Instability of CW RFQ with High Beam Loading

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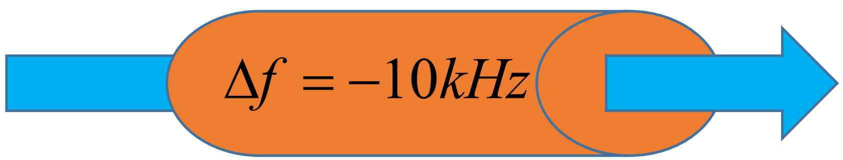
Background

- In the beam experiment on the demo Injector II of the CIADS, when the 10 mA CW beam passed through the RFQ , large power reflection from the power coupler can shut down the generator, due to the interlock system.
- The feedforward is adopted to maintain the amplitude of the fields in the RFQ when the beam passes. Even if , the beam loss in the following SC section is still significant, because the field is smaller than the designed value.
- The RFQ had to be detuned by amount of 10 kHz to minimize the reflection power, therefore, minimize the generator power, which means the optimum detuning of the RFQ under the 10 mA CW beam is 10 kHz.



- This large optimum detuning of RFQ is against the previous experience in normal conducting acceleration.

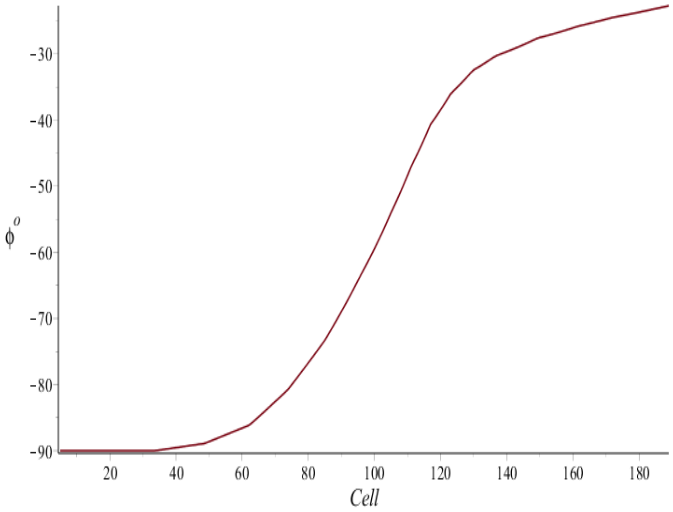
Difficulties in Theoretical Calculation

- There's an equation for calculating the optimum detuning,

$$\tan\psi = -\frac{I_{b,e}R}{(1+\beta)V_c}\sin\phi$$

But it is merely for the case of the single cell cavity.

- For the RFQ , the above expression can't be directly used , because:
  - 1. The synchronous phase are varying vastly along the whole

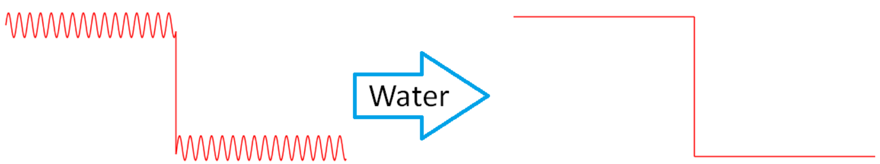


The RFQ of the demo Injector II of CIADS has 186 cells with synchronous phase varies from -90^o to -23^o along the structure.

  - 2. There are two dominant modes in the RFQ
    Monopole Mode & Quadruple Mode

Validity of the Experimental Result

- The validity of experiment can be assured in the following two aspects:
- 1. The RFQ is tuned by the variation of the temperature of the cooling water, large specific heat capacity of the water can filter any transient fluctuation in the eigen-frequency.



The variation is automatically time-averaged to eliminate the variation of the eigen-frequency due to other effects.

- 2. The experiment value of df/dT is consistent with that obtained from the simulation carried by the software ANSYS (For the 3D electromagnetic problem, finite element method can get the eigen-frequency with a very high precision).

$$\left(\frac{df}{dT}\right)_{\exp} \approx \left(\frac{df}{dT}\right)_{sim} \approx 16kHz/{}^{o}C$$

Paradox in Detuning

- For the RFQ without beam, minimum generator power requires,

$$f_0 = f_g$$

- The eigen-frequency of the RFQ with beam has to be tuned to be 10kHz higher to minimize the generator power, which seems to imply that the eigen-frequency of the RFQ has changed by the beam.

$$\begin{cases} f_0' + 10kHz = f_g \\ f_0 = f_g \end{cases} \Rightarrow f_0' = f_0 - 10kHz$$

- According to the Maxwell's equation, the eigen-frequency of the RFQ is solely determined by its shape and size, independent of the beam,

$$f_0' = f_0$$

- The Paradox due to the insufficiency of the widely-used beam loading theory based on the current source model of the beam in explaining the optimum detuning.

