Typology of space charge resonances

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Overview

- Introduction
- Nature of parametric resonances
- The 90 deg stopband (2^{nd} /4^{th} order)
- Higher order
- The 120 deg stopband and beyond (circular machines)
- Conclusion

this talk:

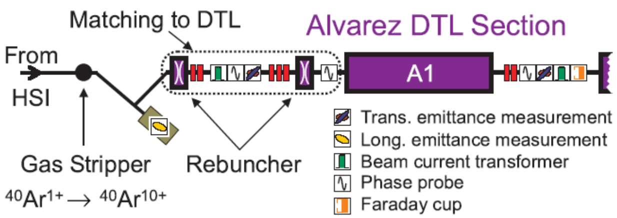
only space charge – **no external nonlinearities**!

no emittance exchange issues

3D - with „fast“ synchrotron periods ~ betatron period

co-worker: O. Boine-Frankenheim

Introductory remarks



✓ 2009: GSI-Unilac-experiment on 90 degree space charge resonance stopband (*L. Groening et al., PRL, 2009*) triggered new studies:

- envelope instability or 4th order resonance? issue studied already in 1980's – now discussed again (more papers in A and B)

Associated questions of interest:

- what are "coherent" space charge resonances? incoherent?
- "collective" versus "single particle" resonances?
- what orders matter? 2nd, 3rd, 4th, ...?
- what is the role of distribution function?
- are these effects important, or only interesting?

External versus parametric resonances

"external"
(magnet or initial space charge multipole):

$$\frac{d^2x}{dt^2} + \omega^2 x = \boldsymbol{\varphi(x,t)}$$

- magnet
- initial space charge multipole

"parametric":

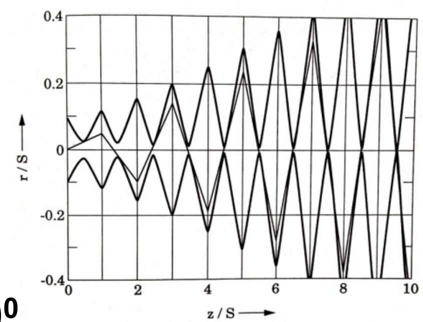
$$\frac{d^2x}{dt^2} + \left(\omega^2 + \boldsymbol{f(t)}\right)x = 0$$

(Hill's equation)

with ***f(t)*** **a system parameter** (length of pendulum, focusing strength etc.) varying periodically with ω_0 → exponential growth for

- $\omega = \frac{n}{2}\,\omega_0$ and n=1,2,3 ...
- **n=1** most significant – called parametric instability, sub-harmonic instability or half-integer 2:1 parametric resonance
- for arbitrarily small initial perturbation x

well-known parametric resonance:
stability of **single particle** motion
$f(t) \sim \varepsilon \cos(\omega_0\, t)$
Mathieu equation
for periodic focusing:
exponential runaway in lattice with **k_{xy}=180^0**



Parametric resonance on **collective** level
2:1 envelope mode instability

$$\frac{d^2}{dt^2}a(t)+K_{0,x}(t)a(t)-\frac{1}{2}\frac{Q}{a(t)+b(t)}-\frac{\varepsilon_x^2}{a(t)^3}=0$$

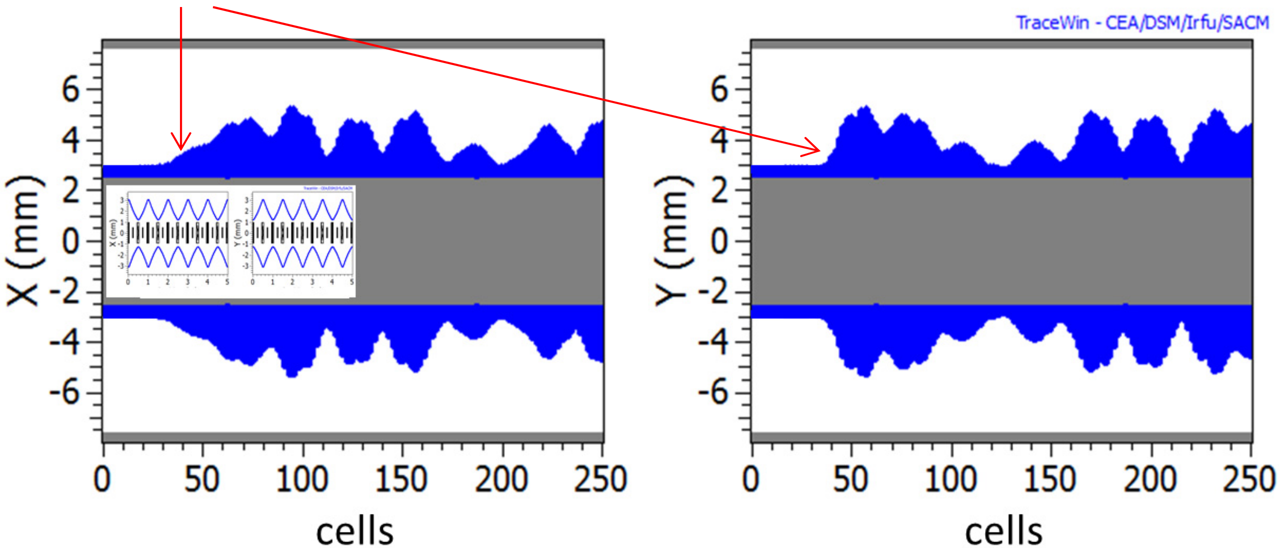
$$\omega_{envelope}\equiv 2k_{0x}-\Delta\omega_{coh}=\frac{\omega_0}{2}=180^0$$

or, in circular notation :

$$2Q_{0x}-\Delta Q_{coh}=N/2$$

SIS18: $Q_{0,v}$=3 !

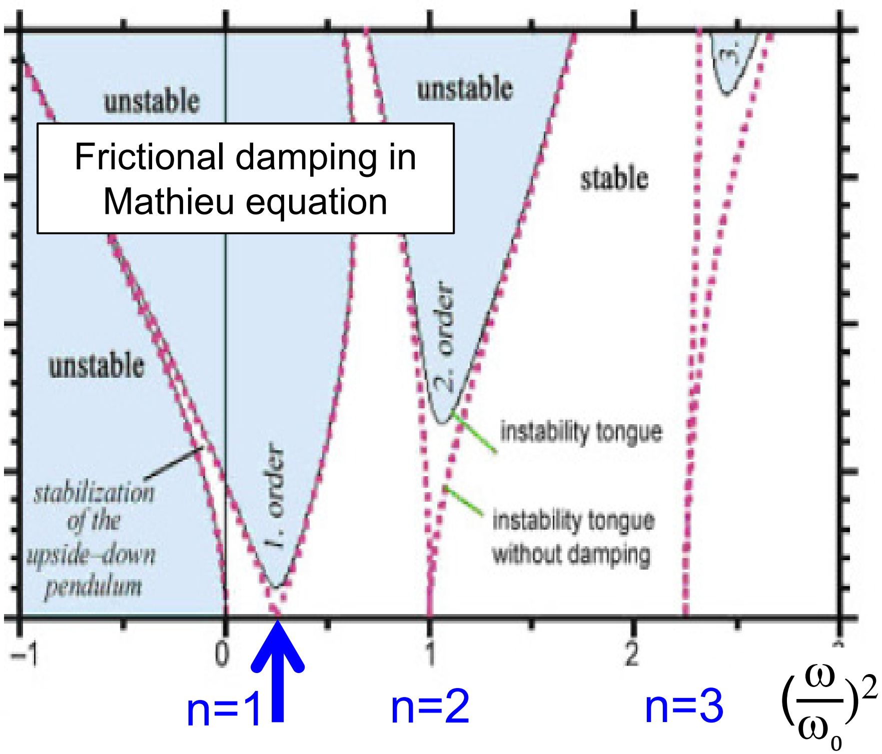
exponential rise



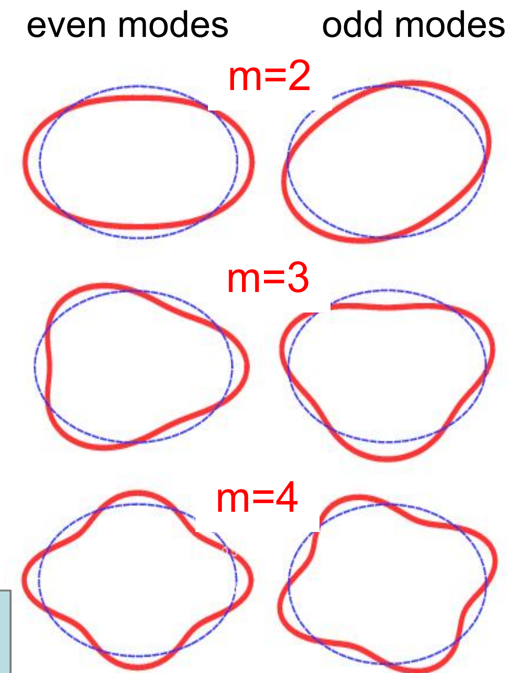
$$k_{0xy}=100^0$$
$$k_{xy}=82^0$$

Parametric resonance of higher order collective eigenmodes

Analogy with damped Mathieu equation with some stabilization?
frictional damping – Landau damping? **Explain damping of some modes?**



$\rightarrow \frac{\omega}{\omega_0/2} = n$ (=1,2,3 ...)



