# Experiments and theory on beam stabilization with second-order chromaticity

**M. Schenk**[1], X. Buffat, L.R. Carver, K. Li, A. Maillard[2], E. Métral

*CERN & [1]EPFL (Switzerland), [2]ENS (France)*

**Acknowledgements**

R. De Maria, S. Fartoukh, G. Rumolo, LHC OP teams

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# Outline

- **Introduction**
  *Transverse Landau damping from detuning with longitudinal amplitude*
- **LHC experiments**
  *Using second-order chromaticity to validate damping mechanism experimentally*
- **Vlasov theory for nonlinear chromaticity**
  *Develop theory to interpret experimental observations analytically*
- **Conclusions**

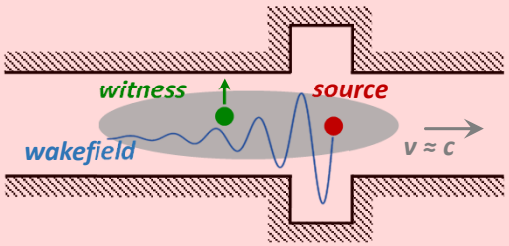
# Introduction

## *Head-tail modes and Landau damping*

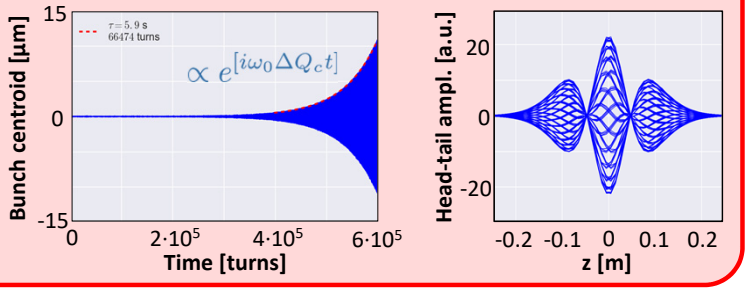
**Problem:** Collective instabilities (Impedance-driven)

1. ***Source* particle interacts** electro-magnetically with **accelerator structure**

   *Creates **wakefield*** that **acts back** on trailing particles ***(witnesses)***

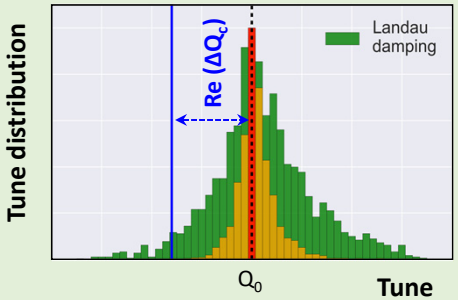



2. **Wake kicks drive (head-tail) instabilities** characterized by ***complex coherent tune shift*** $\Delta Q_c$ and ***associated mode pattern***

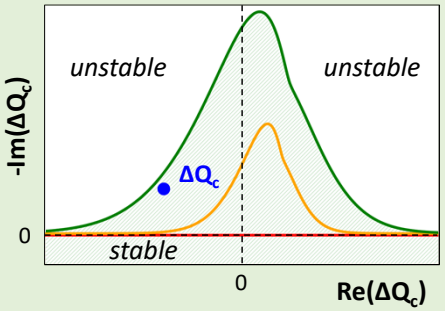



**Possible solution:** Landau damping

1. **Generate large enough incoherent tune spread *ΔQ***




**Stability diagram**




2. **Increases area of stability in complex tune space**

3. **With large enough spread, instability can be suppressed**

# Introduction

*Detuning with longitudinal amplitude*

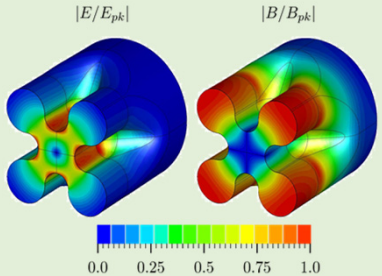
**1 Magnetic octupoles**

$$\Delta Q_x^i = a_{xx} J_x^i + a_{xy} J_y^i$$
$$\Delta Q_y^i = a_{yy} J_y^i + a_{xy} J_x^i$$

Betatron detuning with ***transverse amplitude***

$$\Delta Q_{x,y}^i \left(J_x^i, J_y^i\right)$$

**2 Rf quadrupole[1-4]**
**(*or:* nonlinear chromaticity)**




$$\Delta Q_u^i = a_{uz} J_z^i$$
$$u \in \{x, y\}$$

Betatron detuning with ***longitudinal amplitude***

$$\Delta Q_{x,y}^i \left(J_z^i\right)$$

**Why detuning with longitudinal amplitude?**

- **More efficient:** $\Delta J_z >> \Delta J_{x,y}$ *(LHC design: factor $10^4$ – $10^5$ at 7 TeV)*[1,5]
- **Geometric emittance further reduced** for future **high brightness, high energy** beams
- **Not affected by manipulations in transverse planes** (e.g. halo cleaning)

