Independent Hamiltonian

- Start with a Hamiltonian $$H = \frac{p_x^2}{2} + \frac{p_y^2}{2} + K(s)\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + V(x, y, s)$$
- Choose s-dependence of nonlinear potential V such that *H* is time-independent in normalized variables $$z_N = \frac{z}{\sqrt{\beta(s)}},$$ $$p_N = p\sqrt{\beta(s)} - \frac{\beta'(s)z}{2\sqrt{\beta(s)}},$$

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + \beta(\psi)V\left(x_N\sqrt{\beta(\psi)}, y_N\sqrt{\beta(\psi)}, s(\psi)\right)$$
$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N, \psi)$$

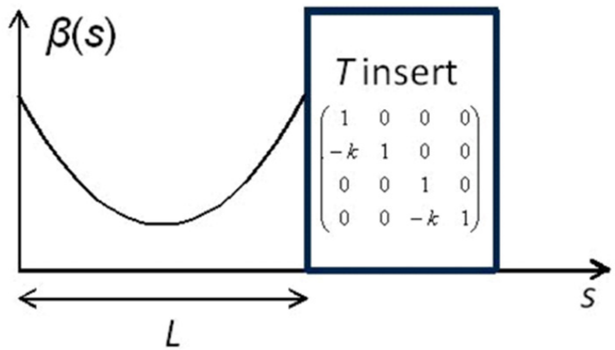
- This results in *H* being the integral of motion
- Note there was no requirement on *V* – can be made with any conventional magnets, i.e. octupoles

Implementation of Time-Independent Hamiltonian

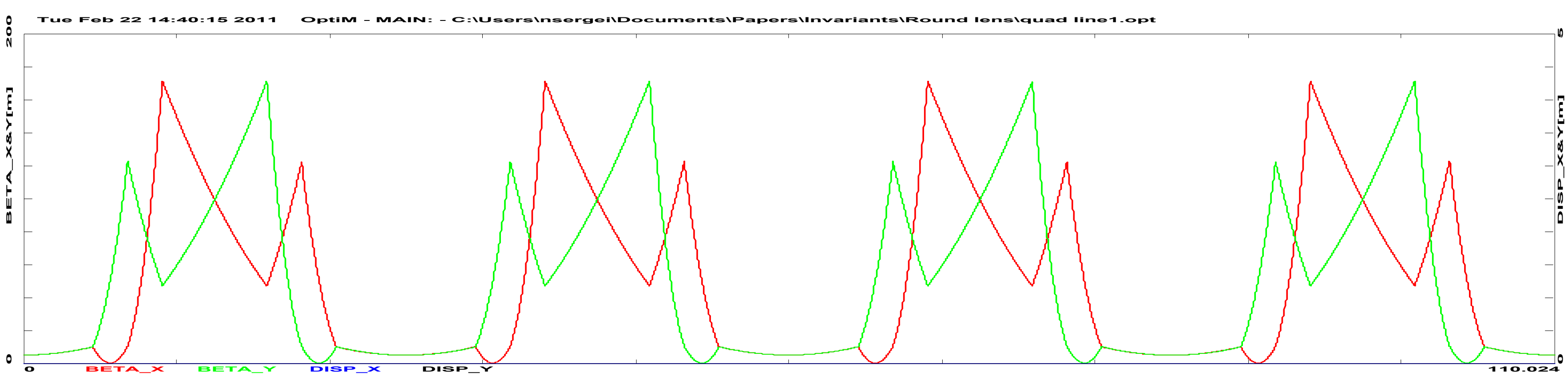
1 Start with a round axially-symmetric *linear* lattice (FOFO) with the element of periodicity consisting of

   a. Drift L

   b. Axially-symmetric focusing block "T-insert" with phase advance n × π



2 Add special nonlinear potential *V(x,y,s)* in the drift

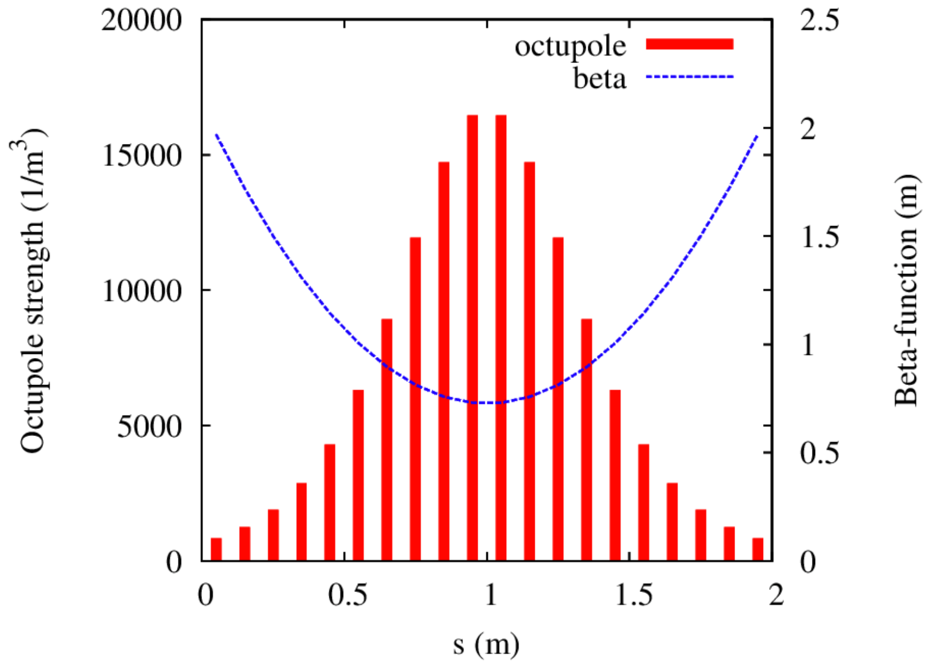


Henon-Heiles Type Systems

- For example, build *V* with Octupoles

$$V(x,y,s)=\frac{\kappa}{\beta(s)^3}\left(\frac{x^4}{4}+\frac{y^4}{4}-\frac{3x^2y^2}{2}\right)$$

