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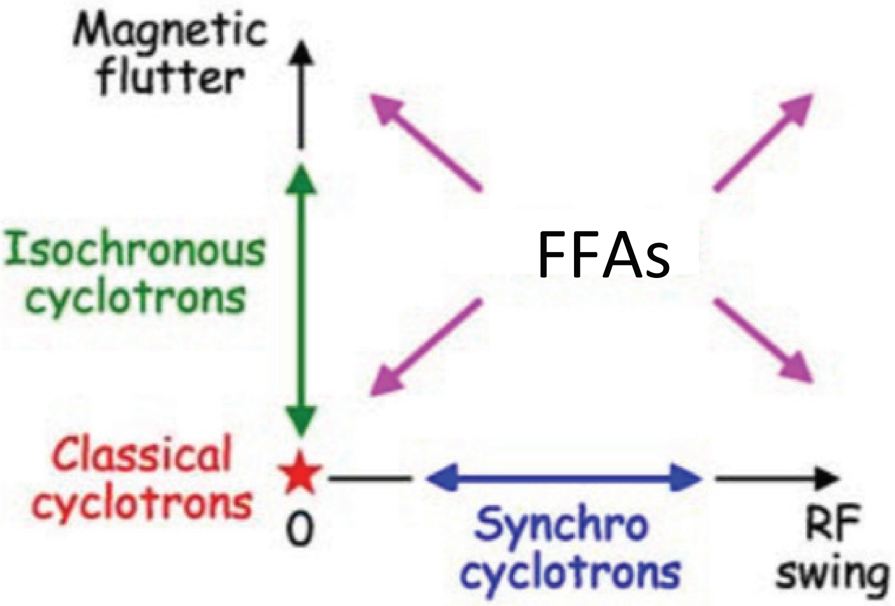
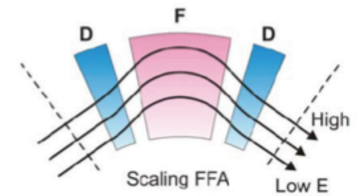
# Fixed Field Accelerators & Space Charge Modeling

Acknowledgements:
FFAG community & Dr. M Haj Tahar

# FFAG -> FFA


Magnetic
flutter
Isochronous
cyclotrons
FFAs
Classical
cyclotrons
0
Synchro
cyclotrons
RF
swing

# Requirements for scaling FFAs




1. the field index k must be constant, where we can define k in terms of the bending radius ρ, the vertical magnetic field By, and its derivative in the horizontal direction x
2. the shape of the particle orbits remains constant as the size of the orbits 'scales' with energy, such that each higher-energy orbit is a geometrically similar enlargement of the lower-energy orbits as described by the following equation, derived by Kolomensky

$$k = -\frac{\rho}{B_y}\frac{\partial B_y}{\partial x}$$

$$\left.\frac{\partial k}{\partial p}\right|_{\theta=\text{const.}} = 0$$

$$\frac{\partial}{\partial p}\left(\frac{\rho_0}{\rho}\right)\bigg|_{\theta=\text{const.}} = 0$$

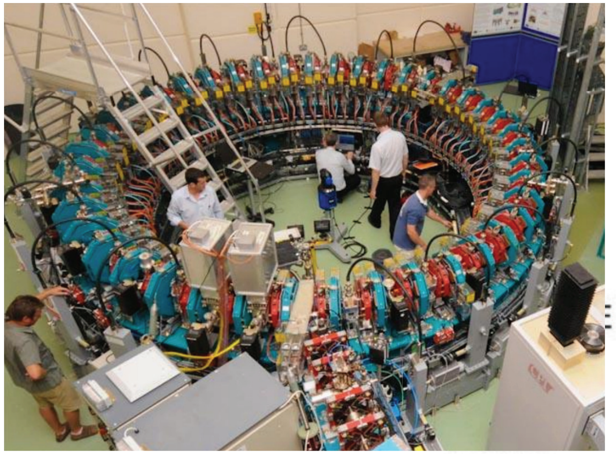
$$B_y = B_0\left(\frac{r}{r_0}\right)^k$$

ν = constant, $\nu_{rf}$ ≠ constant

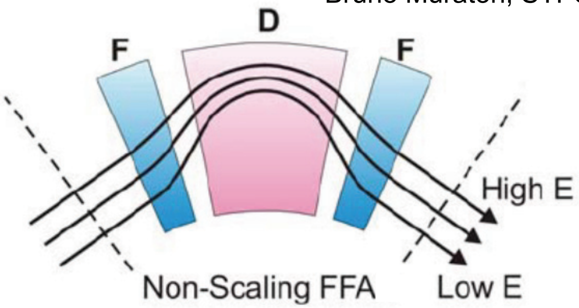
# Non-scaling FFAs

- the 2 conditions from above are relaxed
  - many desirable properties
    - smaller orbit excursions
    - less non-linearity
    - …