# Introduction

## *Rf quadrupole and second-order chromaticity*

- **Direct experimental validation** of **rf quadrupole not possible** at present as no such cavity is available
- ***But:*** **Beam dynamics** is the **same** for **rf quadrupole and second-order chromaticity Q''** at first order

⇨ **Experimental studies with second-order chromaticity can be made**

**Rf quadrupole**

Rf-modulated quadrupole kick leads to betatron detuning with $J_z$

$$\Delta Q^i_{x,y} \propto \pm \cos\left(\frac{\omega}{\beta c} z_i\right) \approx \pm\left[1 - \frac{1}{2}\left(\frac{\omega}{\beta c}\right)^2 z_i^2\right]$$

$$\langle \Delta Q^i_{x,y} \rangle_{T_s} \propto \pm\left[1 - \frac{1}{2}\left(\frac{\omega}{\beta c}\right)^2 \beta_z J_z^i\right]$$

**Second-order chromaticity**

Betatron detuning from relative momentum error $\delta_i = \Delta p_i / p$

$$\Delta Q^i_{x,y} = Q'_{x,y}\delta_i + \frac{Q''_{x,y}}{2}\delta_i^2 + \mathcal{O}\left(\delta_i^3\right)$$

Q'' term leads to

$$\langle \Delta Q^i_{x,y} \rangle_{T_s} = \frac{Q''_{x,y}}{2}\sigma_\delta^2 J_z^i$$

- **Q''** is introduced in a machine by **changing the optics**[7]
- **LHC: main sextupole** families can be used to **enhance Q''** (without affecting Q')

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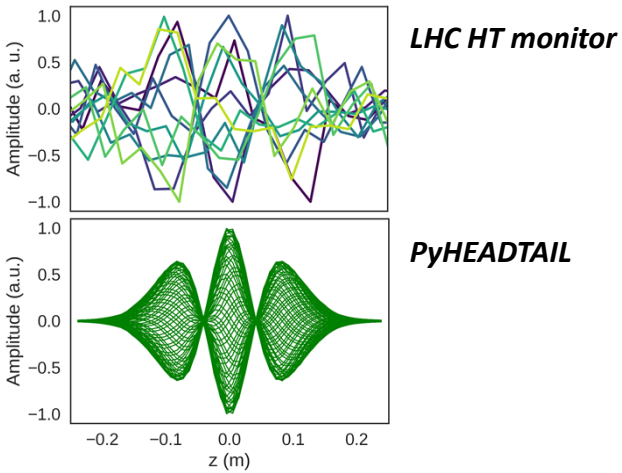
# LHC experiments

*Single-bunch stability at 6.5 TeV* ***without Q''***

***Without Q''***

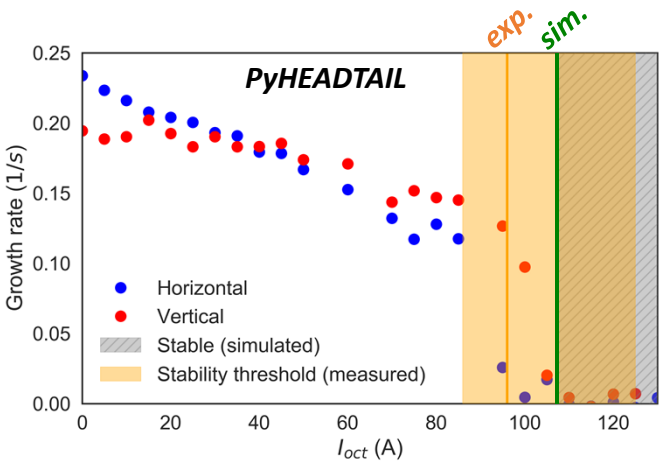
- LHC, 6.5 TeV, transverse feedback system active
- Most prominent instability is **horizontal head-tail mode**[8]
- **Azimuthal** and **radial** mode numbers **l = 0** and **m = 2**
- **Mode patterns** from LHC Head-Tail Monitor and PyHEADTAIL simulations in **good agreement**




- Instability is routinely **mitigated with Landau octupoles**
- **Stabilizing current** determined **experimentally** and in PyHEADTAIL **simulations**[9]

