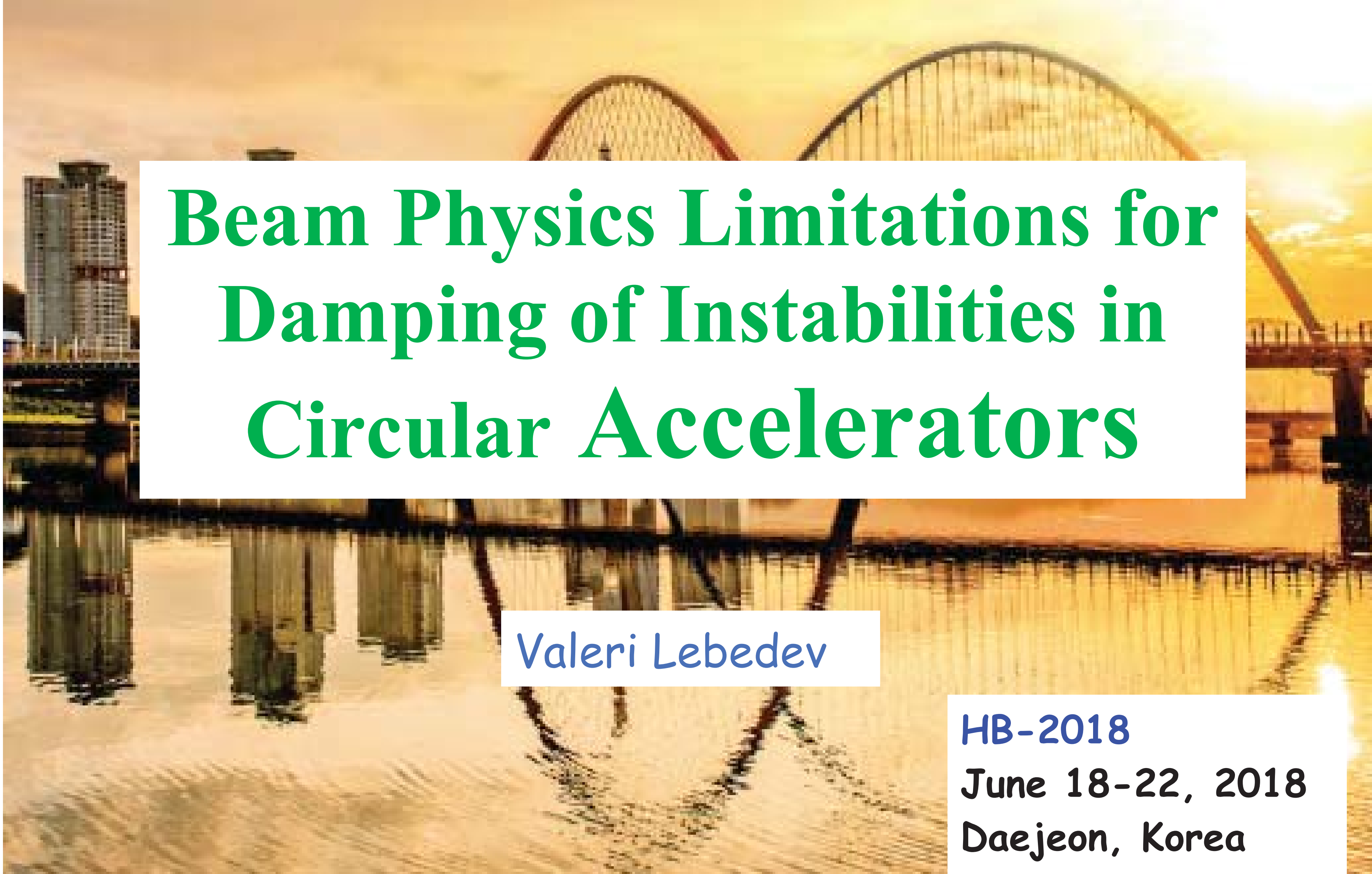

Beam Physics Limitations for Damping of Instabilities in Circular Accelerators

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Fermilab

Objectives

- To discuss delusions which disappeared with more work
  - Some delusions were discarded during this talk preparation
- To share accumulated experience

Talk Outline

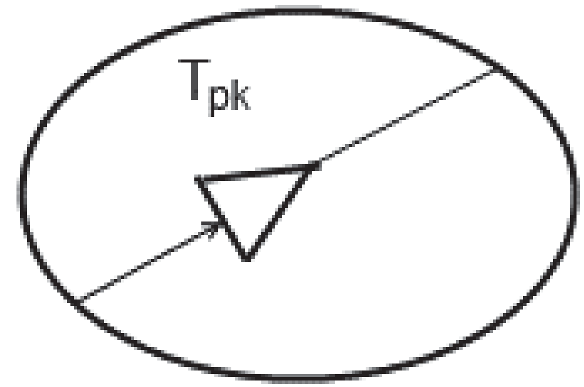
- Causality and correction of damper transfer function
- Emittance growth suppression by a damper
- Limitations on the FB system gain
- Analog preprocessing and postprocessing in digital FB systems
- Effects of x-y coupling on damping