$$U=\kappa\left(\frac{x_N^4}{4}+\frac{y_N^4}{4}-\frac{3y_N^2x_N^2}{2}\right)$$

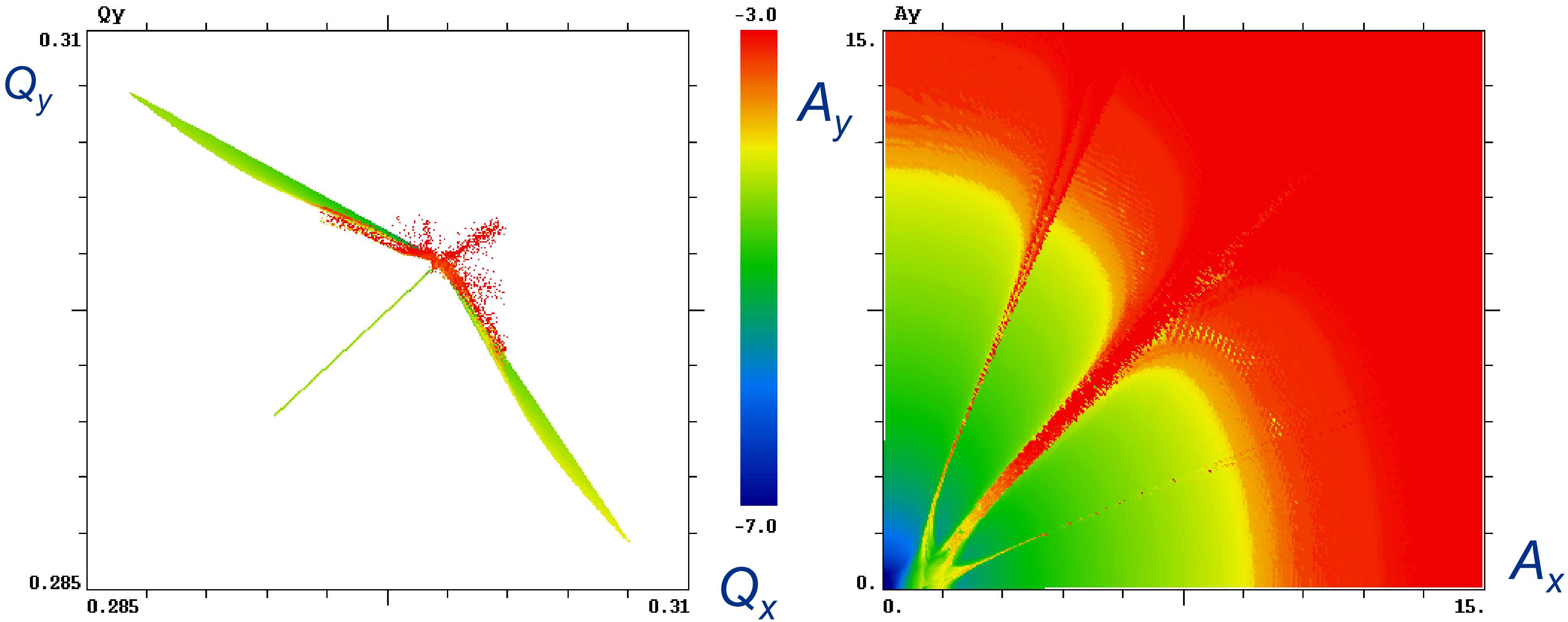


$$H=\frac{1}{2}(p_x^2+p_y^2)+\frac{1}{2}(x^2+y^2)+\frac{k}{4}\left(x^4+y^4-6x^2y^2\right)$$

- Only one integral of motion – *H*
- Tune spread limited to ~12% of Q_0

S. Antipov, S. Nagaitsev, A. Valishev, JINST 12 (2017) no.04, P04008

Performance of Octupole Henon-Heiles System



- While Dynamic Aperture is limited, the attainable tune spread is large ~0.03 – compare to ~0.001 created by LHC octupoles
- By implementing such system, one can already gain in beam immunity to coherent instabilities by 10-30 times

2nd Integral of Motion – Special Potential

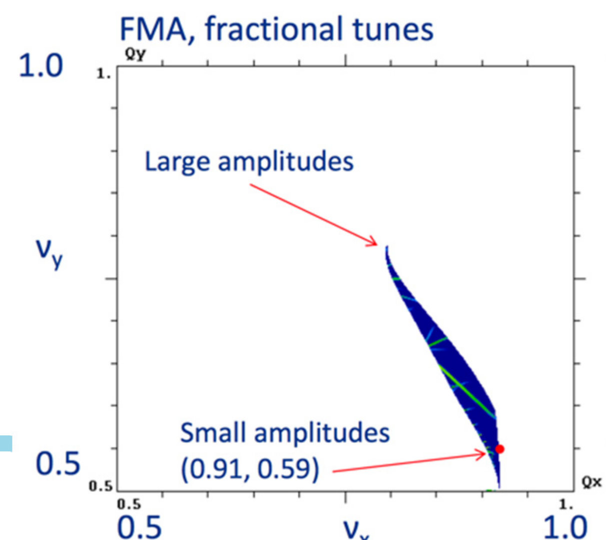
- Find potentials that result in the Hamiltonian having a second integral of motion quadratic in momentum
  - All such potentials are separable in some variables (cartesian, polar, elliptic, parabolic)
  - First comprehensive study by Gaston Darboux (1901)
- Darboux equation