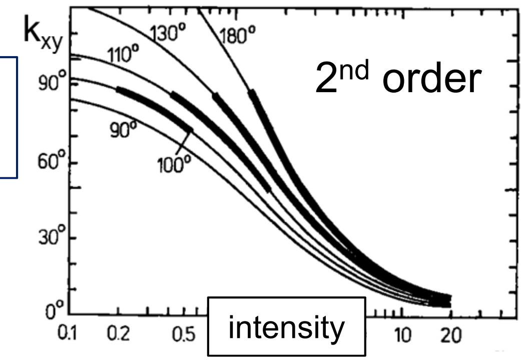
m: mode order **n**: parametric order

$\omega = m k_{0xy} - \Delta\omega_{coh} = \frac{n}{2}\omega_0$ $\quad m Q_{0xy} - \Delta Q_{coh} = \frac{n}{2} N$

Collective parametric resonances in all orders
based on analytical KV-theory (1983) – called "instabilities"

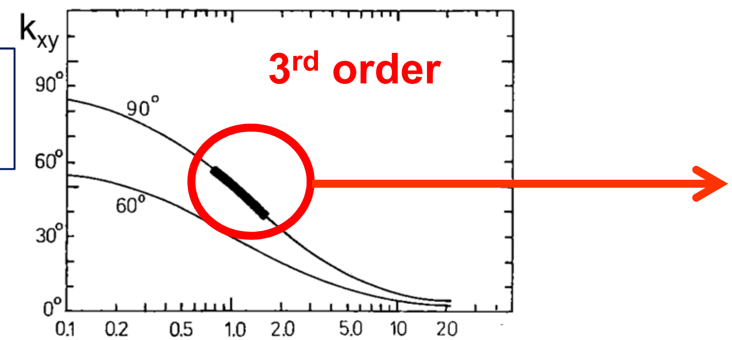
- KV-theory: **driving mode not present** in initial distribution (noise)
- Modes "pumped" parametrically
- Modes of **different order by nature independent** (2nd order **not** subharmonic of 4th etc.)

$$\mathbf{2k_{0xy}} - \Delta\omega_{coh} = \frac{1}{2}\,\omega_0$$



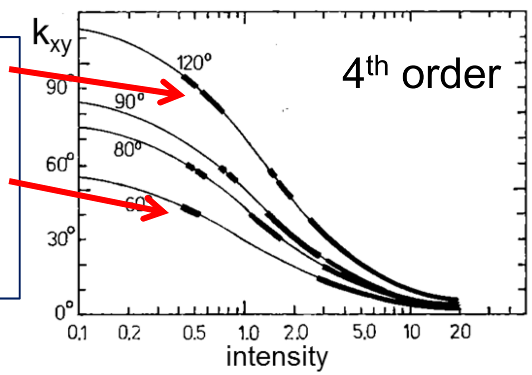
"Self-consistent analytical Vlasov perturbation theory of KV-beam in periodic focusing"
I. H., L.J. Laslett, L Smith, I. Haber, Part Accel., 1983

$$\mathbf{3k_{0xy}} - \Delta\omega_{coh} = \frac{1}{2}\,\omega_0$$



$$\mathbf{4k_{0xy}} - \Delta\omega_{coh} = \omega_0 = 360^0$$

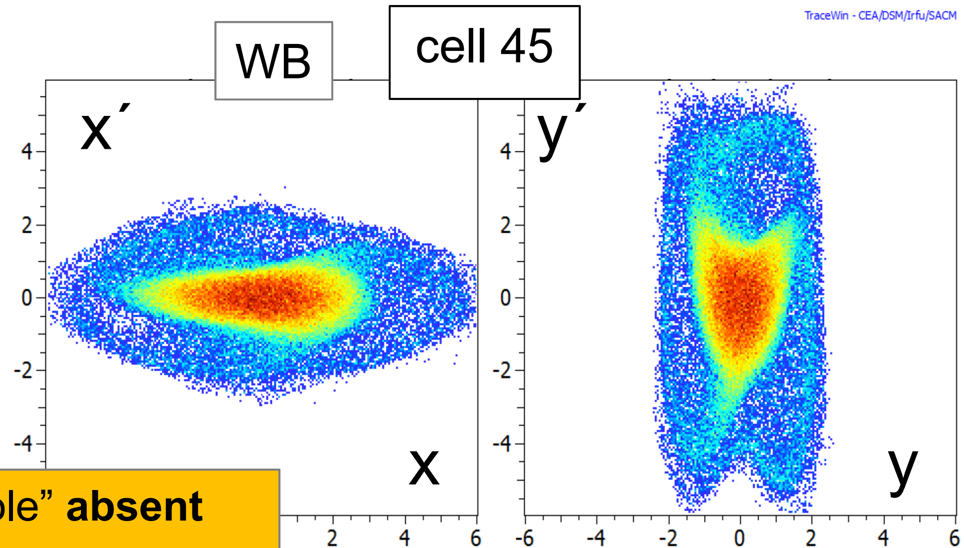
$$\mathbf{4k_{0xy}} - \Delta\omega_{coh} = \frac{1}{2}\,\omega_0 = 180^0$$



3^{rd} order parametric resonance (60 deg stopband)
again half-integer 2:1 type = instability

$$3k_{0xy}-\Delta\omega_{coh}=\frac{1}{2}\,\omega_0=180^0 \qquad 3Q_{0xy}-\Delta Q_{coh}=\frac{1}{2}\,N$$

$k_{0x,y} = 90^0$
$k_{xy} = 41^0$
$k_{0z} = 50^0$

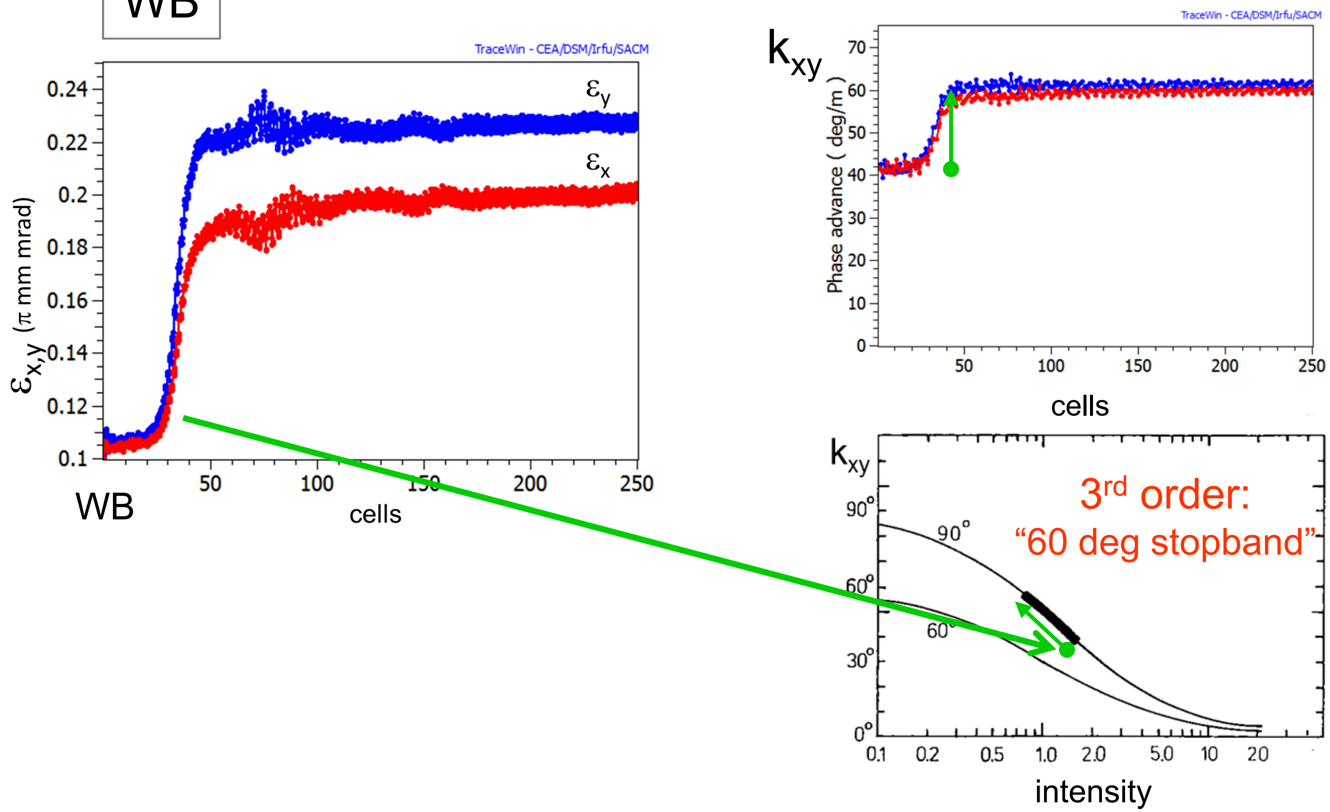


- ✓ space charge “sextupole” **absent** initially
- ✓ grows by pumping of “noise”
- ✓ **coherent motion in x-x’ and y-y’**

