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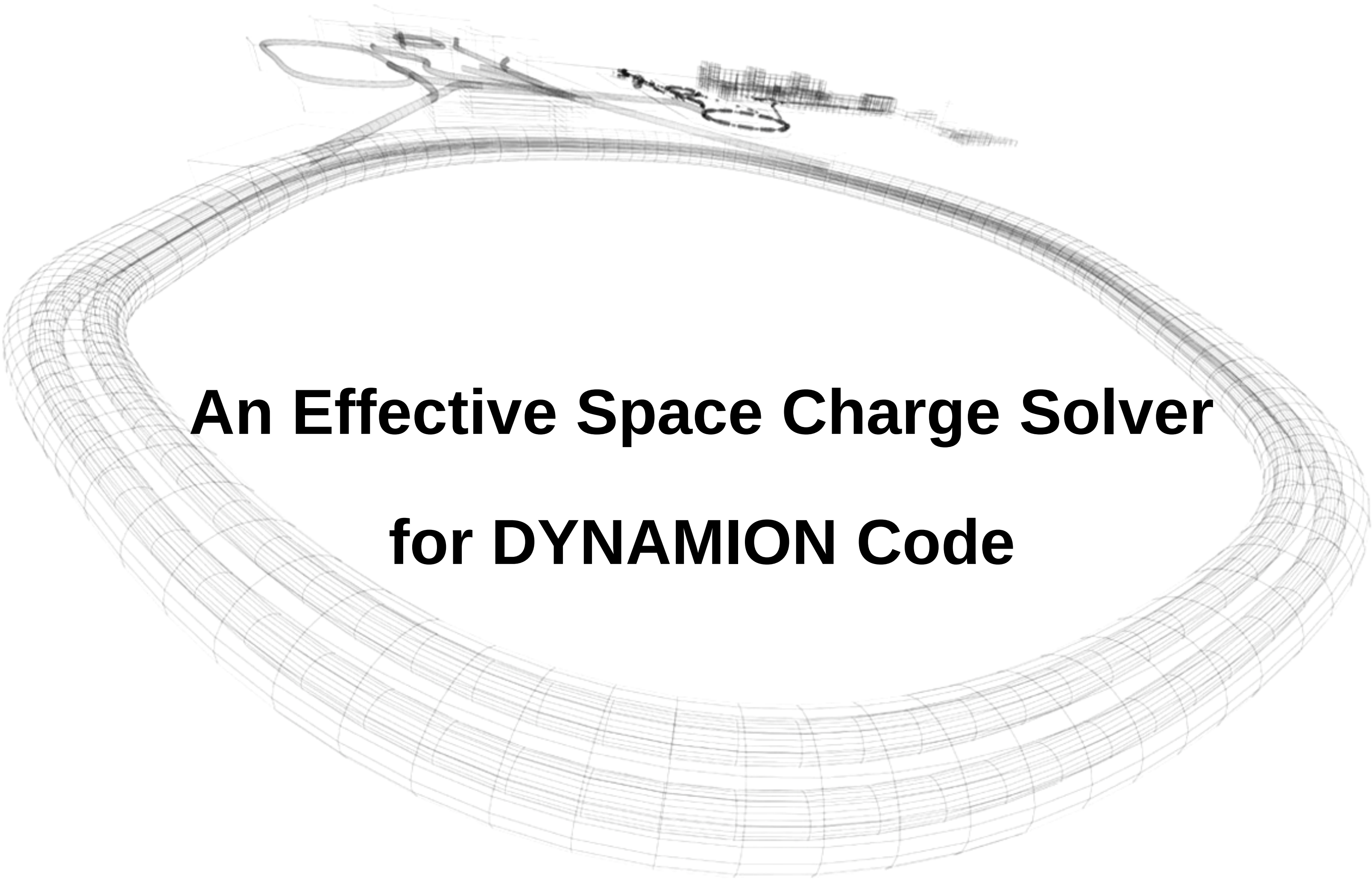
An Effective Space Charge Solver for DYNAMION Code



GSI

- Introduction
- General space charge algorithms based on the effective method of the integral calculation
- Implementation of the space charge algorithms into DYNAMION code
- Conclusions