Two Models of the Beam

- The interaction between the beam and the cavity can be viewed in the following two ways, which will result to the two equivalent model of the beam.

Inducing Electromagnetic Fields → Current Source   Or   Absorb Energy from the Fields → Impedance

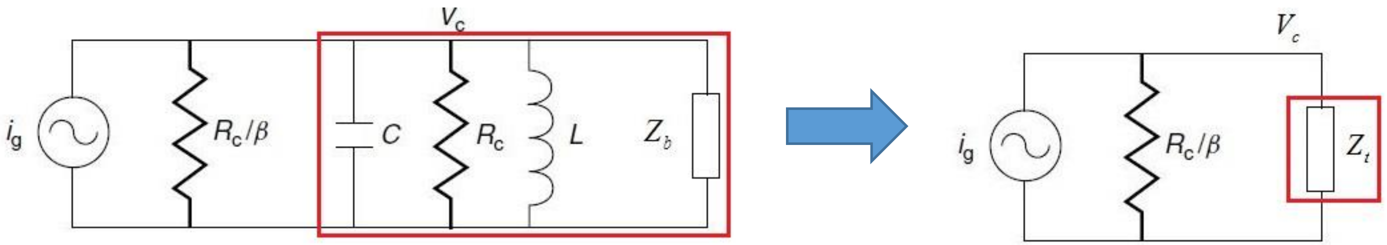
- To obtain the physical picture of the optimum detuning , the beam has to be modeled as an impedance.
- For the Linac, negative synchronous phase results to a capacitive impedance model of the beam,

$$\begin{cases} R_b = \dfrac{I_b}{V_c}\cos\phi \\ C_b = -\dfrac{I_b}{V_c\omega}\sin\phi \end{cases}$$

where ϕ is the impedance angle, which can be reduced to the synchronous phase if there is only one dominant electromagnetic mode existing in the cavity.

Beam-Cavity System

- The impedance of the beam is in parallel with that of the cavity, then according to the circuit theory, the two impedance can be equivalent to one, enlightening that the beam and the cavity can be treated as a whole system.



- Minimizing the generator power means,

$$f_t = f_g$$

- Beam-inducing detuning,

$$\Delta f = f_t - f_0$$

which can be calculated by the following expression,

$$\Delta f = -\frac{f I_b}{4V_c}\left(\frac{r}{Q}\right)\sin\phi$$

Relation between the two Detuning

- From the derivation below , it can be seen the beam-inducing detuning is equal to the optimum detuning.
- Because

$$\Delta\omega = -\frac{\omega I_b}{4V_c}\left(\frac{r}{Q}\right)\sin\phi \quad \Leftrightarrow \quad Q_0\frac{\Delta\omega}{\omega} = -\frac{I_b r}{4V_c}\sin\phi \quad \Leftrightarrow \quad 2Q_0\frac{\Delta\omega}{\omega} = -\frac{I_b R}{V_c}\sin\phi$$

$$\Leftrightarrow \quad \frac{2Q_0}{1+\beta}\frac{\Delta\omega}{\omega} = -\frac{I_b R}{(1+\beta)V_c}\sin\phi \quad \Leftrightarrow \quad 2Q_L\frac{\Delta\omega}{\omega} = -\frac{I_b R}{(1+\beta)V_c}\sin\phi$$

$$\Leftrightarrow \quad \tan\psi = \frac{I_b R}{(1+\beta)V_c}\sin\phi$$

- In the current model of the beam, the effective current of the beam $I_{b,e}$ is the mirror current of the beam current; while in the impedance model here,

  I_b is simply the beam current itself, then $I_b = -I_{b,e}$

- Therefore,

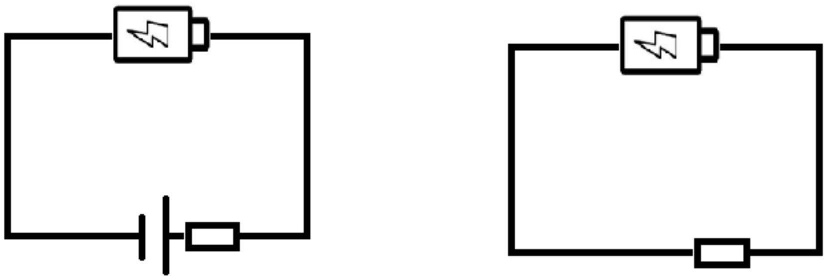
$$\tan\psi = -\frac{I_{b,e}R}{(1+\beta)V_c}\sin\phi = -\frac{P_b}{(1+\beta)P_c}\tan\phi$$

which is the expression for optimum detuning, if the impedance angle is replaced by the synchronous phase.