3^{rd} order parametric instability cont'd
rms tune k_{xy} dynamically migrating through stopband

$$3k_{0xy} - \Delta\omega_{coh} = \frac{1}{2}\,\omega_0$$

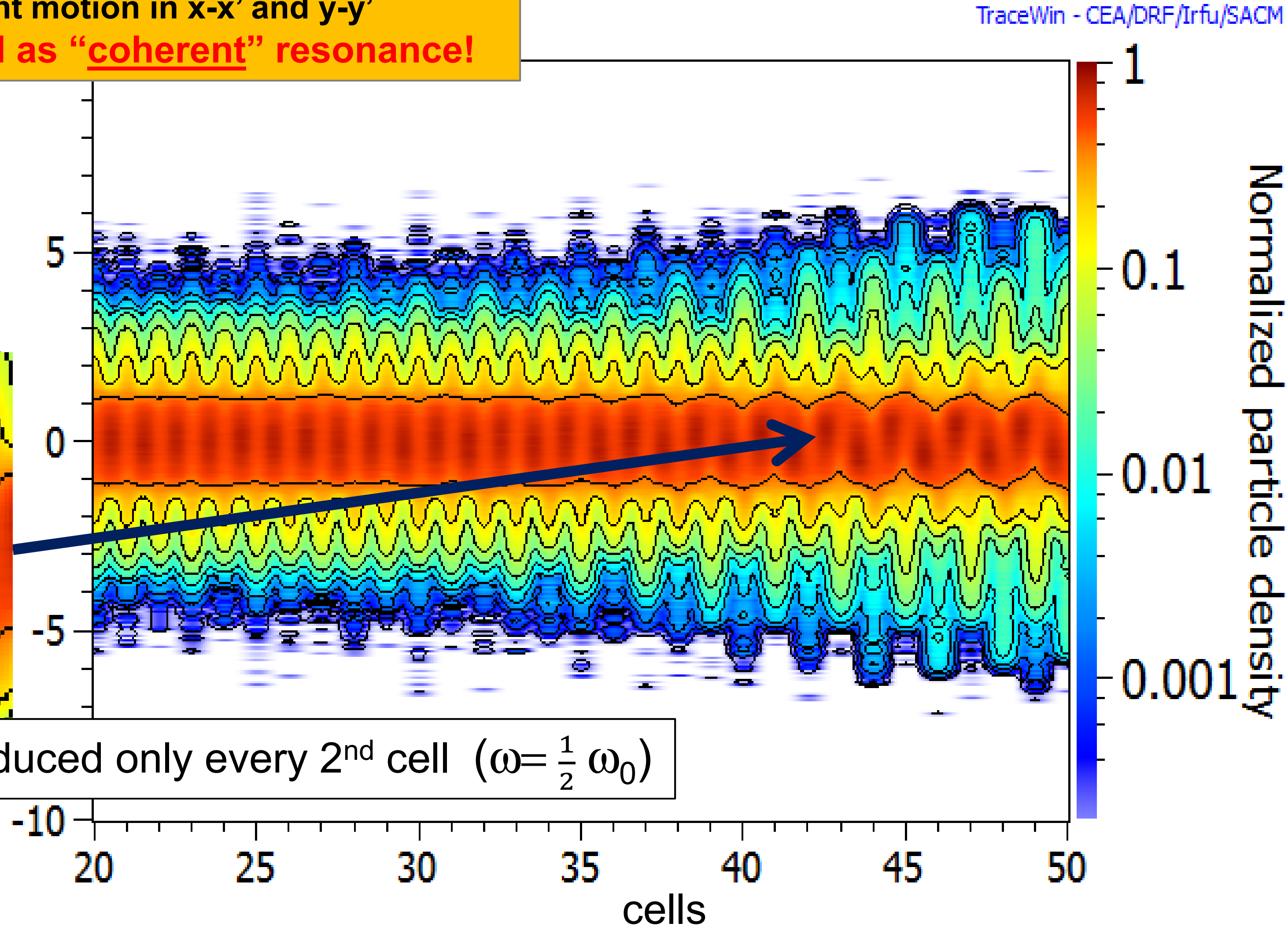
WB

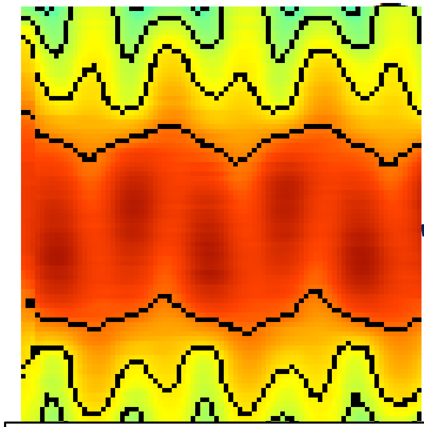


$k_{0x,y} = 90^0$
selfconsistent:
$k_{xy} = 41^0 \rightarrow 60^0$
$k_{0z} = 50^0$

"Coherent resonance" effect clearly seen here (WB)

- ✓ **correlated coherent motion in x-x' and y-y'**
- ✓ **→ characterized as "coherent" resonance!**



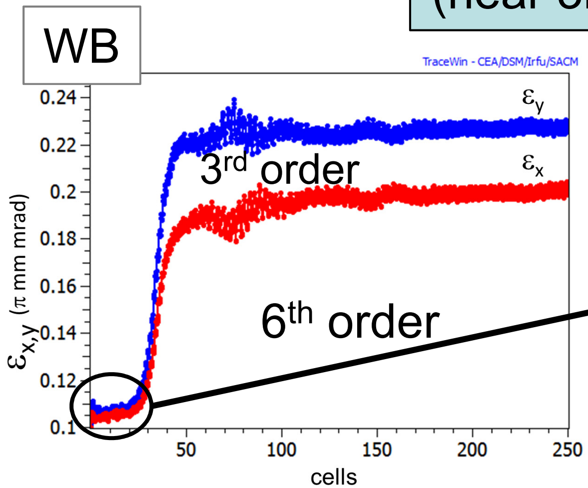


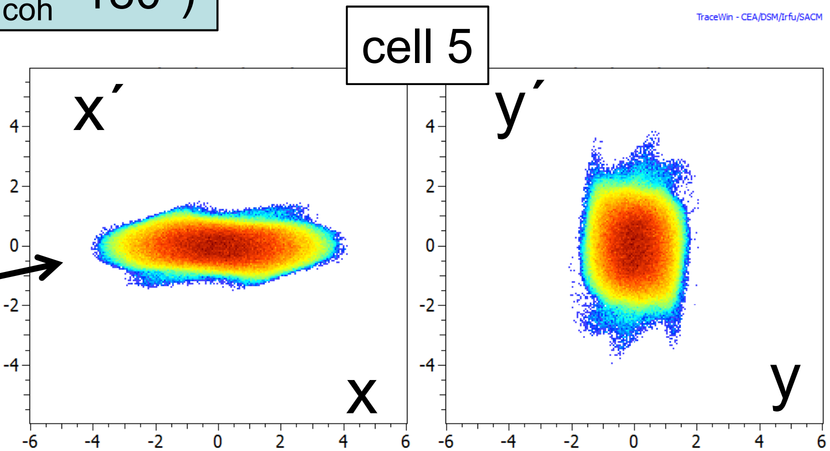
coherence: reproduced only every 2nd cell ($\omega = \frac{1}{2}\,\omega_0$)

preceded by: 6th order space charge resonance

nearly overlaps with 3rd order – although **independent** resonance

$6(k_{0xy}-\Delta k_{inc}) = \omega_0 = 360^0$
(near $3k_{0xy}-\Delta\omega_{3,coh}=180^0$)