$\nu \neq$ constant, $\nu_{rf}$ = constant



Bruno Muratori, STFC



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# Previous Studies

PRL **97**, 104801 (2006) PHYSICAL REVIEW LETTERS week ending 8 SEPTEMBER 2006

**Fundamental Limit of Nonscaling Fixed-Field Alternating-Gradient Accelerators**

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*GSI, Darmstadt, D64291, Germany*
(Received 14 March 2006; published 6 September 2006)

*Systematic* nonlinear space-charge resonances may cause substantial emittance growth in the non-scaling fixed-field alternating-gradient (FFAG) accelerators. To avoid systematic nonlinear space-charge resonances, the phase advance of each nonscaling FFAG cell must avoid $\pi/2$ and $\pi/3$. Using multi-particle numerical simulations, we empirically obtain a minimum tune ramp rate vs the systematic 4th order space-charge resonance strength. We also find that the emittance growth obeys a simple scaling property when the betatron tunes cross the linear half-integer and sum resonances.

DOI: 10.1103/PhysRevLett.97.104801 PACS numbers: 29.27.−a, 41.75.−i, 52.59.Sa

**New Journal of Physics**
The open−access journal for physics

**Emittance growth mechanisms for space-charge dominated beams in fixed field alternating gradient and proton driver rings**

**S Y Lee[1,2,3], G Franchetti[2], I Hofmann[2], F Wang[1] and L Yang[1]**
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Published 28 November 2006
Online at http://www.njp.org/
doi:10.1088/1367-2630/8/11/291

- rms space-charge model to study emittance growth
- mechanisms due to systematic space-charge resonances and random linear errors in accelerators
- the systematic space-charge resonance that can be factorized into a geometric factor and a beam envelope function
- A lot of the synchrotron and cyclotron single particle theory is applicable

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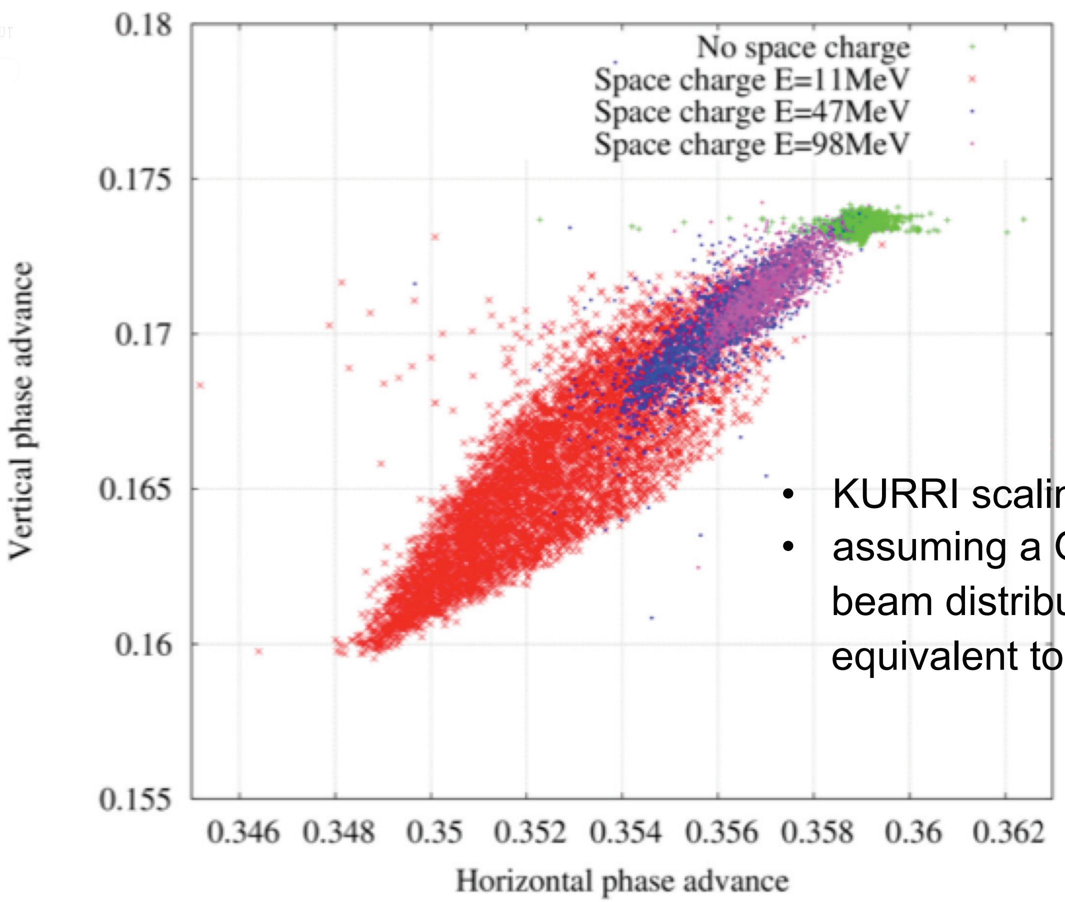
# Updated numerical tools