Standard Space Charge Approaches

Poisson Solvers

analytical solutions

for simple models

particle – particle

(+ special routine to avoid artificial particle collisions)

grid methods

FFT, PIC-solvers

Standard Space Charge Approaches

Poisson Solvers

analytical solutions

for simple models

particle – particle

(+ special routine to avoid artificial particle collisions)

grid methods

FFT, PIC-solvers

Advanced methods

study of the particular cases for

fast and precise integral calculations

Studied Models

study of the particular cases for faster and more precise integral calculations

Ellipsoidal bunch

- axi-symmetrical bunch

analytical method

- arbitrary axis

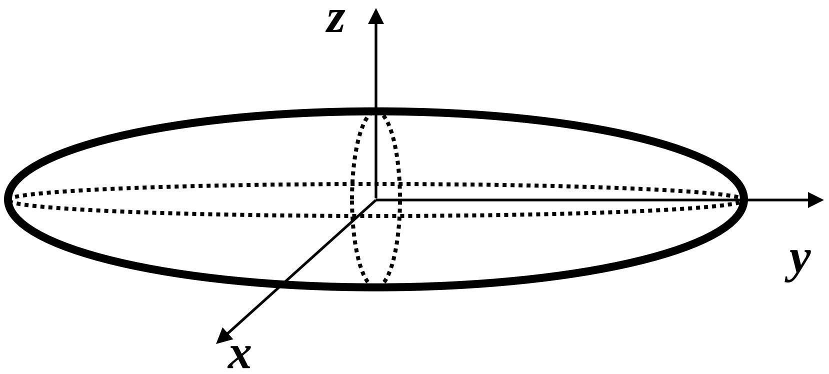
numerical method

The bunch of the arbitrary elliptical cross section

semi-analytical method

EPAC – 04,
ICAP - 06
Orzhekhovskaya, Franchetti

Space Charge of the Ellipsoidal Bunch



a,b,c – **bunch axis**

$t = x^2/a^2 + y^2/b^2 + z^2/c^2$

$$\rho(x,y,z) = \frac{Q}{4\pi abc} n(t),$$

$$\int_0^\infty n(r^2) r^2 dr = 1$$

Kellogg's formulae

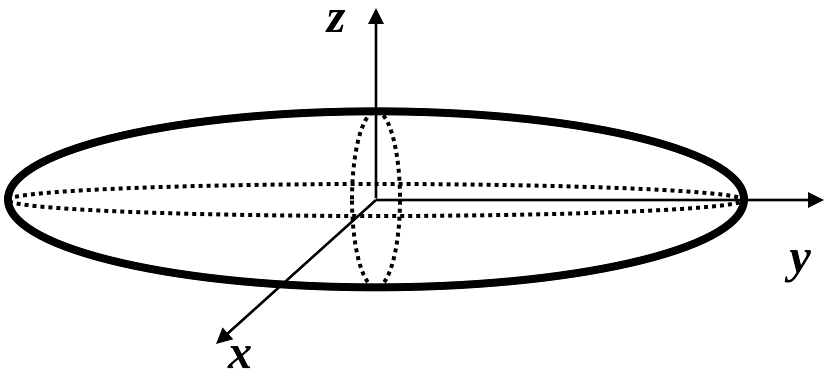
$$E_x = \frac{Q}{2} x \int_0^\infty \frac{n(T)ds}{(a^2+s)^{3/2}(b^2+s)^{1/2}(c^2+s)^{1/2}},$$

$$E_y = \frac{Q}{2} y \int_0^\infty \frac{n(T)ds}{(a^2+s)^{1/2}(b^2+s)^{3/2}(c^2+s)^{1/2}},$$

$$E_z = \frac{Q}{2} z \int_0^\infty \frac{n(T)ds}{(a^2+s)^{1/2}(b^2+s)^{1/2}(c^2+s)^{3/2}},$$

$$T = \frac{x^2}{a^2+s} + \frac{y^2}{b^2+s} + \frac{z^2}{c^2+s}$$

Space Charge of the Ellipsoidal Bunch



Many particles Many steps:

NOT EFFECTIVE !!!

Kellogg's formulae

$$E_x = \frac{Q}{2} x \int_0^\infty \frac{n(T)ds}{(a^2+s)^{3/2}(b^2+s)^{1/2}(c^2+s)^{1/2}},$$

$$E_y = \frac{Q}{2} y \int_0^\infty \frac{n(T)ds}{(a^2+s)^{1/2}(b^2+s)^{3/2}(c^2+s)^{1/2}},$$

$$E_z = \frac{Q}{2} z \int_0^\infty \frac{n(T)ds}{(a^2+s)^{1/2}(b^2+s)^{1/2}(c^2+s)^{3/2}},$$

