

# **Transverse Schottky spectra and beam transfer functions of coasting ion beams with space charge**

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# Outline

- FAIR and SIS-18
- Schottky diagnostics and beam transfer functions
  - Effect of linear space charge
- Measurement of space-charge effects
- Simulation of space-charge effects
- Summary

# FAIR at GSI

FAIR: experiments with high quality and high intensity beams

**SIS-18** becomes booster

Increase of

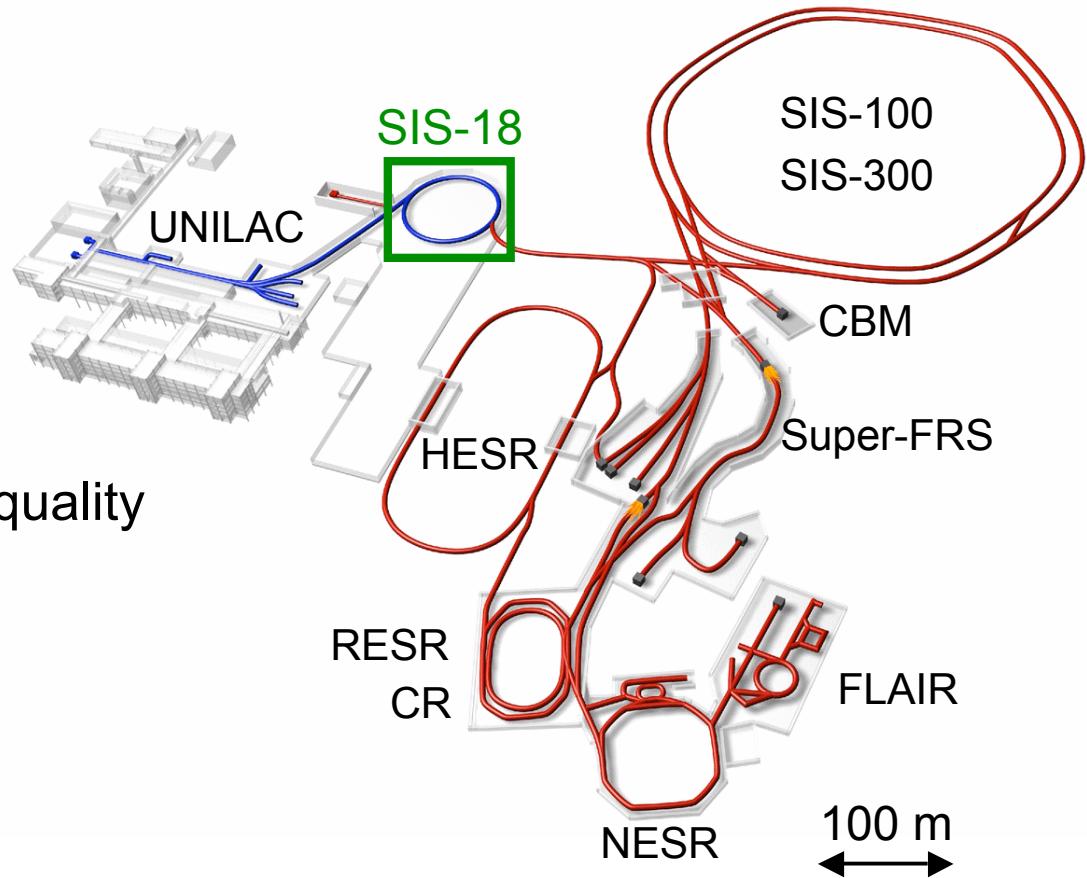
beam intensity

Arise of collective effects

→ Degradation of beam quality  
