HEAD-TAIL BUNCH DYNAMICS WITH SPACE CHARGE

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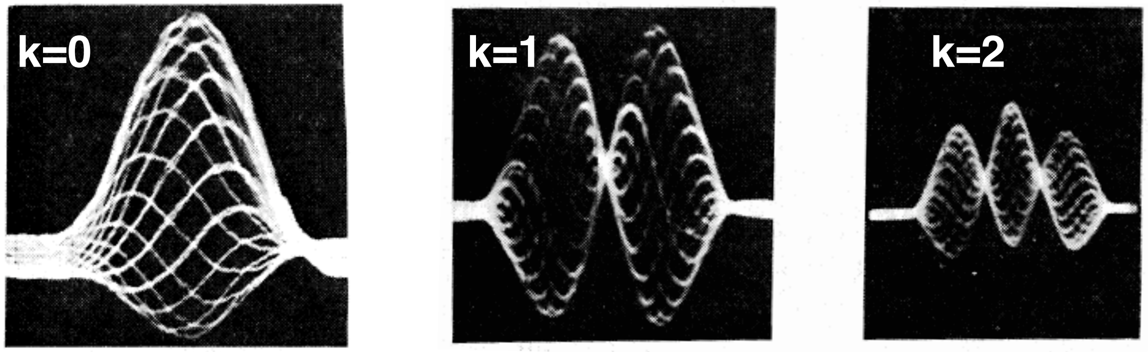
INTRODUCTION



PS Booster 1974

Head-Tail Instability

SIS100 of FAIR: the major concern at the injection plato

modified by Space Charge (ΔQ_{SC} up to 0.25)

$$\Delta Q_{sc}(\tau) = \frac{\lambda(\tau) r_p R^2}{\gamma^3 \beta^2 Q_0 a^2}$$

not considered by classical (Sacherer) theories

here: single-bunch instability, below the coupling threshold

AIRBAG BUNCH

a non-realistic bunch, but a very useful model
M.Blaskiewicz, PRSTAB **1**, 044201 (1998)

square-well (barrier) potential ➔ constant line density ➔ constant ΔQ_{sc}

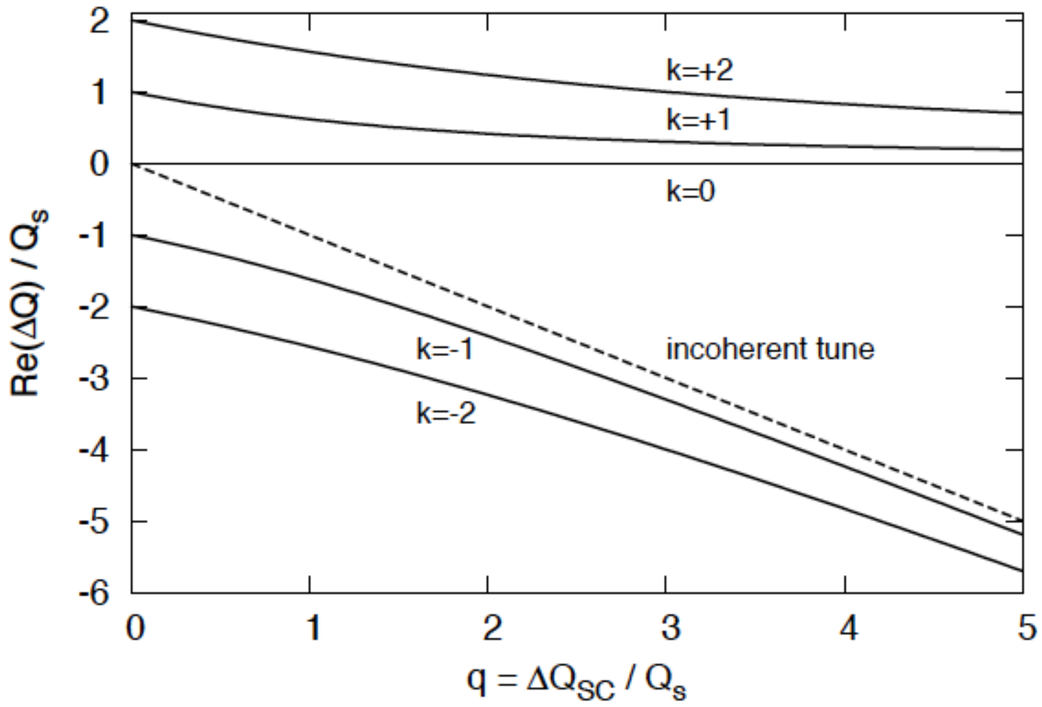
longitudinal distribution $f(p) \propto \delta(p - p_0) + \delta(p + p_0)$

Assumptions: rigid flows, $Q_0 \gg |\Delta Q|$, arbitrary space charge

without wake: $$\Delta Q = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2}$$ "+" for $k \geq 0$

space charge parameter

$$q = \frac{\Delta Q_{sc}}{Q_s}$$



GAUSSIAN BUNCH

Recent analytic works:
A.Burov, PRSTAB **12**, 044202 (2009); A.Burov, PRSTAB **12**, 109901(E) (2009)
V.Balbekov, PRSTAB **12**, 124402 (2009)

OUR APPROACH: PARTICLE TRACKING SIMULATIONS

PIC codes: PATRIC
O.Boine-Frankenheim, V.Kornilov, Proc. of ICAP2006 (2006)

HEADTAIL
G.Rumolo and F. Zimmermann, PRSTAB **5**, 121002 (2002)