$$T = \frac{x^2}{a^2+s} + \frac{y^2}{b^2+s} + \frac{z^2}{c^2+s}$$

Space Charge of the Ellipsoidal Bunch

Polynomial representation

$$n(t) = \sum_{n=0}^{N} c_n t^n$$

$t = x^2/a^2 + y^2/b^2 + z^2/c^2$

Kellogg's formulae

$$E_x = \frac{Q}{2} x \int_0^\infty \frac{n(T)\,ds}{(a^2+s)^{3/2}(b^2+s)^{1/2}(c^2+s)^{1/2}},$$

$$E_y = \frac{Q}{2} y \int_0^\infty \frac{n(T)\,ds}{(a^2+s)^{1/2}(b^2+s)^{3/2}(c^2+s)^{1/2}},$$

$$E_z = \frac{Q}{2} z \int_0^\infty \frac{n(T)\,ds}{(a^2+s)^{1/2}(b^2+s)^{1/2}(c^2+s)^{3/2}},$$

$$T = \frac{x^2}{a^2+s} + \frac{y^2}{b^2+s} + \frac{z^2}{c^2+s}$$

NOW EASY TO CALCULATE!

$$E_x = \frac{Q}{2} x \sum_{l=0}^{N} c_l \sum_{i+j+k=l} \frac{l!}{i!\,j!\,k!} x^{2i} y^{2j} z^{2k} I_{i+1,j,k}$$

Semi-Analytical space charge Solver (SAS)

$$n(t) = \sum_{n=0}^{N} c_n t^n$$

$$E_x = \frac{Q}{2} x \sum_{l=0}^{N} c_l \sum_{i+j+k=l} \frac{l!}{i!\,j!\,k!} x^{2i} y^{2j} z^{2k} I_{i+1,j,k}$$

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$$I_{i,j,k} = \int_0^{\infty} \frac{1}{(a^2+\xi)^{1/2+i}(b^2+\xi)^{1/2+j}(c^2+\xi)^{1/2+k}} d\xi$$

Gaussian Quadrature (or Analytical Solution)

Semi-Analytical space charge Solver (SAS)

$$n(t) = \sum_{n=0}^{N} c_n t^n$$

Error < 0.1%

$$E_x = \frac{Q}{2} x \sum_{l=0}^{N} c_l \sum_{i+j+k=l} \frac{l!}{i!\,j!\,k!} x^{2i} y^{2j} z^{2k} I_{i+1,j,k}$$

$$I_{i,j,k} = \int_0^{\infty} \frac{1}{(a^2+\xi)^{1/2+i}(b^2+\xi)^{1/2+j}(c^2+\xi)^{1/2+k}} d\xi$$

Semi-Analytical space charge Solver (SAS)

$$I_{i,j,k} = \int_0^\infty \frac{1}{(a^2+\xi)^{1/2+i}(b^2+\xi)^{1/2+j}(c^2+\xi)^{1/2+k}} d\xi$$

Integral does not depend on (x,y,z) → **Can be calculated ONCE for all particles**

MUCH FASTER field calculations

$$E_x = \frac{Q}{2} x \sum_{l=0}^{N} c_l \sum_{i+j+k=l} \frac{l!}{i!\,j!\,k!} x^{2i} y^{2j} z^{2k} I_{i+1,j,k}$$

Applications of the Algorithms

HIPPI Project

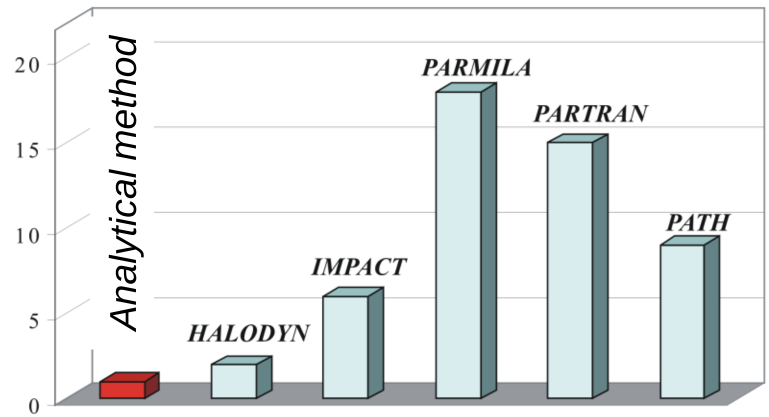
***H**igh **I**ntensity **P**ulsed*
***P**roton **I**njector*

Perfect coincidence !

benchmarking of the linac codes

DYNAMION (*ITEP*, *GSI*),
HALODYN (University of Bologna),
IMPACT (*LANL*, *LBNL*),
LORASR (*IAP*),
PARMILA (*LANL*),
PARTRAN (*CEA*),
PATH (*CERN*),
TOUTATIS (*CEA*)

