Scaling Laws for the Time Dependence of Luminosity in Hadron Circular Accelerators Based on Simple Models of Dynamic Aperture Evolution

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Research supported by the High Luminosity LHC project

Abstract

In recent years, models for the time-evolution of the dynamic aperture have been proposed and applied to the analysis of non-linear betatronic motion in circular accelerators. In this paper, these models are used to derive scaling laws for the luminosity evolution and are applied to the analysis of the data collected during the LHC physics runs. An extended set of fills from the LHC proton physics has been analysed and the results presented and discussed in detail.

Definition of Luminosity

Luminosity is defined as $L = \Xi\, N_1\, N_2$,

where $\Xi = \dfrac{\gamma_r f_{\text{rev}}}{4\,\pi\epsilon^* \beta^*\, k_b} F(\theta_c, \sigma_z, \sigma^*)$

is nearly constant as a first order approximation (excluding levelling and dynamic-β effects, and ignoring emittance blow-up).

We fit our model to the normalised integrated Luminosity data:

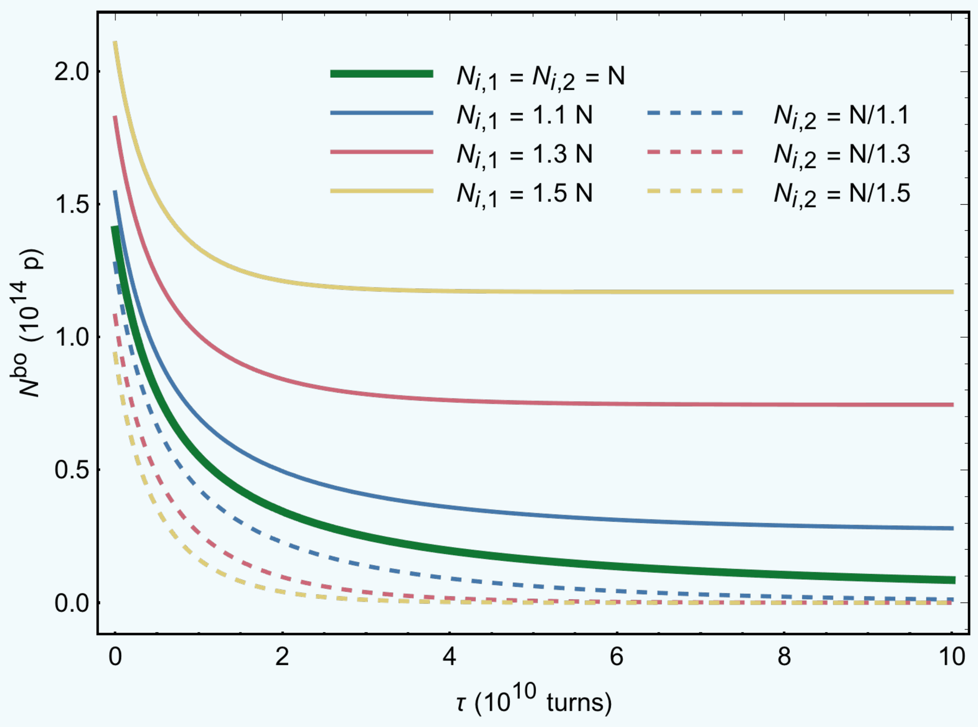
$$L_{\text{norm}}(\tau) = \frac{L_{\text{int}}(\tau)}{L_{\text{int}}(\infty)} = \frac{\varepsilon}{N_i\,\Xi}\int_1^{\tau} d\tilde{\tau}\, L(\tilde{\tau})$$

