Classification of Space-Charge Resonances and Instabilities

Dong-O Jeon
Institute for Basic Science

HB2018
June 21, 2018

Drowned in a swamp of terms...?

Space-charge mechanisms

- There are two families of space-charge mechanisms, and yet they need to be differentiated: instabilities and resonances.

- Instabilities: a.k.a. parametric resonances, coherent resonances, coherent instabilities, parametric instabilities …
- Resonances: a.k.a. (single) particle resonances, incoherent resonances …

- Both families are loosely called “resonances”.
- Many names for the same thing … → confusing even to experts.
- It is beneficial to differentiate the two families of mechanisms.

Instabilities

- Instabilities of a KV distribution were reported in the early literatures, and the 2nd order instability is widely known as "the envelope instability".

- These instabilities of the beam envelope are also called parametric resonances.
- They are parametric resonances of the envelope equation:

$$x'' + k(s)x - \frac{\varepsilon^2}{x^3} - \frac{K(s)}{x} = 0$$

where x is the beam envelope not the particle coordinate.

- They are parametric resonances of the beam envelope.
- Are they resonances of the beam particle? No.

Resonances

- Resonances are well known in circular accelerators. In fact, they are resonances of the beam particle.

- Particle resonances were discovered in high intensity linear accelerators in 2009.
- described by a particle Hamiltonian.

- Space-charge resonances and instabilities may look alike in the phase space!

Resonances

- Resonances are well known in circular accelerators.
  In fact, they are resonances of the beam particle.

- Particle resonances were discovered in high intensity linear accelerators in 2009.
- described by a particle Hamiltonian.

- Space-charge resonances and instabilities may look alike in the phase space!

- However, there is a fundamental difference between resonances and instabilities!!

What is the difference?

Instabilities (or parametric resonances) of **beam envelope** ⇨ No resonance frequency component

Instabilities of the beam envelope → no fixed points in phase space

- Instability of KV distribution was first found by Haber (1979).
- Instabilities of envelope equation were studied analytically by Hofmann et al (1983).
  - 2^{nd}, 3^{rd}, 4^{th} order envelope instabilities have been observed.

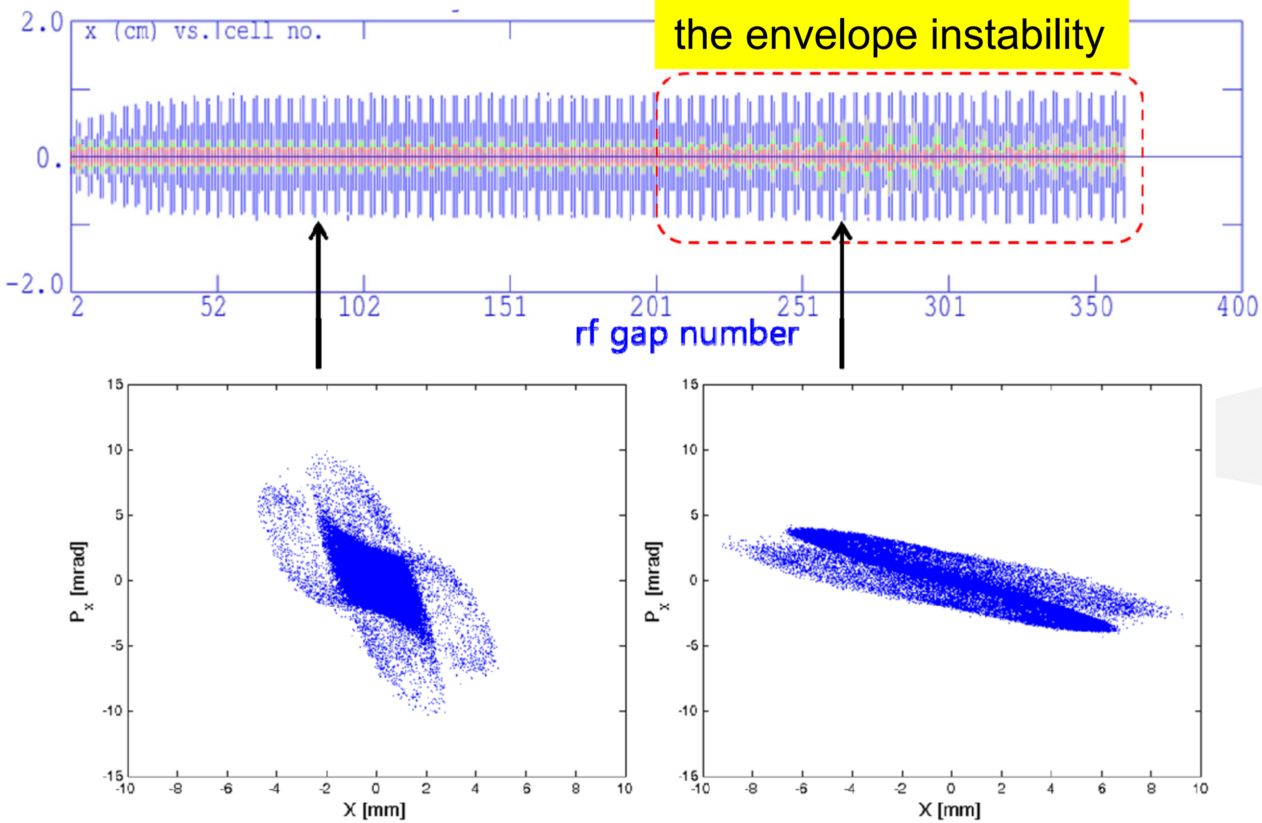
Resonances (or particle resonances) of **beam particle** ⇨ Yes resonance frequency component

Resonances of the beam particle → fixed points in phase space

- $4\sigma = 360^\circ$ 4^{th} order resonance was found by Jeon et al (2009) and verified experimentally by Groening et al (2009).
- $6\sigma = 720^\circ$ 6^{th} order resonance was found (2015).
- 8^{th}, 10^{th} order resonances were found by Hofmann (2016).

Instabilities of the beam envelope
a.k.a. parametric resonances or envelope instabilities

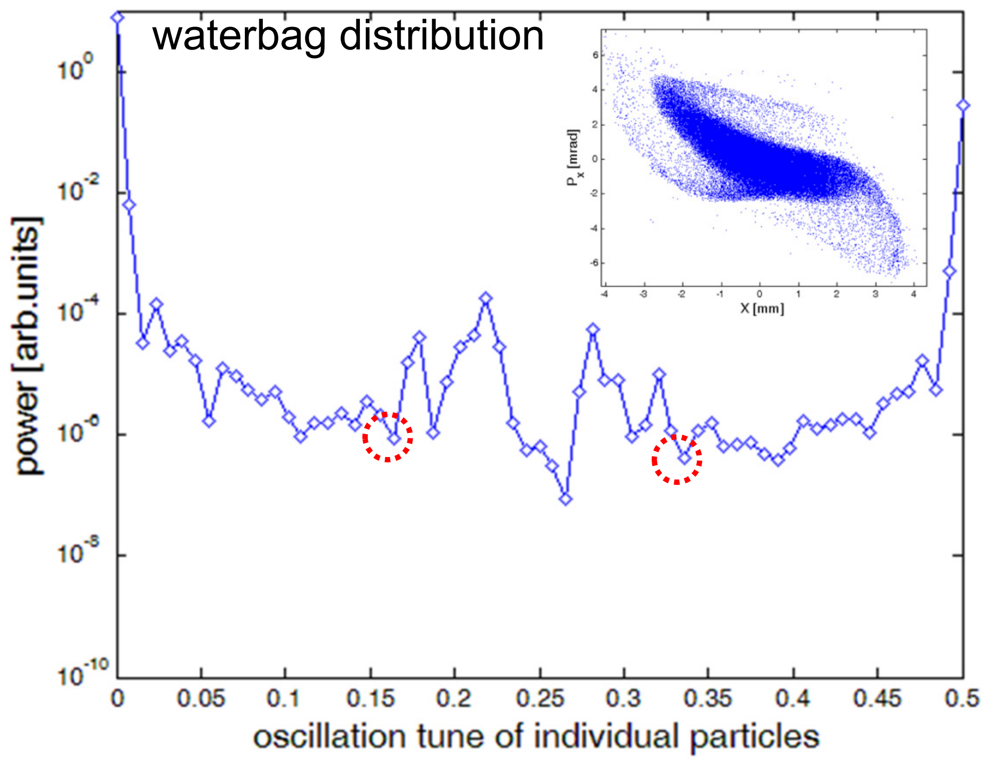
2^{nd} order envelope instability
for high intensity linear accelerators



- $2\sigma_o - \Delta\sigma_{2,coh}$ = 180° second order instability for a constant- σ_o lattice with σ_o = 100° and σ = 70° with Gaussian distribution.
- Observed for KV, Gaussian, waterbag distributions.
- The envelope instability is excited following the 4^{th} order resonance for a constant-σ_o lattice.

3^{rd} order envelope instability
for high intensity linear accelerators

Jeon et al., NIM A **832** (2016) 43

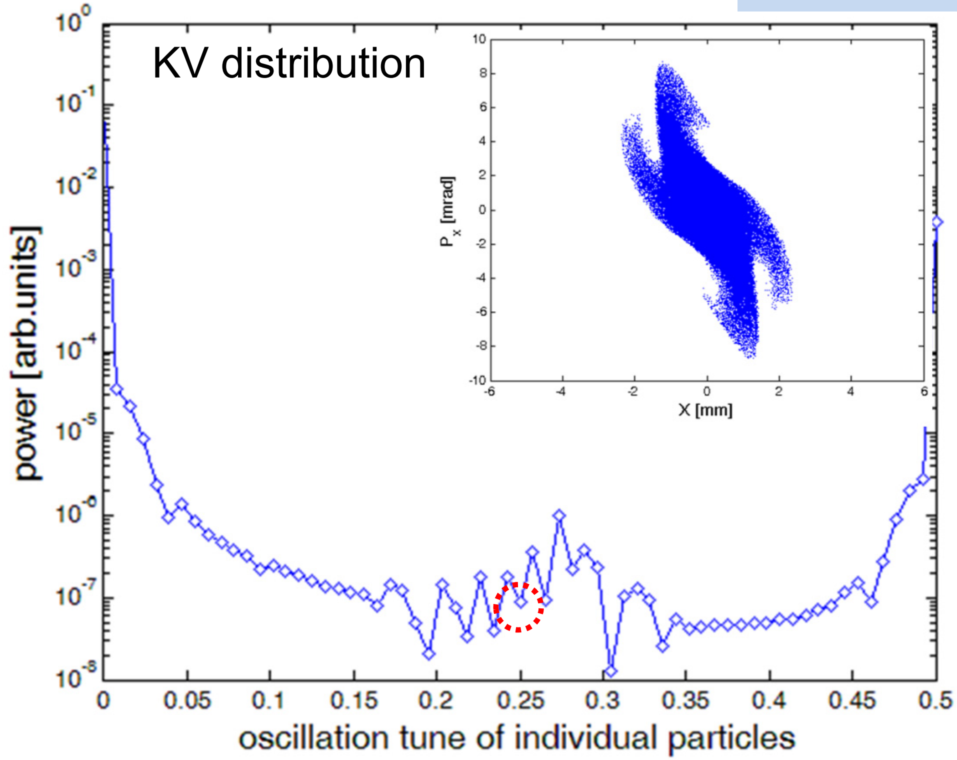


- $3\sigma_o - \Delta\sigma_{3,coh} = 180°$ third order instability for a constant- σ_o lattice $\sigma_o = 92°$ and $\sigma = 40°$ (90 mA beam).
- Observed for KV and waterbag distributions, but no for Gaussian distribution.
- Not a resonance: no resonance peaks around 1/3 or 1/6 in the FFT spectrum.

Institute for Basic Science

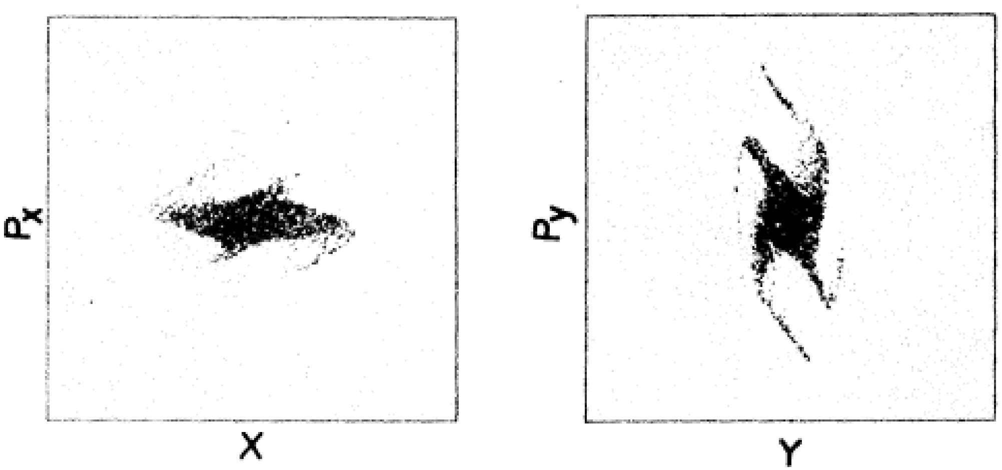
4^{th} order envelope instability
for high intensity linear accelerators

Jeon, J Korean Phys Soc **72**, 1523 (2018)

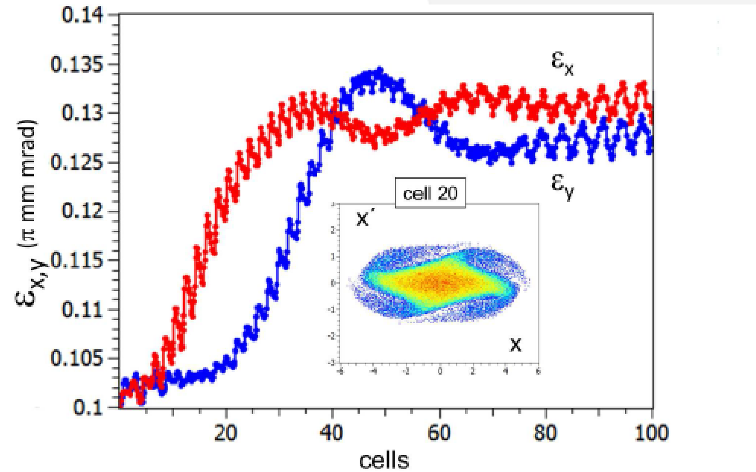


- $4\sigma_o - \Delta\sigma_{4,coh} = 2\cdot180°$ fourth order instability for a lattice with $\sigma_o = 112°$ and $\sigma = 85°$
- Observed only for a KV distribution.
- Not a resonance: no resonance peak around 1/4 = 90°/360 in the FFT spectrum.