CPU - TIME (in units)



10^5 particles

Axi-symmetric Ellipsoidal Bunch Static Case

GSI

Applications of the Algorithms

MICROMAP

Giuliano Franchetti
GSI (Darmstadt)

Library of codes for beam dynamics simulation in circular accelerators

- analytical method for the 2D elliptical cross section or for the axi-symmetrical ellipsoidal bunch
- semi-analytical methods for the bunches of an arbitrary elliptical cross section
- grid method for the 2D cross section
- high speed and high accuracy of calculation
- beam dynamics simulations and beam losses investigations for SIS18 and FAIR

The multiparticle DYNAMION code

DYNAMION

ITEP (Moscow)
GSI (Darmstadt)

Advanced code for beam dynamics simulation in linear accelerators

- created in ITEP (1985)
- developed in collaboration ITEP and GSI since 1991
- end-to-end beam dynamics simulations for linacs
- was used for simulations in different laboratories

- particle – particle method for space charge calculations with special routine to avoid artificial particle collisions
- PIC solver (ITEP) is under tests

Interpolation of the charge density

$$n(t) \rightarrow \sum_{n=0}^{N} c_n t^n$$

$t = x^2/a^2 + y^2/b^2 + z^2/c^2$

Minimization of the maximum interpolation error for polynoms of order N

Chebyshev nodes

$$t_k = \frac{1}{2} + \frac{1}{2}\cos\frac{(2k+1)\pi}{2N+2}$$

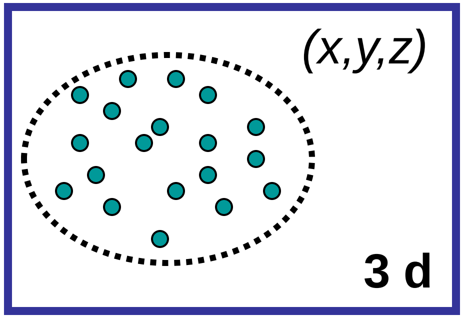
$k = 0, 1, \ldots, N$

N ~ 20 !

EPAC – 04, Orzhekhovskaya, Franchetti

Implementation into DYNAMION code

reconstruction of the charge density from the discrete particle coordinates

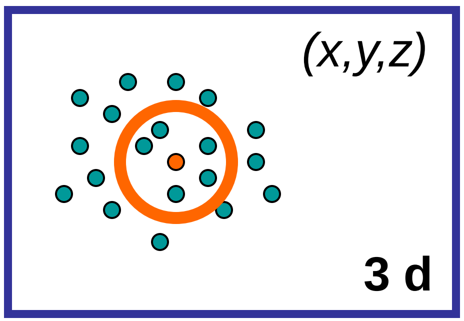


$$t = x^2/a^2 + y^2/b^2 + z^2/c^2$$

$$0 \leq t \leq 1$$

Implementation into DYNAMION code

reconstruction of the charge density from the discrete particle coordinates



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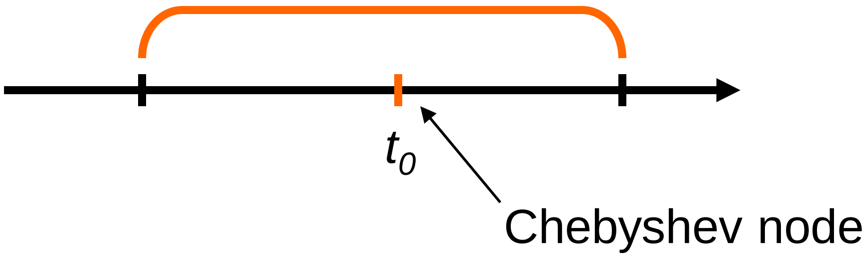

$n\,(t_0)$ - ?

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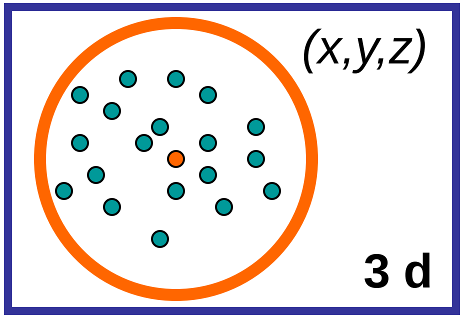