Causality

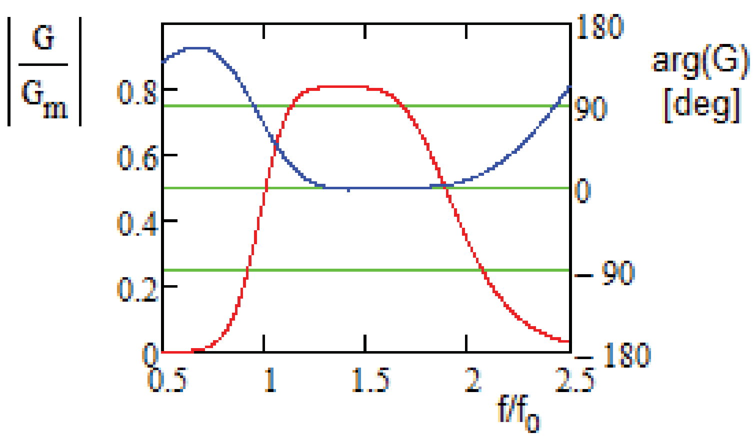
- Causality results in that for typical amplifier the amplitude and phase responses are related (Kramers - Kronig relations)
  - However, there is no causality limitation in a damper
  - Shorter cable can make signal coming ahead of bunch (particle)

⇨ Amplitude and phase responses can be controlled independently but it requires additional time

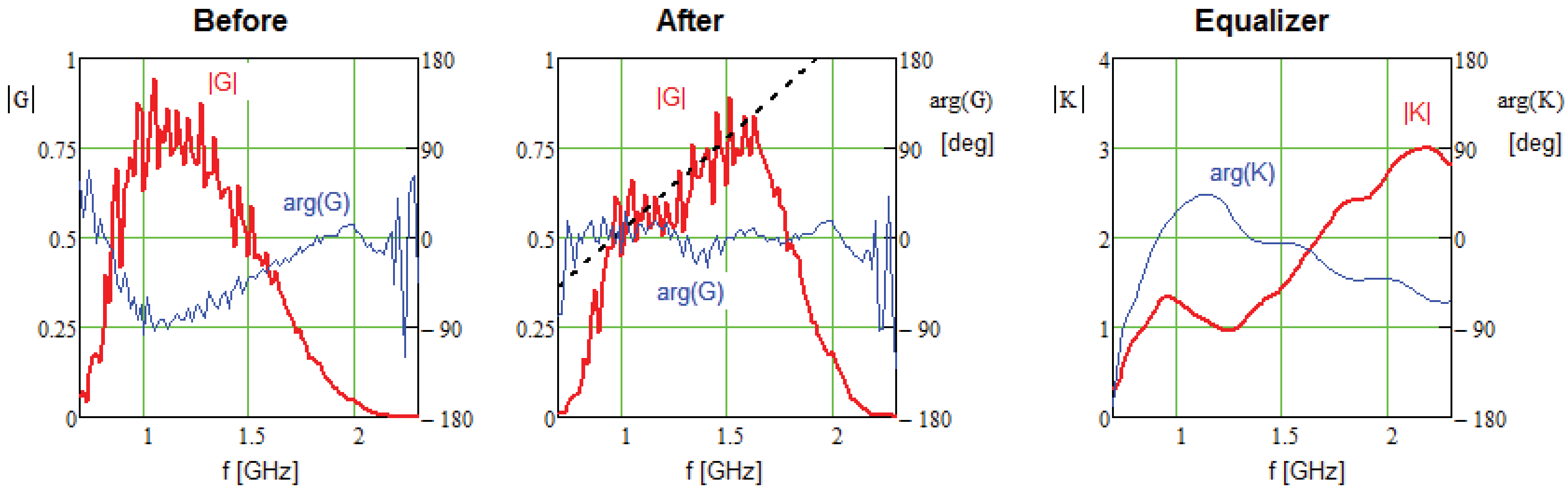
- Main limitations for digital filtering are the same but digital filtering adds additional flexibility to a system



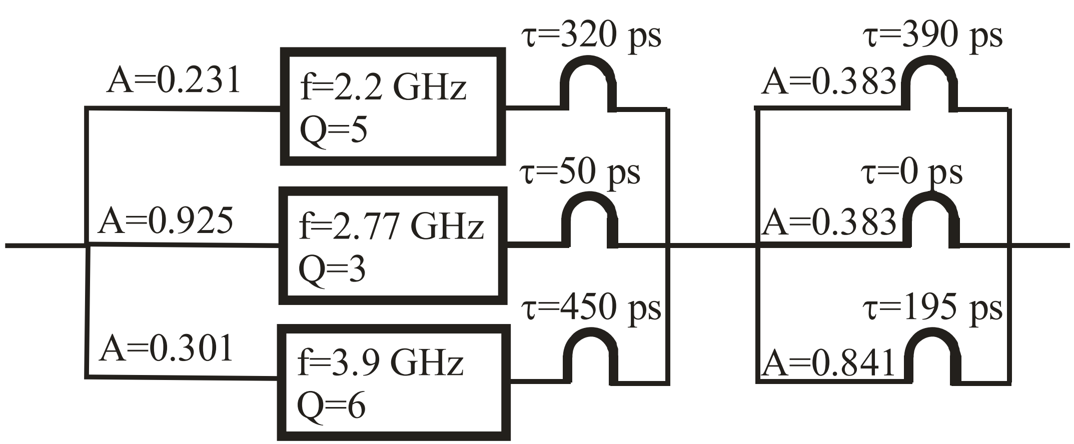
Phase in the band center is corrected by negative delay