4th order envelope instability
for high intensity linear accelerators



Courtesy of Haber and Maschke,
PRL **42**, 1479 (1979)
KV distribution
σ_o = 90° and σ = 30°



Courtesy of Hofmann (HB2016)
Waterbag distribution
σ_o = 70° and σ = 35°

- $4\sigma_o - \Delta\sigma_{4,coh}$ = 180° fourth order instability
- Observed for KV and waterbag distributions.

Applying KV instabilities to non-KV beams

- Beam envelope equation was derived for a KV distribution by Kapchinskij and Vladmirskij.
- The envelope equation was extended to any charge distribution with elliptical symmetry by Sacherer, noting that second moments of any particle distribution ⟺ linear part of the force.
- Vlasov-Poisson-equation approach relying on a KV distribution is also subject to similar limitations.

- One-to-one correlation between instabilities of KV and non-KV distributions may be limited .
- The 3rd and 4th instabilities have been observed only for waterbag distributions (non-KV).
- No high order instabilities have been observed for Gaussian distributions.
- This suggests the possibility that high order instabilities may not be observable for real beams.

Instabilities

- Beam envelope becomes identical to itself when the particle makes 180° phase advance.
- → Instability condition is $m\sigma_o - \Delta\sigma_{m,coh} = n180°$.
- → Mathieu-type instabilities.
- Called "half integer resonance" by some.

- But half integer resonances known in circular accelerators are $2\sigma = n360^o$.
- Particle resonance condition $m\sigma = n360^o$ comes from the Fourier expansion of the Hamiltonian.

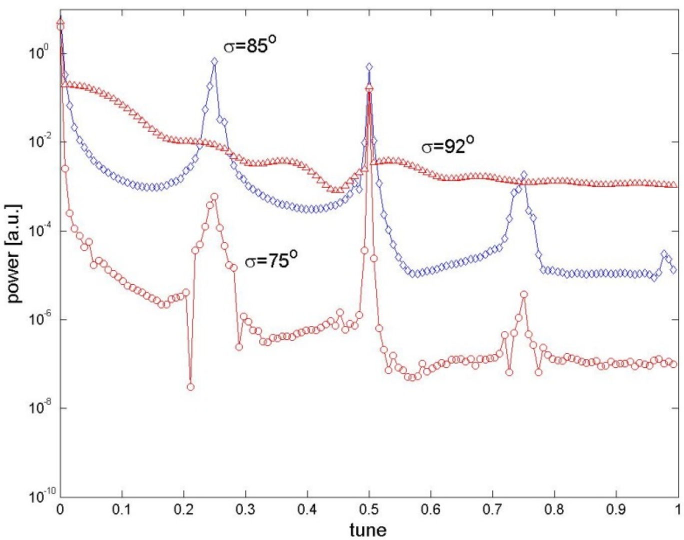
- Terminologies of two different worlds got mixed.

Resonances of the beam particle
a.k.a. (single) particle resonances,
incoherent resonances

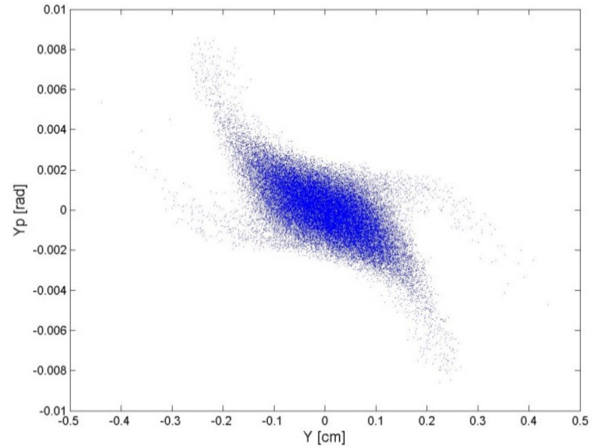
4^{th} order resonance
Prediction of the resonance

Jeon et al, PRST-AB **12**, 054204 (2009)

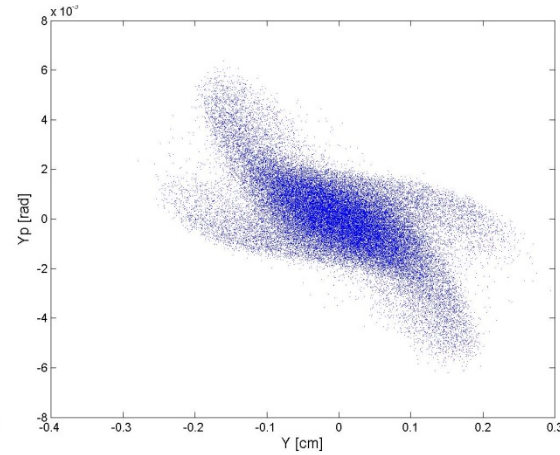
Frequency Spectrum



Cross from below



Cross from above



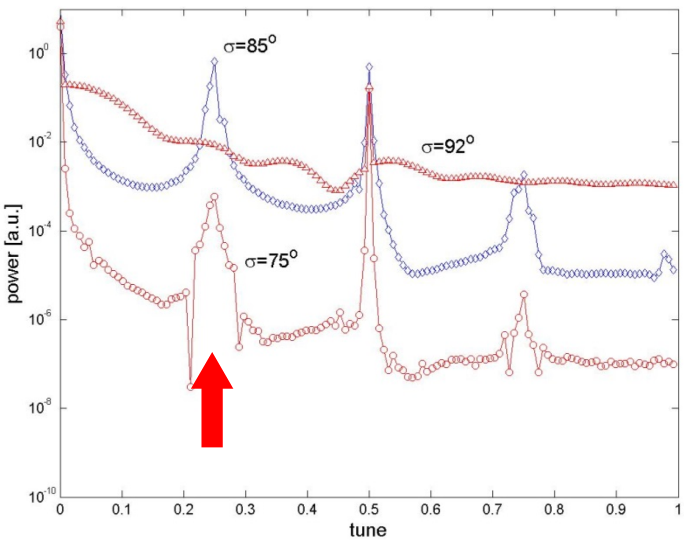
- The 4^{th} order resonance of the beam particle was discovered in high-intensity linear accelerators in 2009.

- Stable fixed points do exist and their properties are observed.
  - The **resonant frequency component is observed** at the tune 1/4 = 90°/360°.
  - Behavior difference depending on whether to cross the resonance “from above” or “from below” due to stable fixed points.

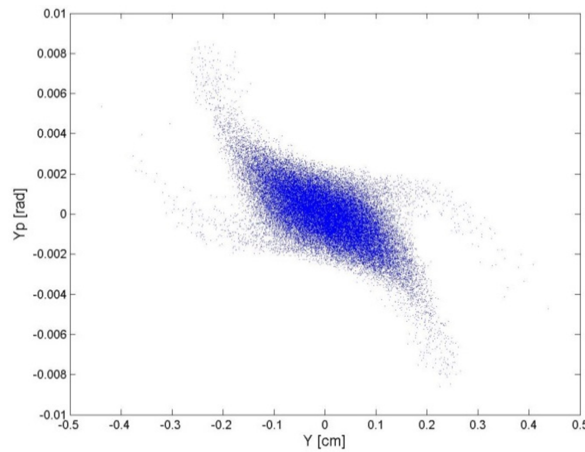
4^{th} order resonance
Prediction of the resonance

Jeon et al, PRST-AB **12**, 054204 (2009)

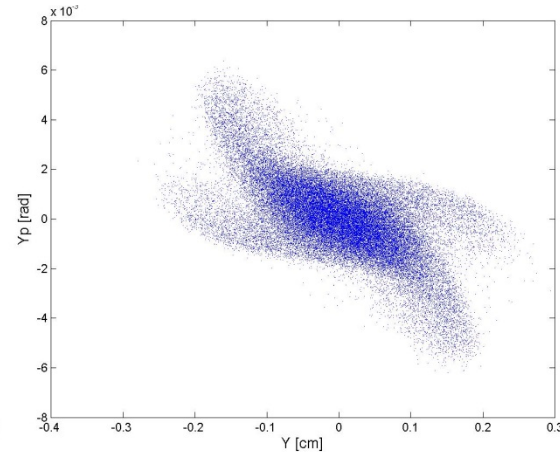
Frequency Spectrum



Cross from below



Cross from above

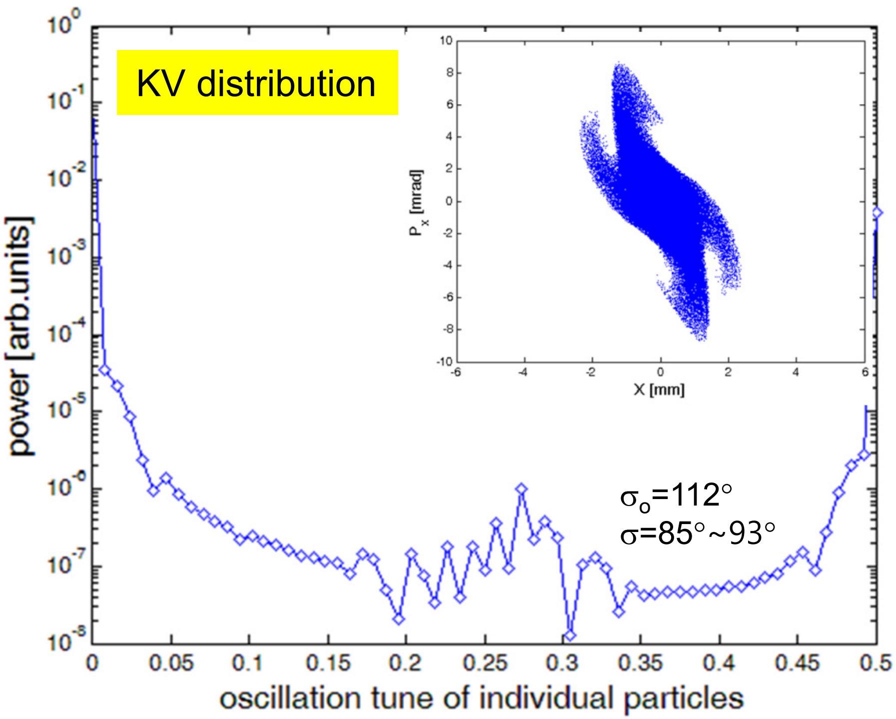


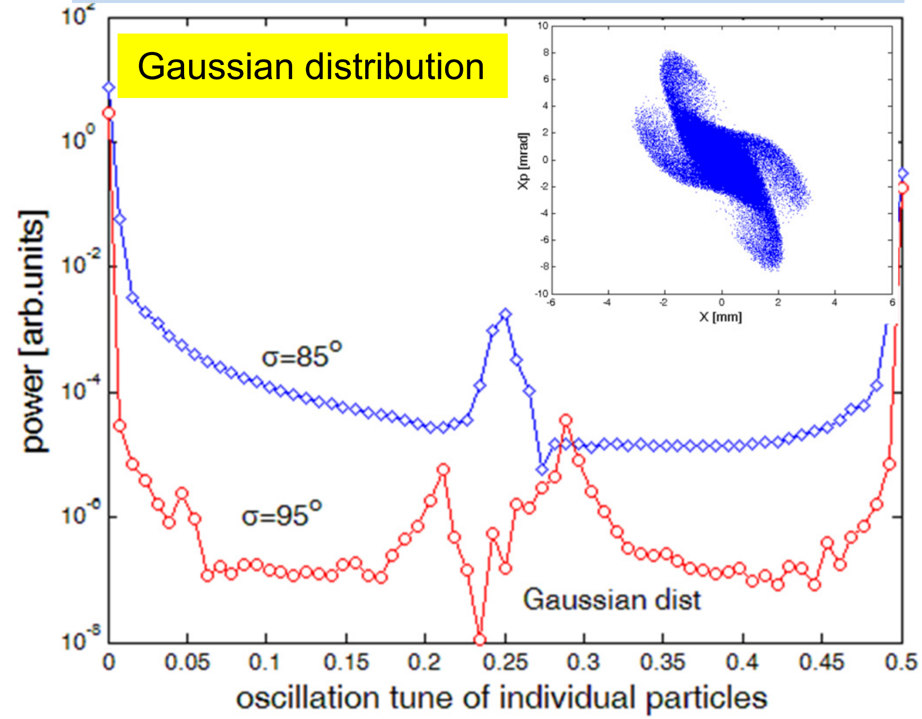
- The 4^{th} order resonance of the beam particle was discovered in high-intensity linear accelerators in 2009.

- Stable fixed points do exist and their properties are observed.
  - The **resonant frequency component is observed** at the tune 1/4 = 90°/360°.
  - Behavior difference depending on whether to cross the resonance "from above" or "from below" due to stable fixed points.

Appearance may be deceiving!