- ✓ 6th order space charge potential **present** initially
- ✓ negligible coherent motion →
- ✓ **“single particle” (incoherent) resonance”**

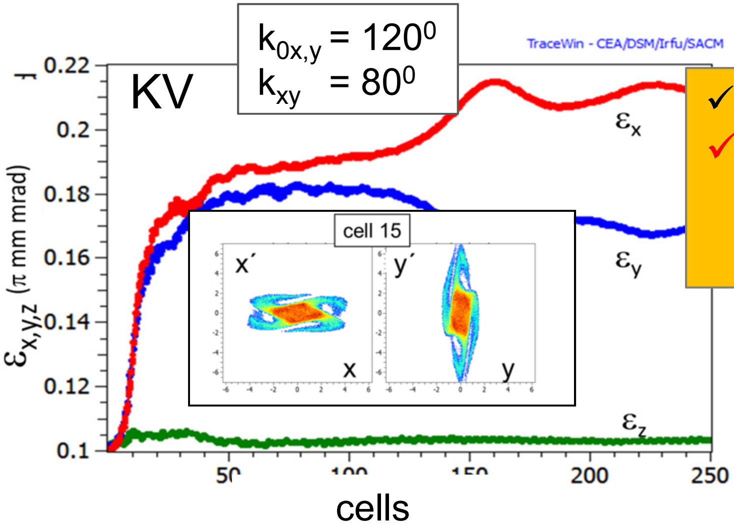
Role of *distribution* function
for space charge resonances?

- in KV distribution beam:
  - only parametric resonances/instabilities exist
  - in all orders
  - but initial single particle motion is linear
- "realistic distributions:
  - how many parametric resonances really exist?
  - how many nonlinear single particle resonances?
  - how to distinguish?

The "90 deg" stop-band

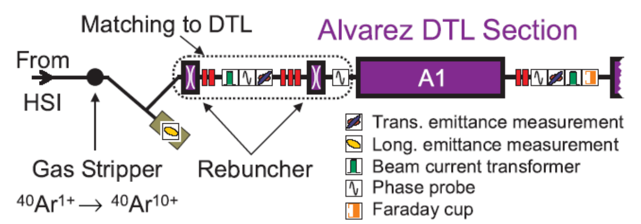
"old" topic – theory "revived" since 2009 GSI-UNILAC experiment

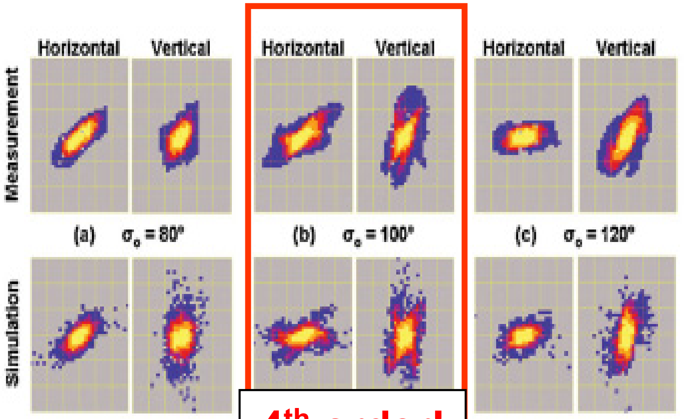
$2k_{0xy}-\Delta\omega_2 = 180^0$? or $4k_{0xy}-\Delta\omega_4 = 360^0$? what do we expect?



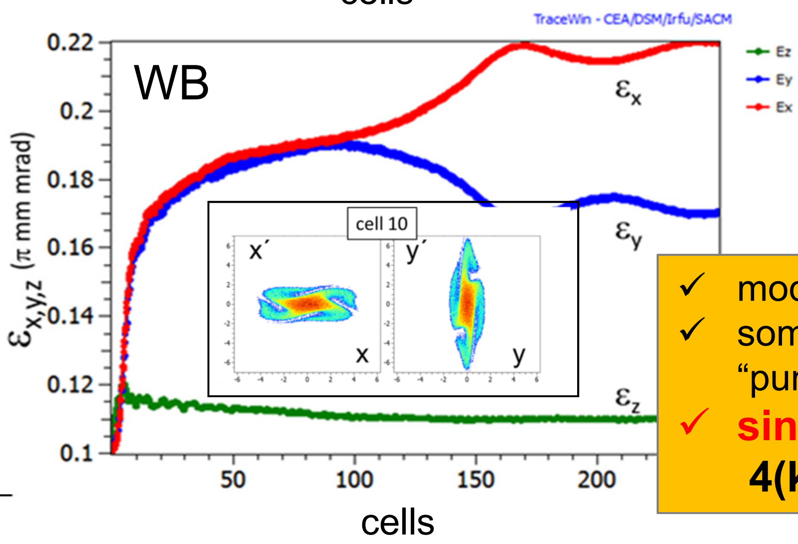
- ✓ mode **absent** initially
- ✓ **purely parametric resonance**

$4k_{0xy}-\Delta\omega_{coh}=360^0$





4th order!

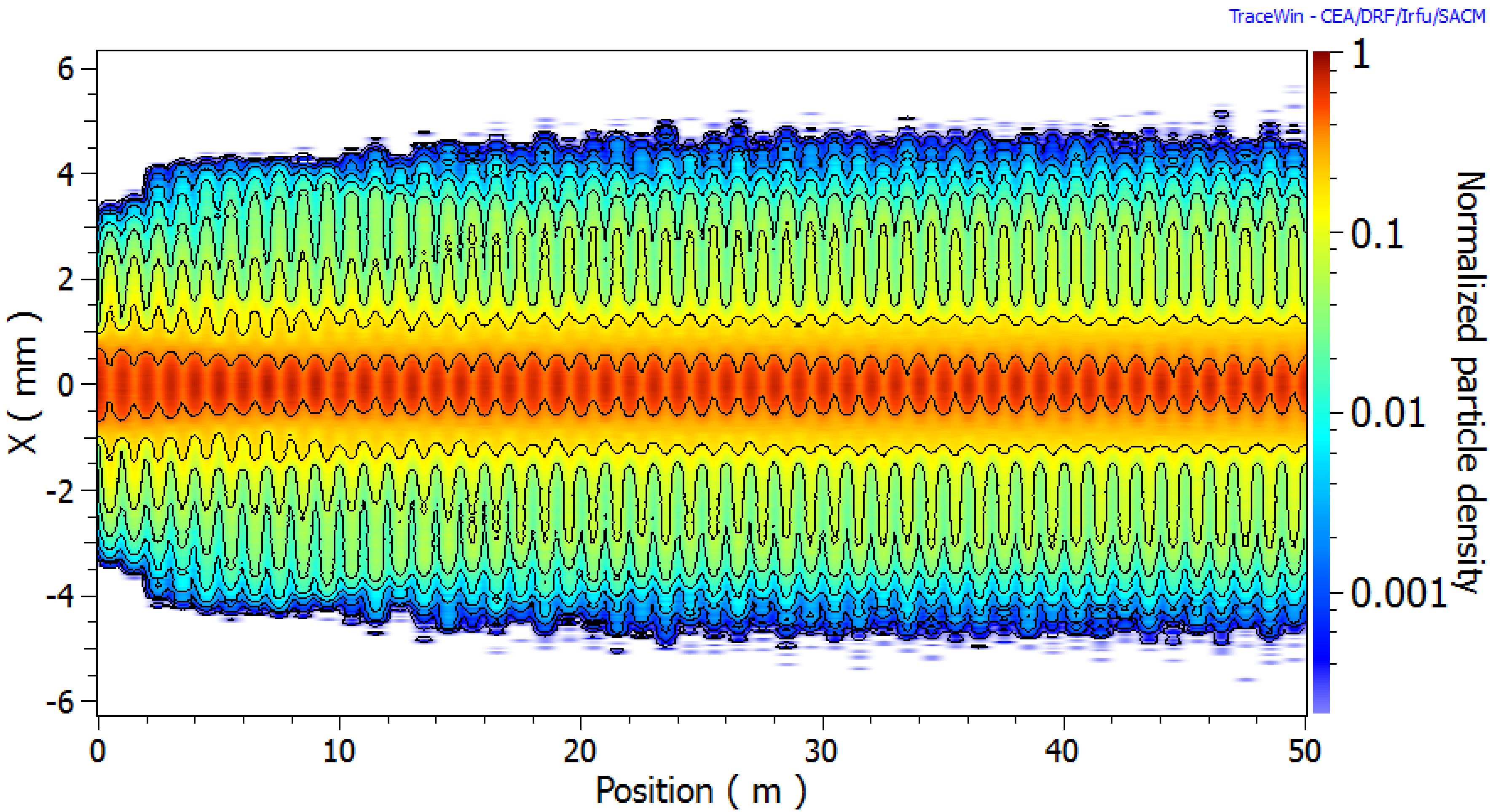


- ✓ mode **present** initially
- ✓ somewhat enhanced by parametric "pumping"
- ✓ **single particle resonance**