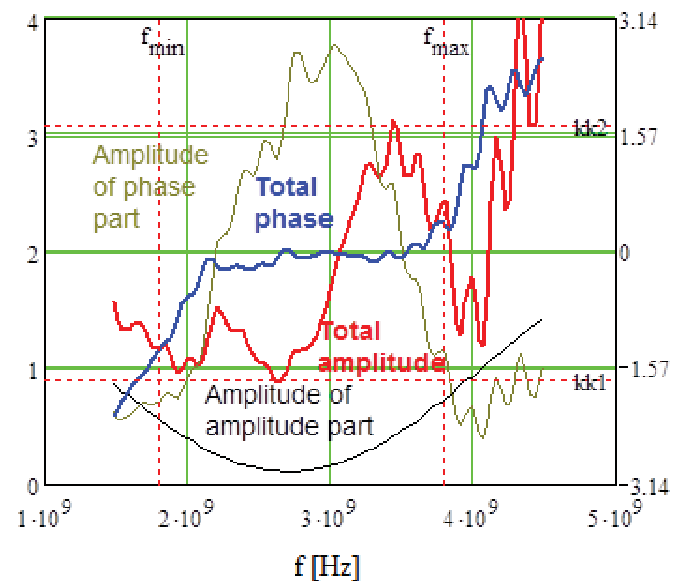


Equalizers for Stochastic Cooling



Correction of transfer function for Recycler





Equalizer for Accumulator Stacktail

Analog Circuit to Create 1/sqrt(ω) Gain Dependence

- Often ⊥ multi-bunch instability is driven by resistive wall impedance

  $$\Rightarrow \quad \lambda \propto 1/\sqrt{\omega}$$

  - That determined the choice of this example

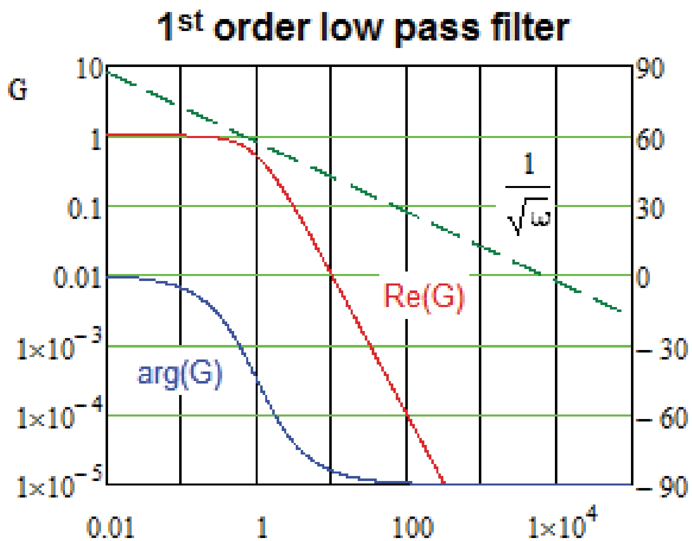
$\tau_1 := 1 \quad \tau_2 := 0.02$

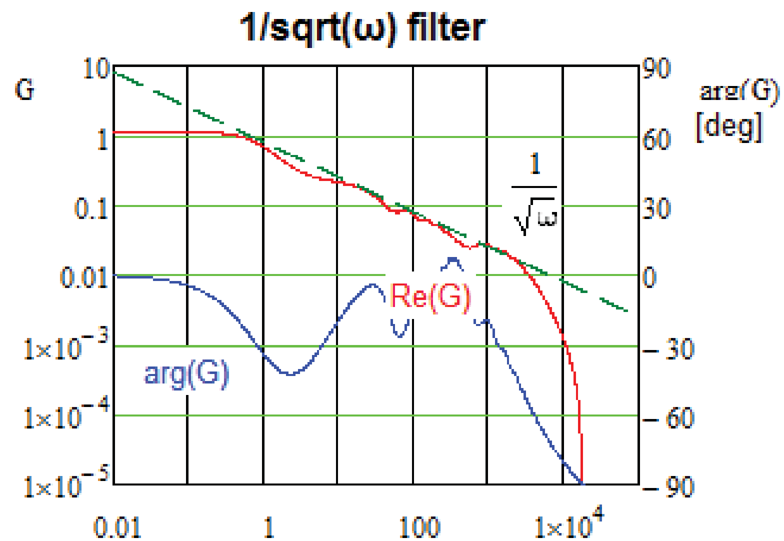
$\tau_3 := 0.02 \quad \tau_4 := 0.004 \quad \tau_5 := 4\cdot10^{-4} \quad \tau_6 := 8\cdot10^{-6}$

$$G(\omega) := \frac{1}{1 + i\cdot\omega\cdot\tau_1}$$

$$G_{c1}(\omega) := \frac{\exp(.1i\cdot\omega\cdot\tau_1)}{(1 + i\cdot\omega\cdot\tau_1)\cdot(1 + i\cdot\omega\cdot\tau_2)} \qquad G_{c2}(\omega) := \frac{0.11\cdot\exp(0.01i\cdot\omega\cdot\tau_1)}{(1 + i\cdot\omega\cdot\tau_3)\cdot(1 + i\cdot\omega\cdot\tau_4)}$$

$$G_{c3}(\omega) := \frac{0.03\cdot\exp(.00i\cdot\omega\cdot\tau_1)}{(1 + i\cdot\omega\cdot\tau_5)\cdot(1 + i\cdot\omega\cdot\tau_6)} \qquad G_c(\omega) := G_{c1}(\omega) + G_{c2}(\omega) + G_{c3}(\omega)$$