Jeon, J Korean Phys Soc **72**, 1523 (2018)





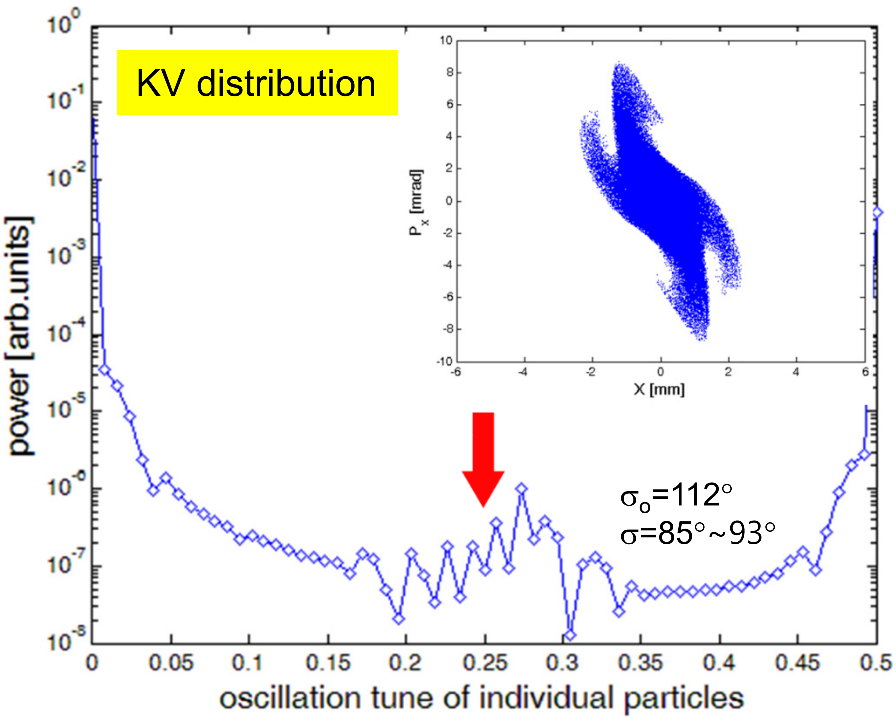
4^{th} order envelope instability
KV distribution

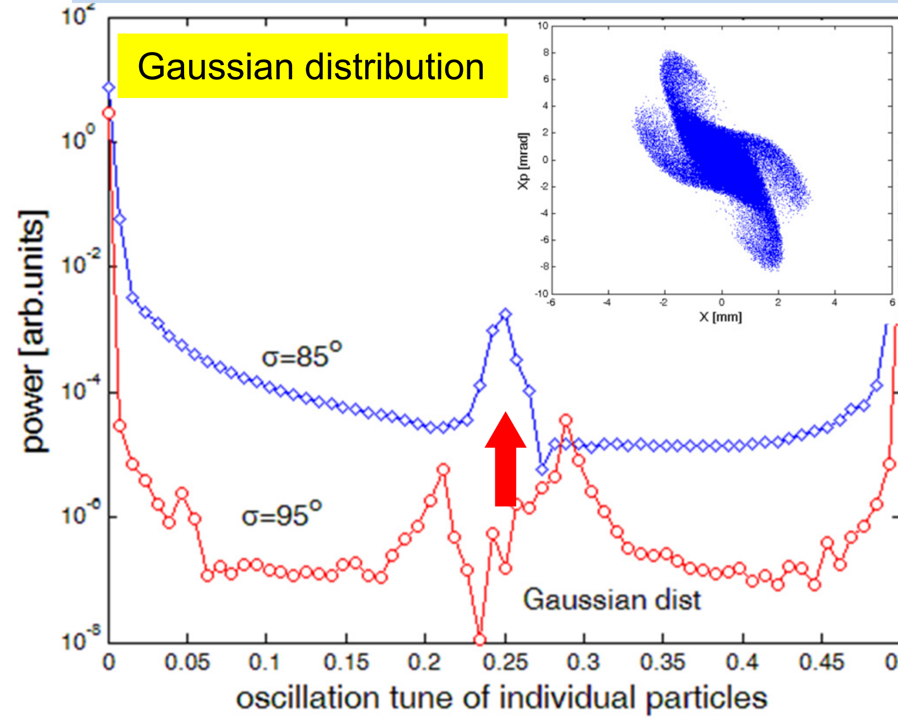
4^{th} order resonance
Gaussian distribution

- Instability and resonance, their appearances in the phase space may look alike. But they are completely different mechanisms.
- No resonance frequency component is observed for the 4^{th} order instability of a KV distribution.

Appearance may be deceiving!

Jeon, J Korean Phys Soc **72**, 1523 (2018)





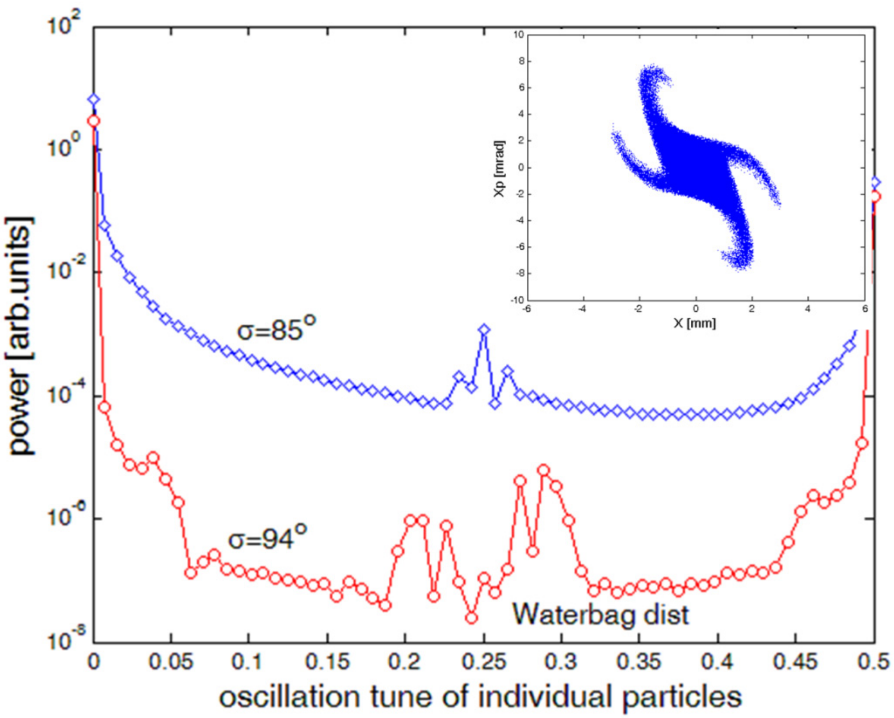
4th order envelope instability
KV distribution

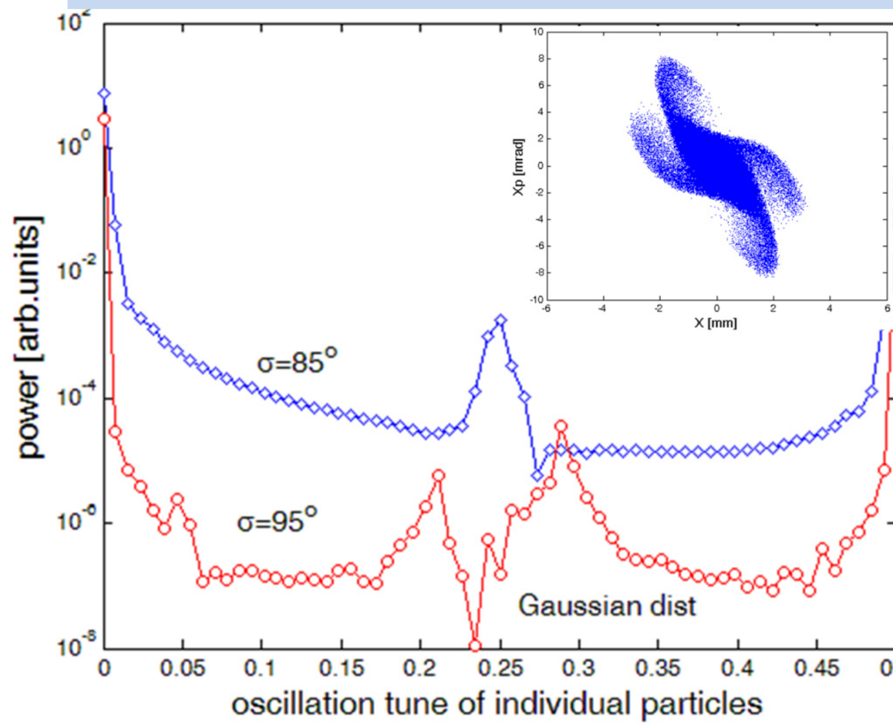
4th order resonance
Gaussian distribution

- Instability and resonance, their appearances in the phase space may look alike. But they are completely different mechanisms.
- No resonance frequency component is observed for the 4th order instability of a KV distribution.

Resonance frequency peak

Jeon, J Korean Phys Soc **72**, 1523 (2018)





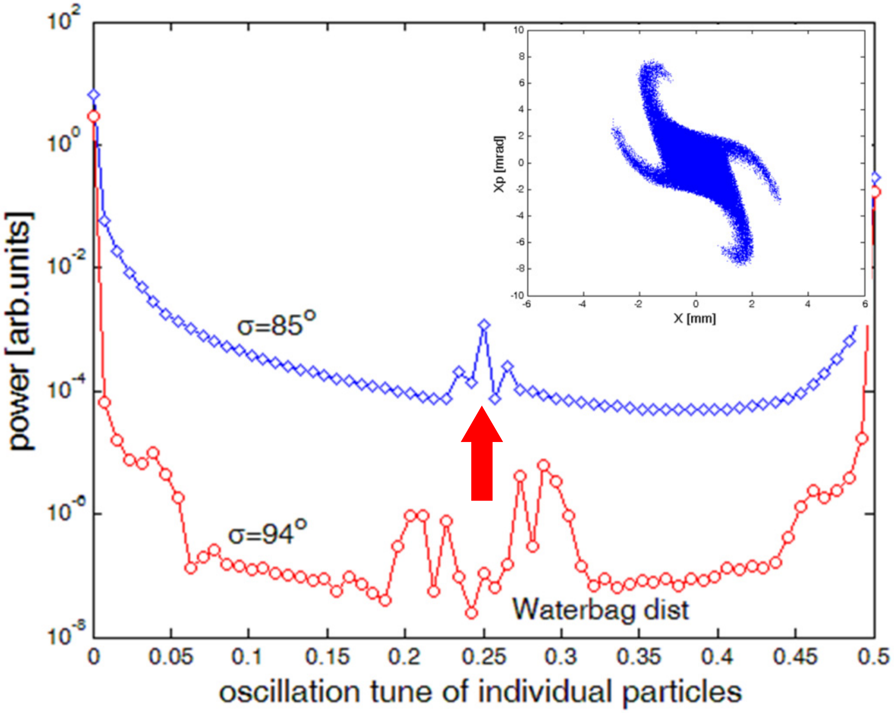
4th order resonance
Waterbag distribution

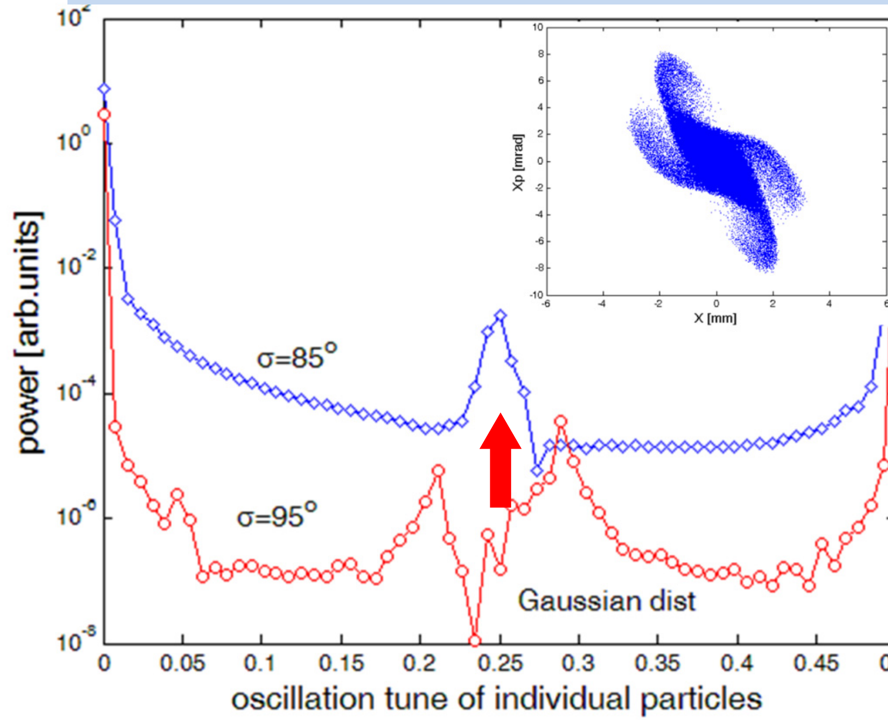
4th order resonance
Gaussian distribution

- Clear resonance frequency peak at 1/4 = 90°/360° is observed for non-KV beam distributions.
- The 4th order resonance was **verified in the two experiments**.

Resonance frequency peak

Jeon, J Korean Phys Soc **72**, 1523 (2018)





4^{th} order resonance
Waterbag distribution

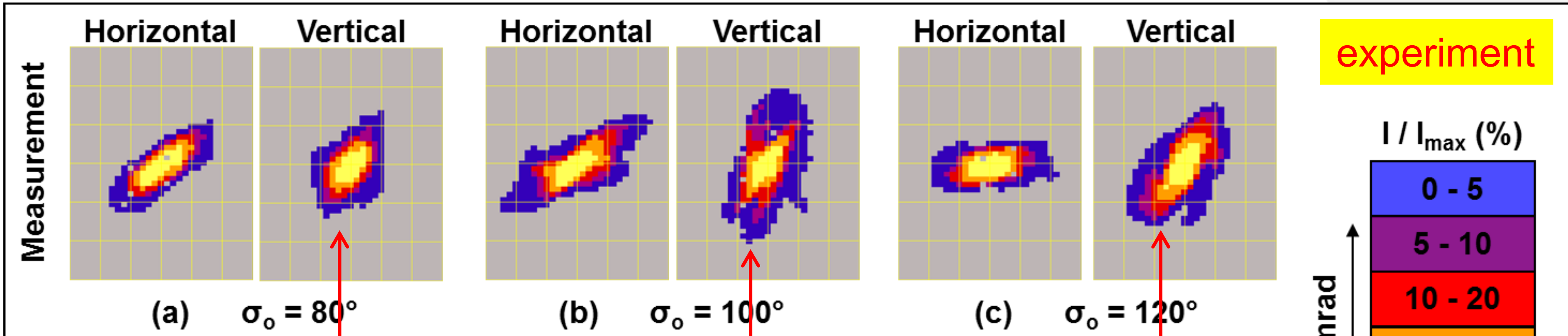
4^{th} order resonance
Gaussian distribution

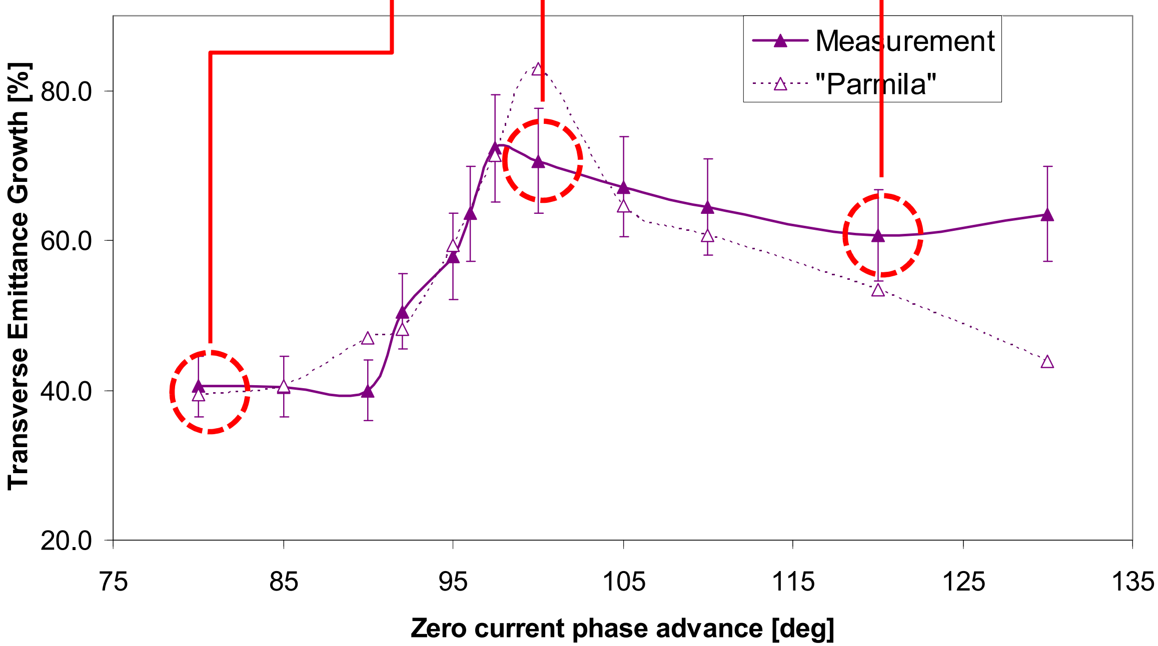
- Clear resonance frequency peak at 1/4 = 90°/360° is observed for non-KV beam distributions.
- The 4^{th} order resonance was **verified in the two experiments**.