$4(k_{0xy}-\Delta k)=360^0$ more suitable

GSI

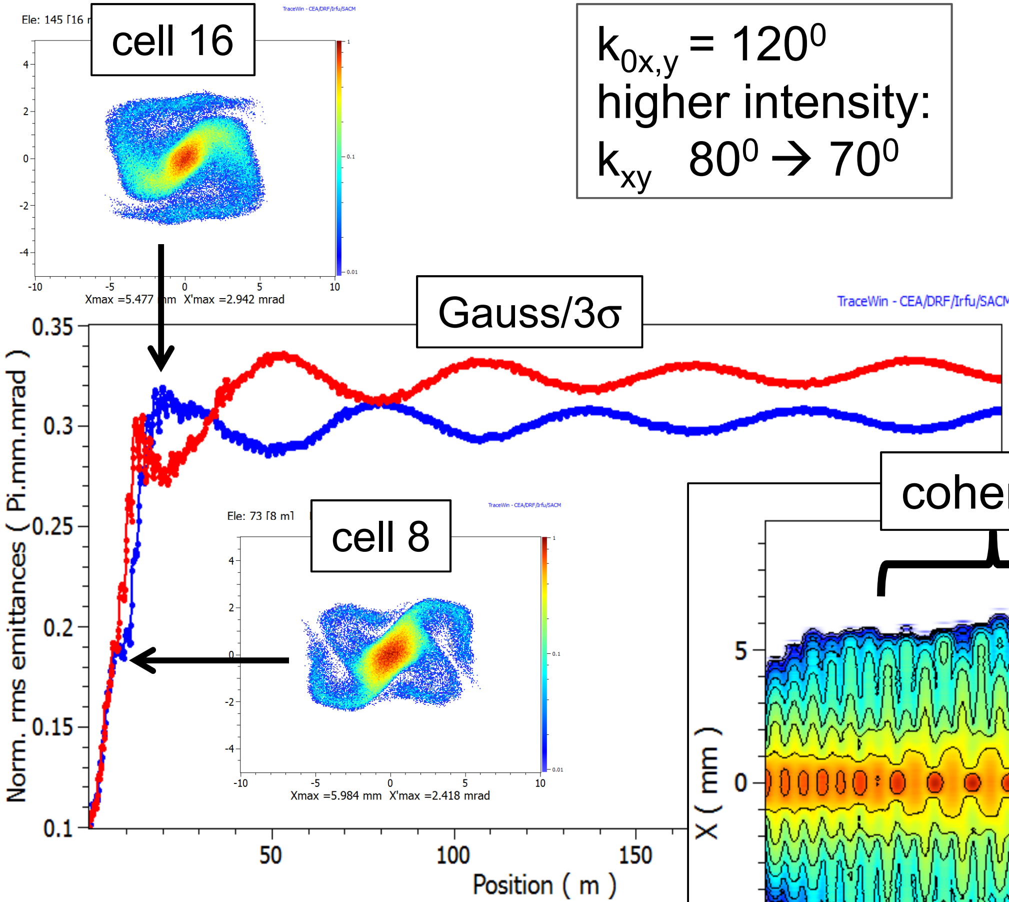
Density evolves incoherently
frozen space charge with emittance update should be ok

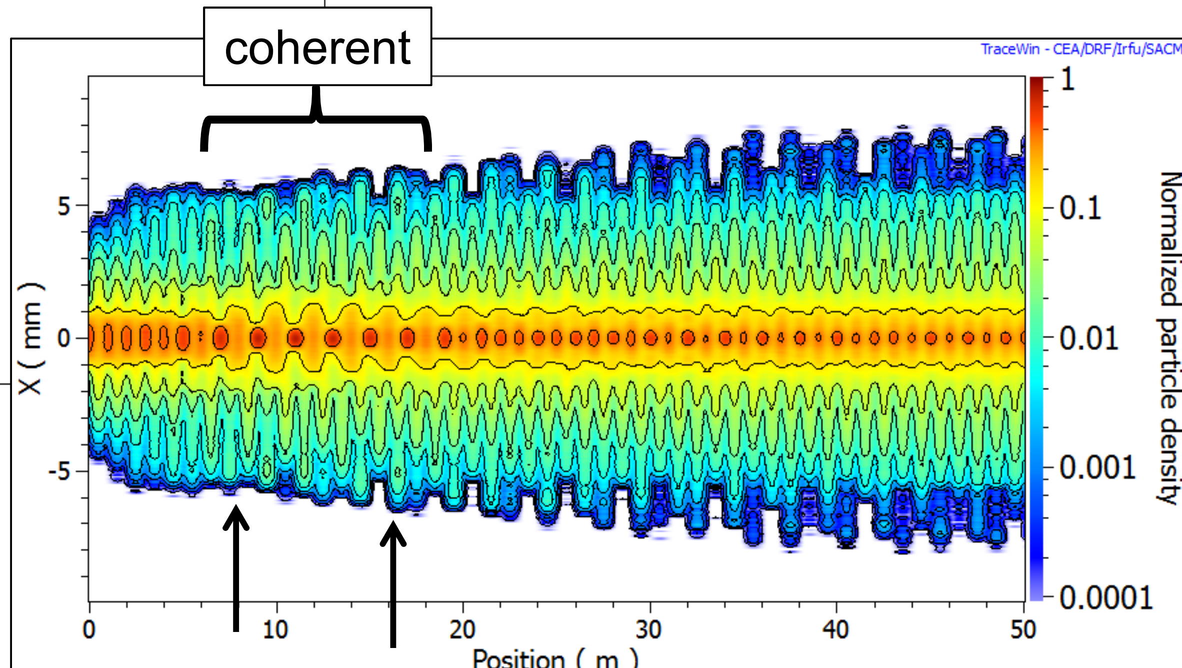


90 degree stopband – higher intensity

$k_{0x,y} = 120^0$
higher intensity:
k_{xy} $80^0 \rightarrow 70^0$

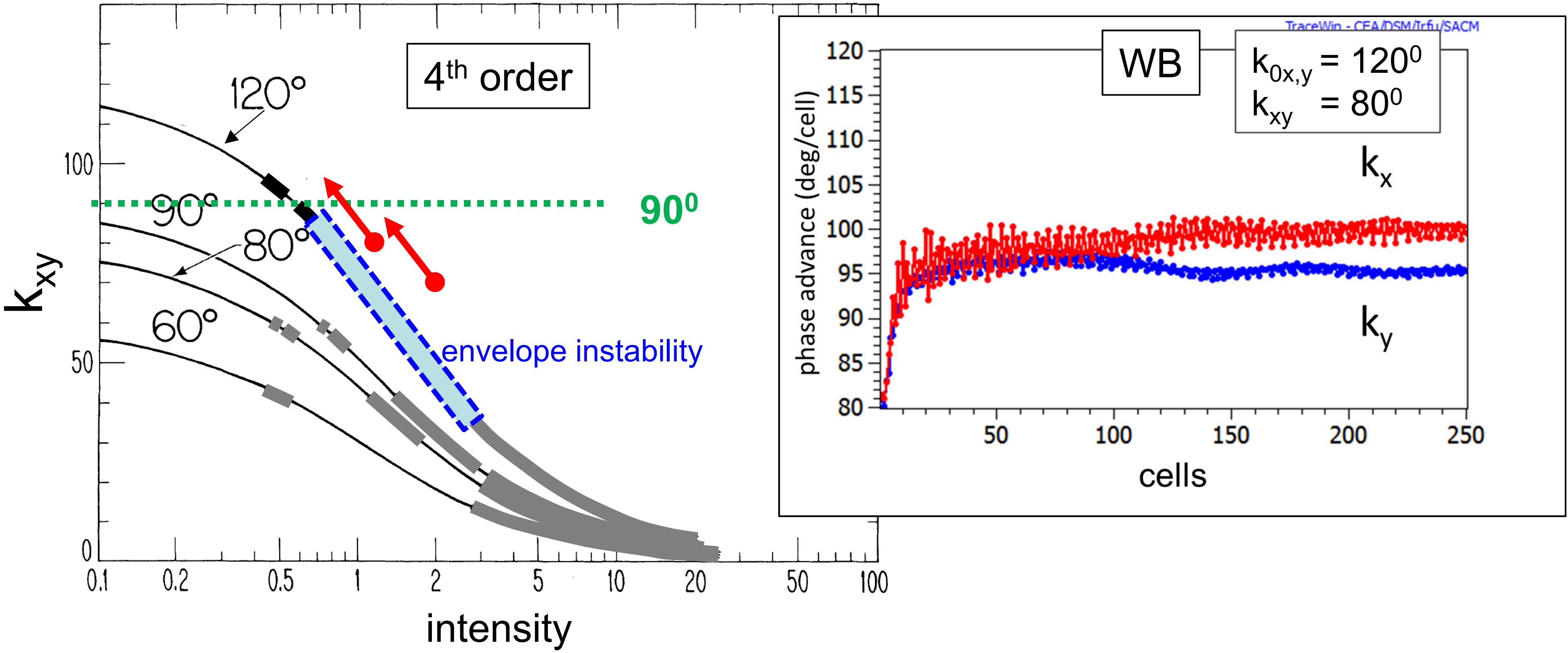
2^{nd} order
(envelope instability)
dominates over 4^{th} order