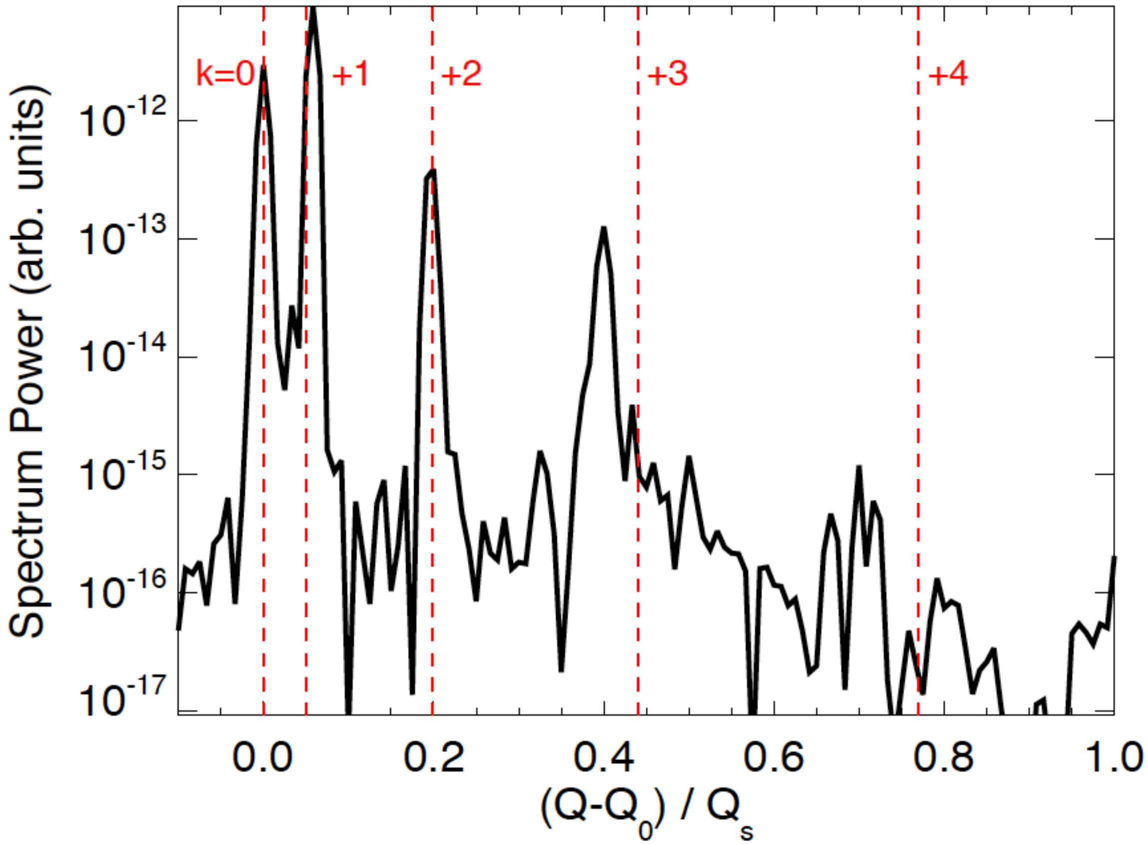
this work: “semi-frozen” electric field, homogeneous transverse profile,
the space charge implementation verified using the airbag theory
V. Kornilov and O. Boine-Frankenheim, Proc. of ICAP2009, San Francisco (2009)
Gaussian bunch: Gaussian line density, Gaussian momentum distr.

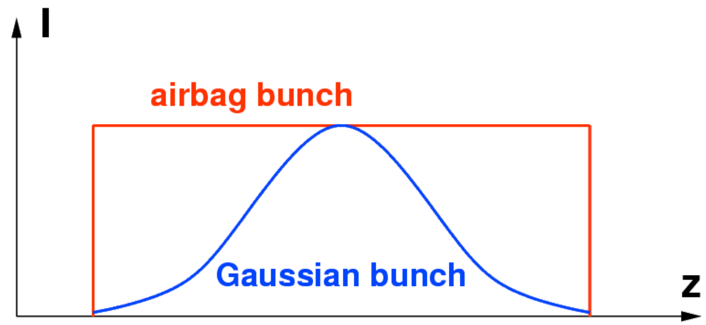
GAUSS-BUNCH WITH SC

Simulations for the Gaussian bunch
Spectrum example for q=20,
red dashed line: the airbag theory

space charge parameter

$$q = \frac{\Delta Q_{\rm sc}}{Q_{\rm s}}$$





Tune shifts are well predicted by the air-bag theory, especially for stronger SC

Bunch form seems to be not very important!

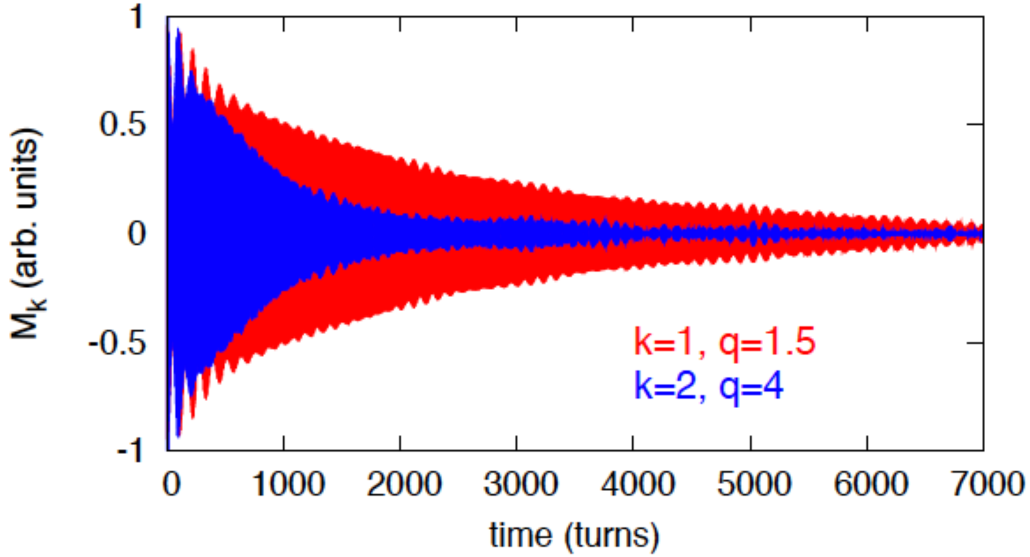
LANDAU DAMPING DUE TO SC

Landau damping in bunches exclusively due to the effect of space charge discussed in [Burov 2009], [Balbekov 2009]

$$\boxed{\Delta Q_{\mathrm{sc}}(\tau) = \frac{\lambda(\tau) r_p R^2}{\gamma^3 \beta^2 Q_0 a^2}}$$