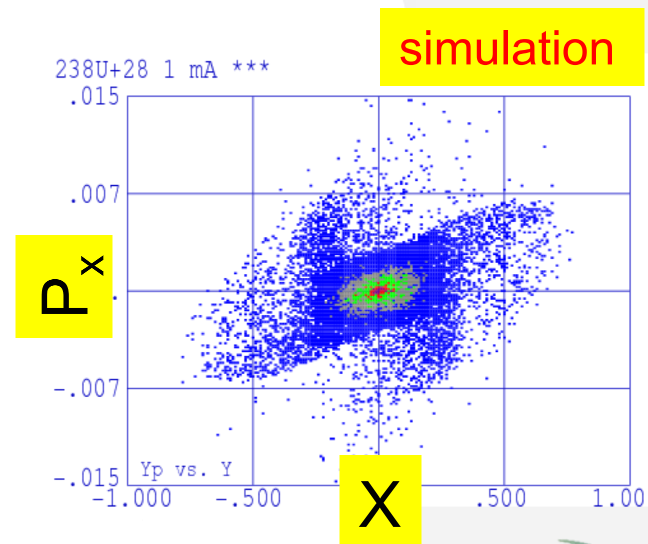
Experiment 1 of the 4th order resonance

using GSI UNILAC

Groening et al., PRL **102**, 234801 (2009)

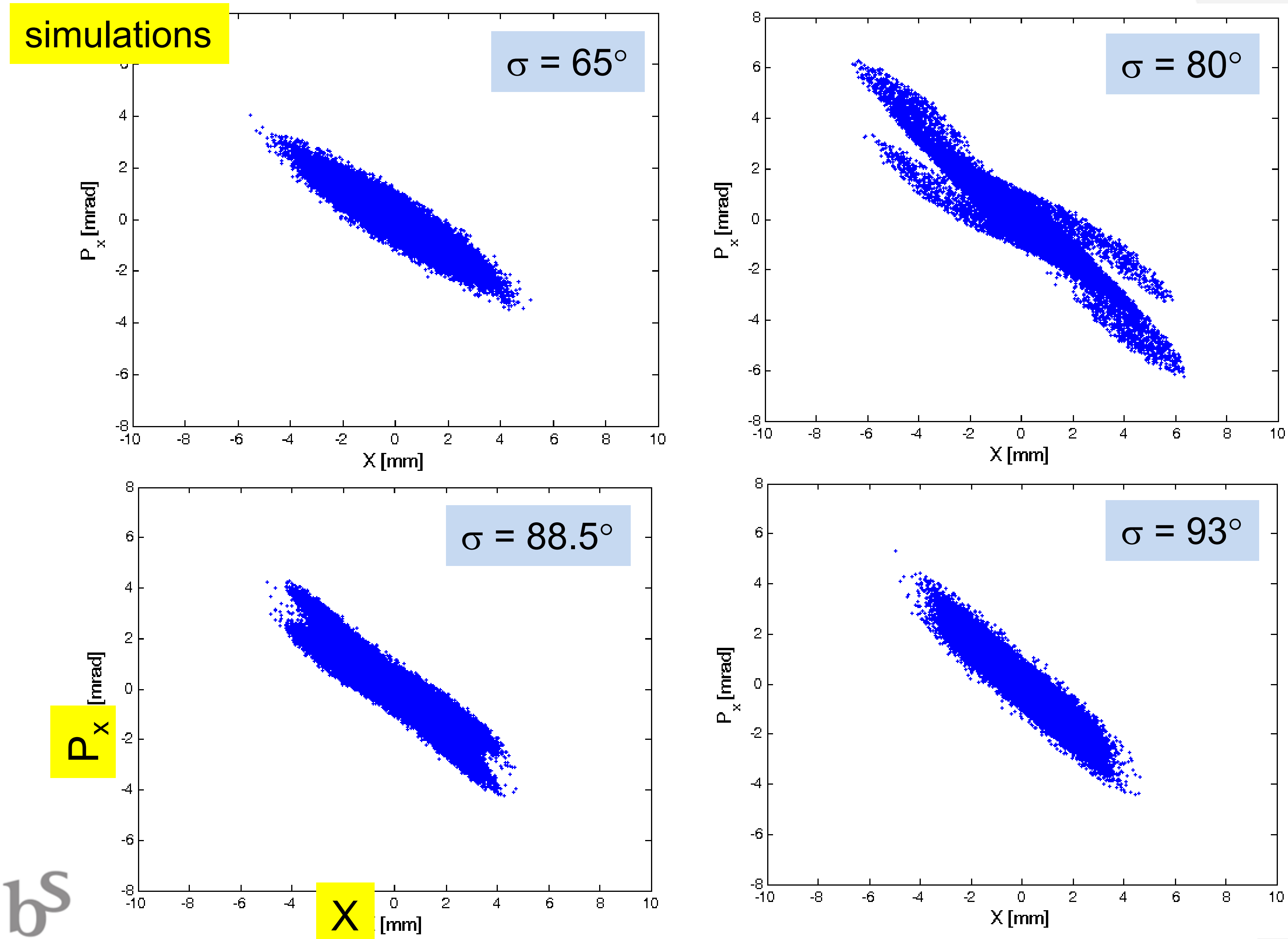






Experiment 2 of the 4th order resonance

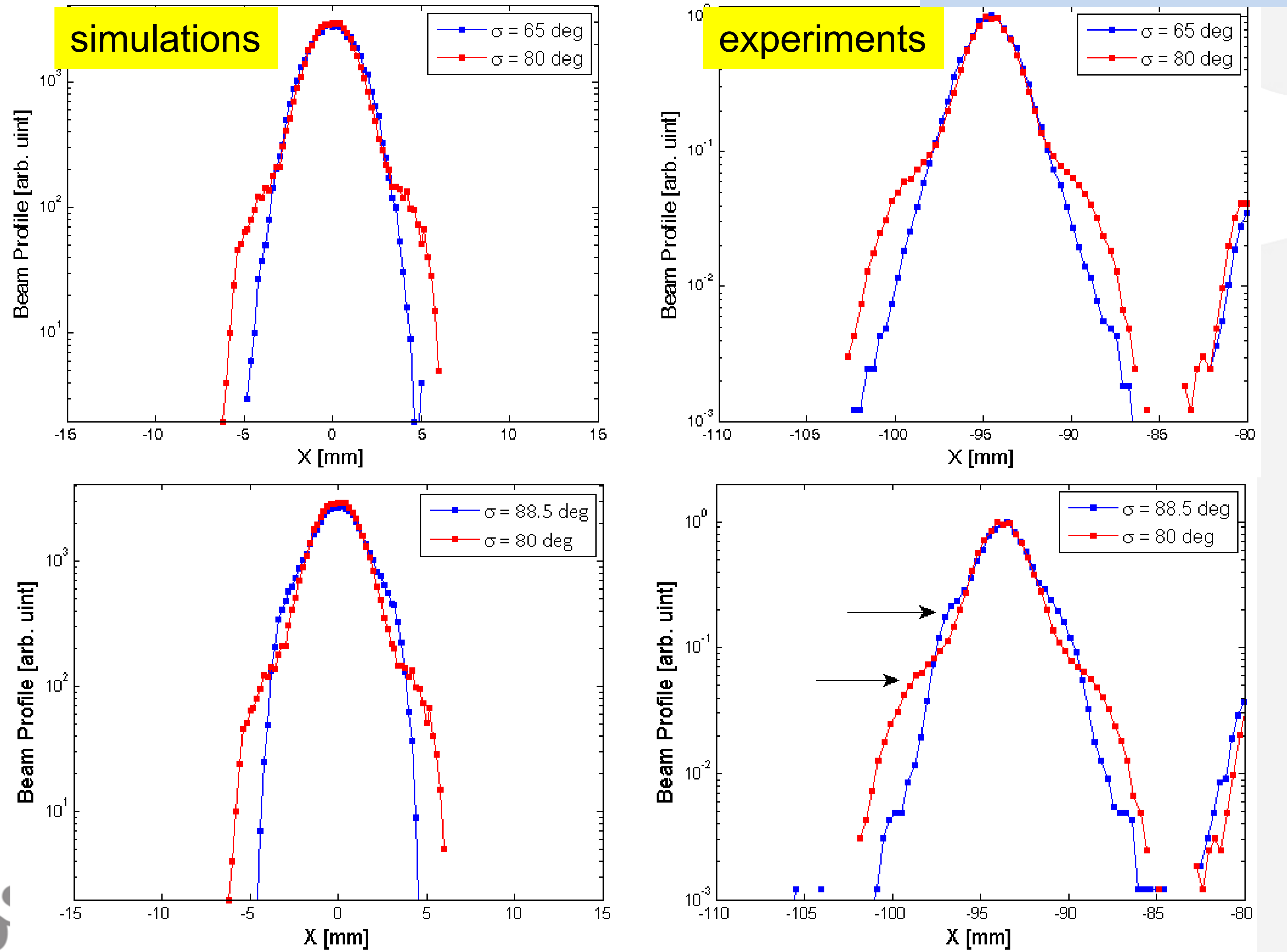
SNS linac, Simulations



Experiment 2 of the 4th order resonance

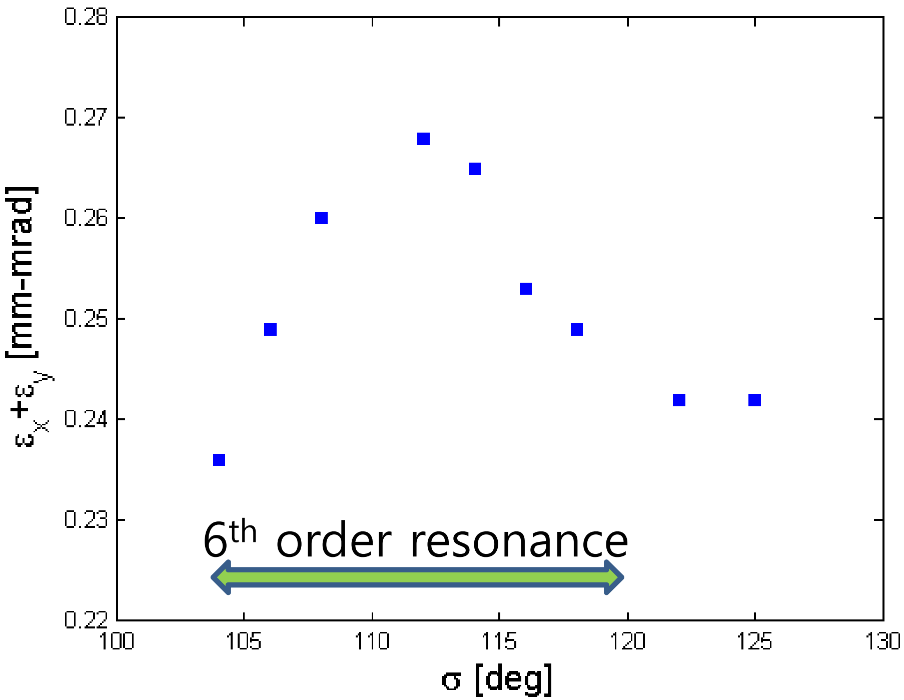
SNS linac, Experiment

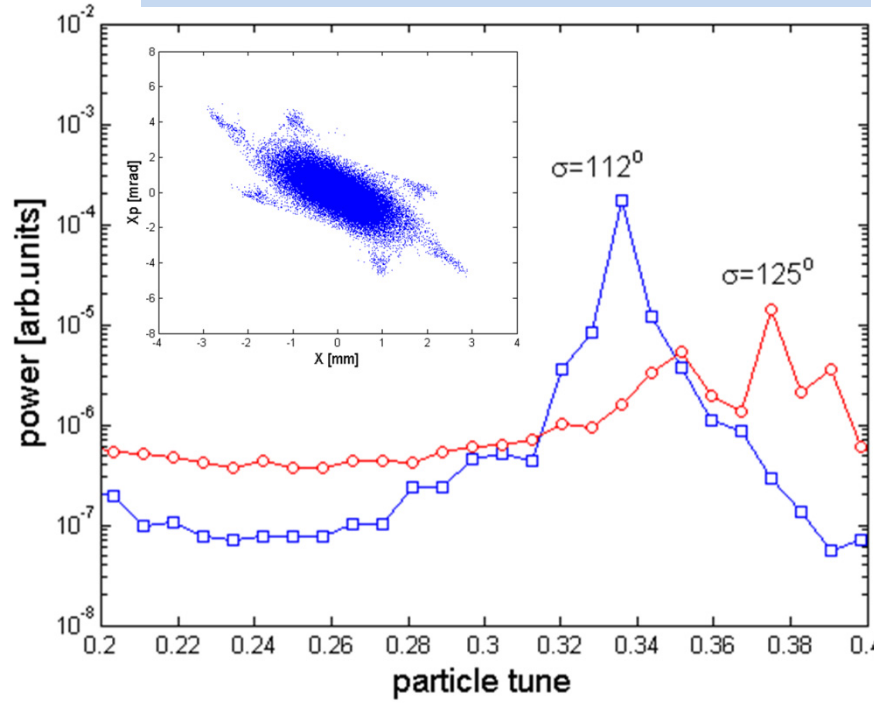
Jeon, PRAB **19**, 010101 (2016)



6th order resonance
for high intensity linear accelerators

Jeon et al., PRL **114**, 184802 (2015)