- **ZGOUBI** is a 3D particle tracking code that solves the non-linear equation of motion using truncated Taylor expansions of the field and its derivatives up to the 5th order
- recent **FFA frozen space-charge** works including algorithmic improvements to ZGOUBI is covered in Haj Tahar thesis, in greater detail
- the analytic expression of the particle distribution does not change, hence the analytical solution of the self-induced electric fields does not change.

Malek Haj Tahar. High power ring methods and accelerator driven subcritical reactor application. Nuclear Experiment [nucl-ex]. Université Grenoble Alpes, 2017. English. <NNT : 2017GREAY004>. <tel-01611525>




- KURRI scaling FFA:
- assuming a Gaussian beam distribution (2D) equivalent to a KV beam.

# COSY Infinity

- Fast Multipole Method (FMM) for Space Charge
- the method relies on an automatic multigrid-based decomposition of charges in near and far regions together with the use of high-order differential algebra
- given an ensemble of N particles, the method allows the computation of the self-fields of all particles on each other with a computational expense that scales as O(N)
- rigorous estimates are obtained using remainder-enhanced DA methods. All high-order multipoles of the space charge fields are also available, necessarily for the computation of high-order transfer maps and all resulting aberrations
- some space charge FFA modeling is ongoing but not yet published

# Illustration of the FMM kernels

P2M (Particle-to-Multipole),

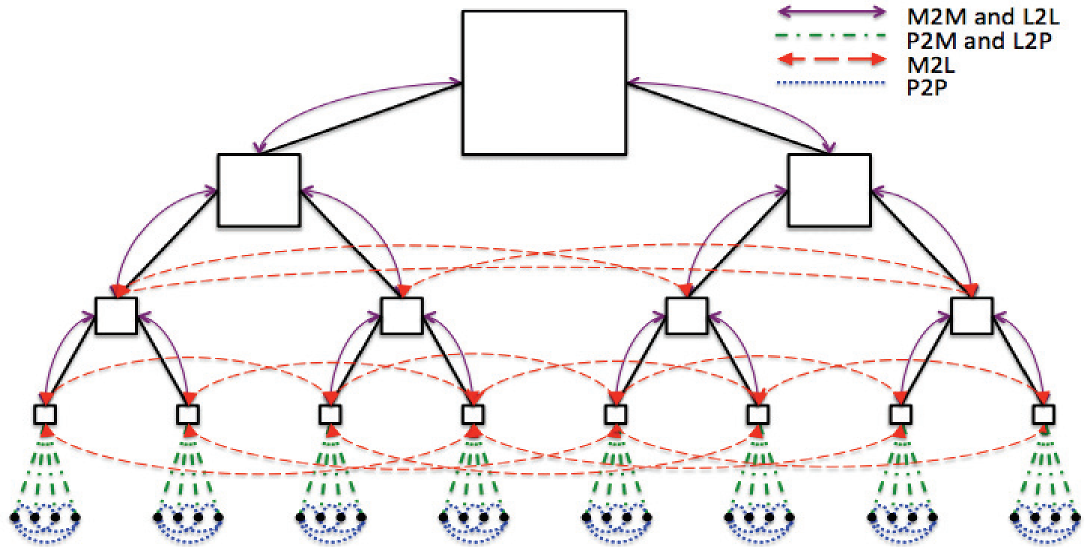
M2M (Multipoleto-Multipole),

M2L (Multipole-to-Local),

L2L (Local-to-Local),

L2P (Local-to-Particle), and

P2P (Particle-to-Particle).