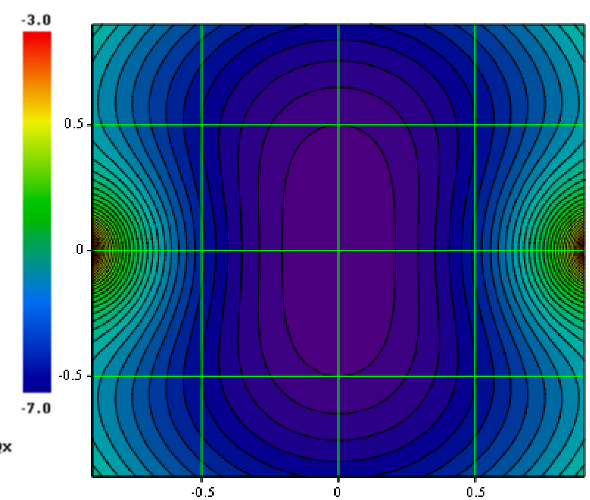
$$xy\left(U_{xx}-U_{yy}\right)+\left(y^2-x^2+c^2\right)U_{xy}+3yU_x-3xU_y=0$$

  - General solution was found, which satisfies the Laplace equation (*Phys. Rev. ST Accel. Beams 13, 084002, 2010*)

$$U(x,y)=\frac{f(\xi)+g(\eta)}{\xi^2-\eta^2}$$

$$f_2(\xi)=\xi\sqrt{\xi^2-1}\left(d+t\,\mathrm{acosh}(\xi)\right)$$
$$g_2(\eta)=\eta\sqrt{1-\eta^2}\left(b+t\,\mathrm{acos}(\eta)\right)$$





Common Features of these Solutions

1. One begins from a conventional linear lattice accelerator (specially designed and carefully controlled)
2. The special nonlinear element (Laplacian or E-Lens) is added to make the lattice integrable.

+ Such approach does not require much new technology – a significant portion of accelerator circumference is made with common quadrupoles and dipoles

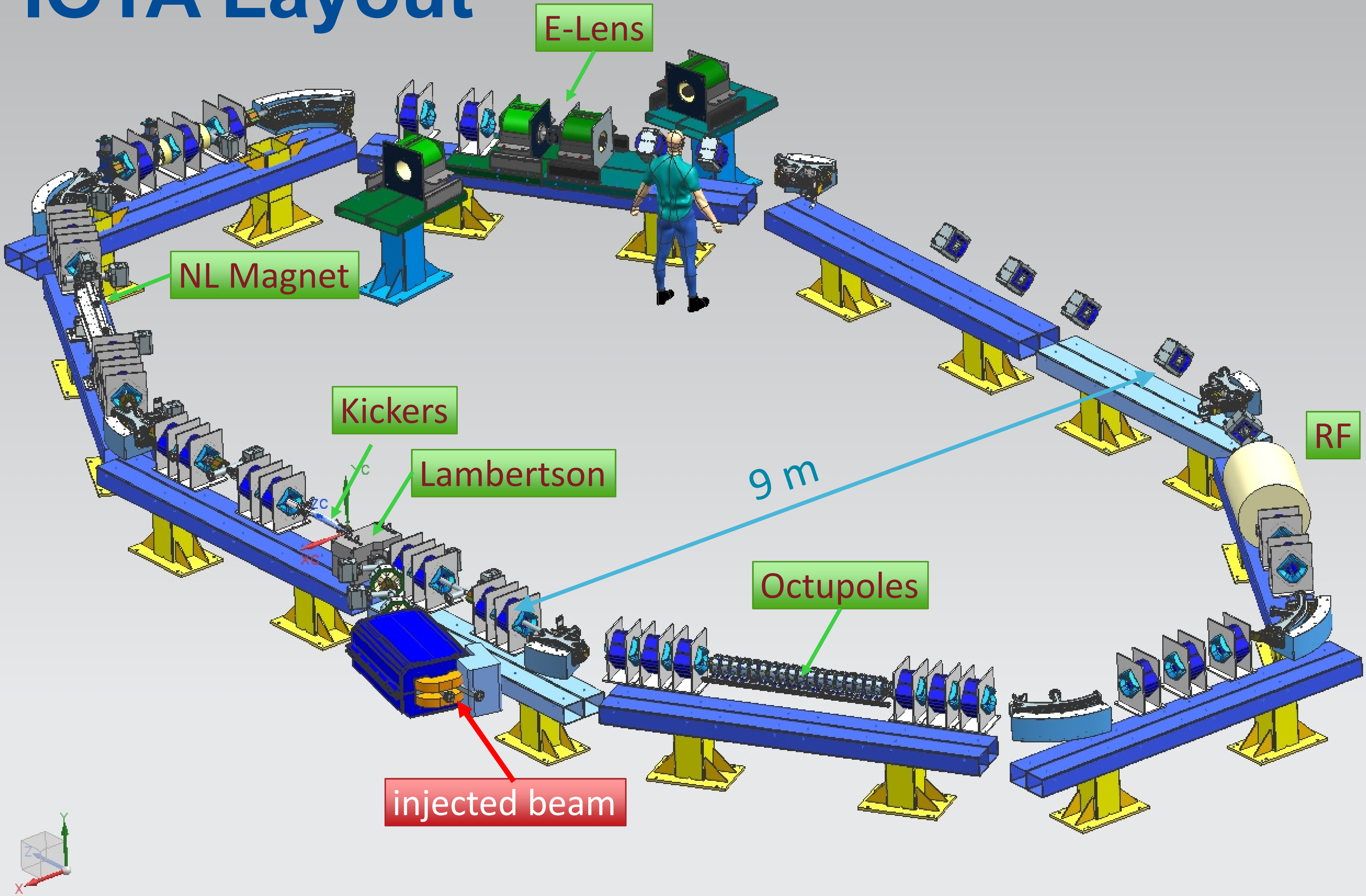
− All approaches rely on the accelerator arc lattice being linear and precise in phase advance and beta-functions
  – Chromaticity, space-charge, nonlinearities – all ruin this requirement