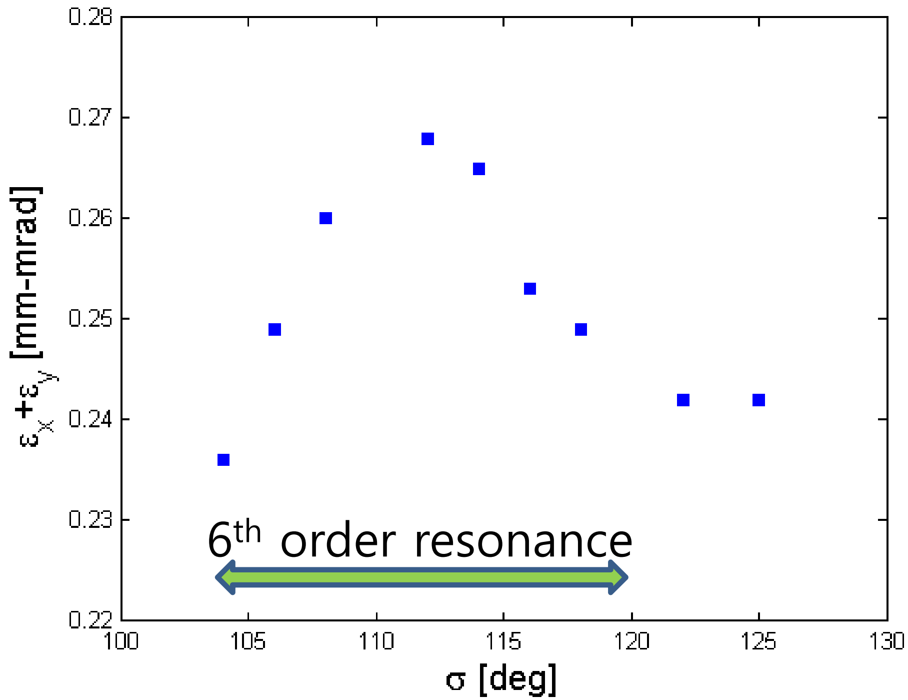


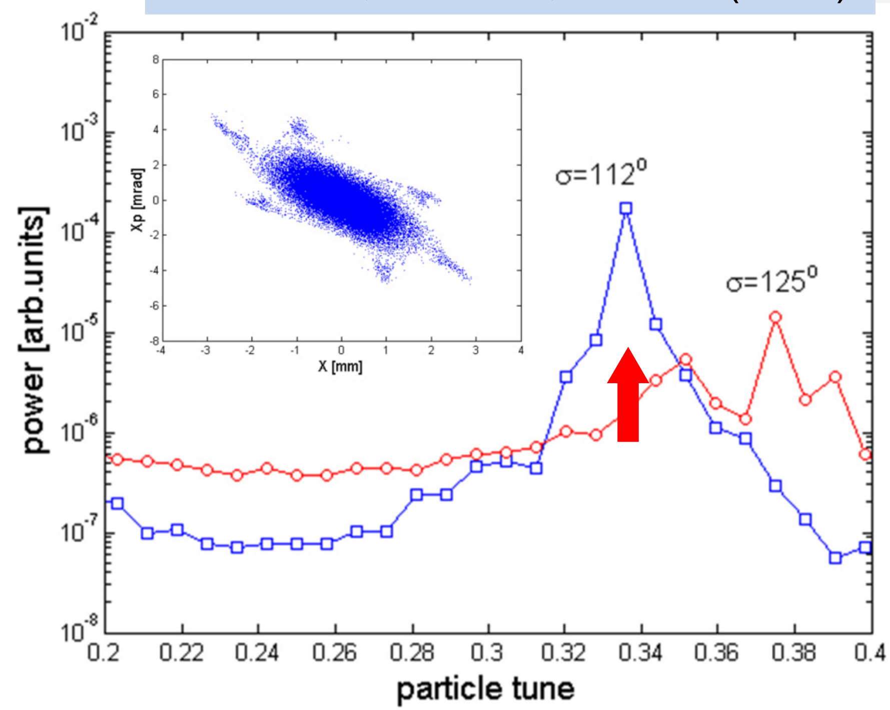


- $6\sigma = 720°$ sixth order resonance for $\sigma < 120°$.
- No resonance effects for $\sigma > 120°$ (Hamiltonian property).
- Frequency analysis shows a peak at $1/3 = 120°/360°$.
- Result of the perturbation of $2\sigma = 360°$ and $4\sigma = 360°$ resonances.

6^{th} order resonance
for high intensity linear accelerators

Jeon et al., PRL **114**, 184802 (2015)

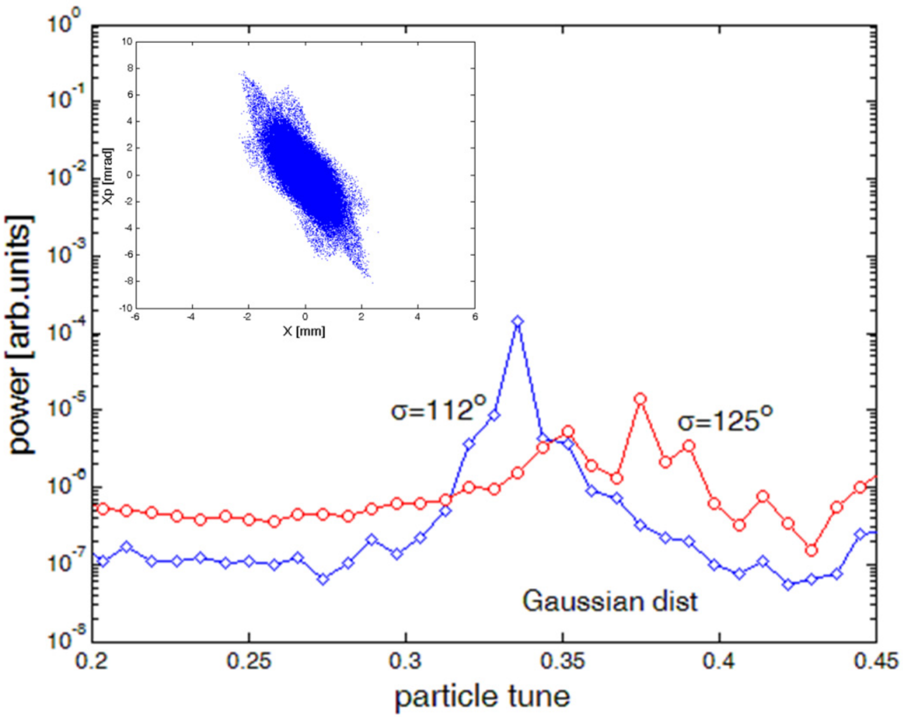


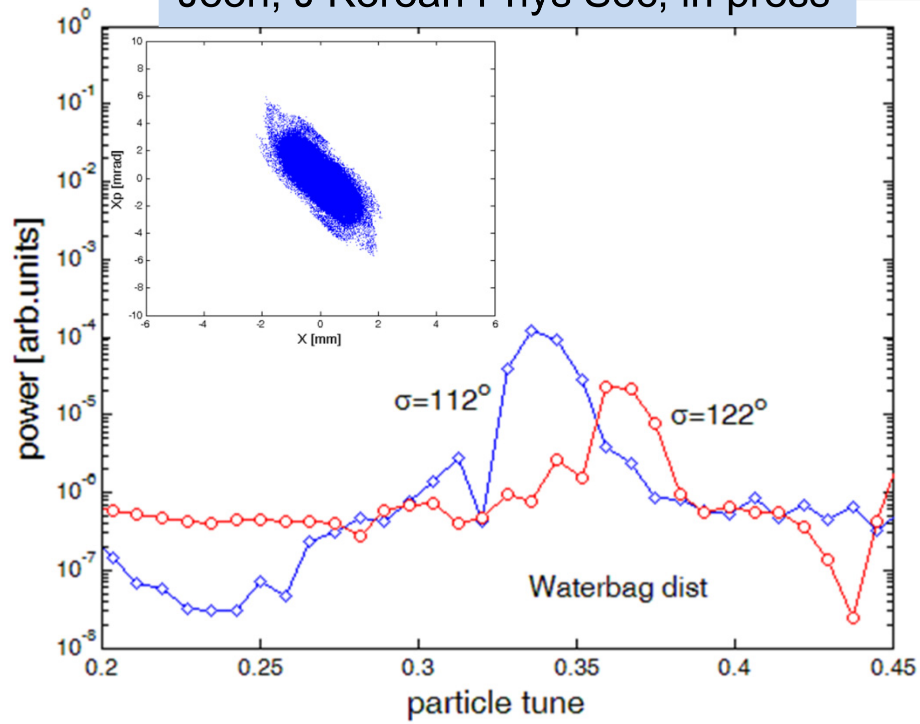


- $6\sigma = 720°$ sixth order resonance for $\sigma < 120°$.
- No resonance effects for $\sigma > 120°$ (Hamiltonian property).
- Frequency analysis shows a peak at $1/3 = 120°/360°$.
- Result of the perturbation of $2\sigma = 360°$ and $4\sigma = 360°$ resonances.

6^{th} order resonance
for high intensity linear accelerators

Jeon, J Korean Phys Soc, in press





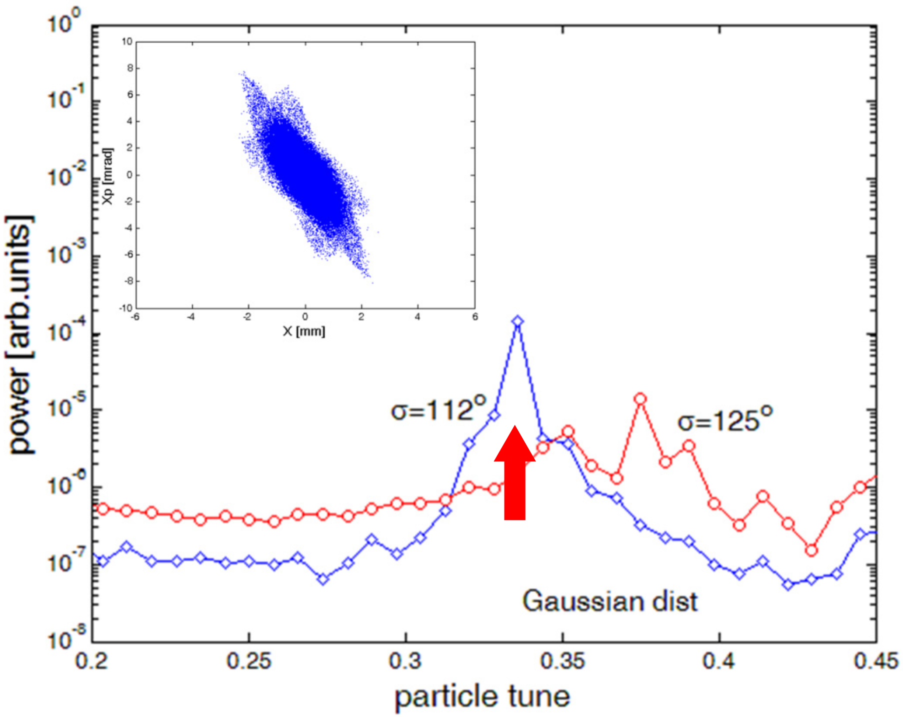
6^{th} order resonance
10-emA Gaussian distribution

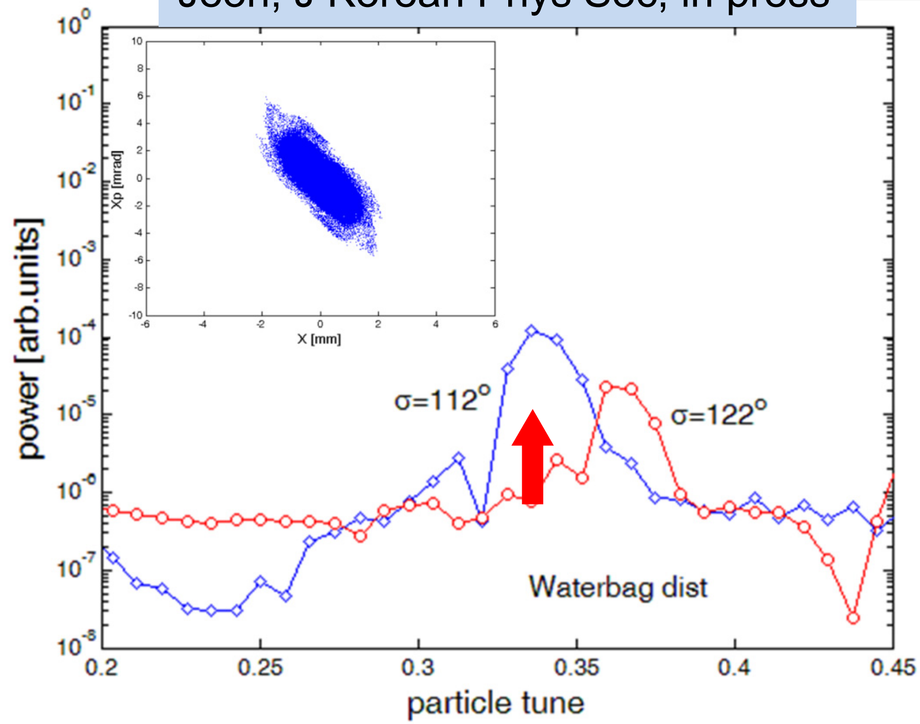
6^{th} order resonance
20-emA waterbag distribution

- Resonance frequency peak at 1/3 for lattice < 120° for non-KV beams.
- No resonance frequency peak for > 120°.

6^{th} order resonance
for high intensity linear accelerators

Jeon, J Korean Phys Soc, in press





6^{th} order resonance
10-emA Gaussian distribution

6^{th} order resonance
20-emA waterbag distribution

- Resonance frequency peak at 1/3 for lattice < 120° for non-KV beams.
- No resonance frequency peak for > 120°.