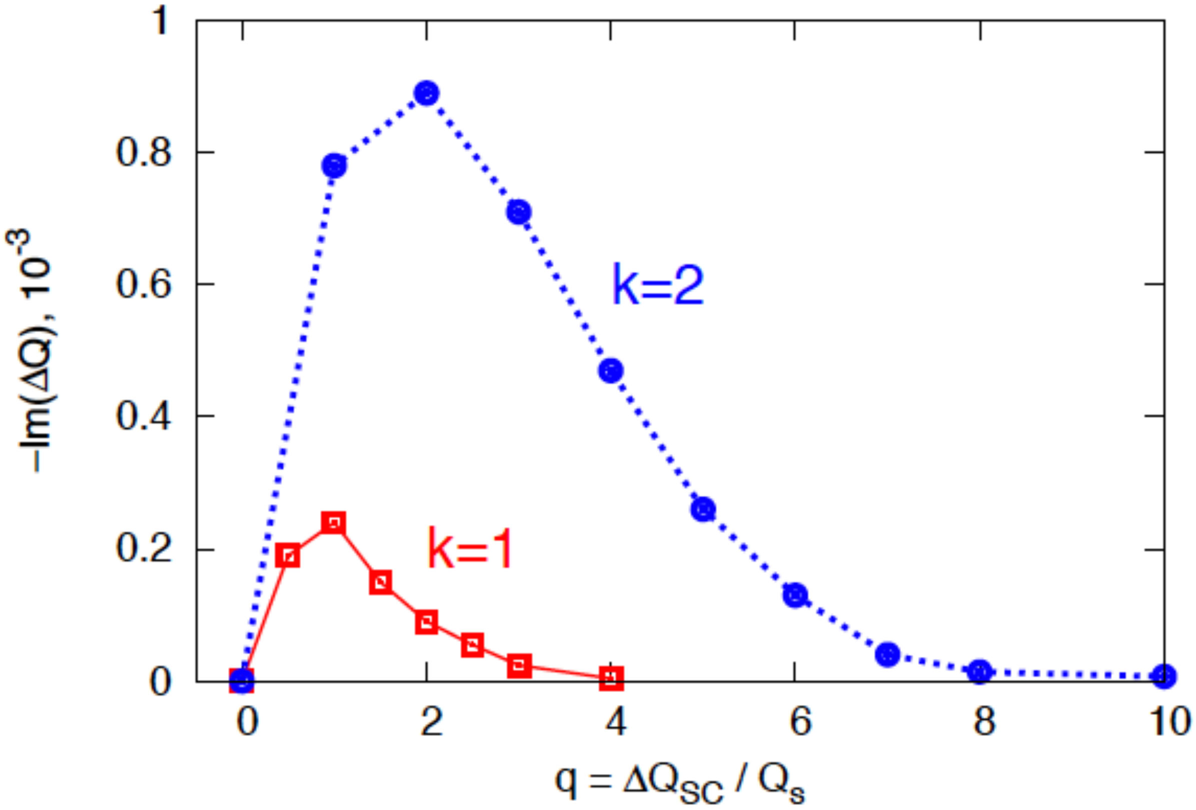
note: in a coasting beam space charge CAN NOT produce Landau damping of its own!

we start a simulation with an initial perturbation of a head-tail mode



$$M_k = \int \overline{x}_{\mathrm{code}} \cos(k\pi\tau/\tau_{\mathrm{b}}) \mathrm{d}\tau$$

LANDAU DAMPING DUE TO SC



Summary of Landau damping simulations:
damping decrement for a Gaussian bunch

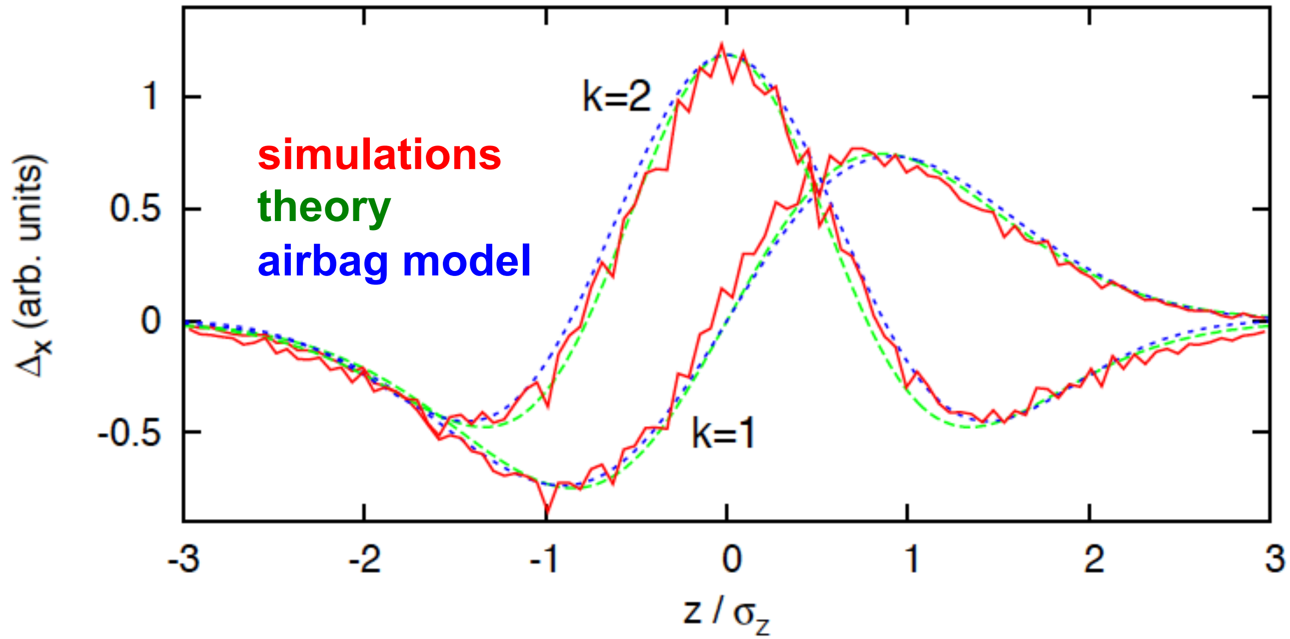
EIGENFUNCTIONS

Eigenfunctions for a Gaussian bunch:
extracted in simulations using Landau damping (at *q*=6)

theory [Burov 2009]: $\bar{y}'' + \nu \exp(-\tau^2/2)\bar{y} = 0$

airbag head-tail eigenfunctions [Blaskiewicz 1998] $\overline{x}_k(\tau) = A_0 \exp(-i\xi Q_0 \tau/\eta) \cos(k\pi\tau/\tau_\mathrm{b})$