$I_{exp} = 96^{+29}_{-10}$ A / $I_{sim} = 107.5 \pm 2.5$ A

- Results **confirm reliability of impedance** and **tracking models**

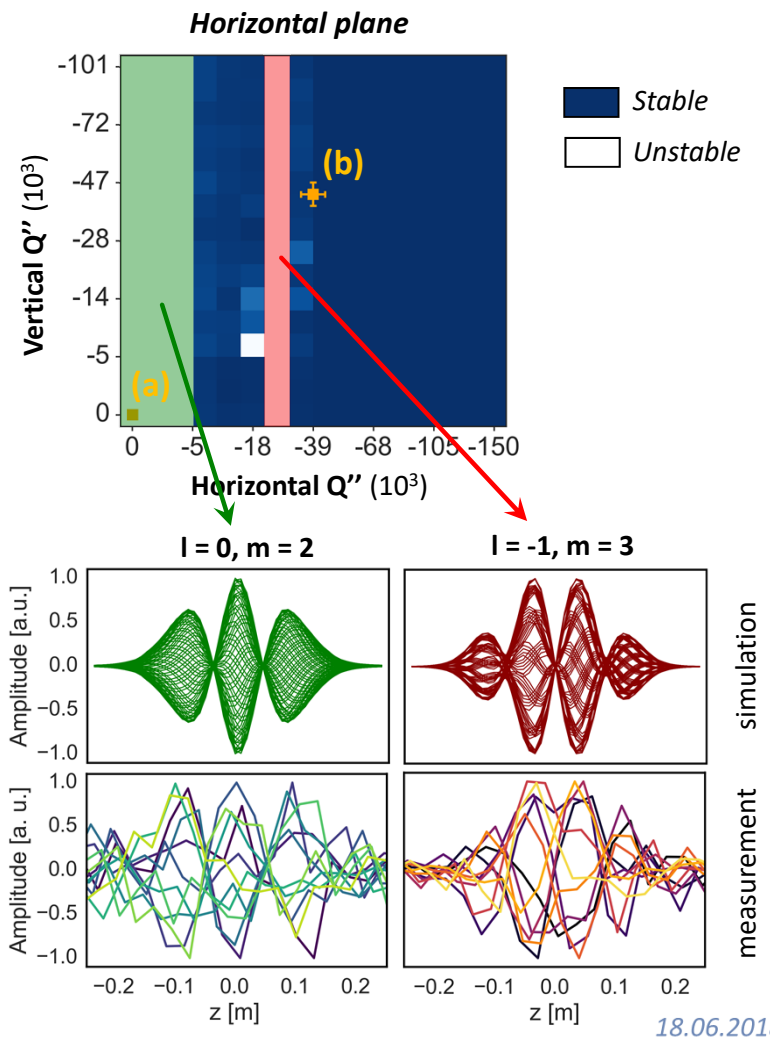


# LHC experiments

## *Q'' study and comparison with PyHEADTAIL*

- **Goal:** Stabilize single bunches at 6.5 TeV with Q''[10,11]
- **PyHEADTAIL predictions**
  Q'' creates large areas of stability interleaved with *two unstable bands* of *different* head-tail modes
- **Q'' experiment: two working points**
  - ***(a)*** ***Without Q''***
    - Octupoles: $I_{exp} = 96^{+29}_{-10}$ A vs. $I_{sim} = 107.5 \pm 2.5$ A
  - ***(b)*** ***Q'' ≈ -4 x 10⁴***
    - Four bunches in the machine
    - Octupoles reduced to 40 A and all four bunches *stable* (Landau damping from Q'')
    - One higher intensity bunch unstable when reducing to 0 A
    - Other three bunches stable *with octupoles off*
    - Instability explained by unstable band next to (b)