Right: an example of analog filter with 1/sqrt(ω) gain

- Method can be used for correction the power amplifier phase response
  - At sufficiently low frequencies it can be done with digital filter

Suppression of Emittance Growth by FB

- Without damping, ⊥ kicks ($\Delta\theta$) result in growth of betatron amplitude

$$\frac{d}{dt}\overline{\Delta x^2} = \frac{f_0}{2}\beta^2\overline{\Delta\theta^2}$$

- For noise with spectral density $S_\theta(\omega)$ we have

$$\frac{d}{dt}\overline{\Delta x^2} = \frac{\beta^2\omega_0{}^2}{4\pi}\sum_{n=-\infty}^{\infty} S_\theta\left((\nu-n)\omega_0\right), \quad \omega_0 = 2\pi f_0$$

where the normalization is: $\overline{\Delta\theta^2} = \int_{-\infty}^{\infty} S(\omega)d\omega$

- Beam decoherence results in emittance growth

$$\boxed{\left(\frac{d\varepsilon}{dt}\right)_0 = \frac{\omega_0{}^2}{4\pi}\sum_k \beta_k \sum_{n=-\infty}^{\infty} S_{\theta_k}\left((\nu-n)\omega_0\right) \xrightarrow[\text{single source}]{\text{white noise}} \frac{f_0\beta}{2}\overline{\Delta\theta^2}}$$

Only resonant frequencies contribute to the emittance growth

- If decoherence is slower than damping, $d\varepsilon/dt$ is suppressed[1]:

$$\boxed{\frac{d\varepsilon}{dt} = \frac{16\pi^2\overline{\Delta\nu^2}}{g^2}\left(\frac{d\varepsilon}{dt}\right)_0, \quad g \gg \sqrt{\overline{\Delta\nu^2}}}$$

where the damping rate in amplitude is: $\lambda = f_0 g/2$

and $\sqrt{\overline{\Delta\nu^2}}$ is the rms tune spread

[1] Particle Accelerators, 1994, Vol. 44, pp. 147-164 and pp. 165-199

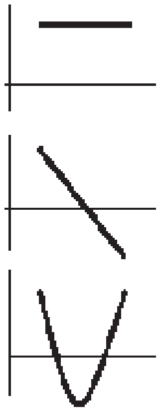
Emittance Growth due to FB System Noise

- Noise in FB system results in additional excitation of betatron oscillations
  - Referencing all FB system noises to the BPM one obtains:

$$\boxed{\frac{d\varepsilon}{dt} \approx \frac{16\pi^2\overline{\Delta\nu^2}}{g^2+16\pi^2\overline{\Delta\nu^2}}\left[\left(\frac{d\varepsilon}{dt}\right)_0 + \frac{f_0 g^2}{2\beta_{pk}}\sigma_{pk}{}^2\right], \quad \sigma_{pk}=\sqrt{\overline{X^2_{BPM}}}}$$

- Suppression of BPM noise by gain reduction with frequency
  - $g \gg \sqrt{\overline{\Delta\nu^2}}$ then effect of FB noise does not depend on gain
  - If at high freq. $g \le \sqrt{\overline{\Delta\nu^2}}$ and external noise is negligible then the BPM noise is suppressed. Actual noise reduction depends on parameters.

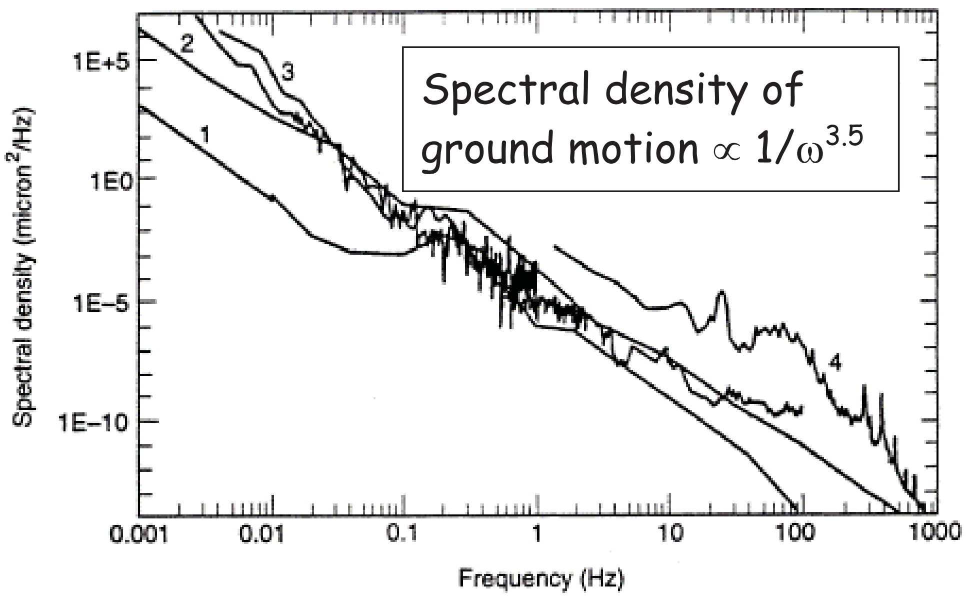
$$\frac{d\varepsilon}{dt} \approx \sum_{n=-n_b/2}^{n_b} \frac{16\pi^2\overline{\Delta\nu^2}}{g_n{}^2+16\pi^2\overline{\Delta\nu^2}}\left[\left(\frac{d\varepsilon}{dt}\right)_n + \frac{f_0 g_n{}^2}{2\beta_p}\frac{\sigma_{bpm}{}^2}{n_b}\right]$$