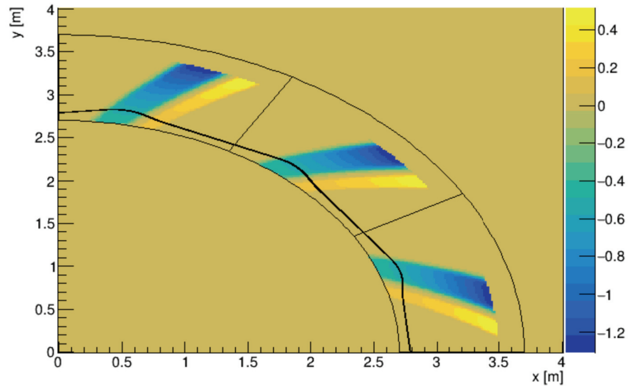


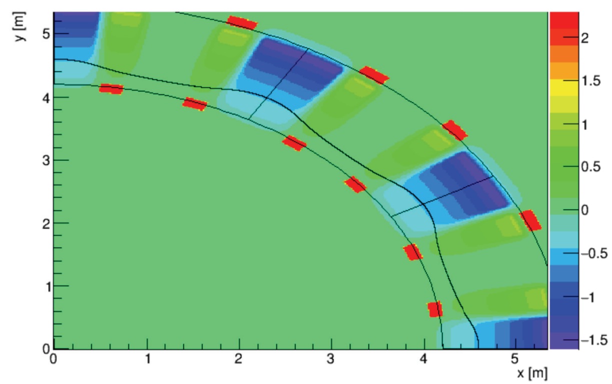

arXiv:1608.02461

# OPAL (open-source https://gitlab.psi.ch/OPAL/src/wikis)

Two methods to model FFAs in OPAL

1. tracking through an analytical field model
2. tracking through a 3D field map




### Acceleration

```
freq:   POLYNOMIAL_TIME_DEPENDENCE, P0= , P1, …
amp:    POLYNOMIAL_TIME_DEPENDENCE, P0= , P1, …
phase:  POLYNOMIAL_TIME_DEPENDENCE, P0= , P1, …
```

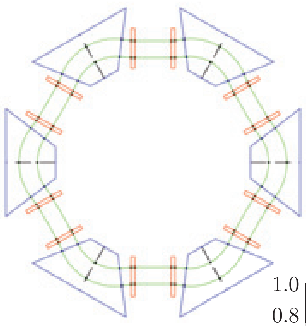
### Space Charge

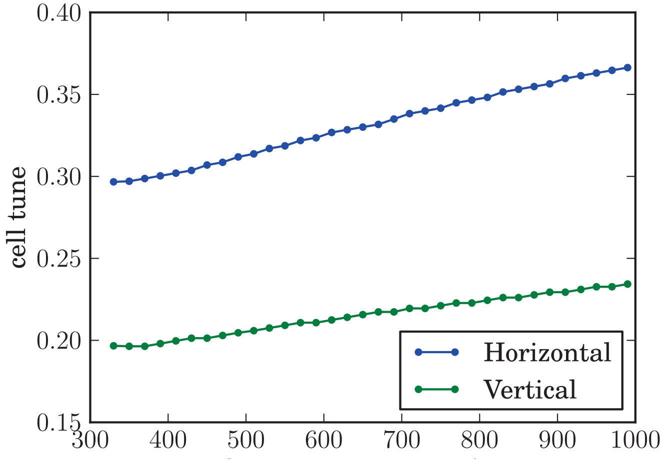
- 3D, FFT, finite difference, AMR (soon)
- massive parallel -> 1:1 possible
  - access **to 4 to 5 σ**
- search for matched distribution with linear sc.
- neighboring bunches

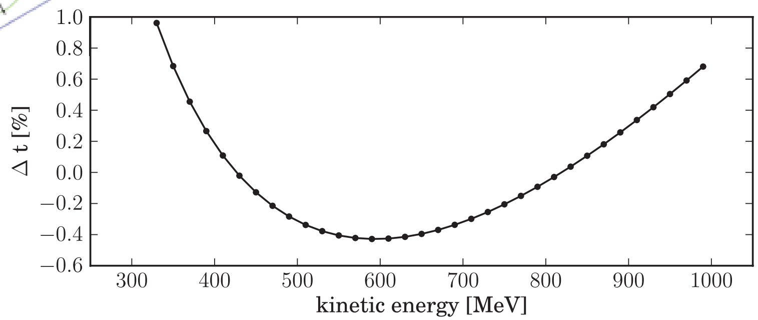
# 1 GeV ADS ring

arXiv:1310.3588

- non-scaling FFA design may be able to support a continuous (CW) beam with far lower peak current than the pulsed alternative.
- a 6-cell non-scaling FFA
- ~ isochronous
- full 3D space charge