**Measured head-tail patterns agree well with simulations**



# LHC experiments

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- **PyHEADT**
  Q'' creat
  *two uns*
- **Q'' expe**
  - *(a) Wi*
    - Oc
  - *(b) Q''*
    - Fou
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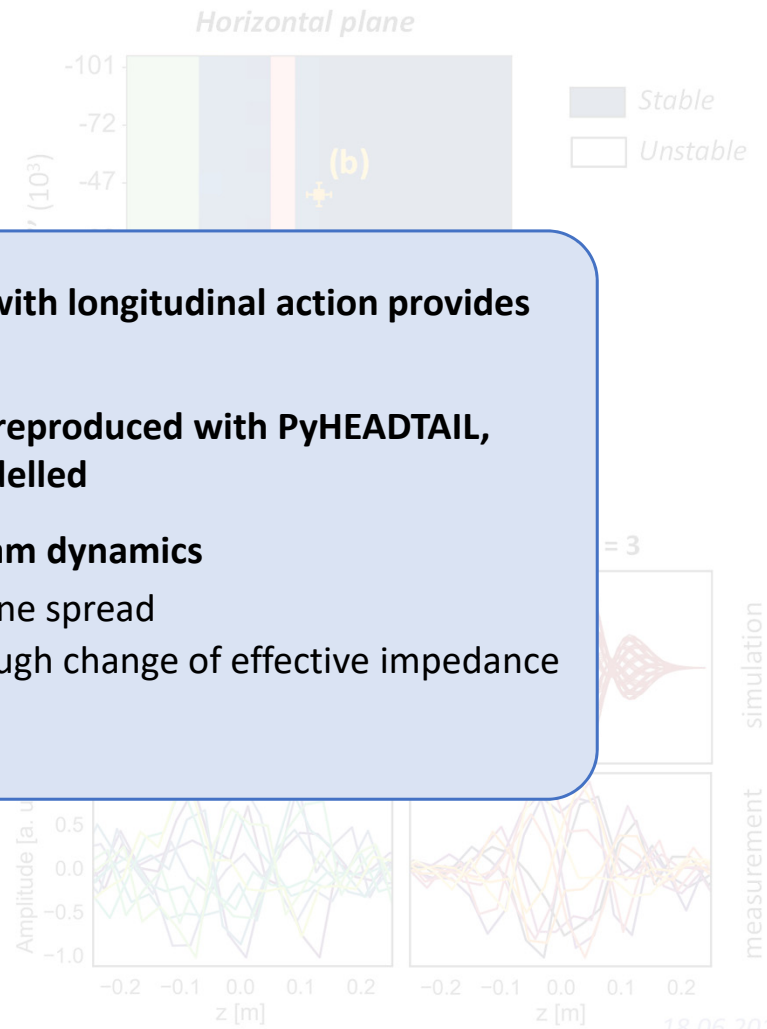
**Measured head-tail patterns agree well with simulations**




- **Experiments demonstrate that detuning with longitudinal action provides beam stability**
- **Details of experimental observations are reproduced with PyHEADTAIL, all the relevant effects are accurately modelled**
- **Q'' (rf quadrupole) has two effects on beam dynamics**
  1. Landau damping from incoherent tune spread
  2. Change of most unstable mode through change of effective impedance

**Can we confirm this analytically?**

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# Vlasov theory for nonlinear chromaticity

*Overview*

**Two effects observed with second-order chromaticity in experiments and simulations**

1. Beam stabilization through Landau damping
2. Change of effective impedance

**Strategy**

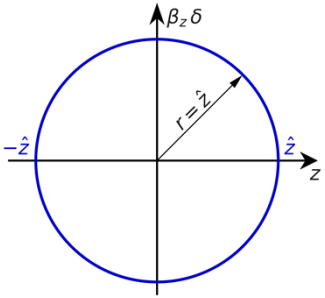
1. **Extend theory to include nonlinear chromaticity $\xi^{(n)}$** up to arbitrary order, starting from **Vlasov equation**

$$\left[\partial_s + \frac{1}{c}\,\omega_\beta(\delta)\,\partial_\theta + \frac{\omega_s}{c}\partial_\phi + \frac{F_y}{E}\partial_{p_y}\right]\Psi = 0 \qquad \text{with} \quad \omega_\beta(\delta) = \omega_{\beta,0}\sum_{k=0}^{m}\frac{\xi^{(k)}}{k!}\delta^k$$

*(Follow A. Chao's[12] derivations for some part, but eigenmodes are different when including nonlinear chromaticity – use different decomposition … discussing only solutions here)*

2. **Airbag bunch: same longitudinal action for all the particles**
   Only experiences change of coherent mode, and *no Landau damping* since no frequency spread possible from detuning with longitudinal action ⇨ two effects decoupled
3. **Gaussian bunch: both effects** are present *(stability diagram theory)*
4. **Compare to tracking simulations and circulant matrix model**

$$g_0(r) \propto \boldsymbol{\delta}(r - \hat{z})$$



# Vlasov theory for nonlinear chromaticity

## *Airbag bunch: formalism with $\xi^{(2)}$*

$$g_0(r) \propto \boldsymbol{\delta}(r - \hat{z})$$

(II) $H_l^p$ describes change of effective impedance via phase terms introduced by all orders of nonlinear chromaticity

- **Linear chromaticity** *(A. Chao, Eq. 6.188)*[12]

$$\Omega^{(l)} - \omega_{\beta,0} - l\omega_s = i\frac{Nq^2c}{2E_0T_0^2\omega_{\beta,0}} \boxed{\sum_{p=-\infty}^{\infty} Z_1^{\perp}(\omega')\ J_l^2\left(\frac{\omega'\hat{z}}{c} - \chi\right)} \quad \textbf{(Eq. I)}$$

$$\chi = \xi^{(1)}\frac{\omega_{\beta,0}\hat{z}}{\eta c}$$
$$\omega' = p\omega_0 + \omega_{\beta,0} + l\omega_s$$

Reduces to Bessel function of first kind if $\xi^{(2)}$ = 0: Eqs. I and II become identical