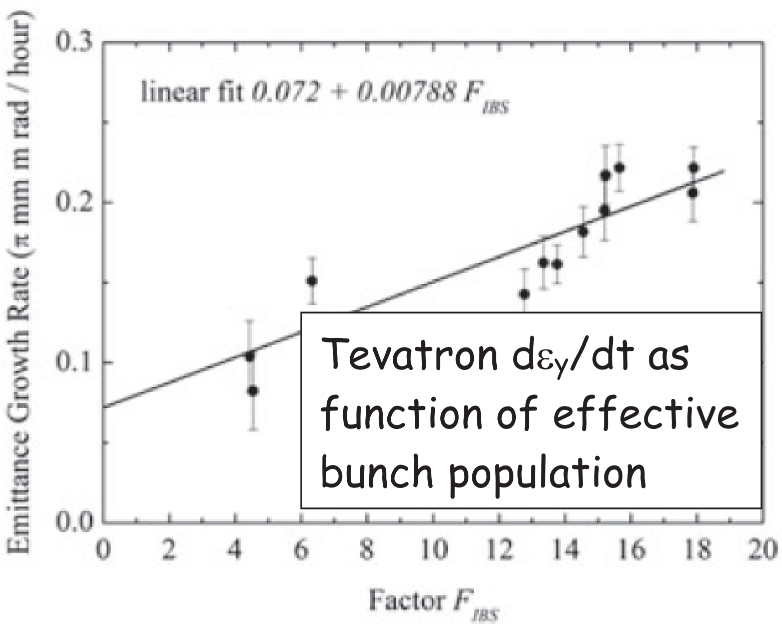


- For head-on collisions in the collider: $\sqrt{\overline{\Delta\nu^2}} \approx 0.2\xi$
- External noise is at low frequencies
  => bunch is kicked as one whole
  - For non-zero chromaticity synchrotron motion changes $\perp$ bunch shape
    => $g > \nu_s$ to prevent suppression of emittance growth

Sources of Emittance Growth

- Fluctuations of bending field
  - Fluctuations of current
  - Oscillations of liners inside SC dipoles (frozen B field), $\Delta B/B \ll 10^{-9}$
- Quad displacement due to ground motion,
- In Tevatron at collisions about 20% of emittance growth is related to field fluctuations
  - Inability to operate at low betatron tune
  - Emittance growth due to scattering on residual gas are excluded by other measurements





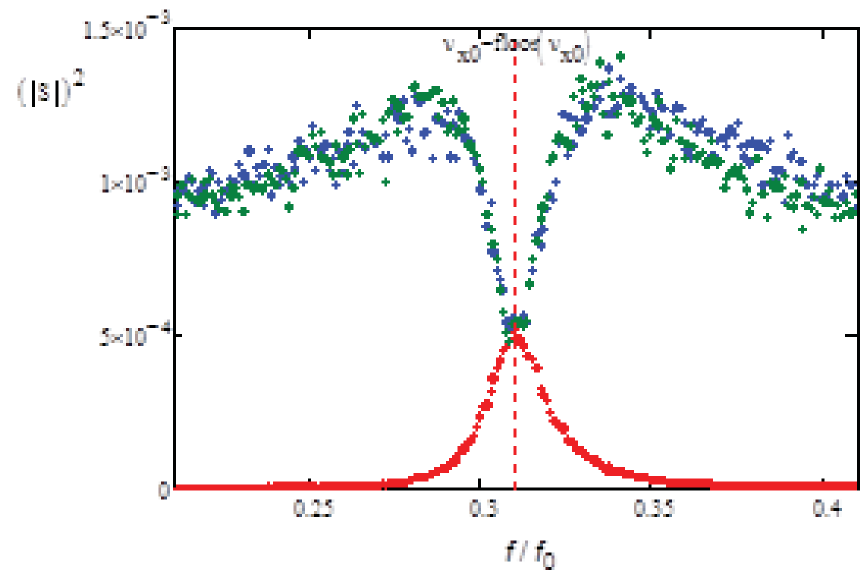
Challenges of Large Hadron Colliders

- With energy increase the revolution frequency comes to audio-frequencies where noise spectral density is high
- LHC "hump" (summer of 2010) resulted in unacceptably large emittance growth + large jumps of emittances
  - These harmful effects were suppressed by gain increase in transverse dampers ($g \gg \xi$)
  - It also required a reduction of damper noise
    - Achieved accuracy of BPMs of about ~ 0.2 - 0.5 μm still produced measurable emittance growth
  - Noisy power supplies were found in about half year
- Problems will grow fast with an increase of machine energy due to coming to even smaller frequencies
- Large size excludes using analog system (digital notch filter)

$$\delta\theta_n = \frac{g_1}{\sqrt{\beta_{p1}\beta_{kick}}} \sum_{k=0}^{K-1} A_k \left(x_{n-k} + \delta x_{n-k} \right), \quad \xrightarrow[\text{condition}]{\text{notch filter}} \quad \sum_{k=0}^{K-1} A_k = 0$$

