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FIXED FIELD ACCELERATORS AND SPACE CHARGE MODELING

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Abstract

The efforts of the Fixed Field Accelerators FFA (formerly known as FFAG accelerators) community to address the high intensity challenge are reviewed. Starting from analytic estimates and linear models for space charge computation, the current possibilities of precise 3D models for start to end modeling are discussed.

HISTORY AND TAXONOMY OF FIXED FIELD ALTERNATING GRADIENT MACHINES

Historical Account

The concept of an FFA is not new. This type of accelerator was invented in the 1950s and 1960s at the same time as the synchrotron was being developed. Much of the early work in developing FFAs was carried out at the Midwestern Universities Research Association (MURA), but only electron FFAs were constructed at the time [1]. More about the history can be found in [2] and the references therein.

Working Principle and Taxonomy

A particular area of recent interest in the field of FFAs is their potential for high-intensity operation, because of their high repetition rate, large acceptance, simpler and cheaper power supplies, and flexibility of the RF acceleration system.

The FFA is a class of circular accelerator that combines properties of both the cyclotron and the synchrotron. It uses a magnetic field which is constant in time, hence the 'fixed-field', together with an increased focusing strength achieved using the 'alternating-gradient' principle [3]. The RF acceleration scheme is usually variable-frequency, but in some specific instances a fixed-frequency system is possible.

Starting with the idea that FFAs are just accelerators which have both a fixed field and alternating-gradient focusing produces a large spectrum of designs. Most FFAs have a very large dynamic aperture. This flexibility of FFA design has only emerged in roughly the last 15 years and the field continues to be a rich source of novel developments.

The Original or 'Scaling' FFA

In 1943 Marcus Oliphant described the idea of the synchrotron as follows:

> Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field ... which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations.

He intended that the magnetic field should be varied temporally and the beam should always follow the same annulus. However, in principle there is no reason why the annulus may not change radius and the field vary spatially rather than temporally. This is the fundamental idea behind the FFA. A large variation of the field with radius will constrain the change in radius of the orbits; this can lead to a larger field increase with radius and more compact orbits than in a cyclotron. This is the original type of FFA, which we now call 'scaling'.

The FFA accelerators, were proposed independently in the early 1950s by Ohkawa in Japan [4], Symon *et al.* in the United States [5], and Kolomensky in Russia [6].

Symon *et al.* proposed:

> A type of circular accelerator with magnetic guide fields which are constant in time, and which can accommodate stable orbits at all energies from injection to output energy.

This relies on introducing sectors with a reversed magnetic field into a cyclotron-like machine, producing strong focusing throughout the energy range. The field may rise rapidly with radius such that the orbits are relatively compact over a large energy range.

The field is arranged in such a way that the increase in gradient with momentum results in the beam experiencing the same focusing independent of radius. This means that the betatron tunes are constant for all orbits. This constant focusing (or constant betatron tune) is ensured if two conditions are met. First, the field index k must be constant, where we can define k in terms of the bending radius ρ, the vertical magnetic field B_y, and its derivative in the horizontal direction x:

$$k = -\frac{\rho}{B_y}\frac{\partial B_y}{\partial x}. \tag{1}$$

Therefore we require

$$\left.\frac{\partial k}{\partial p}\right|_{\theta=\text{const.}} = 0. \tag{2}$$

The second requirement is that the shape of the particle orbits remains constant as the size of the orbits 'scales' with energy, such that each higher-energy orbit is a geometrically similar enlargement of the lower-energy orbits as described by the following equation, derived by Kolomensky [7]:

$$\left.\frac{\partial}{\partial p}\left(\frac{\rho_0}{\rho}\right)\right|_{\theta=\text{const.}} = 0. \tag{3}$$

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If the field meets these two conditions, the FFA is referred to as being of the 'scaling' variety.

To satisfy these requirements, we use a magnetic field that increases with radius. The particular shape of the field is given by the r^k law with a reference radius r_0, where the field increase is characterised by the field index, k:

$$B_y = B_0 \left(\frac{r}{r_0} \right)^k . \tag{4}$$

For details, illustrations and other types such as the spiral FFA we refer to [8].