- Experimental verification is desired – IOTA at Fermilab

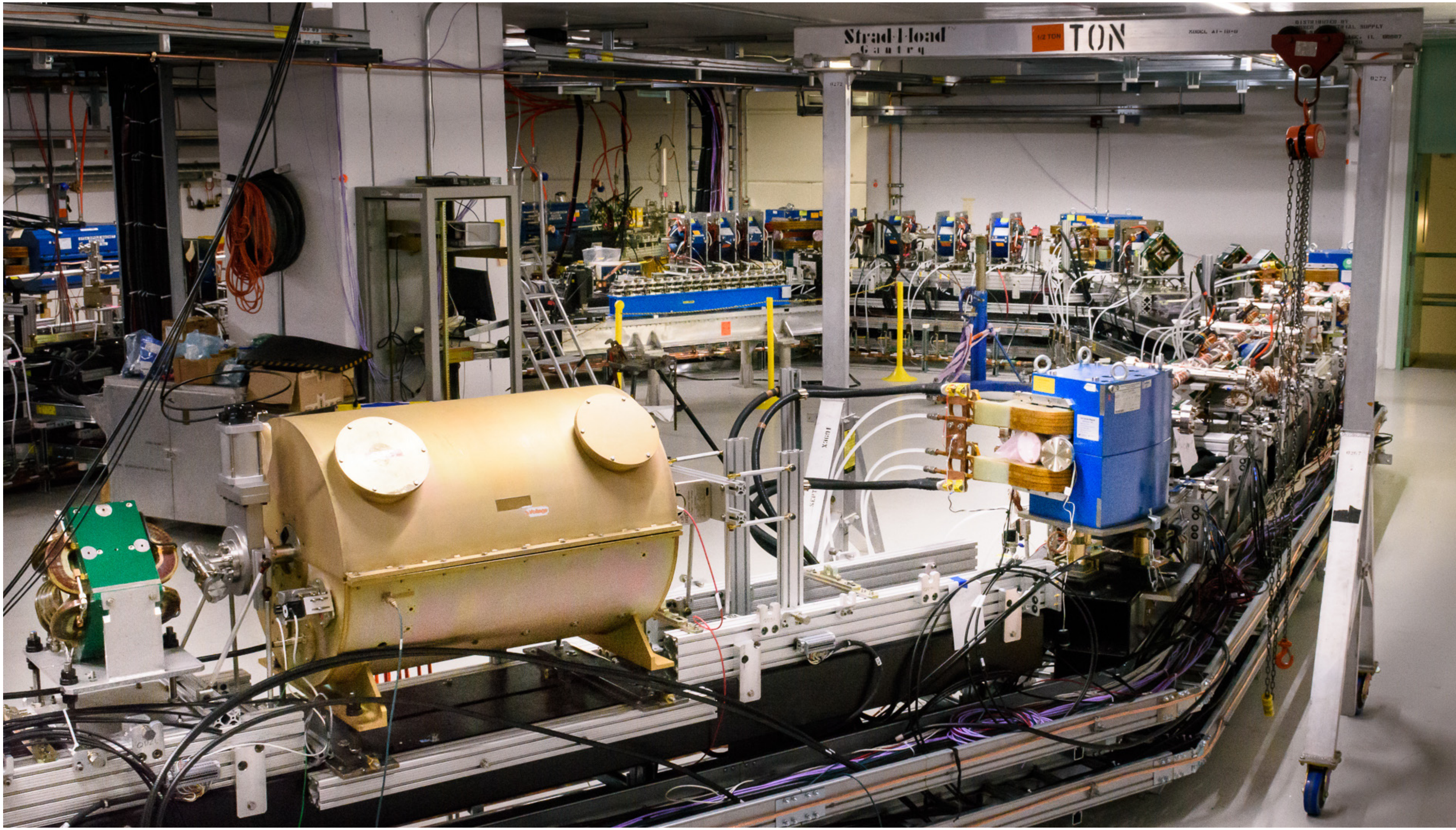
Integrable Optics Test Accelerator

- **Adaptable**
  - Can operate with either electrons or protons
  - Large aperture
  - Easily reconfigurable – quick-change experimental equipment
  - Significant flexibility of the lattice
- **Accurate**
  - Precise control of the optics quality and stability
  - Comprehensive set of precision instrumentation
  - Set up for very high intensity operation (with protons)
- **Affordable**
  - Cost-effective solution, re-use existing parts whenever possible
  - Mostly based on conventional technology (magnets, RF)
  - Balance between low energy (low cost) and research potential