- **Including nonlinear chromaticity**

$$\Omega^{(l)} - \omega_{\beta,0} - l\omega_s - \overset{\textbf{(I)}}{\boxed{\langle\Delta\omega_\beta\rangle_\phi(\hat{z})}} = i\frac{Nq^2c}{2E_0T_0^2\omega_{\beta,0}} \boxed{\sum_{p=-\infty}^{\infty} Z_1^{\perp}(\omega')\overset{\textbf{(II)}}{\boxed{|H_l^p(\hat{z})|^2}}} \quad \textbf{(Eq. II)}$$

**(I)** **Changes real part of $\boldsymbol{\Omega^{(l)}}$** for airbag bunch. For **non-airbag bunch**, term will represent **frequency spread**, lead to **dispersion integral** and **Landau damping** *(non-zero only for even orders!)*

$$\langle\Delta\omega_\beta\rangle_\phi(r) = \frac{1}{2\pi}\int_0^{2\pi} \Delta\omega_\beta(\delta(r,\phi))\, d\phi$$

$$\Rightarrow \langle\Delta\omega_\beta\rangle_\phi = \xi^{(2)}\frac{\omega_{\beta,0}\hat{z}^2}{4\beta_z^2}$$

**(II)** $\boldsymbol{H_l^p}$ **describes change** of **effective impedance via phase terms** introduced by *all orders* of nonlinear chromaticity

$$H_l^p(\hat{z}) = \frac{1}{2\pi}\int_0^{2\pi} e^{il\phi}\, e^{-i\frac{\omega'}{c}\hat{z}\cos\phi}\, e^{-i\frac{\omega_{\beta,0}}{\omega_s\beta_z}\hat{z}\left[\xi^{(1)}(1-\cos\phi) - \xi^{(2)}\frac{\hat{z}}{4\beta_z}\sin\phi\cos\phi\right]}\, d\phi$$

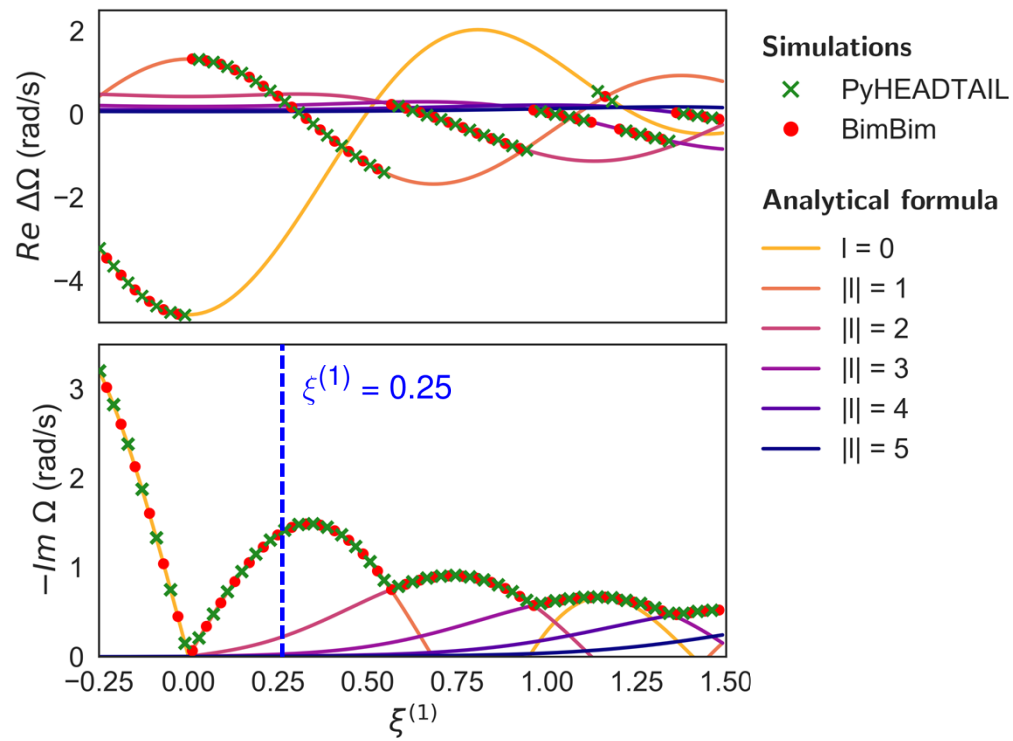
**Reduces** to **Bessel function** of first kind if **$\xi^{(2)}$ = 0: Eqs. I and II** become **identical**

*Here shown up to 2nd order, general definition in proceedings*

# Vlasov theory for nonlinear chromaticity

## ***Airbag bunch:*** *linear chromaticity scan*

- First test with **linear chromaticity** and **$\xi^{(2)} = 0$**
- **Broad-band resonator impedance**
- **Complex frequencies of azimuthal modes up to order |l| = 5**
- **Analytical result** using **Eq. II** *(equivalent to Eq. I for linear chromaticity)*
- **Comparisons** with ***PyHEADTAIL*** **tracking** code show **excellent agreement**
- Same for **circulant matrix model** (*BimBim*)[13]
- **Next: Scan in $\xi^{(2)}$** for fixed **$\xi^{(1)} = 0.25$**