In terms of beam dynamics, it is useful to compare the scaling FFA with a synchrotron. Modern synchrotrons employ the principle of alternating-gradient or 'strong' focusing [9,10], in which alternating focusing and defocusing magnets lead to much stronger focusing forces in the transverse plane than in constant-gradient weak-focusing synchrotrons. This alternating-gradient focusing is also employed in the FFA. The transverse beam dynamics in the FFA is therefore much the same as in the synchrotron, at least for a single orbit or energy, in the sense that we may discuss beta functions, dispersion, and so forth. The difference is that in this case the field is highly nonlinear and these transverse optics functions may vary with radius.

The Non-scaling FFA

The non-scaling FFA allows the strict scaling laws applied in the original scaling FFA to be relaxed. The idea of violating the strict scaling law of the FFA occurred to Kent Terwilliger and Lawrence W. Jones in the 1950s [11], but such a machine was never pursued. Two of the main disadvantages of the original FFA are the highly nonlinear magnetic field required and the large aperture of the magnets and RF arising from the shift of the orbit with energy, which can be up to the order of 0.5–1.0 m. The non-scaling FFA arose from the question "what if we violate the scaling law?" Or, more specifically, "What if we take a line tangent to the scaling law, such that the field is linear with radius?" This radical idea led to the linear non-scaling FFA and was proposed in the 1990s [12].

Linear non-scaling FFA The linear non-scaling FFA is so called because it uses only up to linear focusing elements, that is, quadrupole and dipole fields. When only quadrupoles and no higher-order multipoles are used for focusing, the beam shifts outward with acceleration because of dispersion and is subject to a reduced level of focusing. This is really like considering a synchrotron where we do not ramp the magnets with time. In the scaling FFA, this is avoided by varying the gradient with the momentum and by making the beam pipe wider to allow for the orbits moving. But in the linear non-scaling FFA the scaling law is ignored, which allows the gradient to be increased and the the dispersion function to be reduced reduce even further to reduce the shift of the orbit with momentum. To achieve this, a linear non-scaling FFA lattice may use normal bending with a defocusing 'D' quadrupole and reverse bending with a focusing 'F' quadrupole, and may (or may not, depending on the design) change at high momentum to use the 'D' quadrupole for reverse bending and the 'F' for normal bending, as described in Ref. [13].

One must then ask what happens to the beam dynamics in such an accelerator. One consequence is that the orbits no longer 'scale', so they are no longer geometrically similar at different energies. The orbits can be made much more compact than in the scaling FFA. However, the most dramatic difference is that the betatron tunes are no longer constant with energy. Betatron tunes are usually designed to be kept constant in order to avoid the effects of resonances. In the linear non-scaling FFA, they vary dramatically throughout the acceleration cycle, crossing not just high-order betatron resonances but also integer resonances.

In theory, if the acceleration is fast enough, the beam may be able to cross betatron resonances before they have time to build up, and therefore any amplitude growth effects may be mitigated. How fast this crossing needs to be depends on imperfections and alignment errors in the machine, and clearly necessitates a fast acceleration rate. In fact, the linear non-scaling FFA was proposed in the context of muon acceleration, where very fast acceleration before the muons decay is an absolute requirement. The many questions surrounding the dynamics of such a machine led to the construction of the first non-scaling FFA accelerator, known as EMMA [14].

FORWARD LOOKING PROJECTS IN THE HIGH INTENSITY FRONTIER

At the high intensity frontier, the KURRI collaboration is pursuing an attempt to understand high intensity effects, with dedicated experiments and modelling efforts. The accelerator complex has been operated for ADSR experiments connecting the 100 MeV proton beam line with the research reactor facility. Upgrade plans to higher energies (300 - 500 MeV) includes a new FFA ring which adopts continuous acceleration with fixed frequency (serpentine acceleration). These higher energy beams can also be used for neutron or muon production experiments as well as ADSR studies. On the modelling front, extensive single particle studies, with a variety of codes, have being conducted [15], in order to validate the models. This code comparison, for example, includes tune computation as shown in Figure 1.

Other ambitious projects have been proposed beyond 1 MW of beam power, for example the MERIT proposal of Mori et al. [16] and the innovative design of S. Machida of an *Scaling Fixed-Field Alternating-Gradient Accelerators with Reverse Bend and Spiral Edge Angle* [17]. The latter is actively pursued as a possible replacement of the ISIS facility in the United Kingdom.