$\tau_\mathrm{b} = \frac{4\sigma_z}{R}$

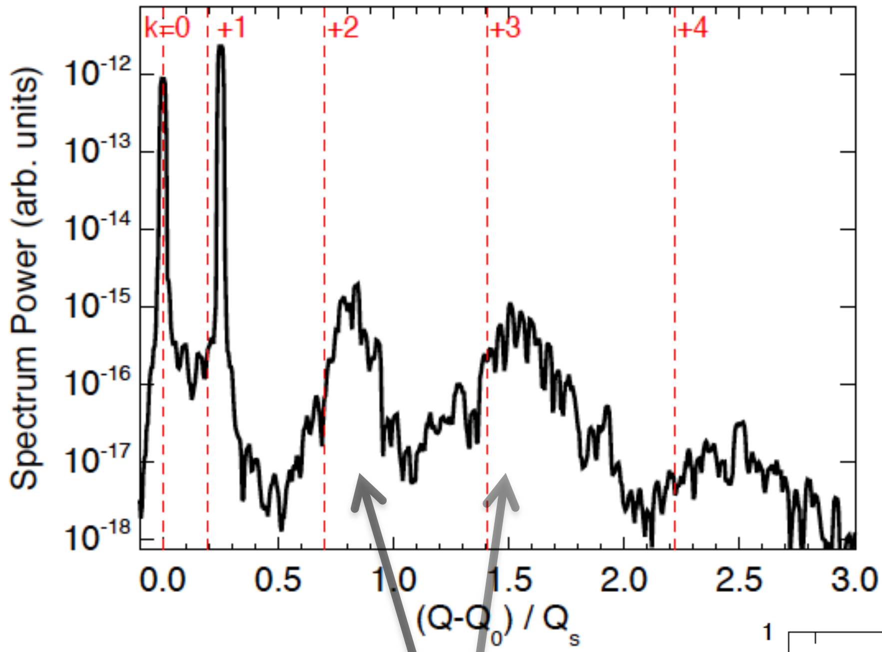


eigenfunctions of a Gaussian bunch are very close to the airbag modes!