IOTA Layout


E-Lens
NL Magnet
Kickers
Lambertson
9 m
RF
Octupoles
injected beam

IOTA Today

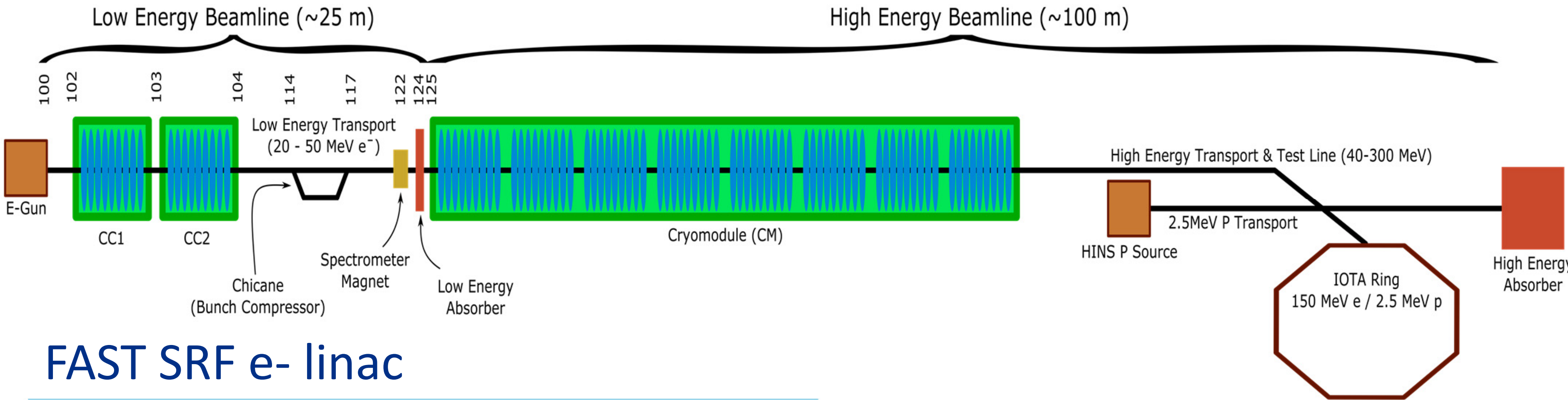


IOTA Parameters

Nominal kinetic energy	e^-: 150 MeV, p+: 2.5 MeV
Nominal intensity	e^-: 1×10^9, p+: 1×10^{11}
Circumference	40 m
Bending dipole field	0.7 T
Beam pipe aperture	50 mm dia.
Maximum b-function (x,y)	12, 5 m
Momentum compaction	$-0.02 \div 0.1$
Betatron tune (integer)	$3 \div 5$
Betatron tune chromaticity	$-15 \div 0$
Transverse emittance r.m.s.	e^-: 0.04 μm, p+: 2μm
SR damping time	0.6s (5×10^6 turns)
RF V,f,q	e^-: 1 kV, 30 MHz, 4
Synchrotron tune	e^-: $2\times10^{-4} \div 5\times10^{-4}$
Bunch length, momentum spread	e^-: 12 cm, 1.4×10^{-4}
Beam lifetime	e^-: 1 hour, p+: 1 min

Fermilab

Fermilab Accelerator Science and Technology FAST facility



FAST SRF e- linac

Bunch charge	up to 2 nC
Gun gradient	45 MV/m
CC1 gradient	26 MV/m
CC2 gradient	15 MV/m
CM2 gradient	up to 31.5 MV/m
Beam energy	150 (up to 300) MeV
Bunch length	6-8 ps (rms)
Pulse length	1 ms
Bunch freq.	3 MHz
Rep. rate	1 Hz (5 Hz)

FAST proton injector

Particles	proton
Kinetic Energy	2.5 MeV
Momentum	69 MeV/c
β	0.073
RF Structure	325 MHz
Beam current	10 mA
Emittance	0.3 mm mrad
Rep. rate	1 Hz
Pulse length	1.5 us
ΔQ_{sc} in IOTA	-0.5

IOTA Science – Nonlinear Optics and Beyond

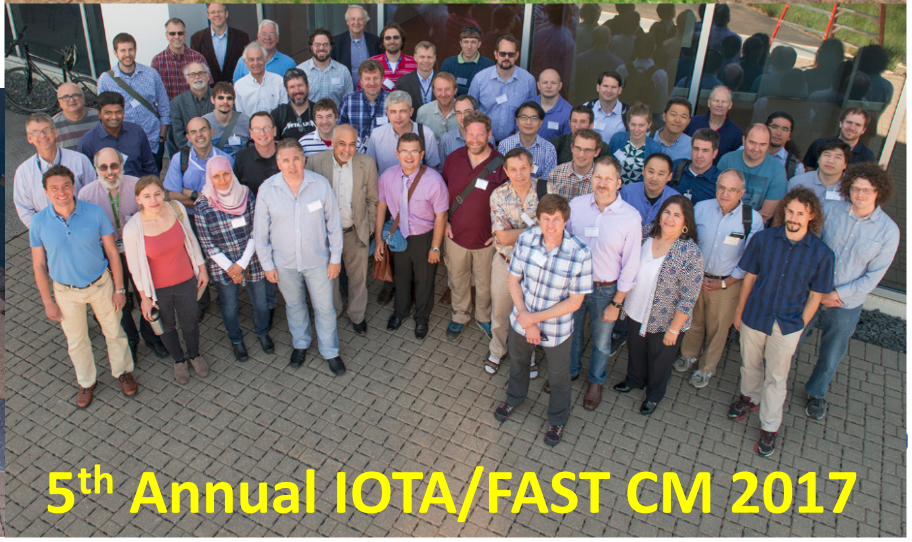
1. **Nonlinear Integrable Optics** – Experimental demonstration of NIO lattice in a practical accelerator
2. **Optical Stochastic Cooling** – Proof-of-principle demonstration J.Jarvis, TUP2WA01
3. **Space Charge Compensation** – Suppression of SC-related effects in high intensity circular accelerators
   - Nonlinear Integrable Optics
   - Electron lenses
   - Electron columns C.S.Park, B.Freemire, WEA2WA04
   - Circular betatron modes
4. **Electron Cooling** – Advanced techniques
5. **Quantum Physics** – single electron wave function