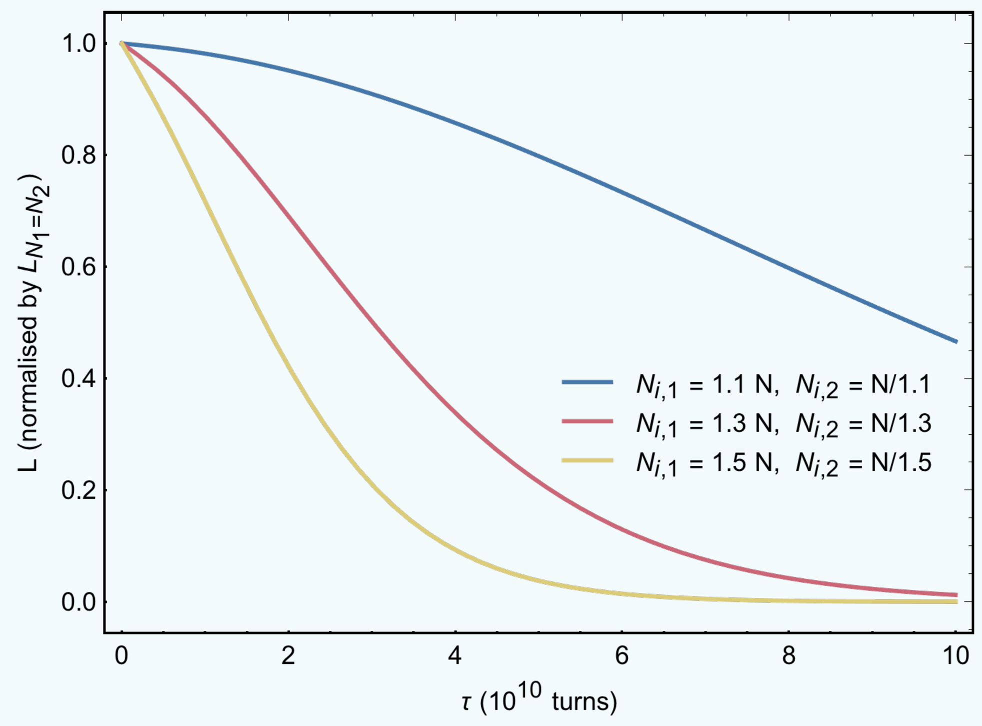
where τ is the number of elapsed turns, related to normal time by $\tau = f_{\text{rev}}\, t + 1$.

Burn-off contribution to the evolution of luminosity

The contribution of the burn-off — the number of protons that are colliding in the experiments — can be easily estimated from the exponential decline differential equation:

$$\dot{N}_1(\tau) = \dot{N}_2(\tau) = -\varepsilon N_1(\tau) N_2(\tau) \qquad \varepsilon = \frac{\sigma_{\text{int}}\, n_c\, \Xi}{f_{\text{rev}}} \sim 10^{-24}$$





Adding pseudo-diffusive effects: evolution of DA

The realistic behaviour is much more complex, e.g. beam-beam and IBS invalidate the above model. We then model all possible pseudo-diffusive effects by assuming that the evolution of the dynamic aperture (DA) is given by:

$$D(\tau) = D_\infty + \frac{b}{[\log \tau]^\kappa}$$

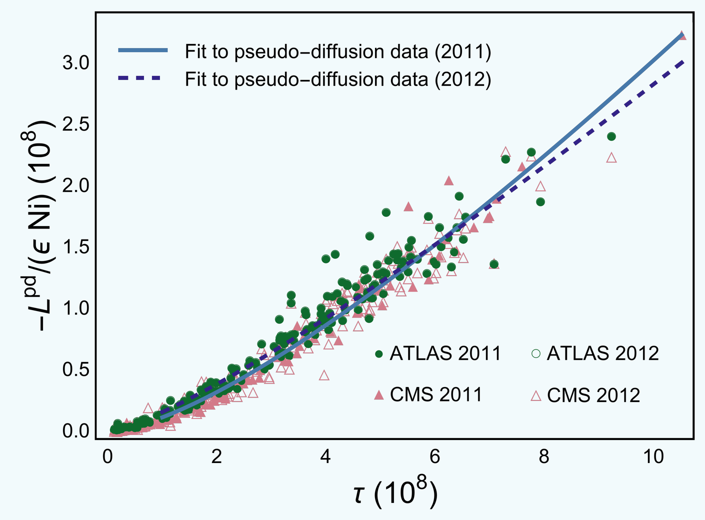
This modifies the previous differential equation into:

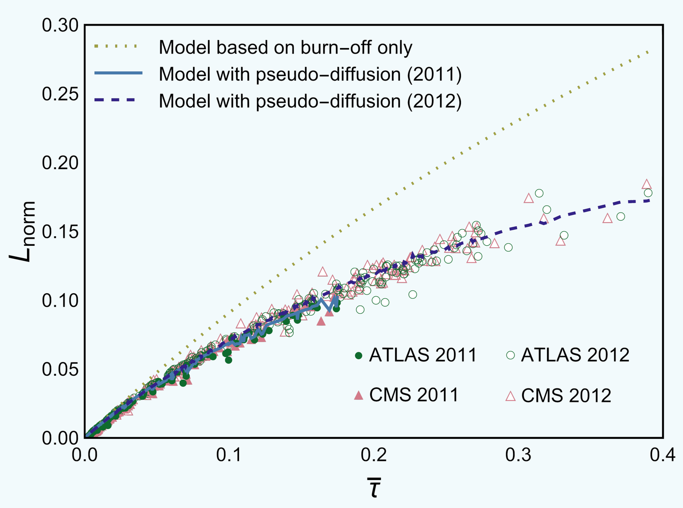
$$\dot{N}(\tau) = -\varepsilon\, N^2(\tau) - \mathcal{D}(\tau) \qquad \mathcal{D}(\tau) = N_i \left(\dot{\mathrm{e}^{-\frac{1}{2}D^2(\tau)}}\right)$$

Expand in orders of ε and write $L_{\text{norm}}(\tau) = L^{\text{bo}}_{\text{norm}}(\tau) + L^{\text{pd}}(\tau)$ where now finally

$$L^{\text{pd}}(\tau) = -\varepsilon N_i \int_1^{\tau} d\tilde{\tau} \left[e^{-\frac{D^2(\tilde{\tau})}{2}} - e^{-\frac{D^2(1)}{2}}\right] \left\{2 - \left[e^{-\frac{D^2(\tilde{\tau})}{2}} - e^{-\frac{D^2(1)}{2}}\right]\right\}.$$

Fit to 2011 and 2012 data



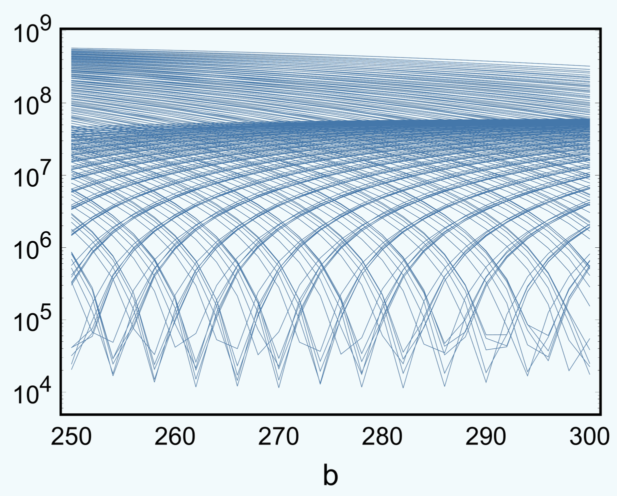


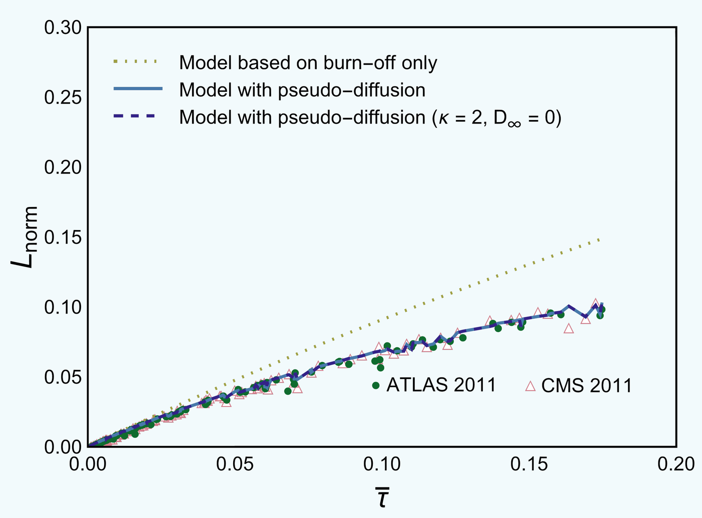
	2011	2012
$R^2_{\text{adj.}}$	97.84%	95.75%
D_∞	-0.43 ± 0.38	0.82 ± 0.52
b	350 ± 150	560 ± 114
κ	1.68 ± 0.16	2.08 ± 0.35