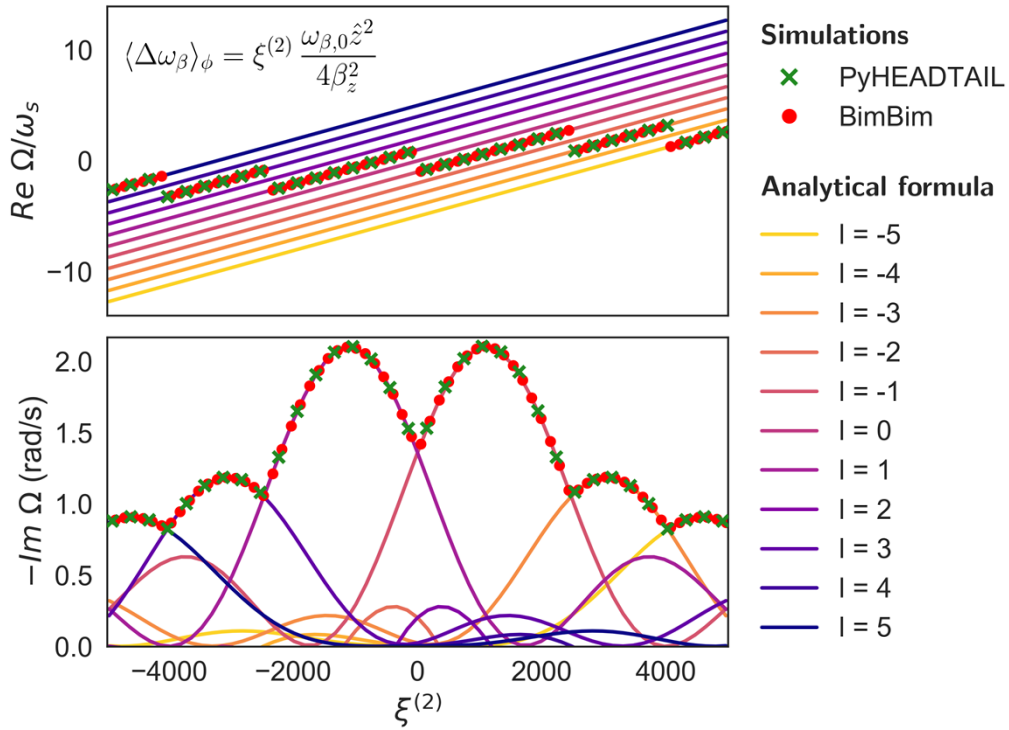


# Vlasov theory for nonlinear chromaticity

## ***Airbag bunch:*** *second-order chromaticity scan*

- **Scan $\xi^{(2)}$** at fixed **$\xi^{(1)}$ = 0.25**
- **Same resonator impedance**
- **Analytical result** using **Eq. II**
- **Modes** indeed **change** with **$\xi^{(2)}$**
- **Real coherent frequency** shows **constant shift** as expected from Eq. II
- Again, **excellent agreement** with **PyHEADTAIL** and **BimBim**

**Formalism works well for the airbag model**



# Vlasov theory for nonlinear chromaticity

## *Gaussian bunch*

- **Eigenvalue equation including nonlinear chromaticity**
  *For the airbag beam before, integration over r was straightforward*

$$\forall l \in \mathbb{Z} \qquad \sigma_p = -i\frac{q^2\omega_s\omega_0}{4\pi\omega_{\beta,0}\eta E_0}\sum_{p'=-\infty}^{\infty}\sigma_{p'}Z_1^{\perp}(\omega')\int_0^{\infty}\frac{r g_0(r)\overline{H_l^{p'}}(r)H_l^p(r)}{\Omega^{(l)}-\omega_{\beta,0}-l\omega_s-\langle\Delta\omega_\beta\rangle_\phi(r)}dr$$

- Assuming a **highly-peaked impedance** only **one term $p_0$** in the sum **contributes** (single-peak approximation).
- We **recognize 'standard' dispersion relation**

$$\forall l \in \mathbb{Z} \qquad 1 = -i\frac{q^2\omega_s\omega_0}{4\pi\omega_{\beta,0}\eta E_0}Z_1^{\perp}(\omega'_{p_0})\int_0^{\infty}\frac{r g_0(r)|H_l^{p_0}(r)|^2}{\Omega^{(l)}-\omega_{\beta,0}-l\omega_s-\langle\Delta\omega_\beta\rangle_\phi(r)}dr \qquad \textbf{(Eq. III)}$$