SPACE CHARGE MODELS

In circular accelerators, candidates for emittance growth are the half-integer stop-band that can perturb the beam envelope function, the Montague resonance, and the sum resonance induced by the random skew-quadrupole field.

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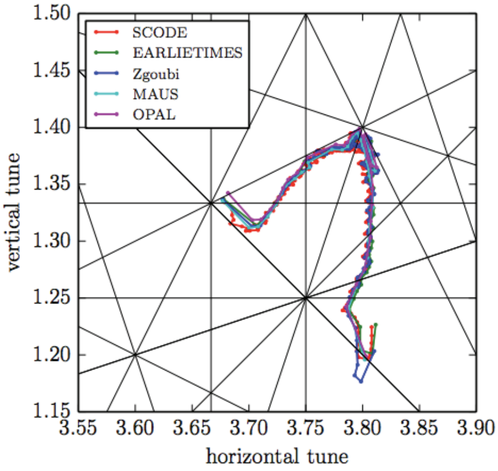

Figure 1: Betatron tune from 11 to 139 MeV of the 150 MeV FFA at KURRI, [15]

In particular, the betatron tunes of non-scaling FFA accelerators may cross many resonances during the beam acceleration. There are a few studies on the effects of resonance crossing in the FFA, which examine only non-systematic resonances, that are in principle correctable [18].

As shown in [19, 20] systematic nonlinear space-charge resonances may cause substantial emittance growth in non-scaling FFA accelerators. To avoid systematic nonlinear space-charge resonances, the phase advance of each non-scaling FFA cell must avoid $\pi/2$ and $\pi/3$.

Lee [19] used 24 FODO cells separated by dipoles. The 4D particle tracking algorithm uses linear maps $M_{1/2}$ for half of the FODO cell, followed by a space-charge kick M_{sc}. The same action is the applied at the end of the cell. The M_{sc} is calculated from a Gaussian charge distribution.

Using multi-particle numerical simulations, Lee et al. empirically obtain a minimum tune ramp rate vs the systematic 4th order space-charge resonance strength.

The emittance growth factor, EGF, is defined as the ratio of final emittance to the initial emittance and, can be expressed as

$$EGF = \exp\frac{\lambda 2\pi g^2}{d\nu/dn}, \qquad (5)$$

where g is the stop-band width, $d\nu/dn$ is the tune-ramp rate, and λ is a constant.

The emittance growth obeys a simple scaling property when the betatron tunes cross the linear half-integer and sum resonances. This limits the betatron tune range and the momentum acceptance for non-scaling FFA accelerators. The EGF is found to obey scaling properties in the linear space-charge tune shift parameter, the tune ramping rate, and stop-band withs of random quadrupole and skew-quadrupole errors.

Basically all theory and single particle models, developed for synchrotrons or cyclotrons and coasting beam analysis including reduced order space charge models can be applied for FFAs as well.

SPACE CHARGE CODES

COSY INFINITY

The Fast Multipole Method (FMM) is used in COSY INFINITY, that allows the computation of space charge effects of arbitrary and large distributions of particles in an efficient and accurate way. The method relies on an automatic multigrid-based decomposition of charges in near and far regions and the use of high-order differential algebra methods to obtain decompositions of far fields that lead to an error that scales with a high power of the order. Given an ensemble of N particles, the method allows the computation of the self-fields of all particles on each other with a computational expense that scales as O(N). Rigorous estimates are obtained using remainder-enhanced DA methods. All high-order multipoles of the space charge fields are also available, necessarily for the computation of high-order transfer maps and all resulting aberrations. Some FFA modelling is ongoing but not yet published.

ZGOUBI

ZGOUBI is a single particle tracking code that solves the non-linear equation of motion using truncated Taylor expansions of the field and its derivatives up to the 5th order. Recent FFA space-space charge works including algorithmic improvements to ZGOUBI is covered in [21], in greater detail.

A frozen space charge model is employed on top of the ZGOUBI model. The assupmtion is, that the analytic expression of the particle distribution remains the same, so that the analytical solution of the self-induced electric fields does not change. For instance, if we employ a KV beam model, then the problem reduces to calculating the edge radii r_x and r_y of the beam elliptical cross section. Besides, if we cut the magnet into m thin slices, one may assume that the beam radii do not change much within each slice and the error scales as $1/m$.