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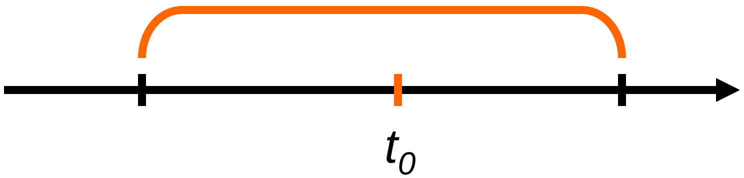
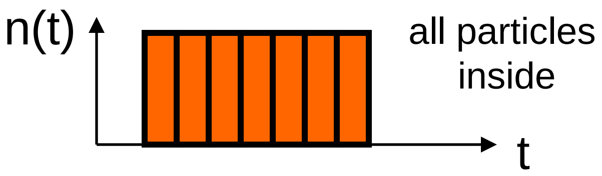


How Large ?

TOO Large



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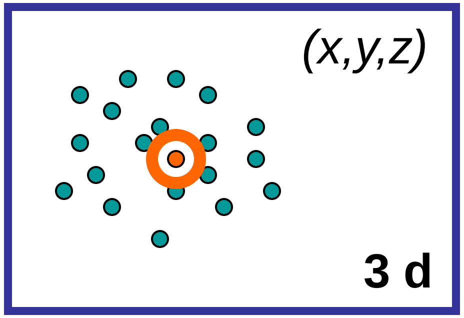
reconstruction of the charge density from the discrete particle coordinates



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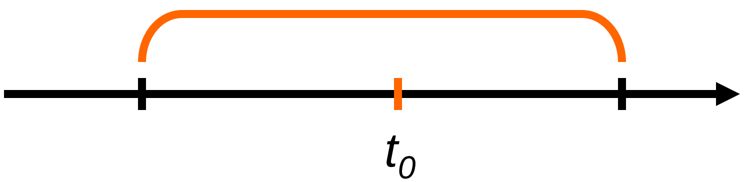

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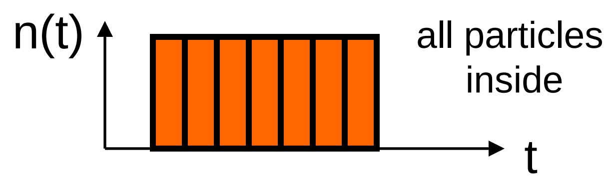
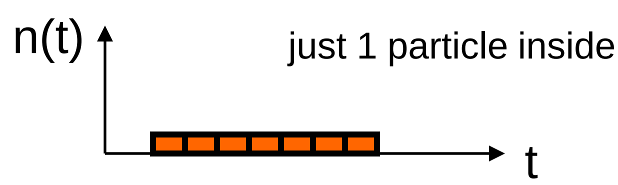
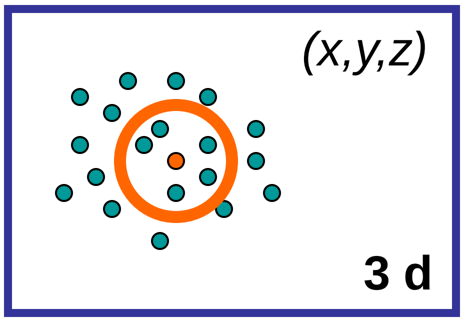


TOO Small



Implementation into DYNAMION code

reconstruction of the charge density from the discrete particle coordinates



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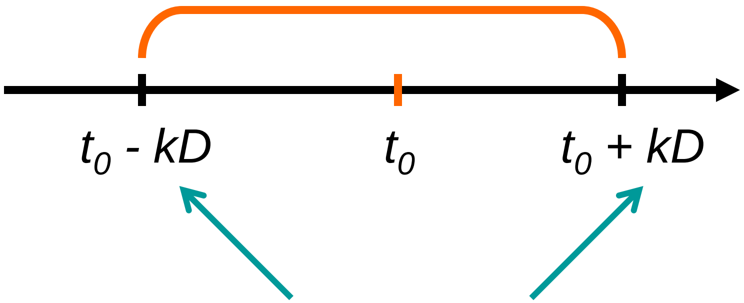
$$0 \le t \le 1$$