Suppression of BPM Noise by Digital Filters

- Can optimally built digital filter reduce sensitivity to BPM errors?
  - The answer is no!!!
    - Simple explanation is that each error is applied K times. Errors are added coherently the same as damping terms.
    - This final answer is correct if the damper is in the linear regime. I.e. the gain is sufficiently small so that FB system is far from instability
    - Formal prove is in: W. Hofle, V. Lebedev et al. IPAC'11
- Measurements of spectrum of BPM signal enables computation of betatron frequency, and damper gain and phasing
- More points are used in filter more sensitive is the damper to a betatron frequency error and smaller maximum gain is achievable $g_{max} \propto 1/K$



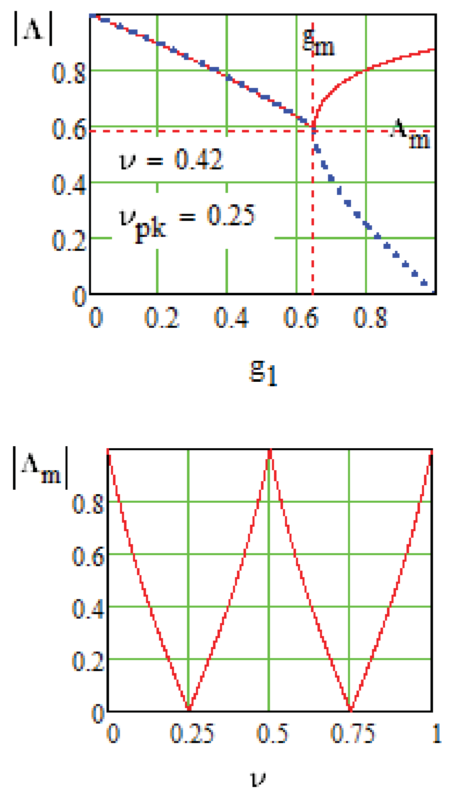
Spectral density of noise for two-BPM LHC system
Red – actual beam motion

Limitations on the FB System Gain

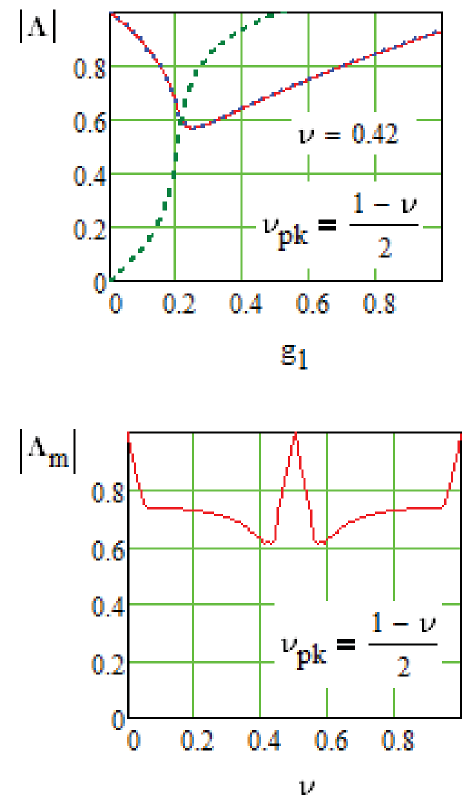
- Large hadron colliders require large FB system gain to suppress $d\varepsilon/dt$

$$\mathbf{x}_{n+1}=\mathbf{M}_{kp}\left(\mathbf{M}_{pk}+\begin{bmatrix}0&0\\0&g\end{bmatrix}\sum_{k=0}^{K-1}A_k x_{n-k}\right)\Rightarrow\left|\begin{bmatrix}1&0\\0&1\end{bmatrix}\lambda-\mathbf{M}-\mathbf{M}_{kp}\begin{bmatrix}0&0\\0&g\end{bmatrix}\sum_{k=0}^{K-1}A_k\lambda^{-k}\right|=0$$

- Usage of larger number of turns for correction computation reduces achievable damping rate; $\lambda_{max}\propto 1/K$.



One-turn system (no notch filter): optimal ν_{pk}=0.25, 2 eigen-values



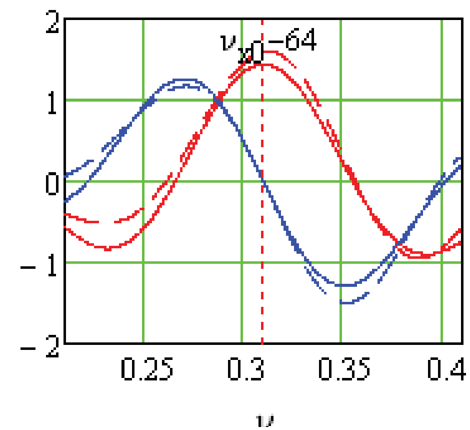
Two-turn system (notch filter): opt. ν_{pk}=(1-ν)/2, 4 eigen-values (Λ_0=0, 3 others are shown)

LHC Transverse Dampers (as in 2011)