In Figure 2 the influence of space charge is studied on a coasting beam at 2 different energies.

OPAL

The Lorentz-Force equation is integrated in time using various integration schemes, as described in [22]. Space charge effects are included in the simulation by specifying an appropriate field solver. By default OPAL does not assume any symmetry i.e. is full 3D. The space charge forces are calculated by solving the 3D Poisson equation with various boundary conditions using a standard or integrated Green's function method, or grid based methods, such as a semi unstructured finite difference scheme [23] or (in the near future) a full adaptive mesh refinement scheme. The image charge effects of the conducting cathode are also included using a shifted Green's function method, or Robin boundary conditions can be applied in the case of the finite difference

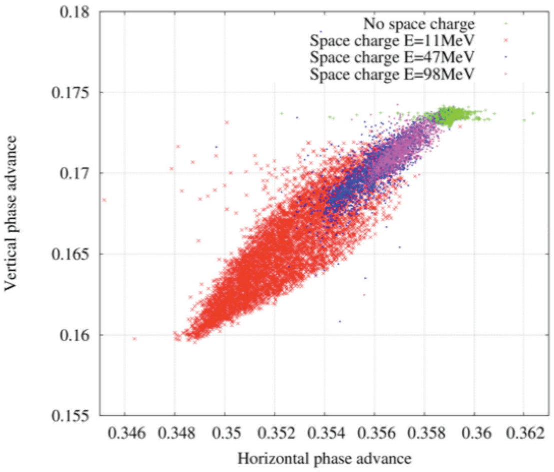

Figure 2: KURRI scaling FFA: $kF = kD = 7.6$, assuming a Gaussian beam distribution equivalent to the KV beam. Space charge tune shift, calculated with a 2D frozen space charge mode, at 11, 47 and 98 MeV [15].

schemes. If more than one Lorentz frame is defined, the total space charge forces are then the summation of contributions from all Lorentz frames, i.e. beams with large energy spread can be modelled.

In order to adequately model FFAs and synchrotrons, a flexible rf-program can be defined by means of a time dependent polynomial, given to each rf-element if necessary.

In order to be able to track trough general rings, a *RingDefinition* element was introduced. This new element contains the main characteristics of a generalised ring, such as harmonic number together with the position of initial (ideal) elements and the position of the reference trajectory. This element can be used in combination with *SBEND3D*, missalignments and and variable rf-cavity elements to make up a complete and realistic ring covering FFAs and synchrotrons.

First results for the very novel type of FFA [17] is available, as shown in Figure 3.

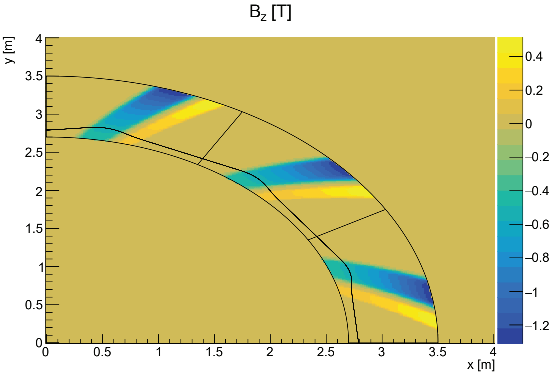

Figure 3: Example of a 12 cell DF spiral type FFA based on [17]. Only 1/4 of the machine is shown.

A last but very interesting feature of OPAL is the ability to search for matched distributions including linear space charge. This feature allows to search for very well defined initial conditions, such that the second order moments Σ of the distribution obey the condition: $\Sigma^f = \Sigma^i$, where i denotes initial and f final. The condition is found exactly one turn [24].

HIGH INTENSITY CHALLENGES

While scaling laws and reduced order models are of utmost importance in the design of every accelerator, when it comes to high intensity this is not enough. High intensity machines such as SNS or the PSI Cyclotrons are limited by losses mainly caused by halo i.e. particles outside of 4 (5) standard deviations of the particle distribution. Estimation and minimisation of halo is the domain of large scale particle tracking with full 3D space charge. On top of these precise models, the full acceleration cycle has to be covered.

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