IOTA Staging

- Phase I with electron beam will concentrate on the academic aspect of single-particle motion stability using e- beams
  - Demonstrate large amplitude-dependent detuning
  - Machine tuning must be at the state of the art level to satisfy integrable optics requirements
  - e- beams best for the purpose
- Phase II – After commissioning with e-, we will move the existing 2.5 MeV proton/H- RFQ to FAST to inject protons into the IOTA ring
  - Opens the way for intense-beam studies

IOTA/FAST Collaboration

- 29 partners
  - Theory
  - Modeling and simulations
  - Experiments at FAST
- At this workshop
  - Radiasoft
  - LBNL
  - NIU



Radiasoft: Equal vertical and horizontal chromaticities help preserve invariants

$$\mathcal{H}_{D-N} = \underbrace{\frac{1}{2}\left(\nu_0 + C_0(\delta)\right)\left(\hat{p}_x^2 + \hat{x}^2 + \hat{p}_y^2 + \hat{y}^2\right) + t\nu_0\mathcal{V}(\hat{x} + D_x(\delta), \hat{y})}_{\mathcal{H}_0} +$$

$$\underbrace{C_\Delta(\delta)\frac{1}{2}\left[\left(\hat{p}_x^2 + \hat{x}^2\right) - \left(\hat{p}_y^2 + \hat{y}^2\right)\right]}_{\Delta\mathcal{H}}$$

$$C_0(\delta) = \frac{C_x + C_y}{2}$$

$$C_\Delta(\delta) = \frac{C_x - C_y}{2}$$

The off-momentum Danilov-Nagaitsev Hamiltonian predicts:

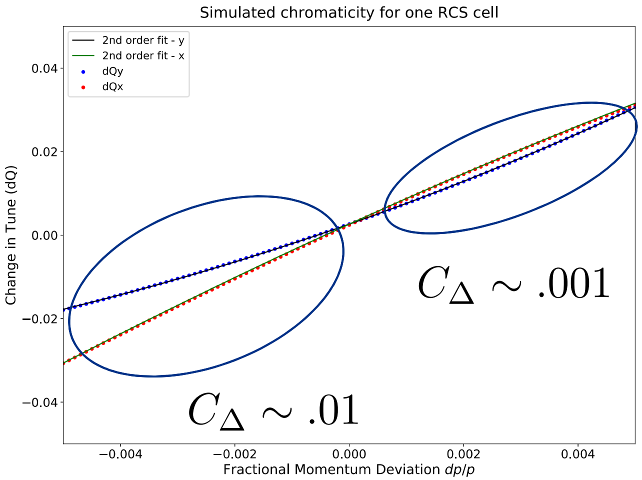
- equal chromaticity will give integrable motion;
- dispersion modifies the invariant, but does not break integrability

Physical intuition suggests:

- adiabatic synchrotron motion will cause the invariants to oscillate with the synchrotron frequency
- the invariants will behave worse when the difference between the chromaticities increases

radiasoft
Fermilab

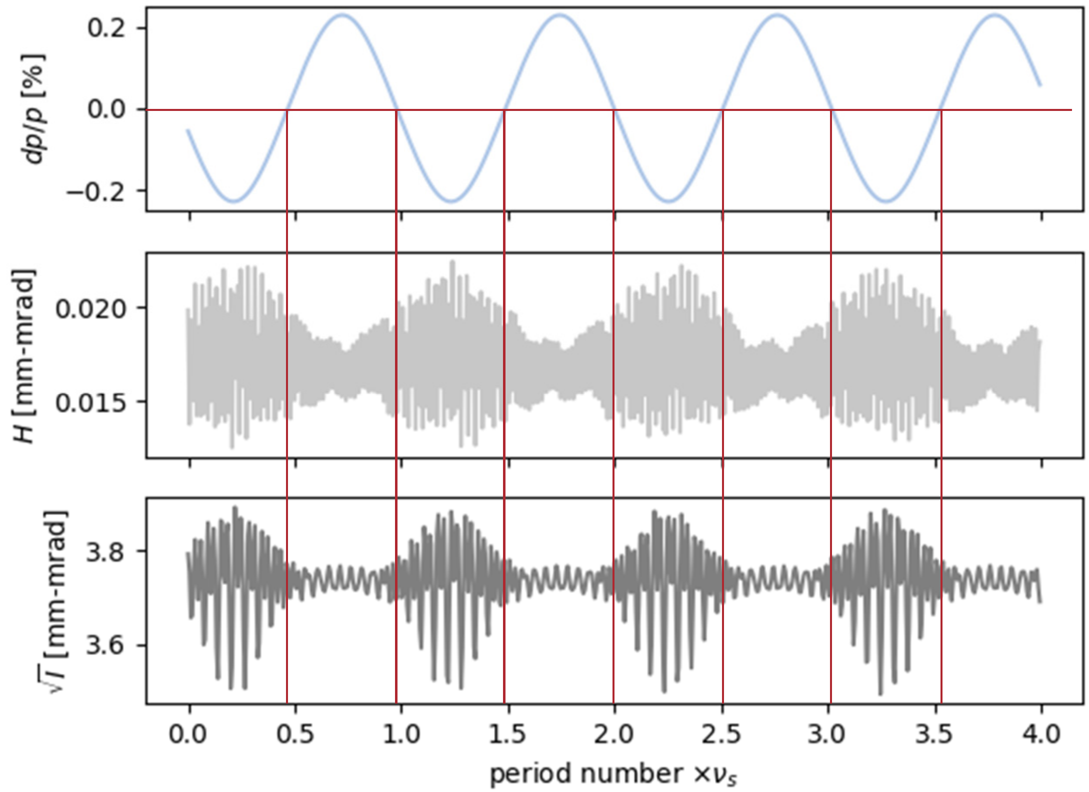
Radiasoft:
Synchrotron motion with chromaticity causes invariants to oscillate at the synchrotron frequency



chromaticity for an integrable RCS lattice, with regions of almost equal and unequal chromaticities