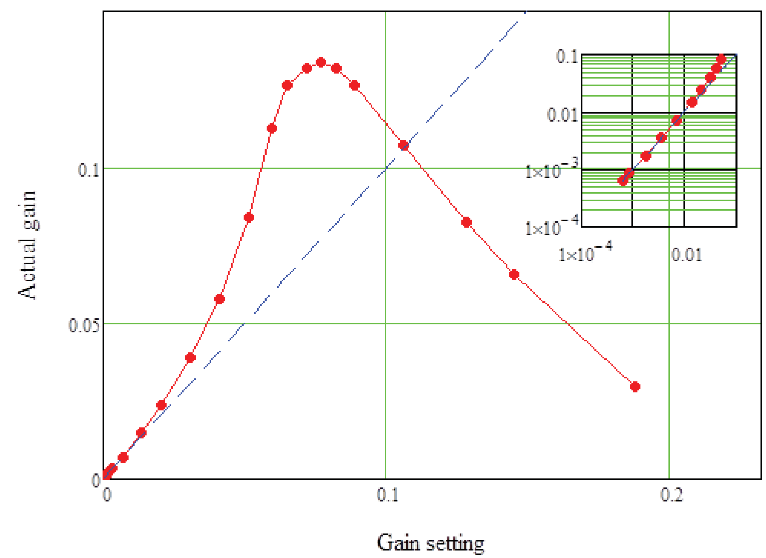
- In 2011 the LHC transverse damper was based on 7^{th} order filter

$$A_k = \left[-\frac{2}{3\pi}\sin\psi \quad 0 \quad -\frac{2}{\pi}\sin\psi \quad \cos\psi \quad \frac{2}{\pi}\sin\psi \quad 0 \quad \frac{2}{3\pi}\sin\psi \right]^T$$

- That significantly reduced the maximum achievable gain of the system and introduced excessive sensitivity to the machine tune
- As far as we understand now such choice did not deliver any advantages
  - In particular, same sensitivity to the BPM noise
- "Reasonable" filter should use 3 turns to accommodate notch filter and arbitrary phase advance between pickup and kicker



Real (red) and imaginary (blue) parts of the gain on the machine tune

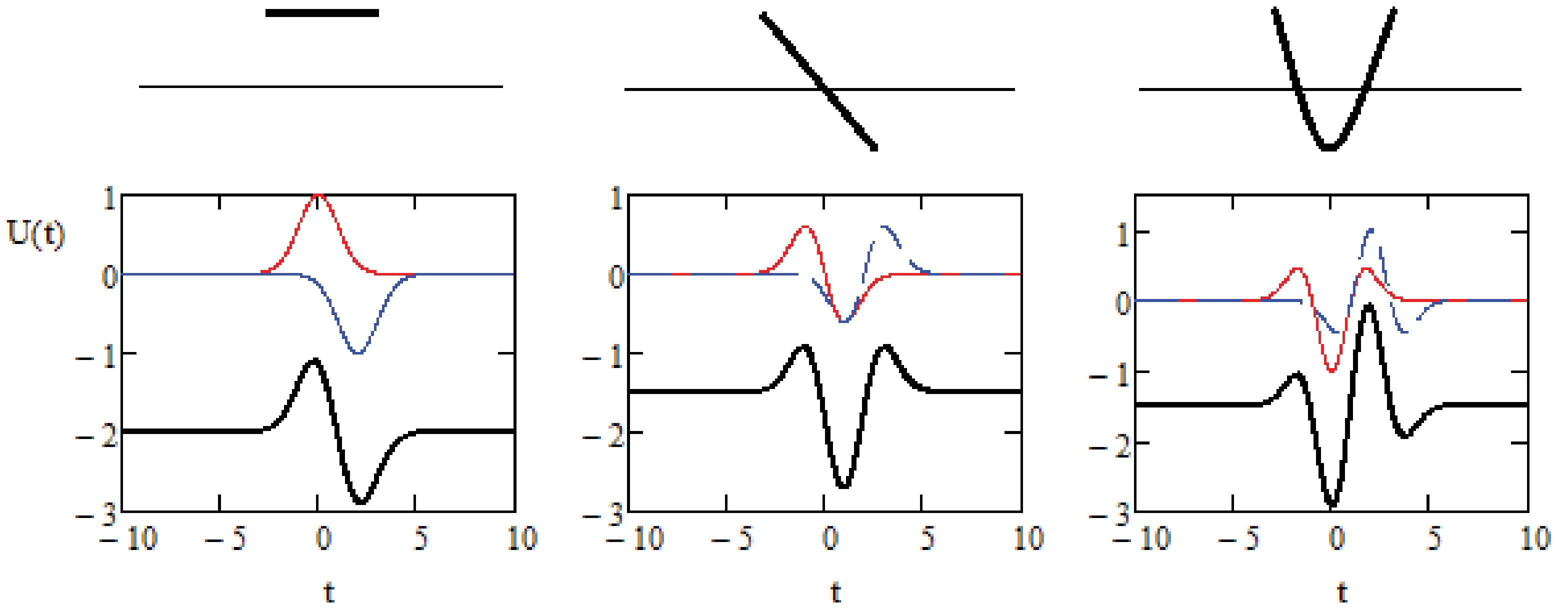


System gain on the gain setting

What does a BPM Measures?