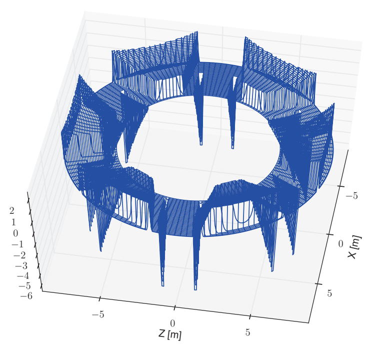


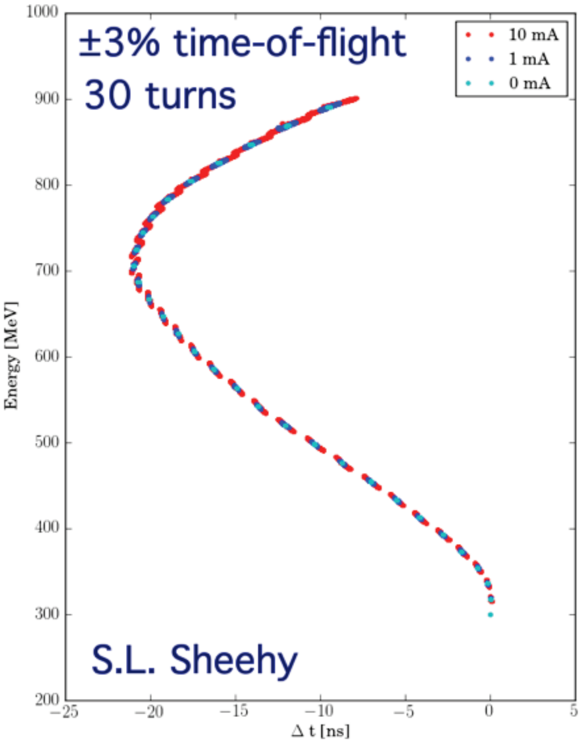



| | 330 MeV | 500 MeV | 1000 MeV |
|---|---|---|---|
| Avg. Radius [m] | 5.498 | 6.087 | 7.086 |
| $\nu_x/\nu_y$ (cell) | 0.297/0.196 | 0.313/0.206 | 0.367/0.235 |
| Field F/D [T] | 1.7/-0.1 | 1.8/-1.9 | 1.9/-3.8 |
| Magnet size F/D [m] | 1.96/0.20 | 2.79/0.20 | 4.09/0.20 |

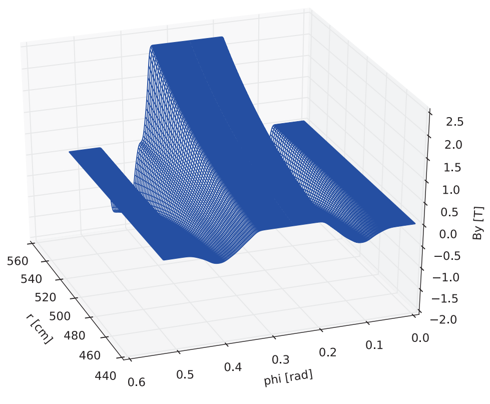
- we used to use CYCLOTRO flavor of OPAL
- this needed a 3D, 360 degree field map
- at that time: only allowed fixed frequency RF
  - single energy studies
  - and serpentine channel
- emittance variation at fixed energy of 330 MeV
- 100 mm long beam at



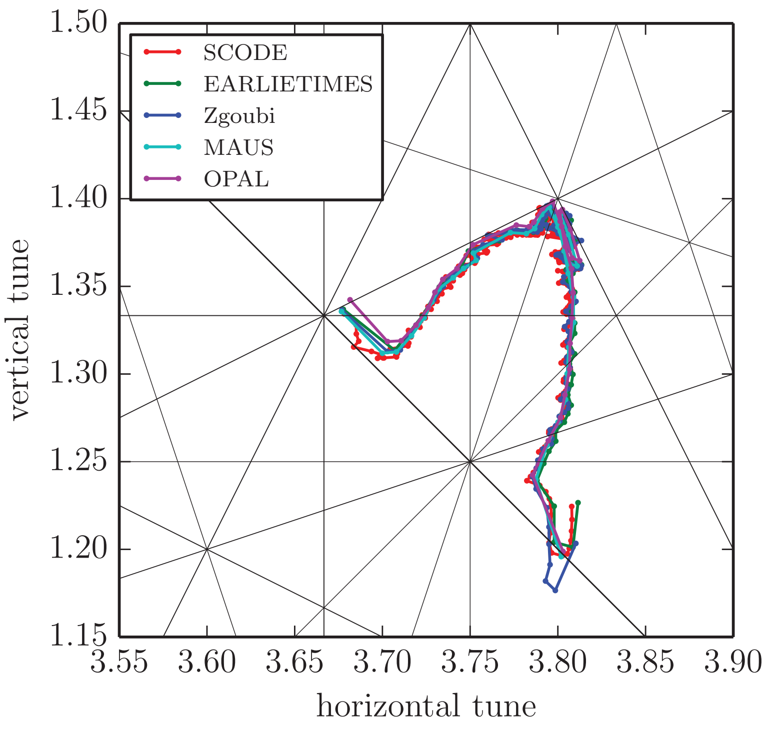



# Ideal KURRI scaling FFA (11-150 MeV)

- previous benchmarking done using real KURRI-FFA field maps. In this case, we wanted a simple model to study dynamics
- generated 3D field maps using Francois' new module in ZGOUBI
- imported to OPAL and other codes, find closed orbits and tune map