Particle Resonances

- The $4\sigma = 360°$ resonance in high intensity linacs was discovered in 2009. [Jeon et al., PRSTAB **12**, 054204 (2009)]

- The $6\sigma = 720°$ resonance was discovered, which was a perturbation of two strong resonances: $2\sigma = 360°$ resonance and $4\sigma = 360°$ resonance. [Jeon et al., PRL **114**, 184802 (2015)]

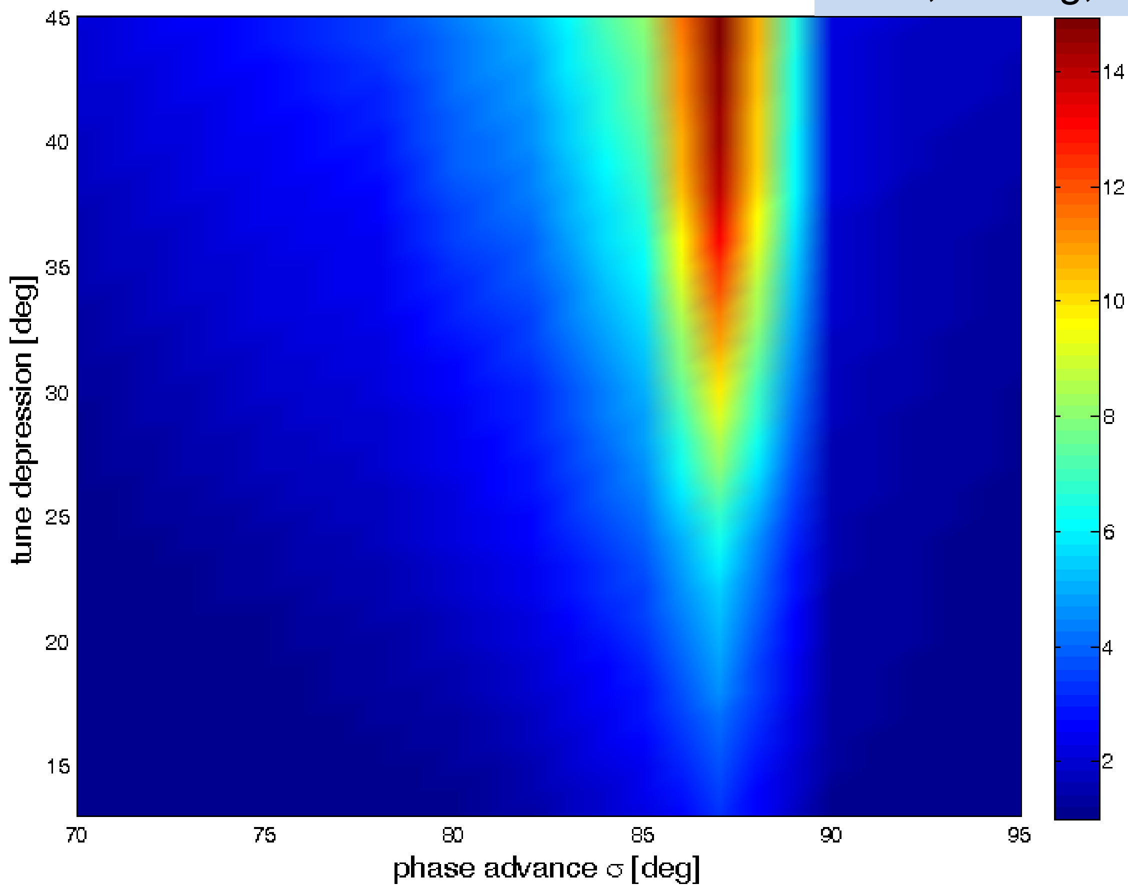
- The $6\sigma = 360°$ resonance was too weak to observe for Gaussian distribution. [Jeon et al., PRSTAB **12**, 054204 (2009)]
- Weak sign was observed for waterbag distribution. [Hofmann et al., PRL **115**, 204802 (2015)]

- Higher order resonances were discovered:
  - $8\sigma = 1080°$ resonance (8:3) = (6:2) ⊕ (2:1)
  - $10\sigma = 1440°$ resonance (10:4) = (8:3) ⊕ (2:1)
    [Hofmann, Proc. of HB2016]

Resonances:
a particle Hamiltonian property

More on 4th order resonance
emittance growth vs σ

Jeon, Hwang, Phys. Plasmas **24**, 063108 (2017)



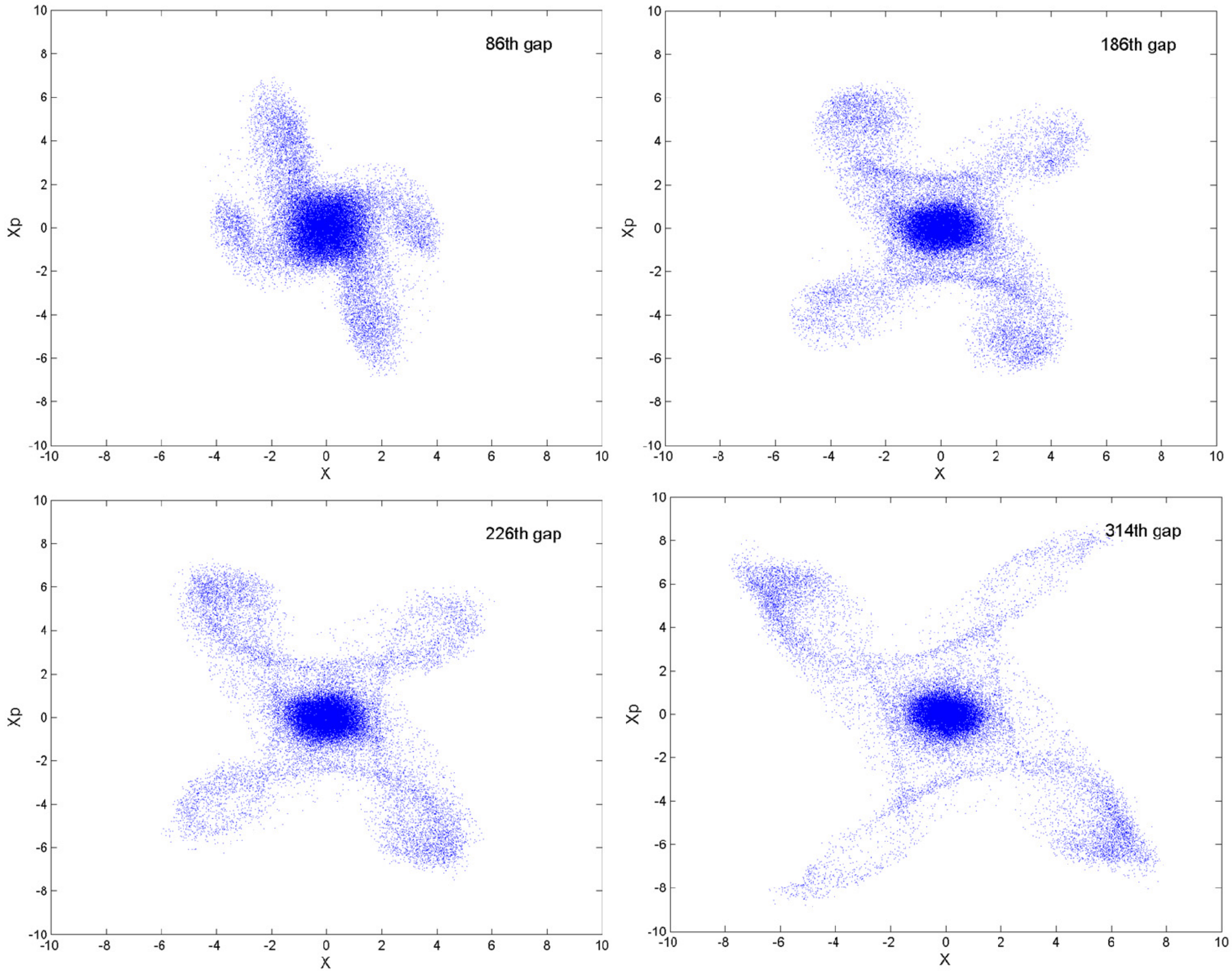
Constant-σ lattices are used for all the data points.

No evidence of the envelope instability is found.

well-matched 3D Gaussian input beam

- Emittance growth factor ($\varepsilon_f/\varepsilon_i$) plot as a function of σ and initial tune depression ($\sigma_o - \sigma$).
- **σ is the relevant parameter** of the 4th order resonance.

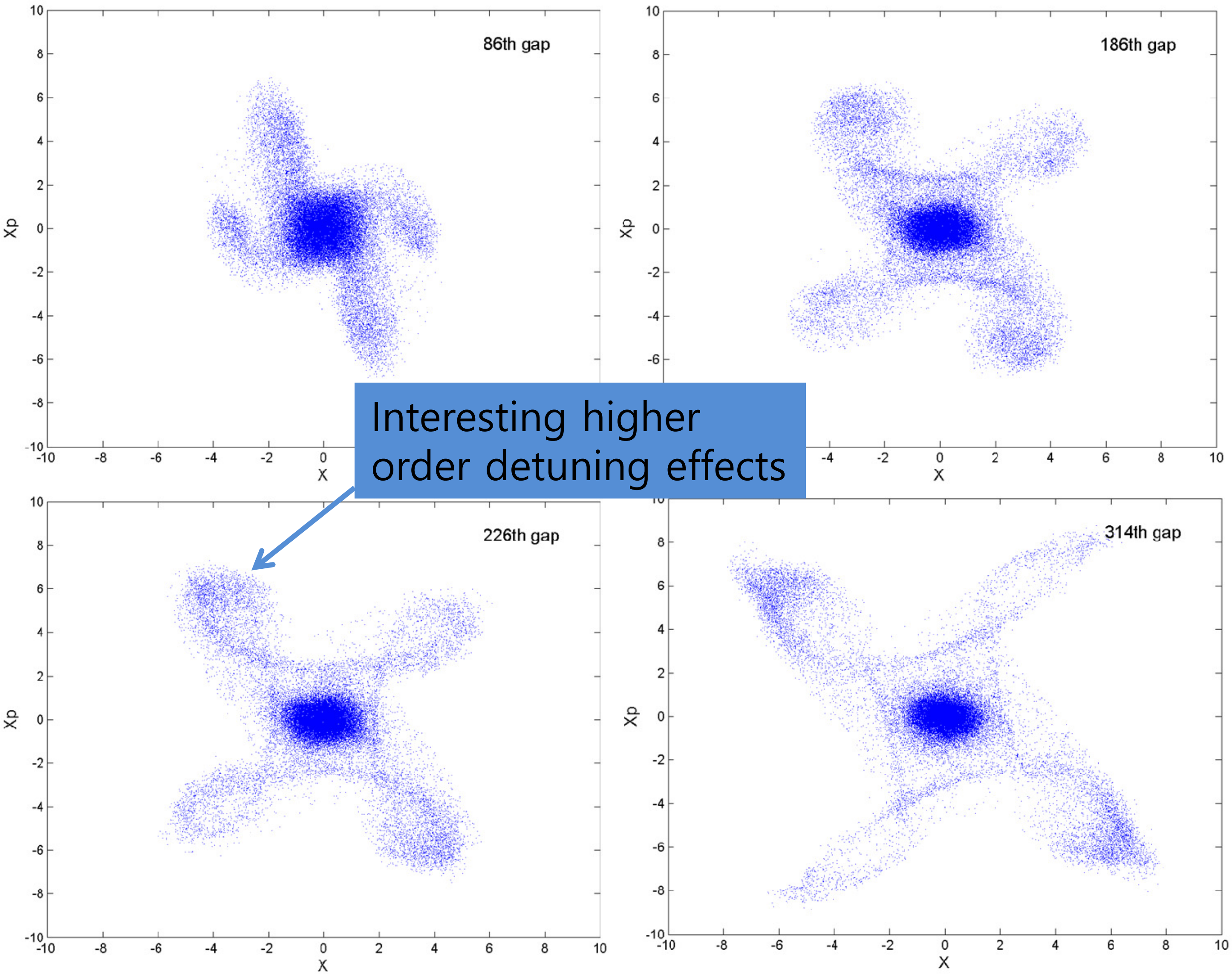
More on 4th order resonance
beam distribution evolution



Input :
well-matched
3D Gaussian beam

30 mA, σ = 87° case

More on 4th order resonance
beam distribution evolution

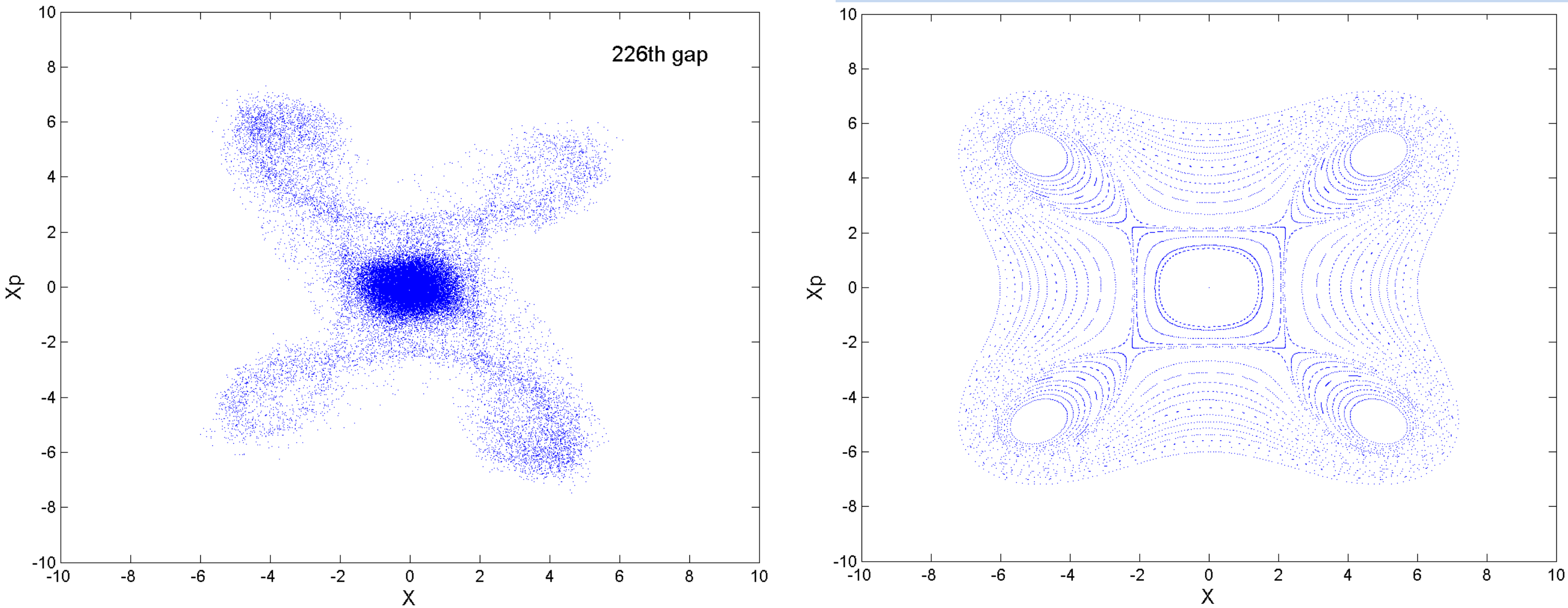


Input :
well-matched
3D Gaussian beam

30 mA, σ = 87° case

More on 4th order resonance
6th order effects

Jeon, Hwang, Phys. Plasmas **24**, 063108 (2017)