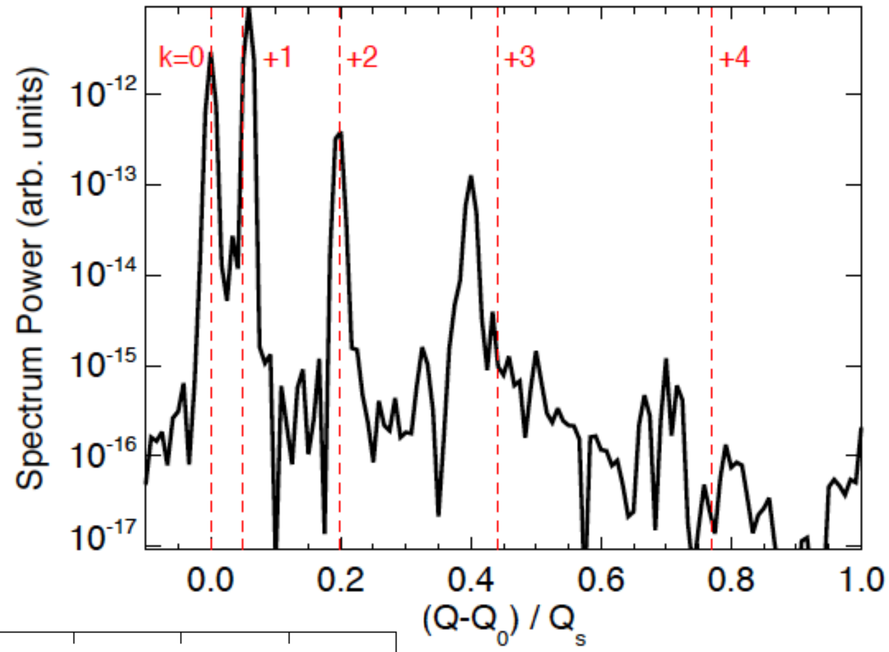
LANDAU DAMPING DUE TO SC

Landau damping can be seen in the bunch spectrum

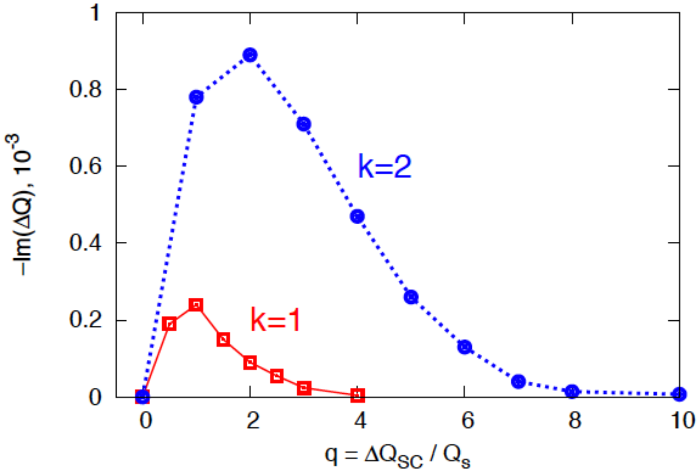
q=5



q=20



the modes *k*=2, *k*=3 suppressed



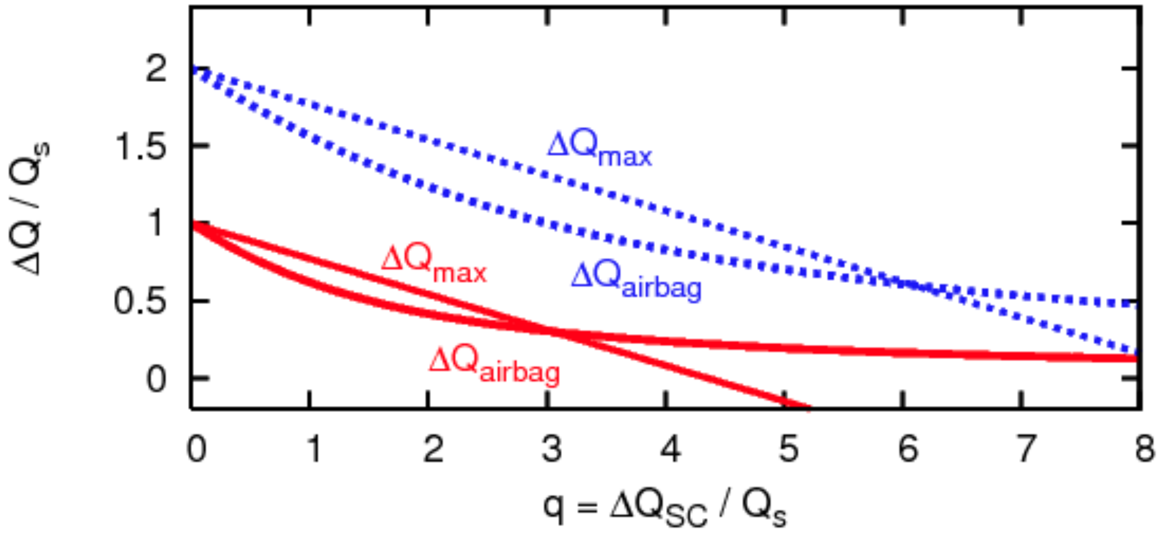
here not

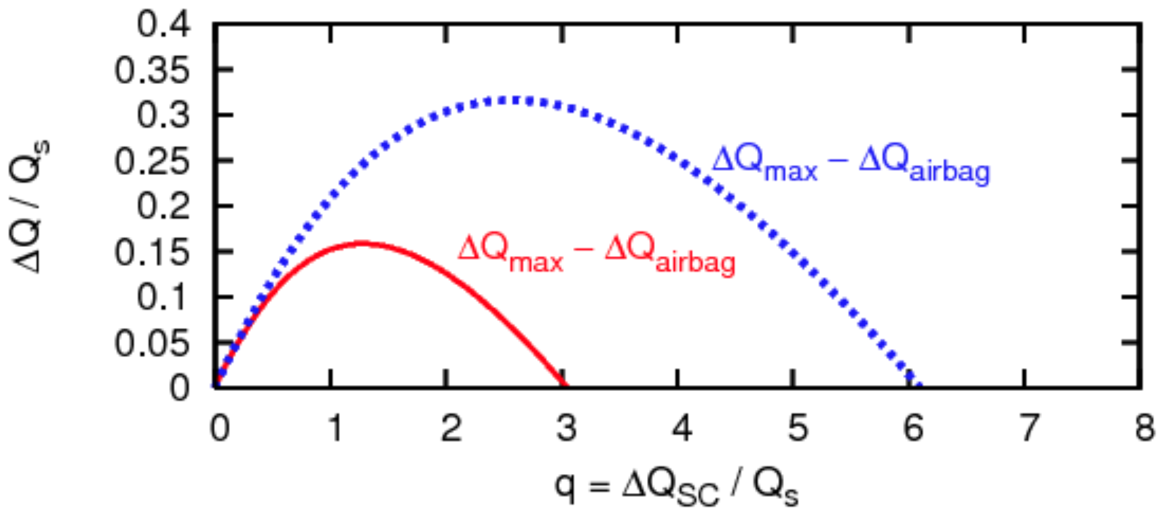
LANDAU DAMPING DUE TO SC

a physical interpretation

upper boundary of the incoherent effective spectrum:
small ΔQ_{sc},
large synchrotron amplitudes

$$\Delta Q_{\max} \approx -0.23 Q_s q + k Q_s$$





red: *k*=1; blue: *k*=2

LANDAU DAMPING DUE TO SC

a physical interpretation

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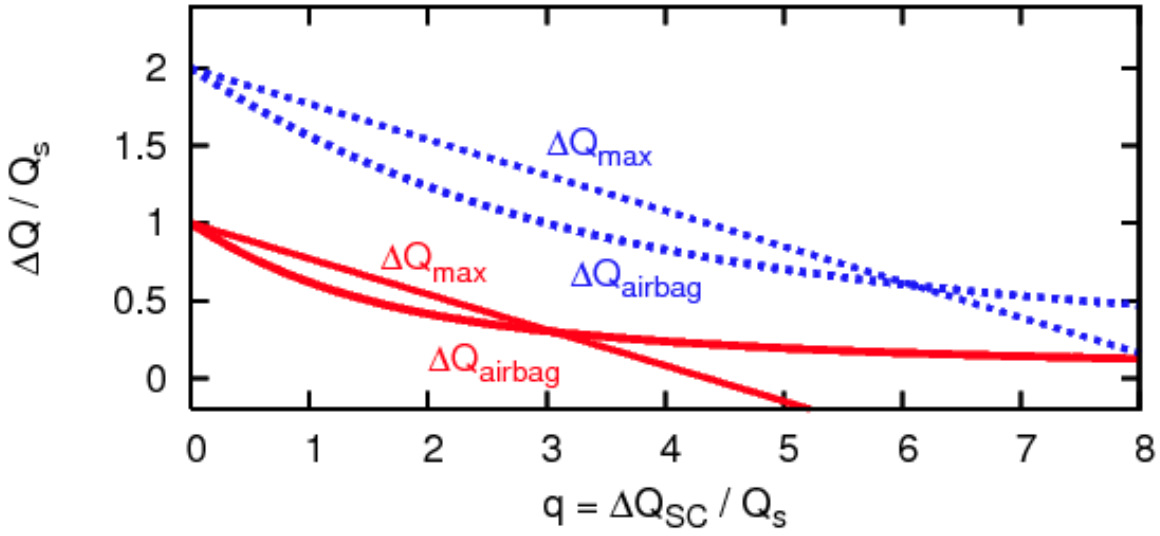
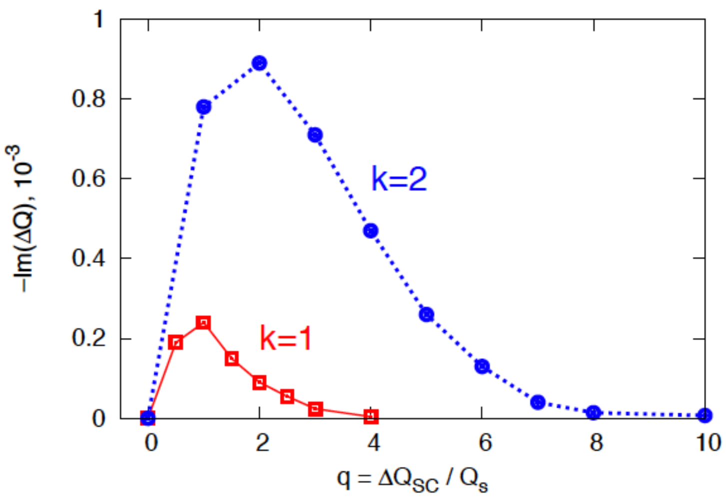
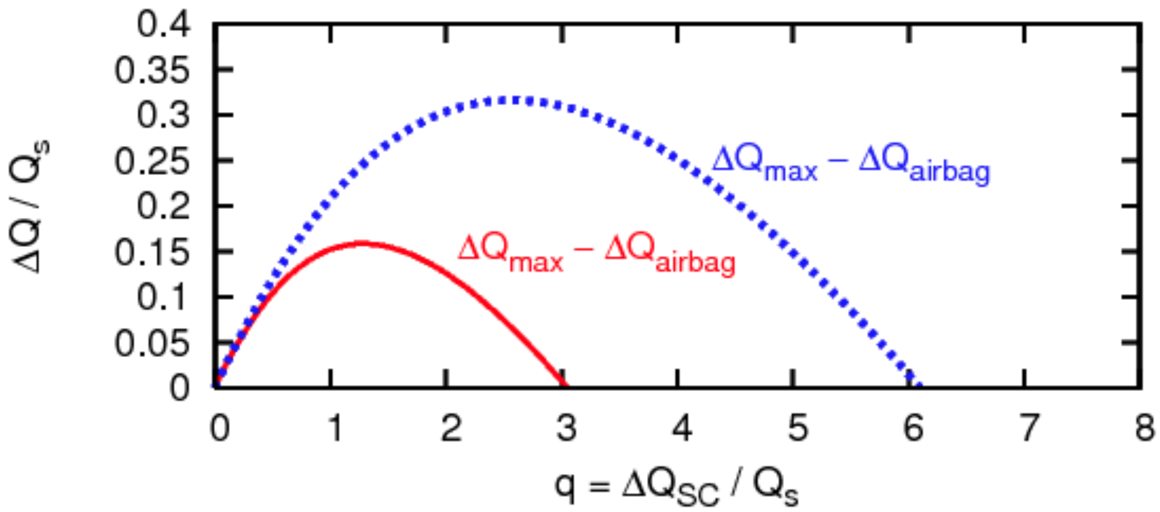