- next step: update to SBEND3D model (using Chris' modules) and incorporate variable frequency RF…
- continue tracking & understand matching/ SC modeling
- re-introduce 'real' field maps



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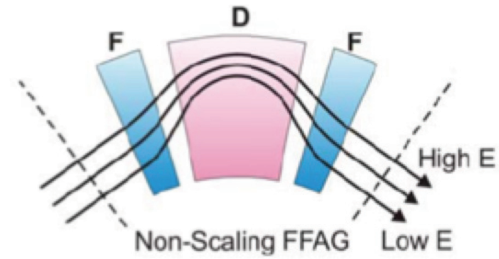
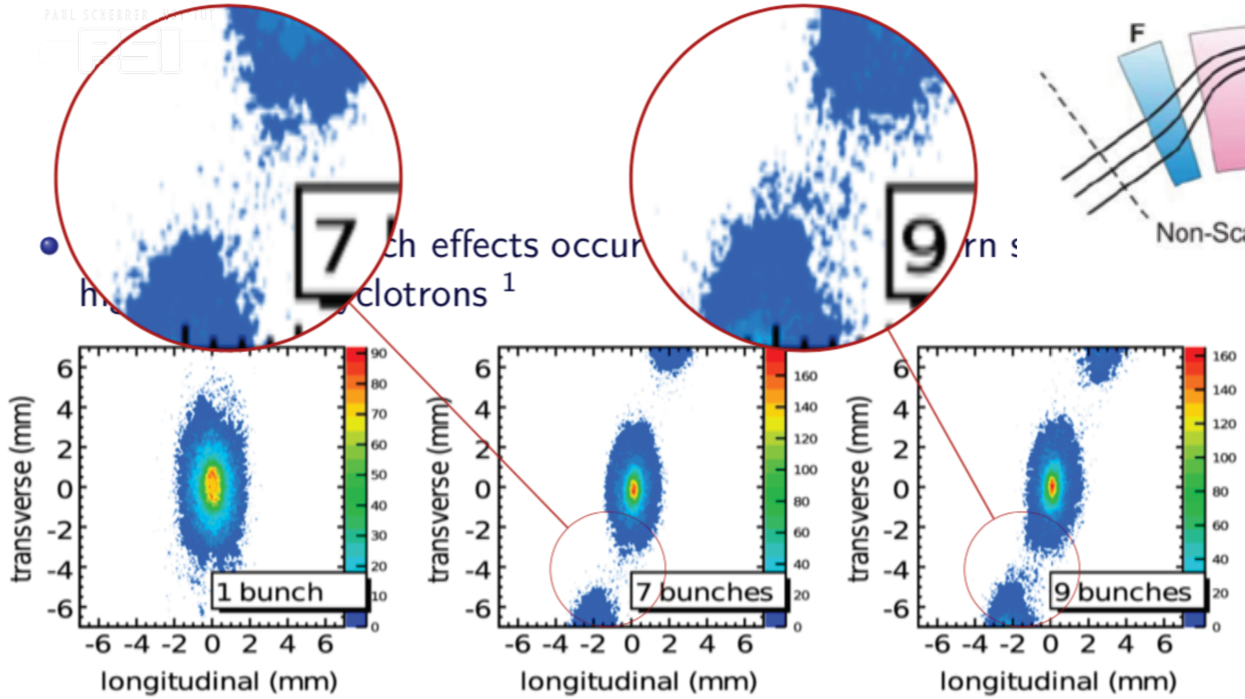
# Final Remarks on MW FFAs and SC Modeling

- MW type of machines FFAs, Cyclotrons etc. are limited by losses only
  - ➢ need to have a full scale mode
    - ◆ start-to-end
    - ◆ full rf-program
    - ◆ 1:1 possible access to 4 to 5 σ of the distribution
    - ◆ for a technical design**, parameter studies** incl. collimator placing is important
    - ◆ multi-objective optimization

- the open-source tool OPAL is toward these capabilities (~ 12 developers)
  - ➢ massive parallel
  - ➢ modern numerical methods
  - ➢ multi-objective optimization
  - ➢ very special FFA capabilities