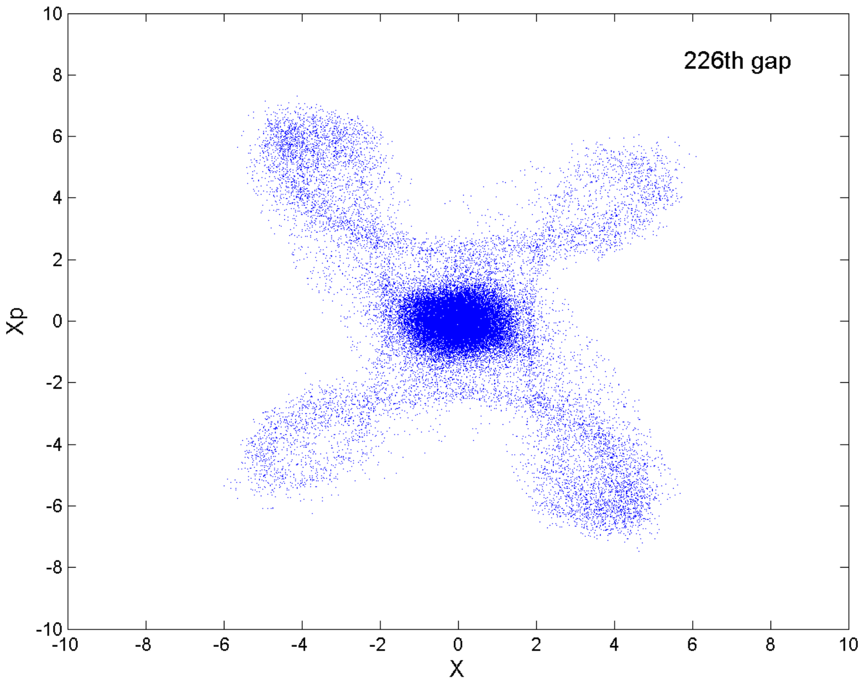
$$H_1 = \left(\nu - \frac{1}{4}\right) I + 5.05 \times 10^{-4}\, I^2 - 4.95 \times 10^{-4} I^2 \cos 4(\phi) - 2.22 \times 10^{-5} I^3$$

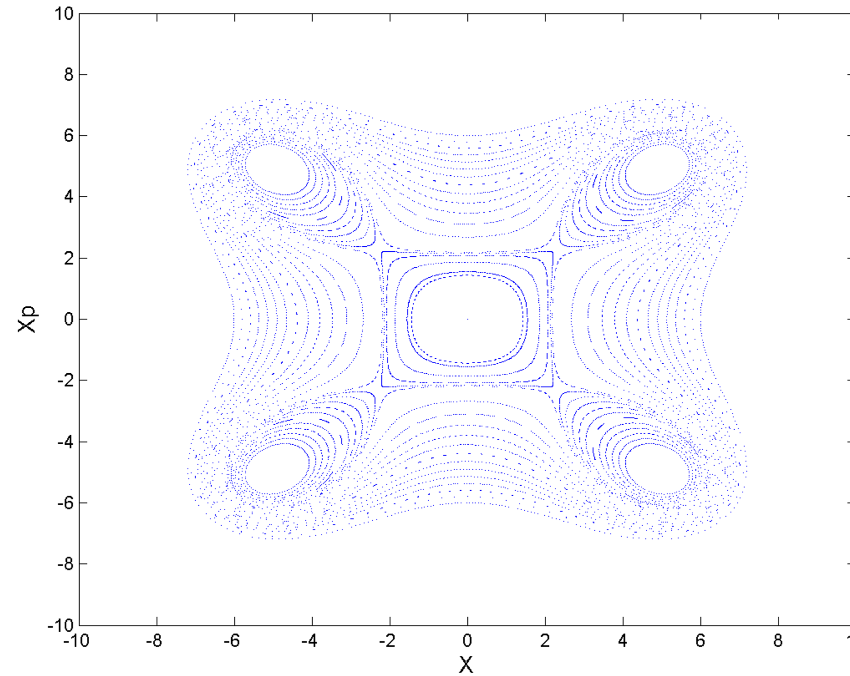
- 4th order resonance develops a four-fold structure that requires a 6th order detuning term.
- The Hamiltonian describes the system well.
- This 6th order term is caused by the redistribution of the beam by the resonance.

More on 4th order resonance

6th order effects

Jeon, Hwang, Phys. Plasmas **24**, 063108 (2017)





$$H_1 = \left(\nu - \frac{1}{4}\right) I + 5.05 \times 10^{-4}\, I^2 - 4.95 \times 10^{-4} I^2 \cos 4(\phi) \boxed{- 2.22 \times 10^{-5} I^3}$$

- 4th order resonance develops a four-fold structure that requires a 6th order detuning term.
- The Hamiltonian describes the system well.
- This 6th order term is caused by the redistribution of the beam by the resonance.

More on 4th order resonance
theory of 2D Gaussian beam

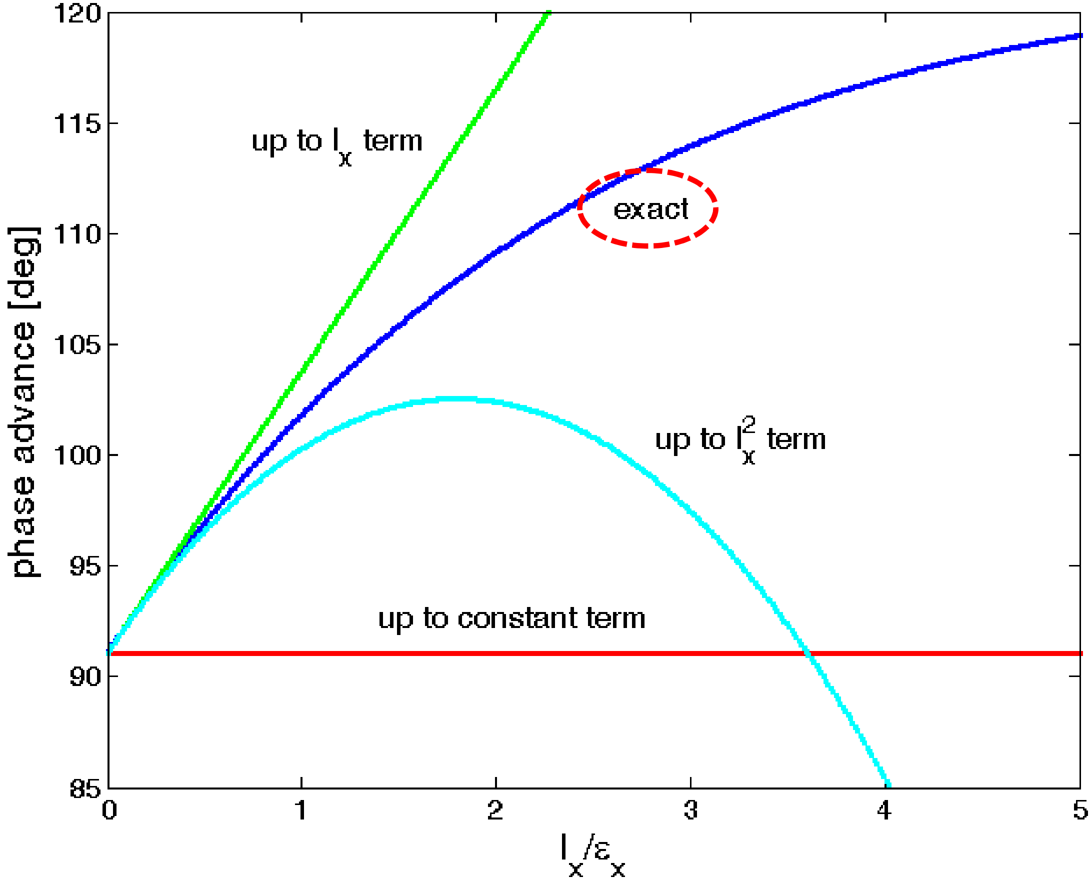
- Analytical formula exists for 2D Gaussian beam.
- Space charge potential is

$$V_{SC} = \frac{K_{SC}}{2}\int_0^\infty dt\,\frac{exp\left(-\frac{x^2}{2\sigma_x^2+t}\right)exp\left(-\frac{y^2}{2\sigma_y^2+t}\right)-1}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)}} = \frac{K_{SC}}{2}\int_0^\infty dt\,\frac{exp\left(-\frac{2\beta_x I_x cos^2\phi_x}{2\sigma_x^2+t}\right)exp\left(-\frac{2\beta_y I_y cos^2\phi_y}{2\sigma_y^2+t}\right)-1}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)}}$$

- Incoherent tune shift becomes:

$$\Delta\nu_x|_{I_y=0} = \oint\frac{ds}{2\pi}\frac{\partial H_v}{\partial I_x} = \frac{K_{SC}}{4\pi}\oint ds\left[-\frac{\beta_x}{\sigma_x(\sigma_x+\sigma_y)} + \frac{2\sigma_x+\sigma_y}{4\sigma_x^3(\sigma_x+\sigma_y)^2}\beta_x^2 I_x - \frac{(8\sigma_x^2+9\sigma_x\sigma_y+3\sigma_y^2)}{48\sigma_x^5(\sigma_x+\sigma_y)^3}\beta_x^3 I_x^2 + \frac{(16\sigma_x^3+29\sigma_x^2\sigma_y+20\sigma_x\sigma_y^2+5\sigma_y^3)}{384\sigma_x^7(\sigma_x+\sigma_y)^4}\beta_x^4 I_x^3 + \cdots\right] \qquad (8)$$

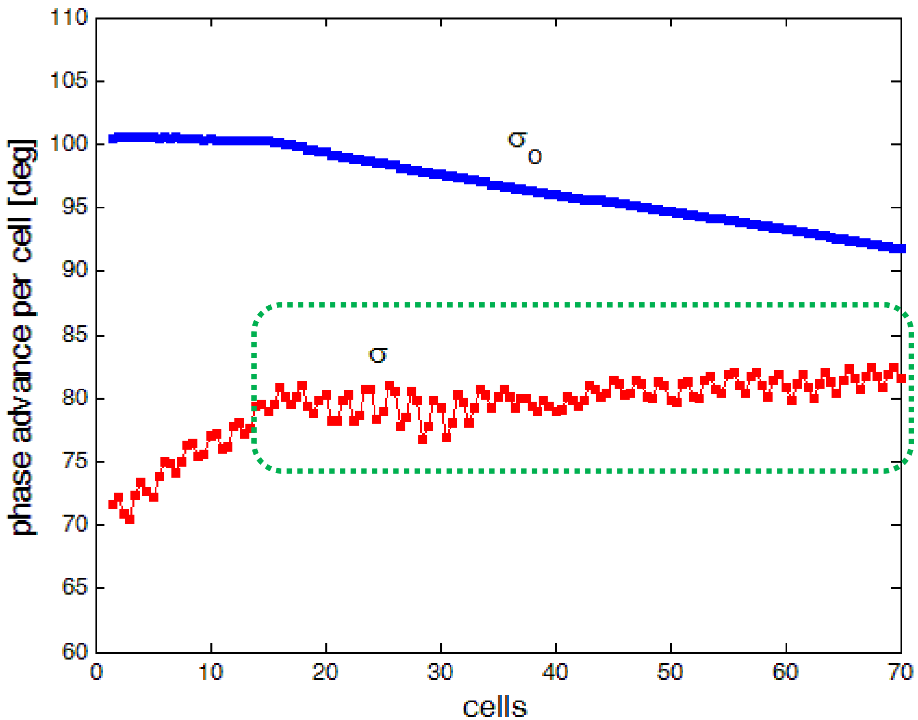
More on 4th order resonance
theory of 2D Gaussian beam

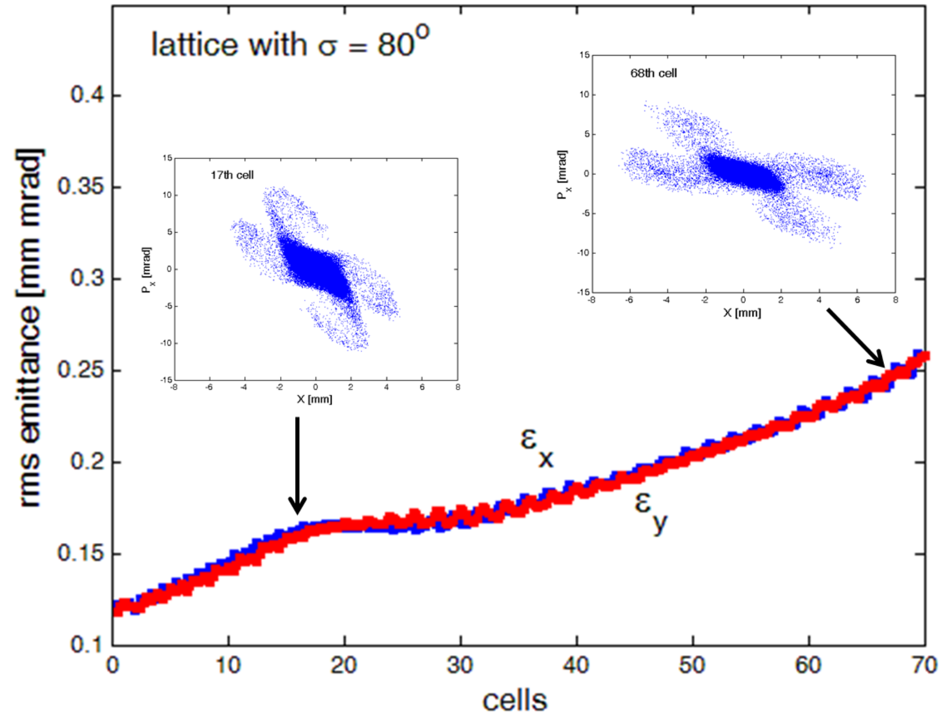


- Particle's phase advance increases monotonically for 2D Gaussian beam, as the oscillation amplitude grows.
- This explains why there is no resonance when $\sigma > 90°$.