Parameter	Value
Periodicity	12
Betatron Tune	21.6
Synchrotron Tune	0.08
Phase-advance over insert	0.3 → 2□
Nonlinear Strength t-value	0.3
Elliptic Distance c-value	0.14 $m^{1/2}$

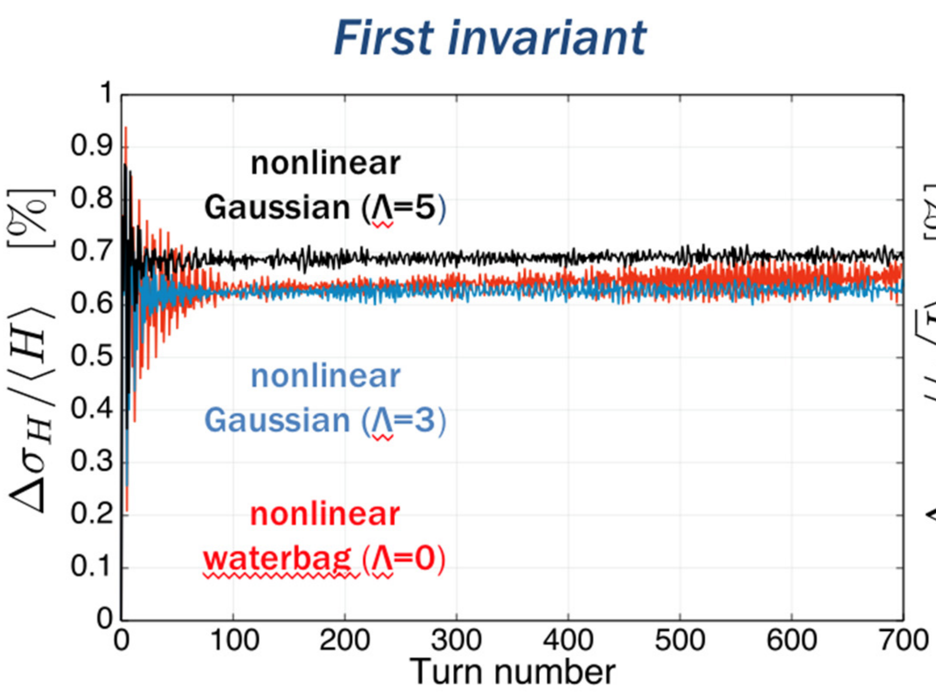
Lattice parameters for an integrable RCS.

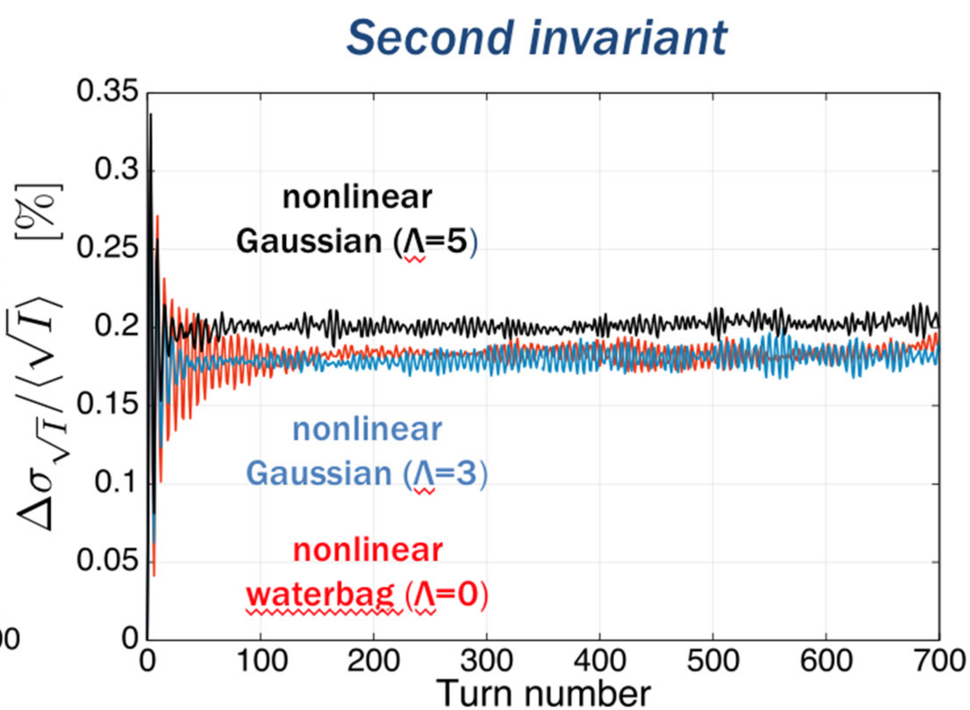


variation of the invariants of motion with the synchrotron tune, with worse behavior in the region of unequal chromaticity

C.Mitchell, LBNL
Tracking in the IOTA Lattice with Space Charge for ΔQ = -0.03: First 700 Turns

Evolution of the standard deviation of the two invariants of single-particle motion for 700 turns.