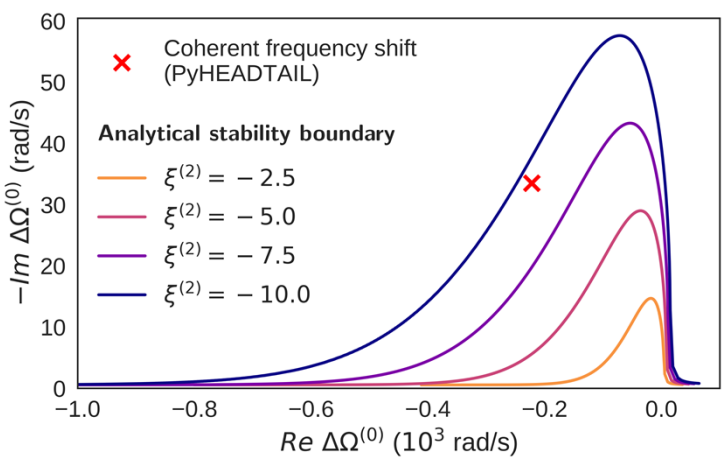
*Integrate numerically to obtain stability boundary diagrams (Landau bypass rule: add iε to denominator)*

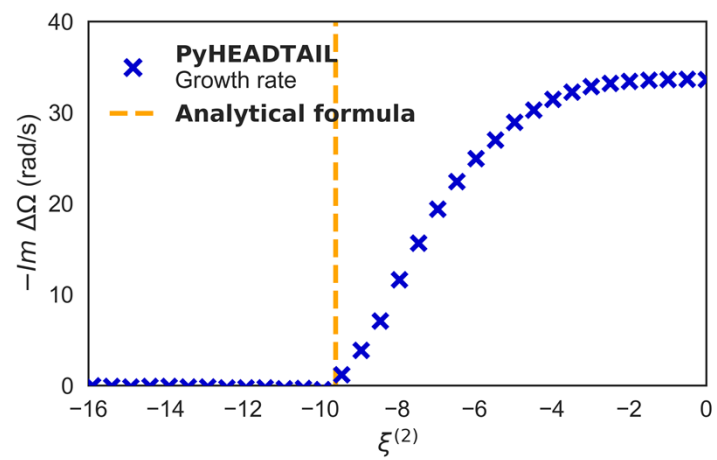
- $H_l^p$ again describes **change of effective impedance from nonlinear chromaticity** (like for airbag bunch)
- **Landau damping** is a result of the ***r*-dependent term** $\langle\Delta\omega_\beta\rangle_\phi(\mathrm{r})$ in the denominator, representing **amplitude-dependent frequency spread**
- **Landau damping** only from **even orders** of chromaticity; **odd orders *do not* produce** amplitude-dependent **frequency spread**

# Vlasov theory for nonlinear chromaticity

## ***Gaussian bunch:*** *benchmarks of theory vs. PyHEADTAIL*

- Highly **narrow-band resonator** impedance (Q = 5 x $10^4$) to fulfil **single-peak approximation**
- Compute **stability boundary diagrams** using Eq. III to **determine stability threshold**
- **Stability diagram** strongly **asymmetric** as a result of **one-sided** frequency **spread**
- Here **negative $\xi^{(2)}$** more suited for stabilization, since mode under consideration has ***Re Ω < 0***
- **Excellent agreement** on **stability threshold** between **PyHEADTAIL** tracking simulations and **stability diagram theory**