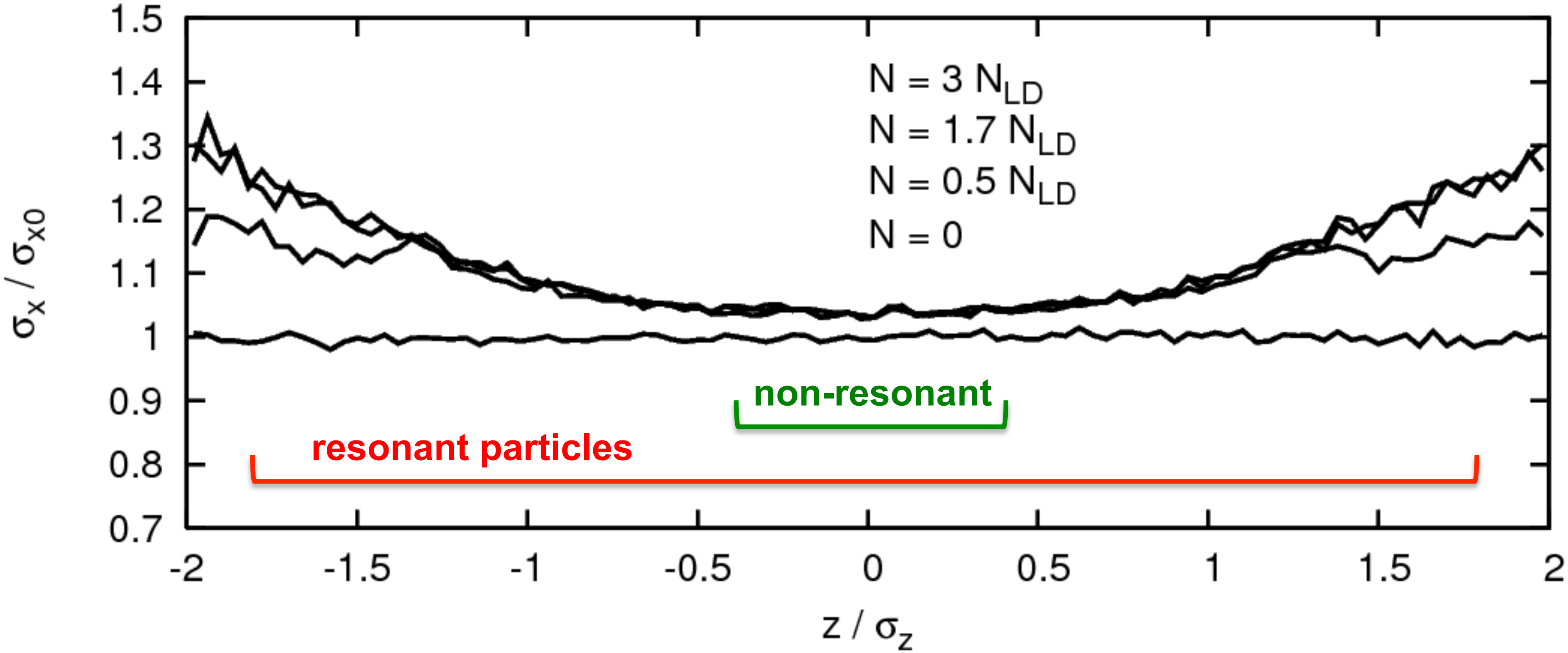


red: *k*=1; blue: *k*=2

LANDAU DAMPING DUE TO SC

Landau damping:
resonant particles with small ΔQ_{sc} have large synchrotron amplitudes, energy transfer happens in the bunch tails

this leads to the local emittance increase:

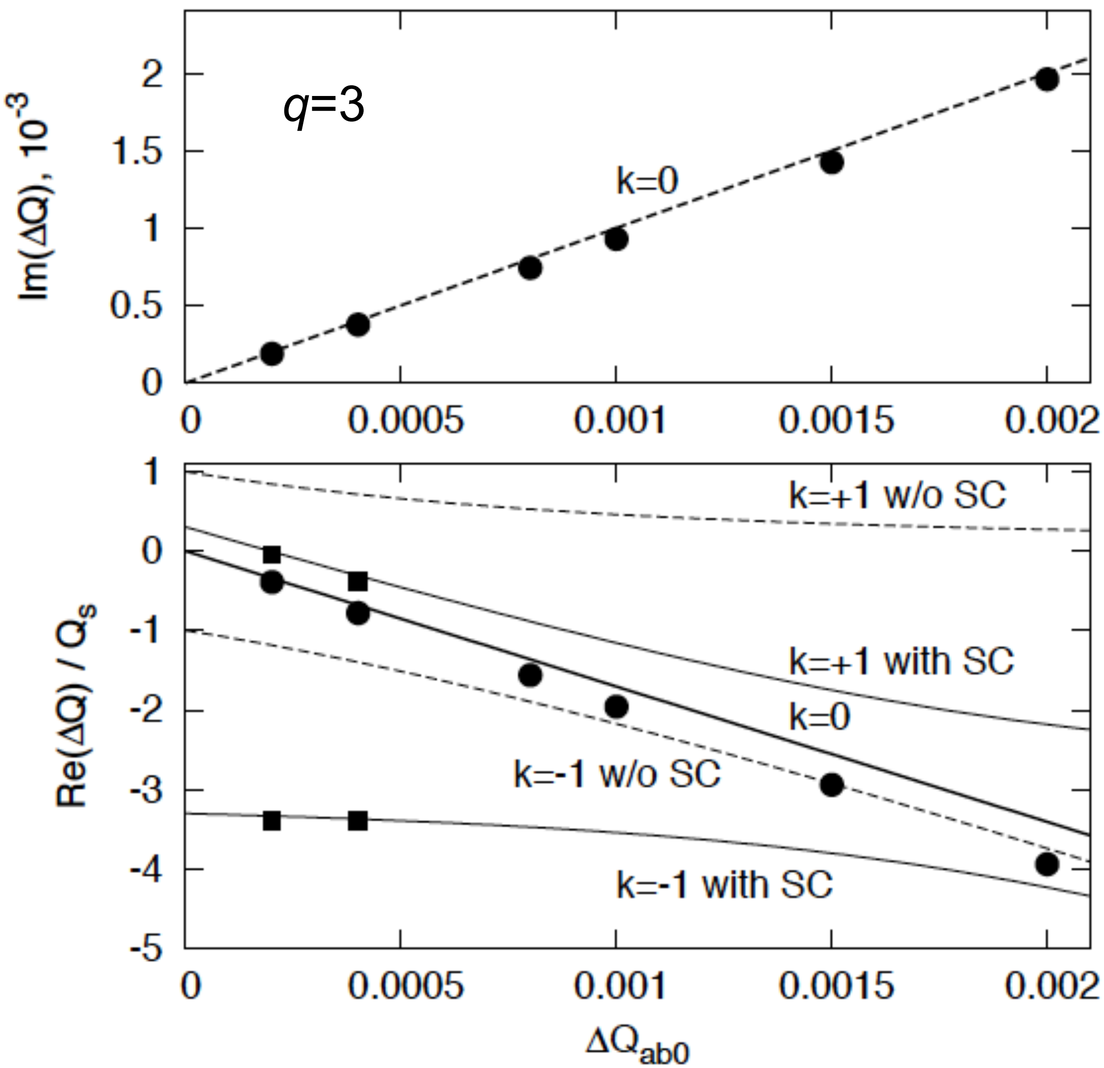


horizontal rms beam size along the bunch, a simulation for k=2, q=3

UNSTABLE AIRBAG BUNCH

airbag bunch:
analytical results
for unstable head-tail modes
[Blaskiewicz 1998]

$$W(\tau) = W_0 \exp(-\alpha\tau)$$
$$\alpha\tau_b \gg 1$$



for k=0
(not affected by space charge)

$$\Delta Q = \Delta Q_0(\alpha/\zeta + i)$$
$$\Delta Q_0 = -\frac{\kappa\zeta}{\alpha^2} W_0$$