- Strip-line BPM out voltage

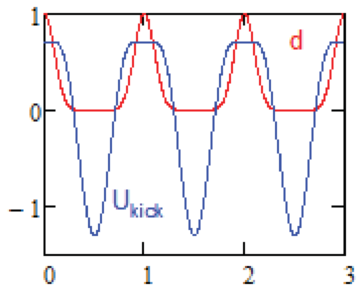
$$U(t) = Z_{cpl}\left(I(t) - I(t - 2L/c)\right), \quad v = c$$



- BPM signals are similar for
  - intra-bunch HOM &
    development of bunch betatron oscillations after uniform bunch kick
- Result of BPM measurements depends on signal treatment before digitization (analog preprocessing)
  - If σ_b << bunch-bunch distance, an analog integration yields center of gravity

Analogue Preprocessing & Postprocessing in Digital FB

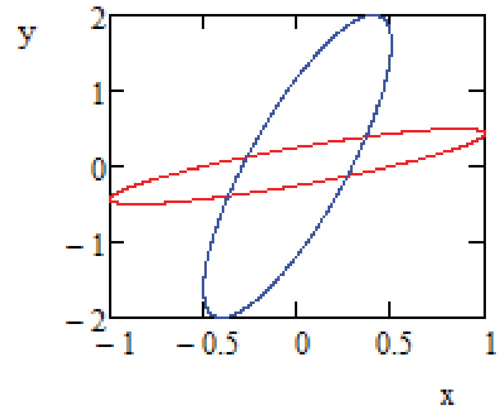
- The following analogue preprocessing methods are usually used:
    - Integration. It may deliver the center of gravity ($\sigma_b \ll L_{bb}$)
    - Mixing with RF + low pass filter ($\sigma_b \ll L_{bb}$)
    - Excitation of oscillator with subsequent digitization's ($\sigma_b \ll L_{bb}$)
    - More than one-point digitization …
  - ♦ Typically all methods may be sensitive to HOMs
- Similar the kick value across bunch may depend on time
  - ♦ That makes non-uniform kicks
  - ♦ May result in additional emittance growth and excitation of HOMs



- Thus, analog preprocessing and post processing affect on the HOM damping/excitation

  In 1st order PT: $\lambda_n \propto -\int X_n(s) U_n(s) ds$
  - ♦ It limits the gain for zero (dipole) mode due to excitation of HOMs
  - ♦ Correctly chosen analog preprocessing and post processing result in a reduction of HOMs excitation and, possibly, damping for some of them
    - It depends on details of each machine
    - It is an area requiring further studies

Effects of X-Y Coupling



- In the course of Tevatron Run II we observed that switching on a one-plane damper could introduce instability in another plane
- The reason of such behavior was strong x-y coupling which could not be completely compensating because of uncontrolled skew-quad components in SC dipoles
- Running dampers for both planes made beam stable

The analysis of the problem can be done similar to a single dimensional case where 2D matrices are replaced by 4-D

$$\mathbf{x}_{n+1} = \mathbf{M}_{kp}\left(\mathbf{M}_{pk} + \mathbf{G}_{x,y}\sum_{k=0}^{K-1} A_k \mathbf{x}_{n-k}\right) \quad \Rightarrow \quad \left|\lambda \mathbf{I} - \mathbf{M} - \mathbf{M}_{kp}\mathbf{G}_{x,y}\sum_{k=0}^{K-1} A_k \lambda^{-k}\right| = 0$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{G}_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{G}_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & g & 0 \end{bmatrix}$$

Perturbation Theory for Symplectic Motion

- In many applications a solution with perturbation theory is sufficient
  - Unperturbed motion $\mathbf{M}\mathbf{v}_j = \lambda_j \mathbf{v}_j$
  - Perturbed motion $(\mathbf{M} + \Delta\mathbf{M})(\mathbf{v}_j + \Delta\mathbf{v}_j) = (\mathbf{v}_j + \Delta\mathbf{v}_j)(\lambda_j + \Delta\lambda_j)$
  - Tune shifts: $\begin{bmatrix} \Delta\nu_1 \\ \Delta\nu_2 \end{bmatrix} = -\frac{1}{4\pi}\begin{bmatrix} \mathbf{v_1}^+ \mathbf{S}\,\Delta\mathbf{M}\,\mathbf{v_1} \\ \mathbf{v}_2^+ \mathbf{S}\,\Delta\mathbf{M}\,\mathbf{v}_2 \end{bmatrix}$
- For damper we have

$$\left| \lambda \mathbf{I} - \mathbf{M} - \mathbf{M}_{kp}\mathbf{G}_{x,y}\sum_{k=0}^{K-1} A_k \lambda^{-k} \right| = 0 \quad \Rightarrow \quad \Delta\mathbf{M}_{1,2} = \mathbf{M}_{kp}\mathbf{G}_{x,y}\sum_{k=0}^{K-1} A_k {\lambda_{1,2}}^{-k}$$

For horizontal damper one obtains (G_x -> G_y for vertical damper)

$$\begin{cases} \Delta\nu_1 = -\dfrac{1}{4\pi}\left(\displaystyle\sum_{k=0}^{K-1} A_k {\lambda_1}^{-k}\right)\mathbf{v_1}^+ \mathbf{S}\,\mathbf{M}_{kp}\mathbf{G}_x\,\mathbf{v_1} \\ \Delta\nu_2 = -\dfrac{1}{4\pi}\left(\displaystyle\sum_{k=0}^{K-1} A_k {\lambda_2}^{-k}\right)\mathbf{v}_2^+ \mathbf{S}\,\mathbf{M}_{kp}\mathbf{G}_x\,\mathbf{v}_2 \end{cases}$$

Conclusions

- Beam physics considerations should be important part of damper design
  - Ignoring them may result in compromised performance and excessive cost

- Damping in the course of slip-stacking is another topic not discussed here but important for support of Fermilab neutrino program.
  It is in a tomorrow morning talk:
  - "High Intensity Proton Stacking at Fermilab: 700 kW Running" R. Ainsworth