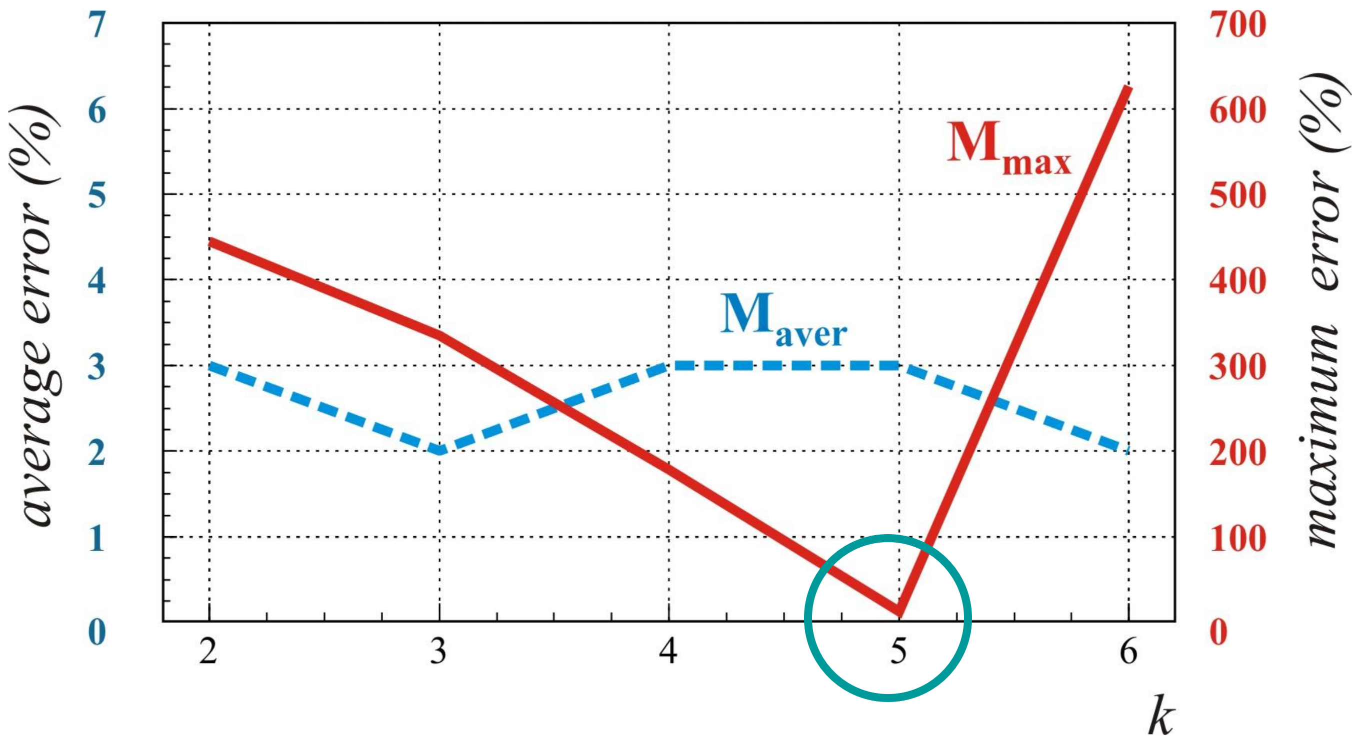
k - ?

N_p particle

$D = 1.0 / N_p$

Implementation into DYNAMION code

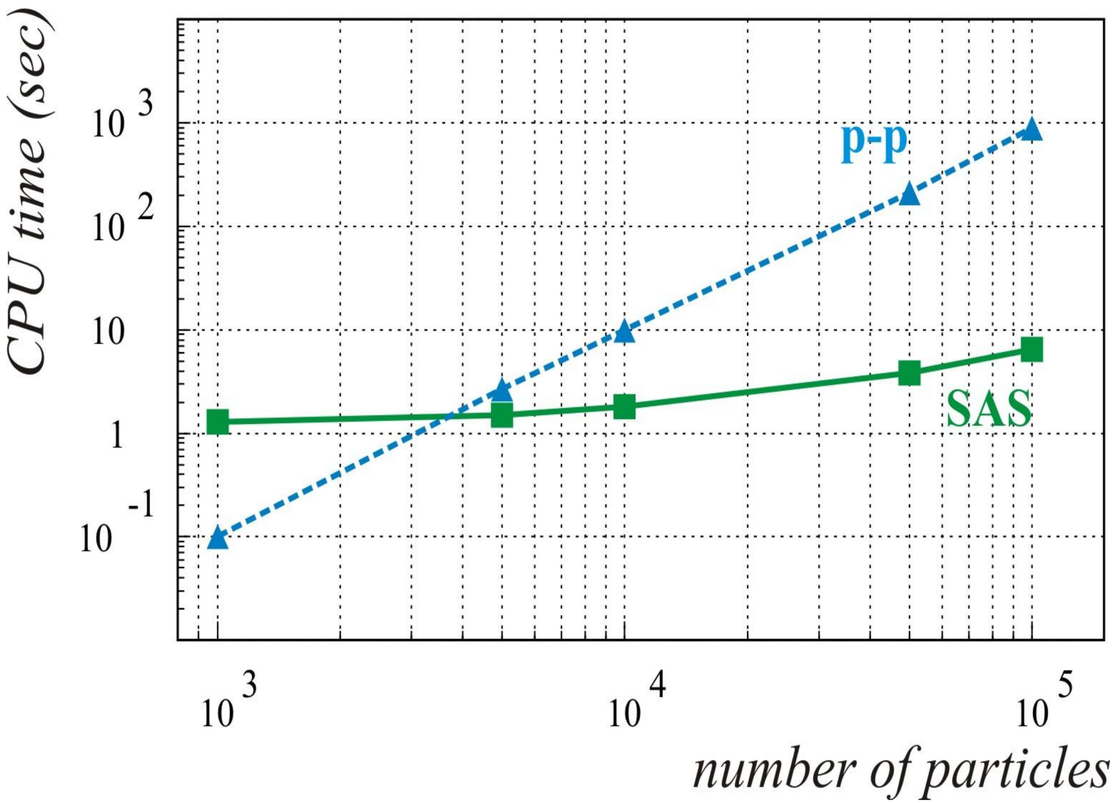
$$M = \frac{|E_x^{\ k}(x,y,z) - E_x^{\ k+1}(x,y,z)|}{E_x^{\ k}(x,y,z)}$$