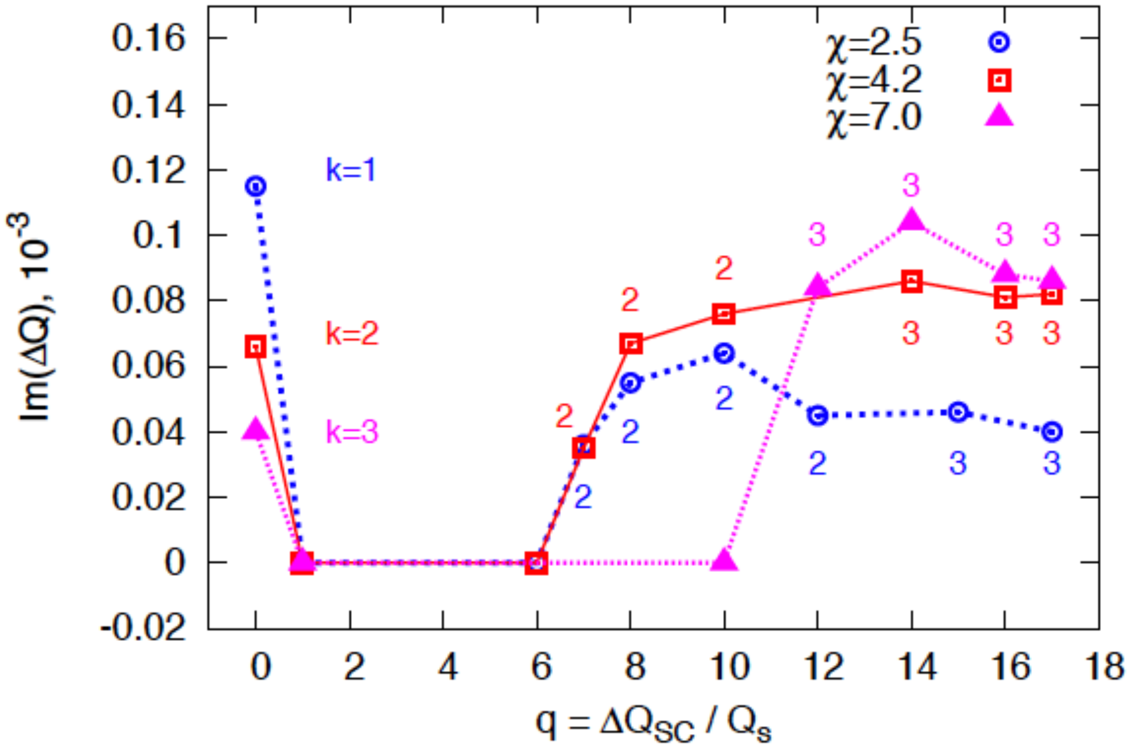
for k>0

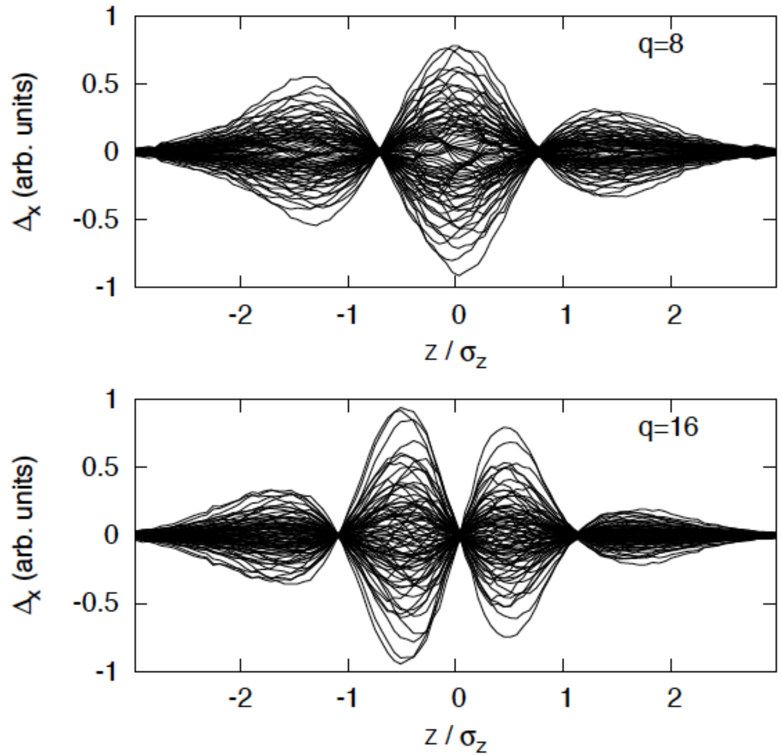
$$\Delta Q = -\Delta Q_{\rm sc} + \frac{\Delta Q_0(\alpha/\zeta + i) + \Delta Q_{\rm sc} \pm \sqrt{[\Delta Q_0(\alpha/\zeta + i) + \Delta Q_{\rm sc}]^2 + 4k^2 Q_s^2[1 - (\Delta Q_0 \pi/2\zeta Q_s \tau_b)^2]}}{2\,[1 - (\Delta Q_0 \pi/2\zeta Q_s \tau_b)^2]}$$

HEAD-TAIL INSTABILITY WITH SC

Simulations for a Gaussian bunch:
growth rates of the most unstable head-tail mode

$\chi=\xi Q_0 \tau_b/\eta=4.2$





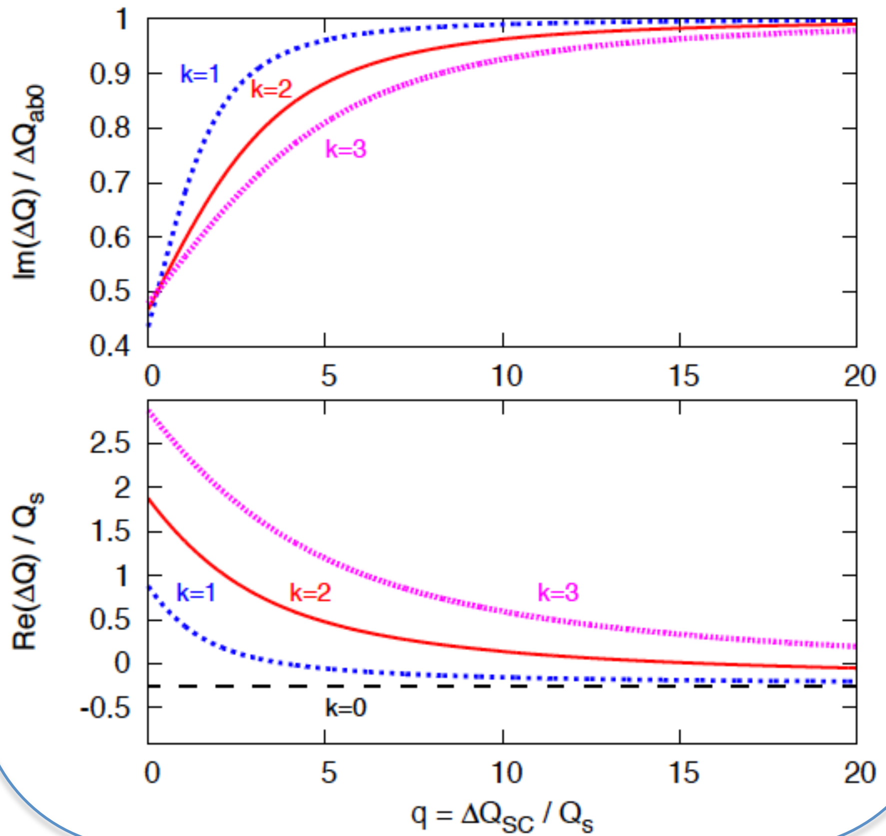
Landau Damping!

wake function
of the thick resistive wall: $$W_{\mathrm{rw}}(z) = -\frac{cL_{\mathrm{rw}}}{b^3}\left(\frac{\beta}{\pi}\right)^{3/2}\sqrt{\frac{Z_0}{z\,\sigma_{\mathrm{rw}}}}$$

HEAD-TAIL INSTABILITY WITH SC

simulations: the growth rates saturate at strong space charge

analytic theory
for the airbag bunch
[Blaskiewicz 1998]



calculations in [Burov 2009]:
treating a wake as a perturbation
➔ diagonal element of the wake operator

$$\Delta Q = \frac{\kappa}{N_{\rm ion}\lambda_0 R}\int_0^{z_b} dz \int_z^{z_b} ds\; W(s-z)\, d_k(s)\, d_k^*(z)$$

$$d_k(s) = \lambda(s)\overline{x}_k(s)$$

provides the growth rate
at strong space charge

comparisons with the simulation results
show a good agreement!

SUMMARY

Landau damping in bunches exclusively due to space charge observed in particle tracking simulations.
The damping strength depends on $q=\Delta Q_{sc}/Q_s$ and on the mode index k.
The energy transfer happens in the bunch tails.

Gaussian bunch: the transverse functions and frequencies are very similar to that prediced by the airbag theory [Blaskiewicz 1998]

Simulations of the head-tail instability with space charge:
Landau damping can stabilize the bunch.
The instability growth rates saturate at strong space charge (compared to [Blaskiewicz 1998] and [Burov 2009]).

Examples:

- CERN PS (E.Métral et al, PAC07)
  $q\sim150$: far above Landau damping, growth rates saturated
- FAIR SIS100
  $q\sim20$: Landau damping might contribute to the stability