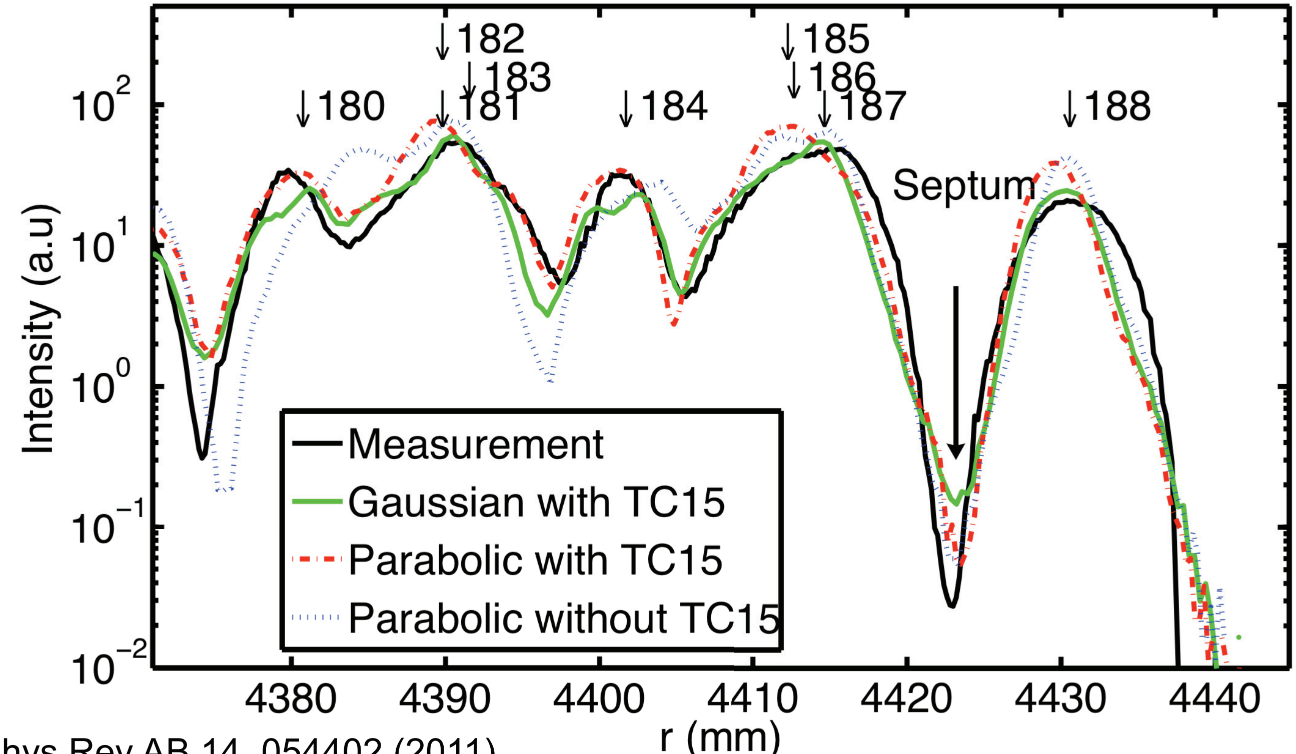



- ch effects occur rn s
h clotrons [1]




[1] Yang, J. J., Adelmann, A., Humbel, M., Seidel, M., and Zhang, T. J. (2010). Beam dynamics in high intensity cyclotrons including neighboring bunch effects: Model, implementation, and application. Phys. Rev. ST Accel. Beams, 13:064201.

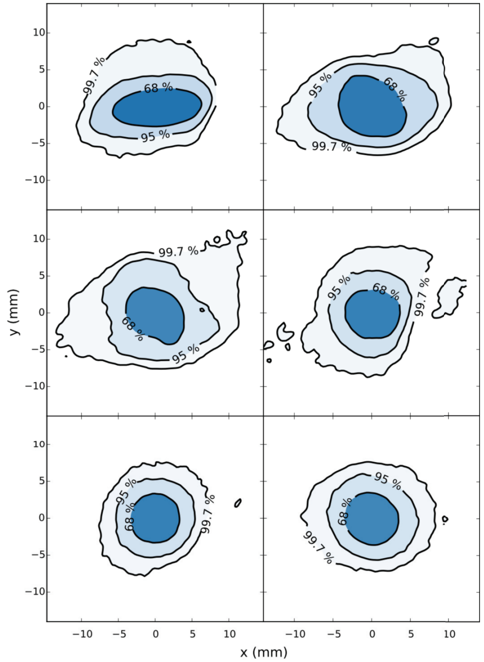
# 4 Orders of Magnitude in dynamical range