k = 5 !

Comparison with p-p method (static case)

Difference in the electric field values is less than 1% !!!



Beam dynamics simulations with DYNAMION code

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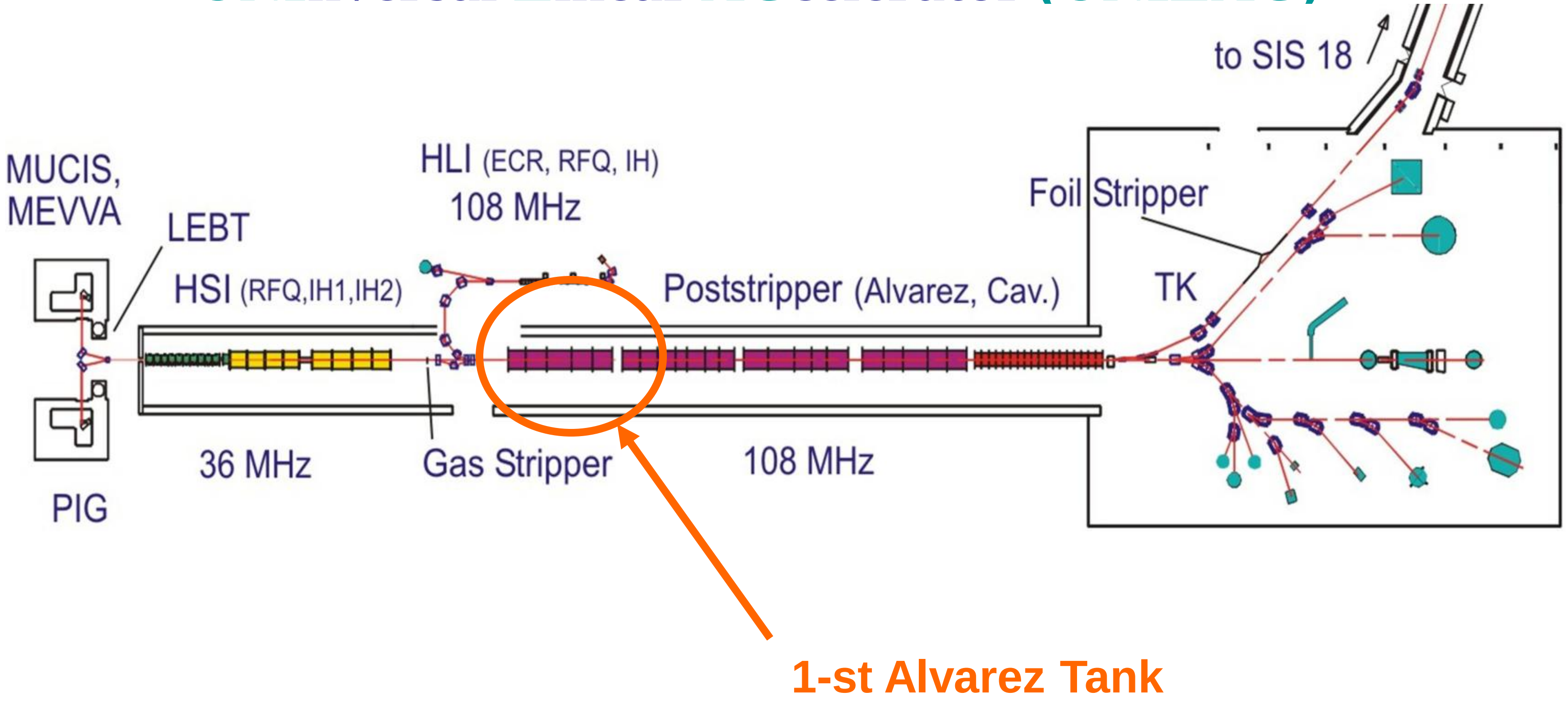
UNInversal Linear ACcelerator (UNILAC)