- **In all cases, <*H*> = 4 mm-mrad. Growth during the initial period of nonlinear mixing and phase space filamentation (turns 1-100) appears to depend only weakly on the cutoff parameter Λ.**
- **The time scale for this initial mixing decreases with increasing Λ, as larger-amplitude particles in the tail contribute to stronger nonlinear damping.**

Beam Physics Experiments at Fermilab

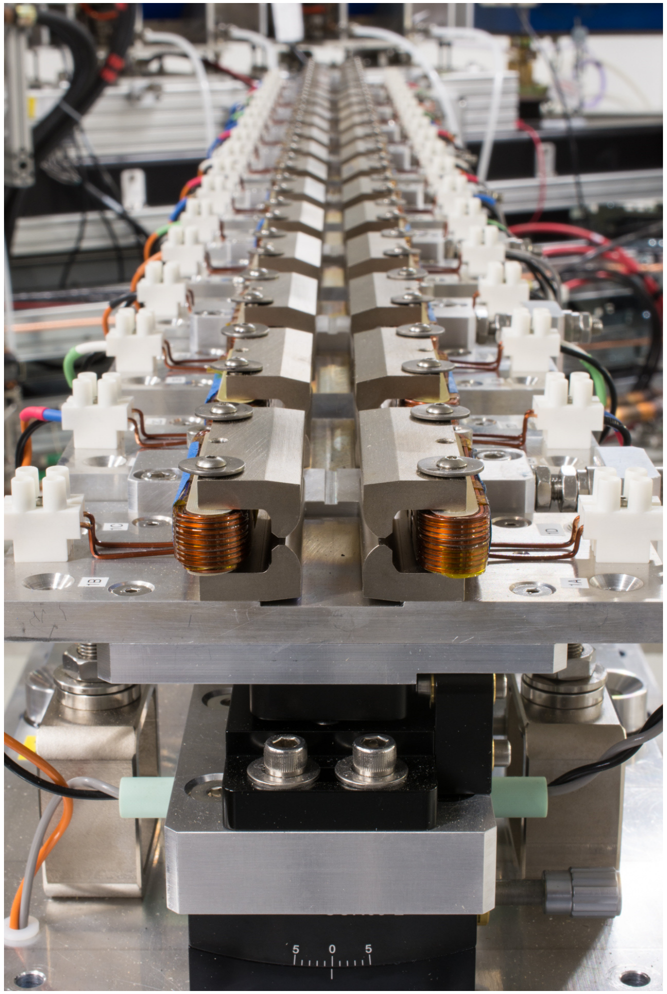
- We invite collaborators to propose high-brightness and high-intensity research that can be carried out at Fermilab's machines
- Studies at main FNAL machines can be organized in 1-2 week blocks during regular operations in 2019
  - Booster
  - Recycler
  - Main Injector
- FAST machines will be available ~6 month/year
  - FAST SRF e- linac
  - IOTA

Summary

- Nonlinear integrable optics may pave the way to higher brightness beams through improved stability of single-particle dynamics as well as stronger Landau damping
- Fermilab’s IOTA is a flexible machine for accelerator R&D
  - Nonlinear integrable optics
  - Space-charge effects and their suppression
  - Optical Stochastic Cooling
  - …
- IOTA is nearing completion – commissioning in Aug. 2018, first research Sept.-Dec. 2018
- We invite collaboration on bright beams making use of Fermilab’s accelerators

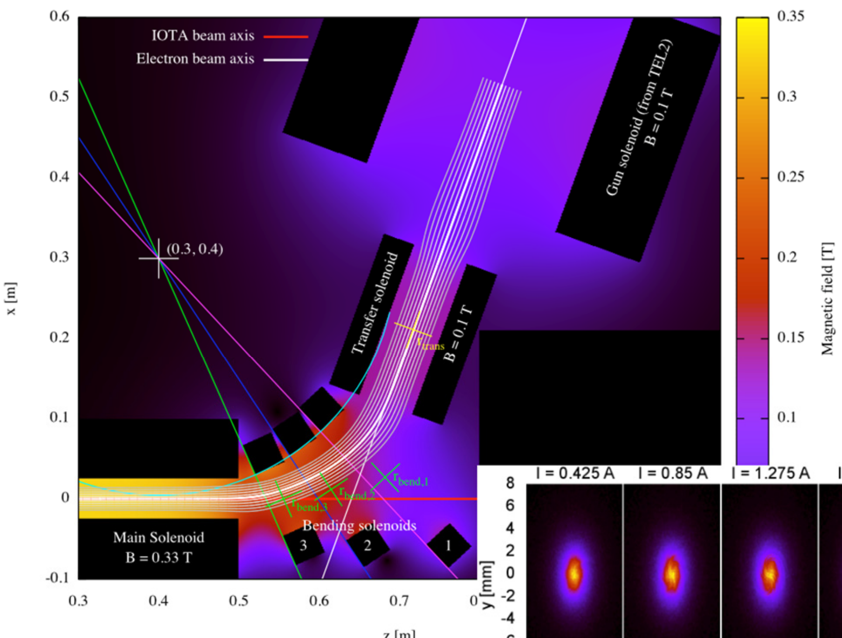
Nonlinear Optics Magnet

RadiaBeam Technologies



IOTA Electron Lens

- Capitalize on the Tevatron experience and recent LARP work
- Re-use Tevatron EL components:
  - Removed TEL-2 gun & collector from Tev tunnel
  - Refurbishment in progress



1. Field-line mapping
2. Single-particle tracking
3. Tracking with space charge
4. Field maps for tracking of circulating beam

Noll and Stancari, arXiv:1511.04507