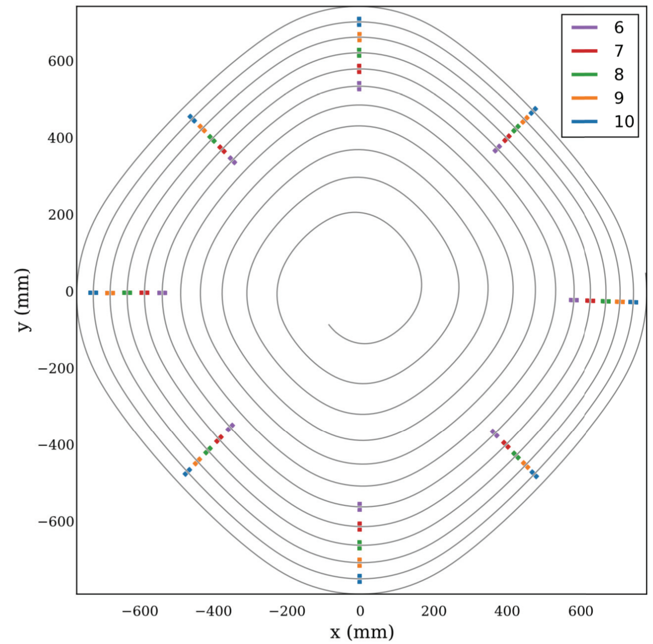



Y. Bi et al. Phys Rev AB 14, 054402 (2011)

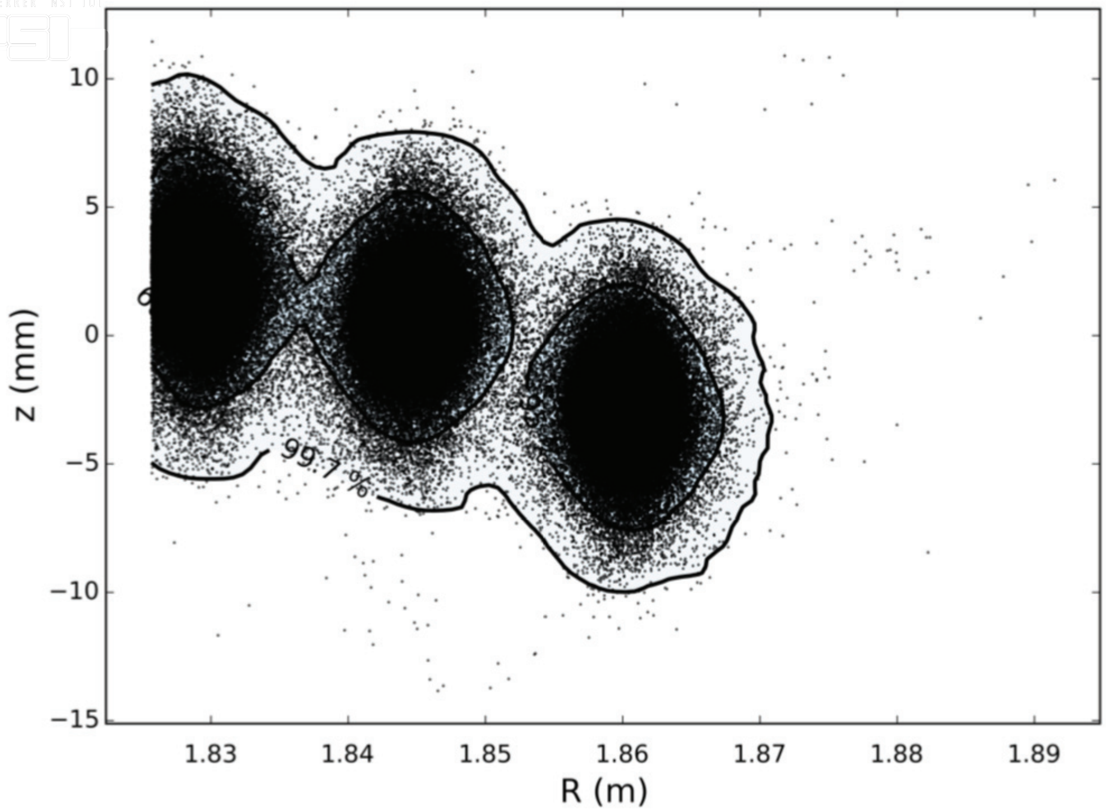
# From IsoDAR Modeling effort relevant for non-scaling FFAs

- placing collimators
- adjusting collimators
- multi objective optim.




Jakob Jonnerby, ETH MSc thesis 2016




Jakob Jonnerby, ETH MSc thesis 2016