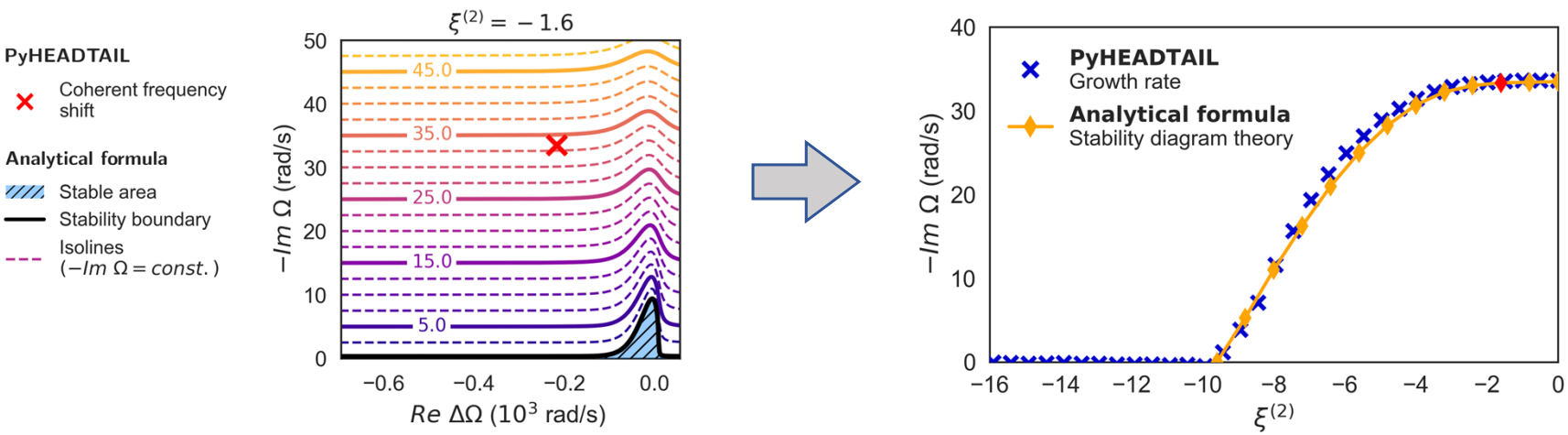


# Vlasov theory for nonlinear chromaticity

## ***Gaussian bunch:*** *benchmarks of theory vs. PyHEADTAIL*

- **Take it one step further**
  Evaluate dispersion relation (Eq. III) for **different values of *iε*** to obtain ***isolines of imaginary part*** and the **distortion of the complex frequency space**
  ⇨ **Instability growth rates at intermediate $\xi^{(2)}$**
- For this particular case, **coherent frequency shift with $\xi^{(2)}$ negligible** (see backup)
- **Theory** works well and shows a **very good agreement with PyHEADTAIL**



# Vlasov theory for nonlinear chromaticity

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- For this pa
- **Theory** wo

- **Second-order chromaticity** (even orders of chromaticity in general) and hence the **rf quadrupole indeed provide Landau damping**
- We have **identified the two effects** that were **observed** in **LHC experiments**

# Conclusions

- Traditional approach for **Landau damping** with **magnetic octupoles less efficient** in future machines
- **Exploring alternative: detuning with longitudinal amplitude** (rf quadrupole / nonlinear chromaticity) from **theoretical, numerical,** and **experimental** points-of-view
- **LHC experiment with Q'' showed two effects**
  1. Landau damping
  2. Change of unstable mode
- **Both effects** are accurately **reproduced** with **PyHEADTAIL**
- **Extended Vlasov formalism for nonlinear chromaticity:** both mechanisms identified analytically
- **Benchmarks against numerical models** show excellent agreement and **validate the theory**
- **Nonlinear chromaticity and rf quadrupole do provide Landau damping**

**Agreement from all points-of-view: theory, simulations, and experiments, demonstrating a good understanding of the involved beam dynamics**

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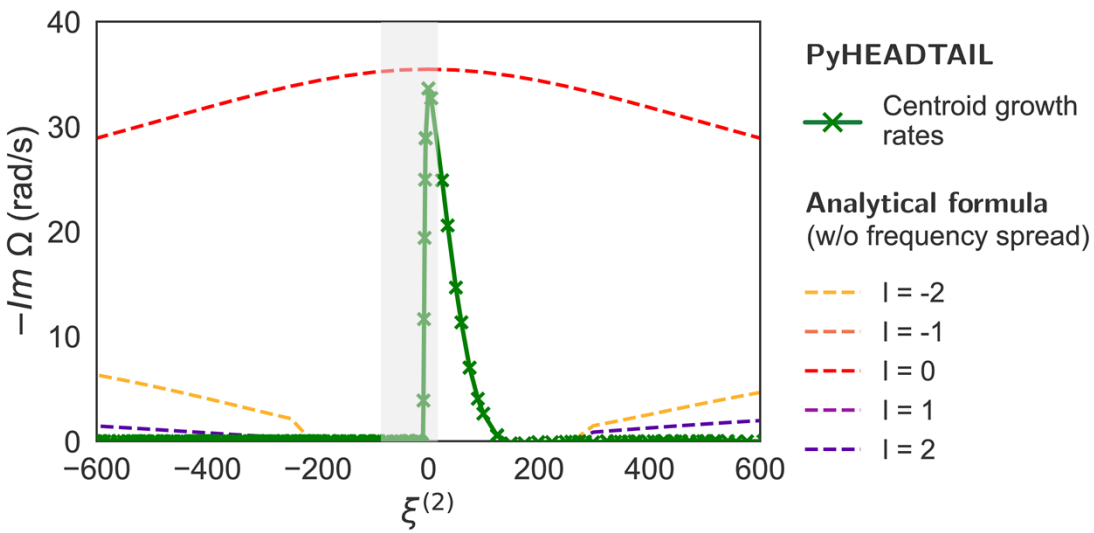
[13] X. Buffat, *Transverse beams stability studies at the Large Hadron Collider*, PhD Thesis, EPFL, Switzerland, 2015.

# Thank you!

# Backup

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