Why?

dynamical "collective" detuning

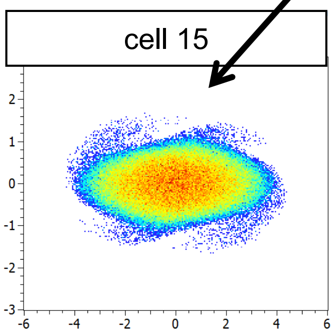


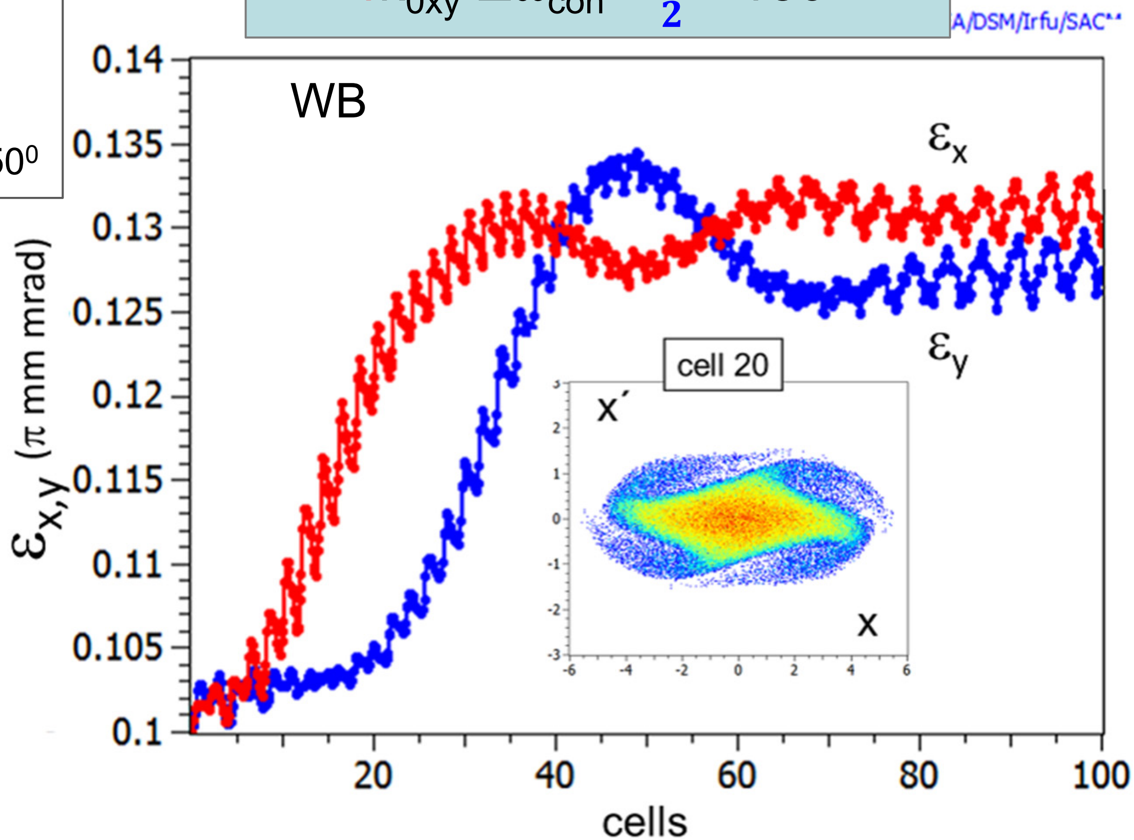
4^{th} order half-integer parametric instability exists

45 degree stopband - seems to be more damped if slow synchrotron period

$k_{0x,y} = 70^0$
$k_{xy} = 35^0$
$k_{0z} = 120^0$
$k_{0z} = 93^0$
$\Delta\varepsilon$: 10x less for $k_{0z} = 50^0$

$$4k_{0xy} - \Delta\omega_{coh} = \frac{\omega_0}{2} = 180^0$$





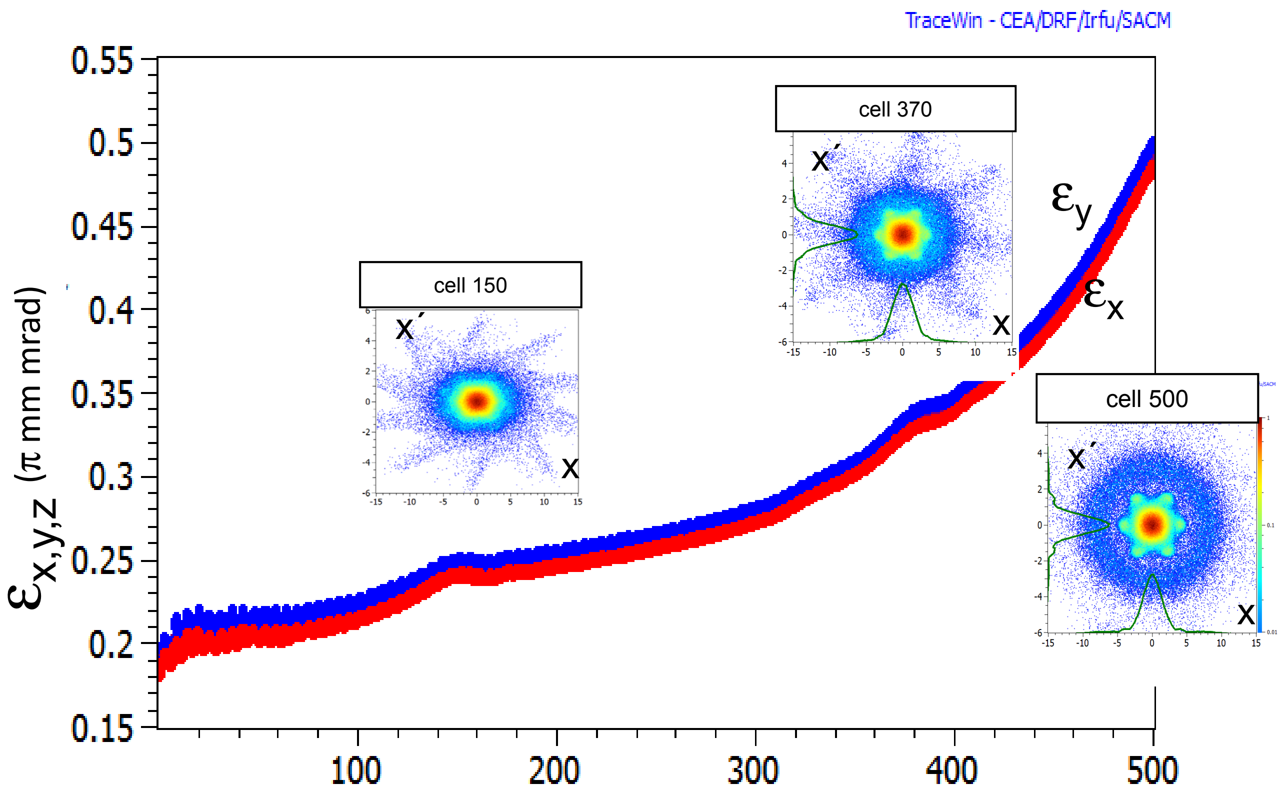
Beyond 120 deg stopband

higher harmonic lattice effects

Gaussian
$\varepsilon_x=\varepsilon_y=\varepsilon_z$

Tune ramp:
$k_{0x,y} = 150^0 \rightarrow 130^0$

$k_{xy} = 140^0 \rightarrow 128^0$
$(k_{0z} = 50^0)$



$k_{0x,y}$:150^0 ....144^0..............................135^0............130^0

half-integer (180^0) parametric resonance not possible
→ found only single-particle (incoherent) resonances

Relevant to circular machine lattices?