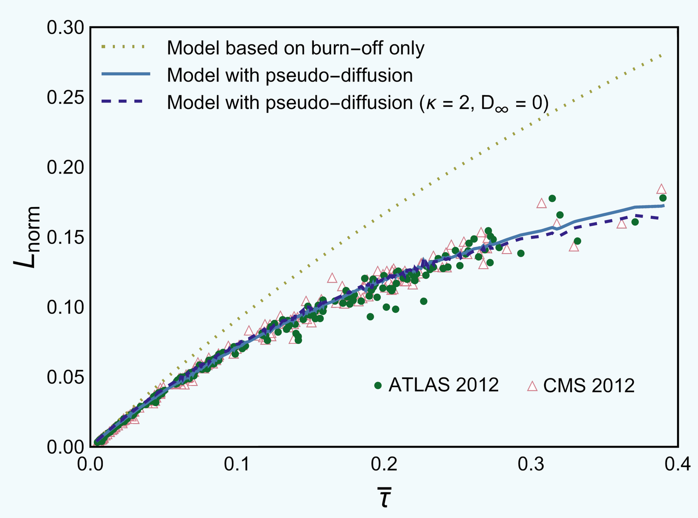
The model delivers an accurate reproduction of the data, with a stark difference when compared to burn-off only.

Degeneracy of the parameter space & fixing κ and D_∞

The fitting algorithm exhibits an approximate degeneracy: the sum of squares Σ^2 has an infinite set of minima. In other words, the parameters are linearly dependent. We investigate this by fixing one parameter and only fitting the two others.







$\kappa = 2$	2011	2012
$R^2_{\text{adj.}}$	97.85%	95.75%
D_∞	-0.03 ± 0.13	0.77 ± 0.13
b	757 ± 49	455 ± 49

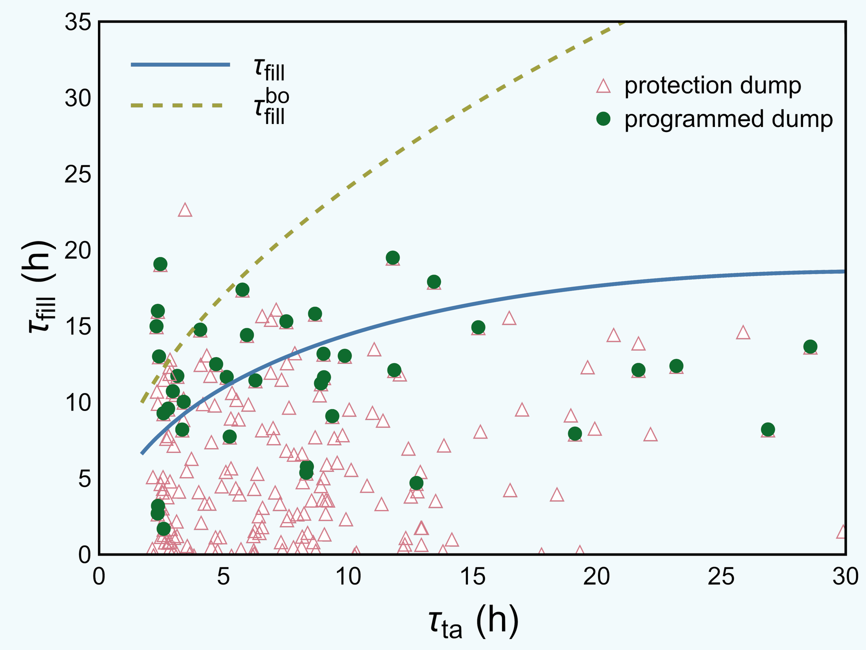
$D_\infty = 0$	2011	2012
$R^2_{\text{adj.}}$	97.85%	97.84%
b	830 ± 370	81 ± 26
κ	2.04 ± 0.13	1.25 ± 0.13

$D_\infty = 0$, $\kappa = 2$	2011	2012
$R^2_{\text{adj.}}$	97.85%	95.15%
b	744.0 ± 1.8	738.2 ± 1.4

The model with $\kappa = 2$ gives rise to an equally good fit as before, while the one with $D_\infty = 0$ even gives a slightly better fit for 2012. Fixing both κ and D_∞ worsens the fits, be it only slightly.

Optimal fill length for 2012

The DA model can be used to calculate the optimal length of a fill, given the turnaround time before that fill.



The fills shown are those in 2012 with a deliberate dump. Their values are close to the optimal fill length when including L^{pd}.

Conclusions & impact

- DA model reproduces luminosity evolution
- similar results for 2011 and 2012
- κ close to theoretical estimate ($\kappa \sim 2$)
- fixing one parameter does not worsen fit
- optimal fill length from DA model
  → clear difference with or without L^{pd}
  → can be used for new algorithms

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