Relation between the Two Theories

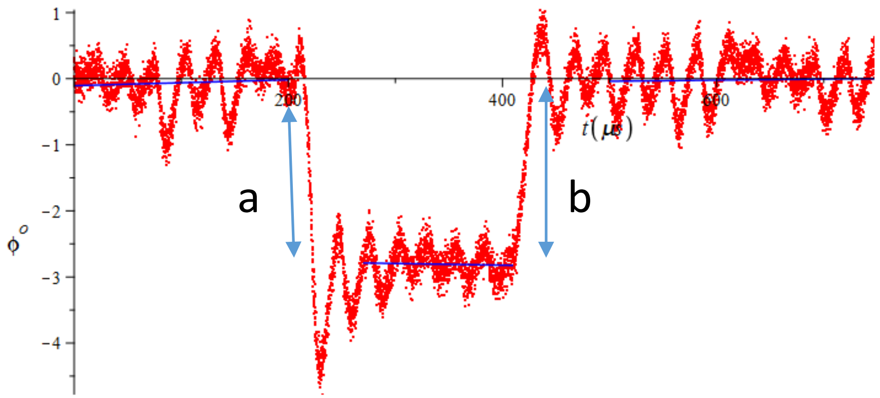
- All the important conclusions obtained from one theory can also be obtained from the other, among which includes the optimum detuning, the optimum coupling and even the general expression for the generator power.

- The two theories are mathematical equivalence when $\mathrm{Re}\left\{\overrightarrow{V_g}+\overrightarrow{V_b}\right\}>0$, which is the case for beam acceleration.
  - Instance of the rechargeable battery.



Effective Synchronous Phase

- Model the RFQ as a single-cell cavity
  - Introducing the notion of effective synchronous phase.
- Validity of the equivalence
  - The variation of the field phase by the beam is almost the same in each cell, which is confirmed by the experiment.



The variation of the synchronous phase of the RFQ from the entrance to exit.

Location	1	2	3	4
$\Delta\phi_a$	-2.82±0.05	-2.85±0.05	-3.06±0.06	-3.01±0.06
$\Delta\phi_b$	-2.86±0.05	-2.90 ±0.05	-3.11 ±0.04	-3.06±0.04

Two-term Potential Approximation

- Two-term potential approximation of the RFQ,

$$\Phi_n(r,\theta,z,t)=\frac{V}{2}\left[A_{0,n}\left(\frac{r}{a}\right)^2\cos(2\theta)+A_{10,n}I_0(kr)\sin(kz)\right]\cos(\omega t+\phi_n)$$

- Effective synchronous phase of the RFQ,

$$\bar{\phi}=-\arccos\left(\frac{\sum_n A_{10,n}\cos\phi_n}{\sum_n A_{10,n}}\right)$$

- With the parameters of the RFQ, we'll obtain $\bar{\phi}\approx-33^o$
- The beam-inducing detuning evaluated with $\bar{\phi}$ is,

$$\Delta f=-\frac{P_b}{(1+\beta)P_c}\tan\bar{\phi}\approx-0.95\text{kHz}$$

which is much smaller than the experiment value.

- This means the contribution from the quadrupole mode can't be neglected when calculating the impedance angle.

Effective RF Phase

- The effective impedance angle can be termed to be the effective RF phase.
- Detailed treatment on the effective RF phase will involve the field integration. For simplicity and consistency. To be consistent with it, we'll estimated the effective RF phase as the following way,

$$\phi_h = -\arctan\left(\frac{\left|\sum_n A_{10,n} \sin\phi_n\right| + \frac{N}{2\sum_n L_n} \sum_n A_{0,n} m_n L_n}{\sum_n A_{10,n} \cos\phi_n} \right)$$

- With the parameter of the RFQ, we'll obtain $\phi_h \approx -78.5^o$
- The beam-inducing detuning evaluated with ϕ_h is,

$$\Delta f' \approx -7.8\text{kHz}$$

- The beam-inducing field phase variation

$$\Delta\phi \approx 4.2^o$$

Thanks for Your Attention!