with high enough phase advances per cell/superperiod

k_{xy} :	120^0	$\sim 120^0$	$\sim 135^0$	$\sim 144^0$
order "m" explored for:	3^{rd} KV	6^{th} Gauss	8^{th} Gauss	10^{th} Gauss
h (lattice harmonic)	1	2	3	4
type	Integer parametric	"single-particle"	"single- particle"	"single- particle"
$m\,k_{xy}$=	360^0	$2x360=720^0$	$3x360=1080^0$	$4x360=1440^0$
mQ_{xy}=	N	2N	3N	4N
SIS18 (N=12): Q_{hor}=	4	4	4.5	4.8

only KV

Categories

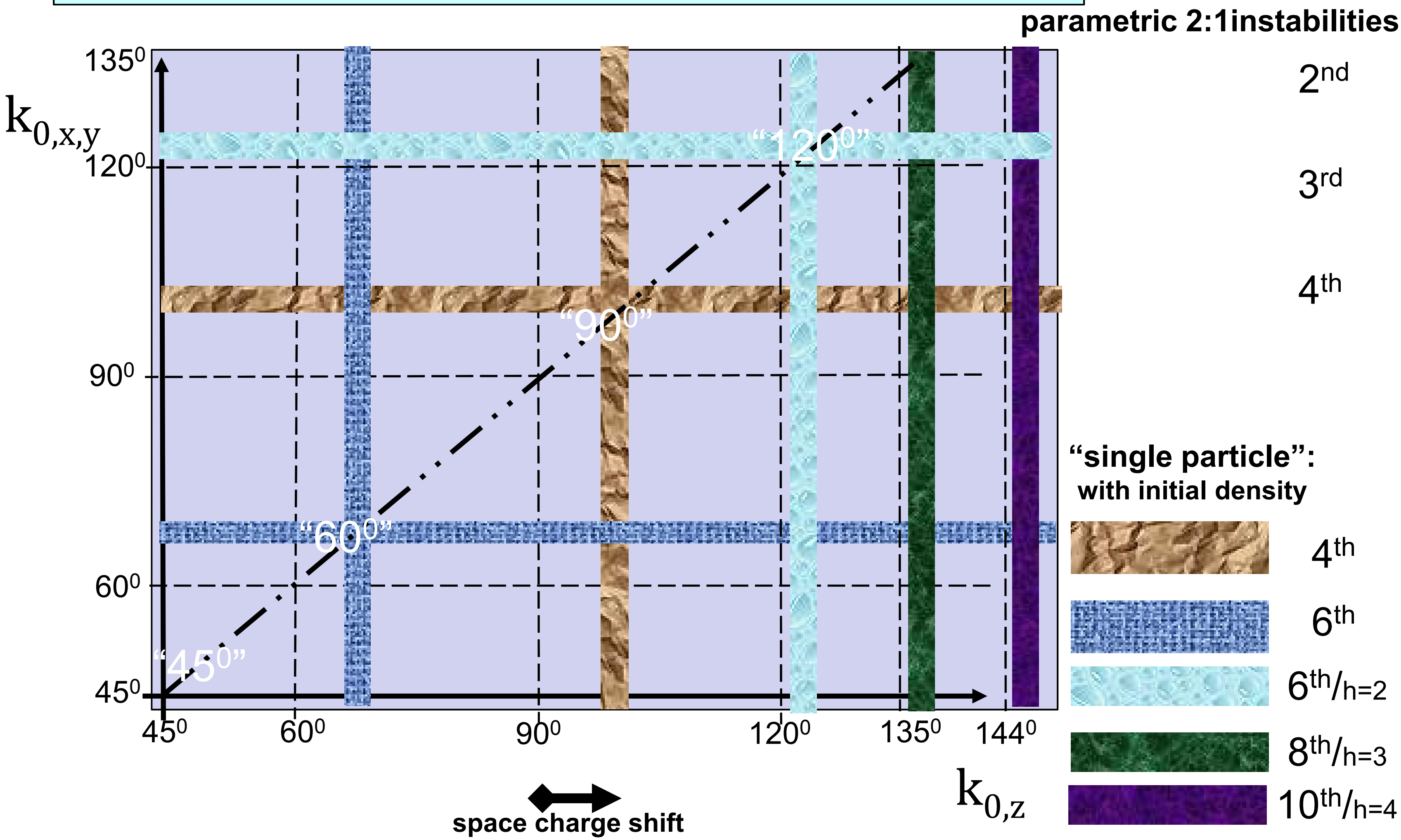
- attempting a "consistent" terminology -

	Single particle resonances error driven | **structure driven**	**Parametric resonances**
Driving term:	magnets | magnets / stationary space charge	collective space charge
Origin:	| non-uniform initial density	arbitrary density eigenmode
Linac	$mk_{xy}=360^0\ h$ (~) **m**: order of resonance **h**: lattice harmonic	$\omega=\omega_0\ \mathbf{n}/2$ $mk_{0xy}=360\ \mathbf{n}/2^0\ +\Delta\mathbf{k_c}$ **n**=1 "half-integer" = strongest parametric case higher only for KV beam (h>1 only KV)
Ring (h>1 higher lattice harm.	$mQ_{xy}=N\ h$ N: number of superperiods per turn; n: lattice harmonic	$\omega=\omega_0/2$ $mQ_{0xy}=N/2\ +\Delta\mathbf{Q_c}$

Summary chart (schematic) with **two groups:**

Group I (shown first): single particle resonances – as in usual circular diagrams
Group II: parametric 2:1 instabilities - not part of " " "

call it **resonance diagram** or **stability chart** ?



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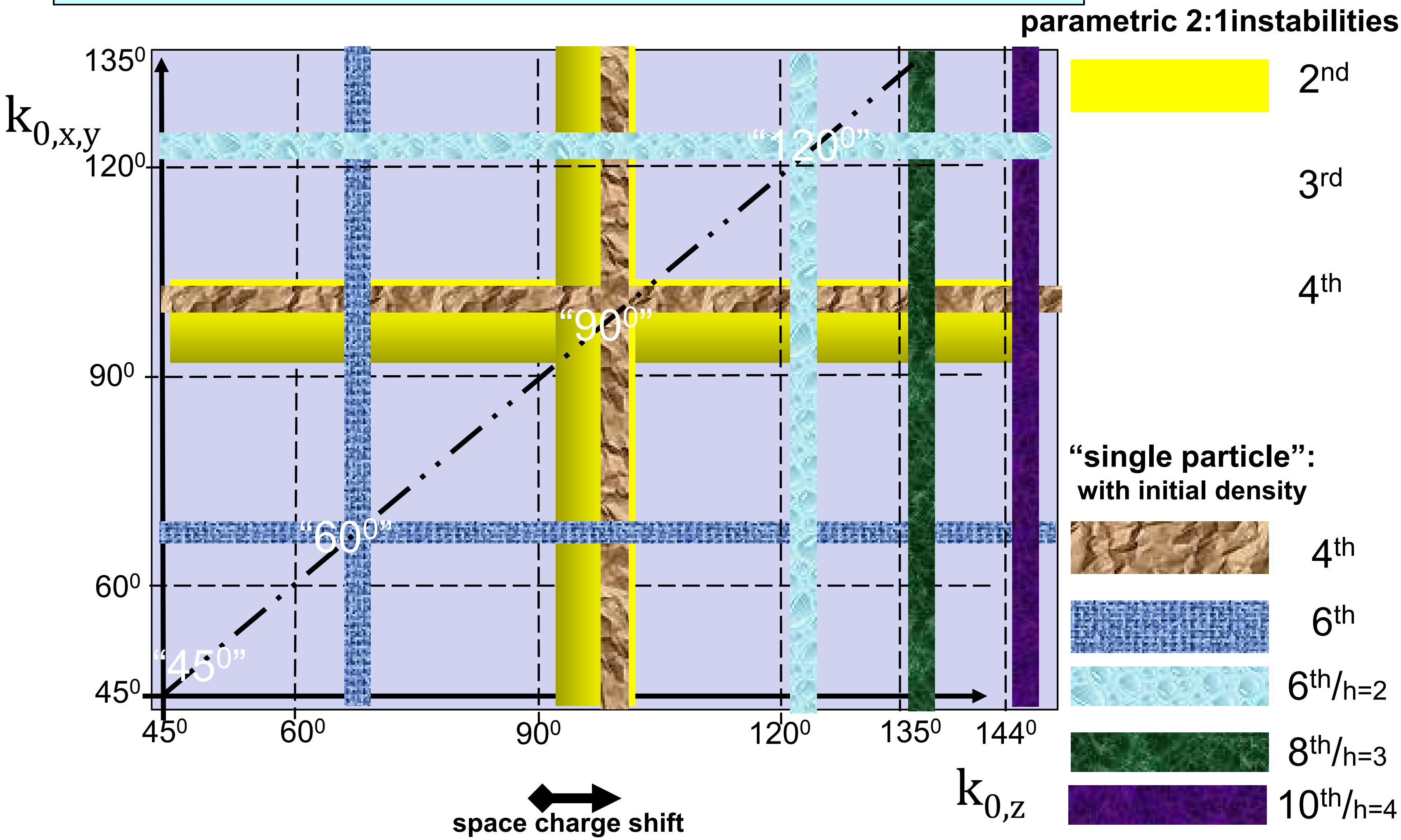