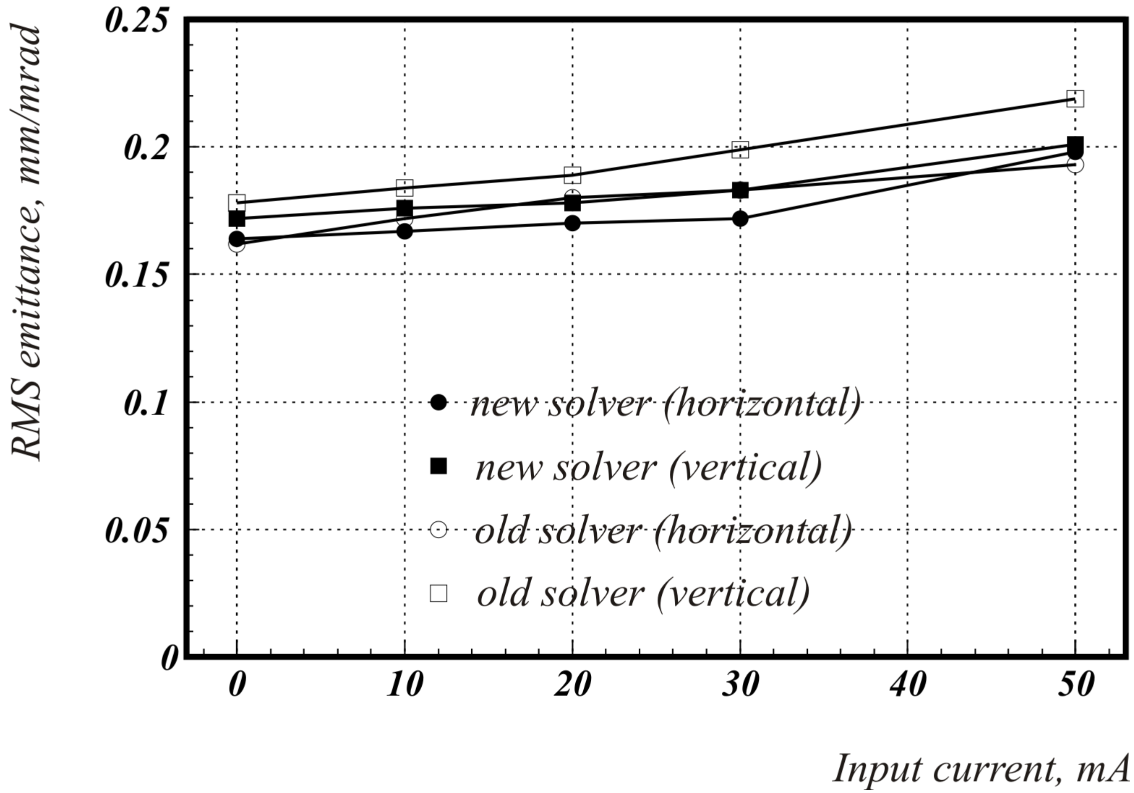


CPU time for beam dynamics simulations (1-st Alvarez tank)

	10^3 particles	10^4 particles	10^5 particles
P-P	10 min	48 hours	-
SAS	1 hour	2 hours	10 hours

Beam dynamics simulations with DYNAMION code

RMS emittance behind 1-st Alvarez Tank of UNILAC



Conclusion

- Fast and precise **S**emi-**A**nalytical Space Charge **S**olver (SAS) is implemented into DYNAMION code

- Good coincidence with other codes for static and dynamics

- Up to 10^6 particles can be simulated

- 2-step scheme:

    1) fast investigation with SAS

    2) final proof with high precision by more time consuming space charge solvers

Outlook

The first results are promising !

- Optimization of the parameters in SAS
- Implementation for bunches of different shape
- Benchmarking of DYNAMION (new space charge solver) with other beam dynamics codes