4^{th} order resonance and 2^{nd} order envelope instability

4^{th} order resonance and envelope instability

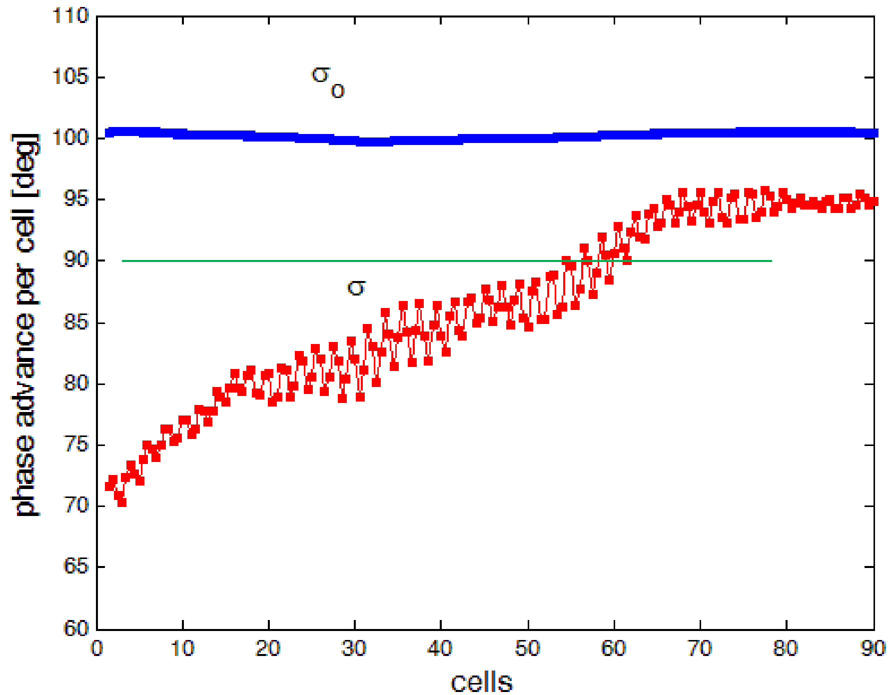


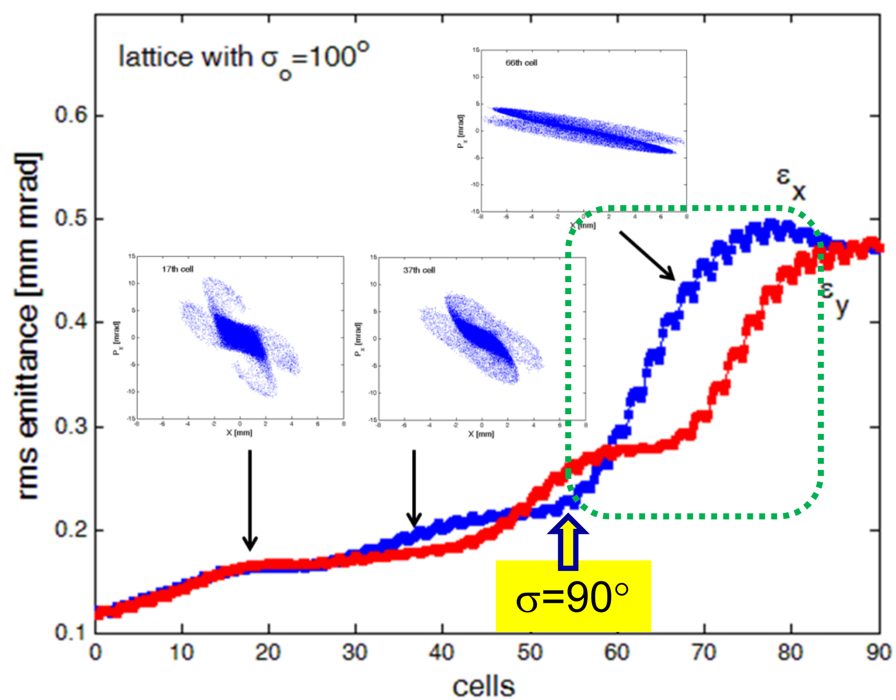


- For a constant-σ lattice, the 4^{th} order resonance dominates over the envelope instability.
- When σ is constant, the 4^{th} order resonance structure persists all the way and the envelope instability is not manifested.

4^{th} order resonance and envelope instability

Jeon et al., NIM A **832** (2016) 43





- For a constant-σ_o lattice, the envelope instability follows the 4^{th} order resonance.
- The envelope instability is manifested after the 4^{th} order resonance disappears when $\sigma > 90°$.
- There is a region where the 4^{th} order resonance is off and the envelope instability is on!

4^{th} order resonance and envelope instability

- Question: Is there a case reporting that the envelope instability develops by itself?
- So far, the envelope instability has been reported following the 4^{th} order resonance (non-KV beam) or the 4^{th} order instability (KV beam) for a constant-σ_o lattice.

- The envelope instability develops from a mismatch.
- The four-fold structure generated by the 4^{th} order resonance presents itself as a mismatch, which can drive the envelope instability when the 4^{th} order resonance is off.

- All the simulations for lattices with $\sigma > 90°$ show neither the 4^{th} order resonance nor the envelope instability.
- The 4^{th} order resonance should not be mistaken for the 4^{th} order envelope instability.

Around 90° phase advance

- There are three mechanisms around 90° phase advance: 4^{th} order resonance, 2^{nd} order envelope instability, and 4^{th} order envelope instability.

- For non-KV distributions (well-matched),
  - 4^{th} order resonance appears first.
  - For a constant-σ lattice, the 4^{th} order resonance persists.
  - For a constant-σ_o lattice, the 2^{nd} order envelope instability follows.

- For KV distributions (well-matched),
  - the 4^{th} order envelope instability appears first.
  - the 2^{nd} order envelope instability may follow depending on conditions.
  - It is interesting that the 4^{th} order envelope instability appears first.

Terminology Suggestion

- Two distinct families of space-charge mechanisms exist:
  - Instabilities (or parametric resonances) of the beam envelope,
  - Resonances of the beam particle.

- Instabilities are instabilities of the beam envelope:
  - more specifically envelope instabilities,
  - a.k.a. parametric resonances (of the envelope equation),
  - but would better be called envelope parametric resonances to distinguish them from particle parametric resonances.

- Resonances are resonances of the beam particle, as known in circular accelerators:
  - would better be called particle resonances,
  - a.k.a. single particle resonances.

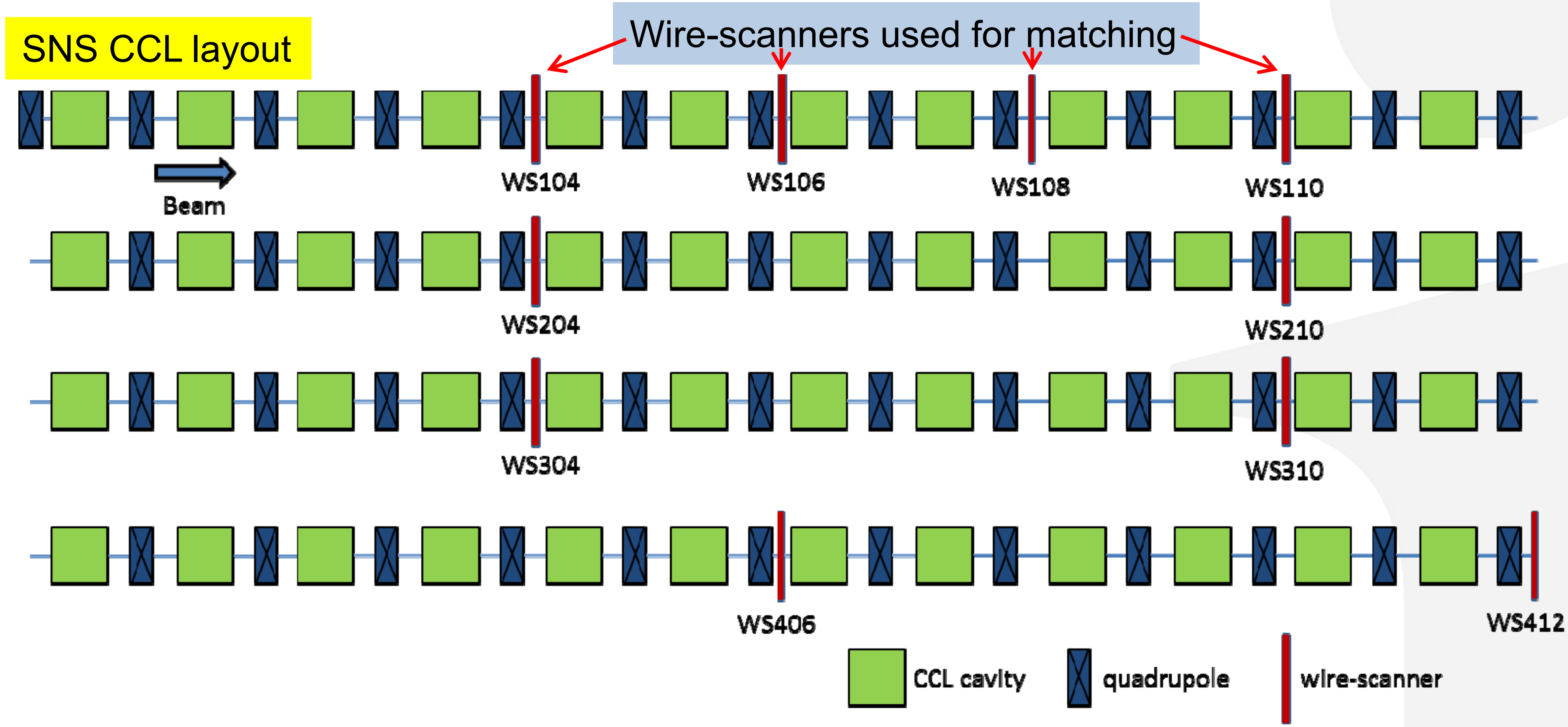
- Resonances
- Particle resonances

- Instabilities
- Envelope instabilities
- Envelope parametric resonances

D. Jeon, Classification of Space-Charge Resonances and Instabilities in High-Intensity Linear Accelerators,
J. Korean Phys. Soc. **72**, 1523 (2018)


Thank you for your attention!
감사합니다

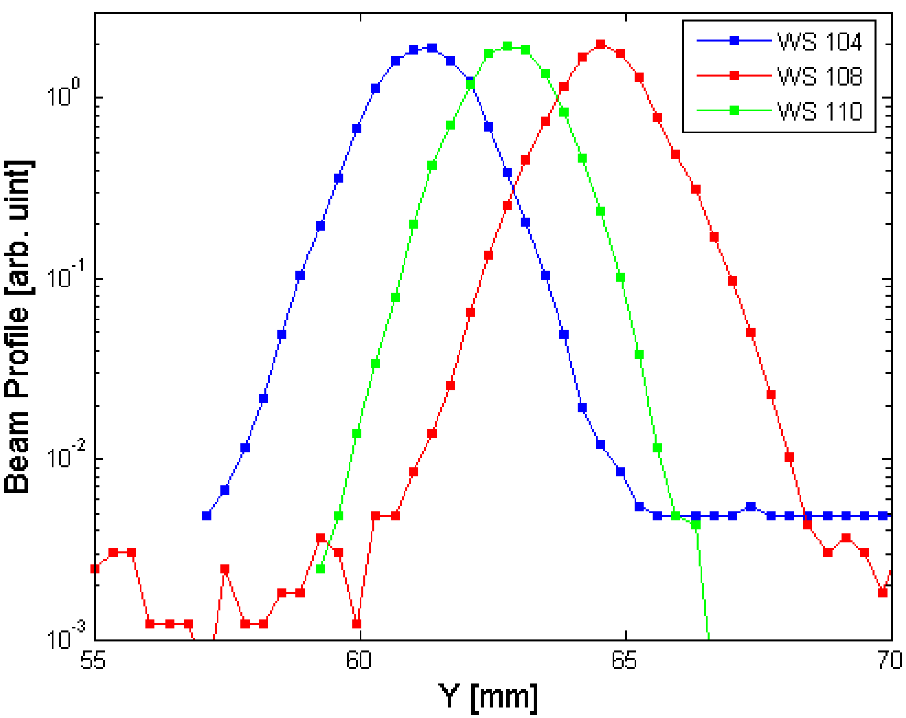
Experiment of the 4th order resonance (II)
using SNS CCL



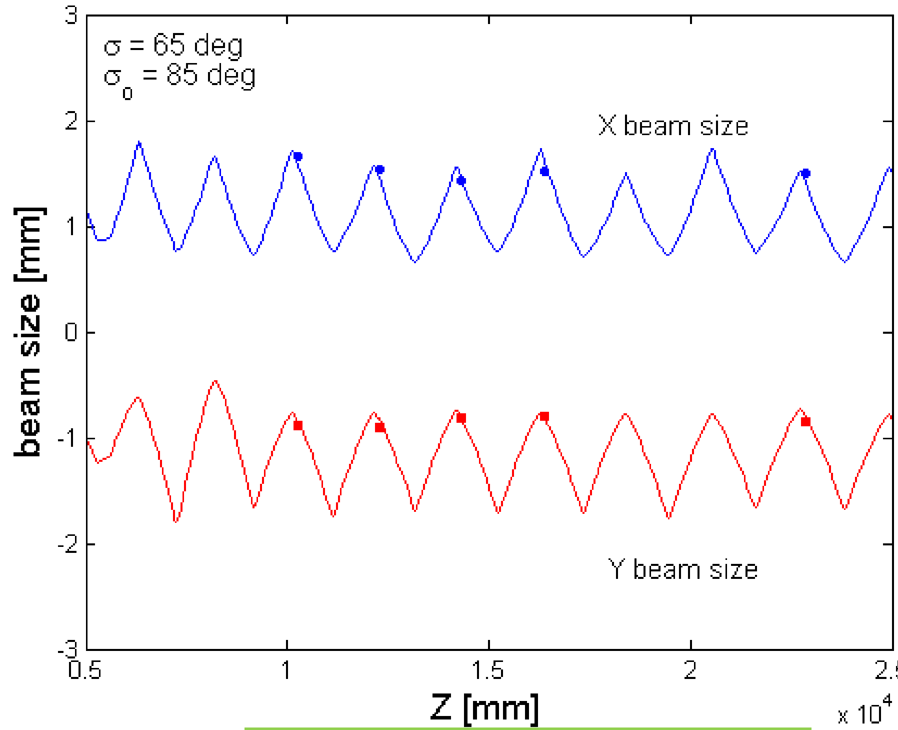
- Schematic layout of the SNS CCL showing the wire-scanners used for the experiment.
- Halo of incoming beams were carefully controlled by matching and the MEBT round beam optics.

Experiment of the 4th order resonance (II)

Halo of incoming beam was minimized



Beam profiles at the CCL entrance



Beam matching to the CCL

- Round beam optics (MEBT) was used to minimize halo formation in the upstream.
- Matching between linac sections was done to avoid the mismatch.
- The beam entering the CCL has little tails.