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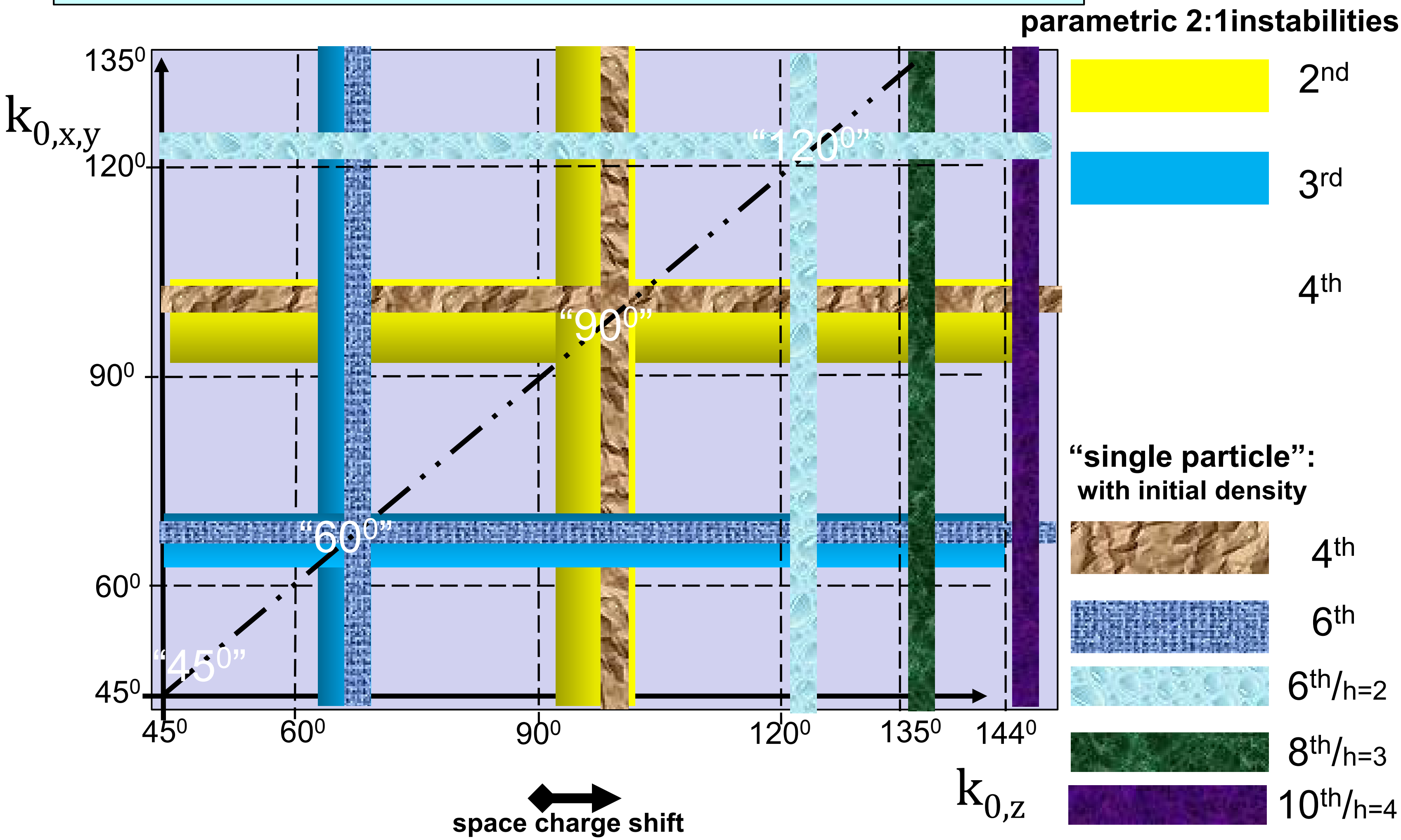

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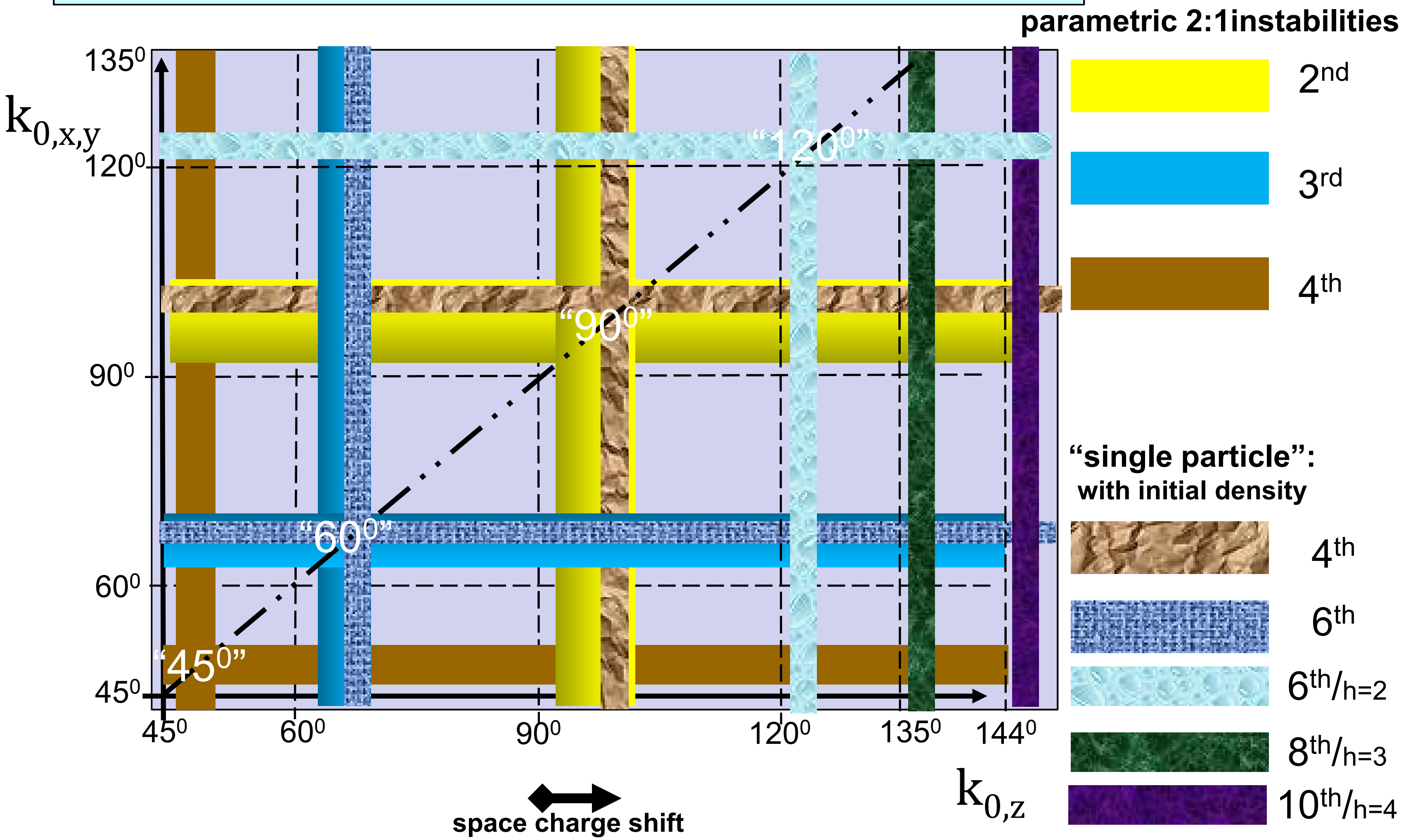

Conclusion

- ✓ **Two main groups** of resonant space charge effects
  - "Single particle" resonances with **driving term in initial space charge** profile – "usual" resonance diagram
  - Parametric **"half-integer"** resonances = instabilities with **driving term pumped from initial noise** – "stability diagram"
- ✓ Parametric resonances characterized by coherence in density - frozen space charge simulation fails!
- ✓ Stimulate **more experiments** to further advance our understanding and come to a more **complete picture** (synchrotron motion?)
- ✓ Analogous discussion on emittance transfer – where also **resonances <u>and</u> instabilities** matter (driven by anisotropy rather than parametrically)

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usual **resonance diagram**