and particle losses

Low energy

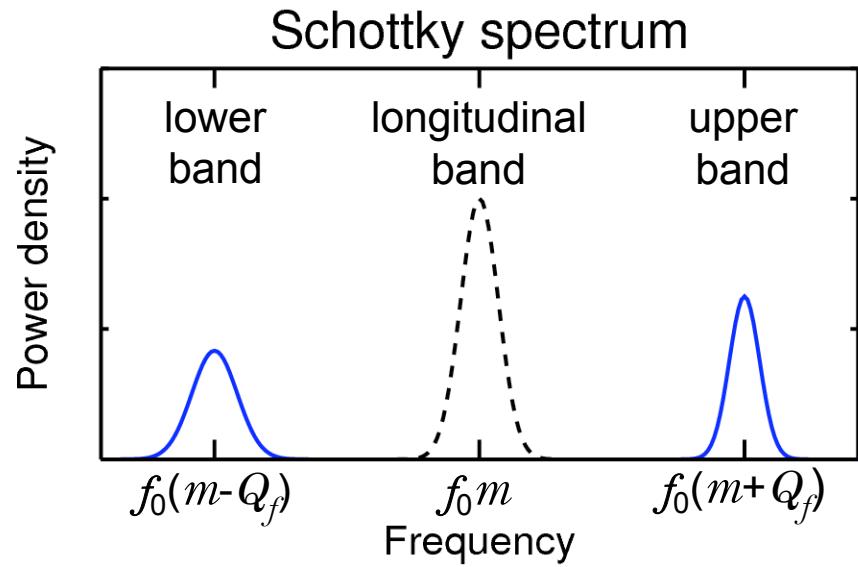
→ strong space charge



P. Spiller, MOIC01

# Low intensity Schottky spectrum

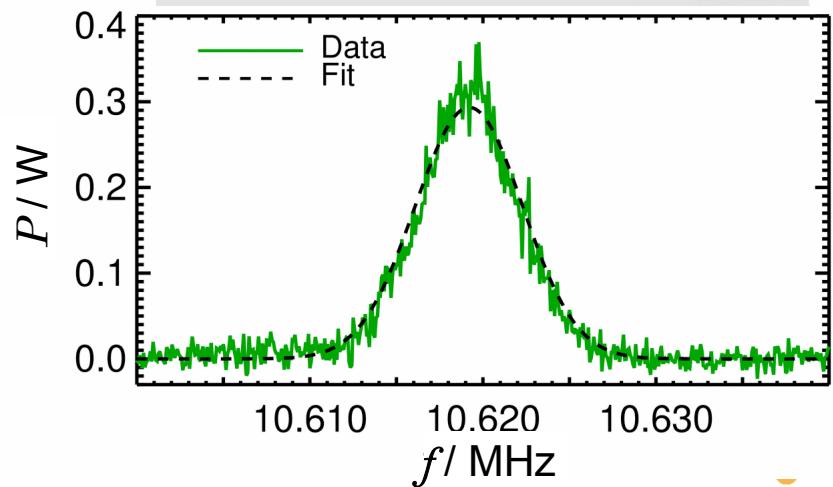
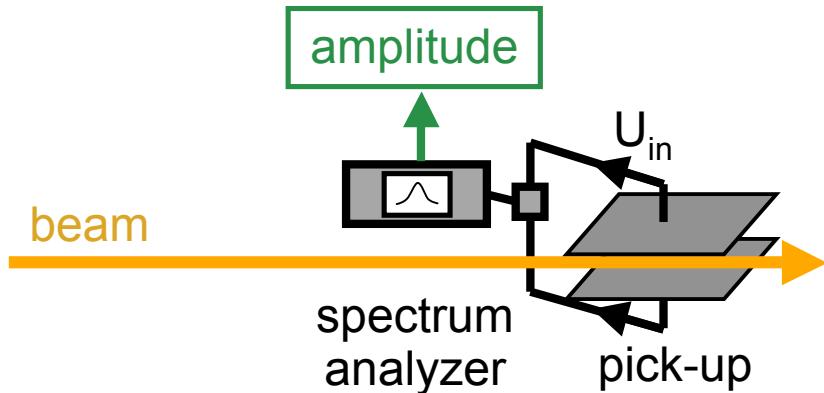
- Based on statistical fluctuations of local beam current and current dipole moment
- Non-destructive measurement of
  - Revolution frequency  $f_0$
  - Fractional tune  $Q_f$
  - Momentum spread
- Features
  - Longitudinal bands peaking at  $f_0 m$
  - Side bands  $P_0(f)$  centered around  $f_0(m \pm Q_f)$
  - Width of sidebands  $\sigma_m^\pm$



# Schottky detection

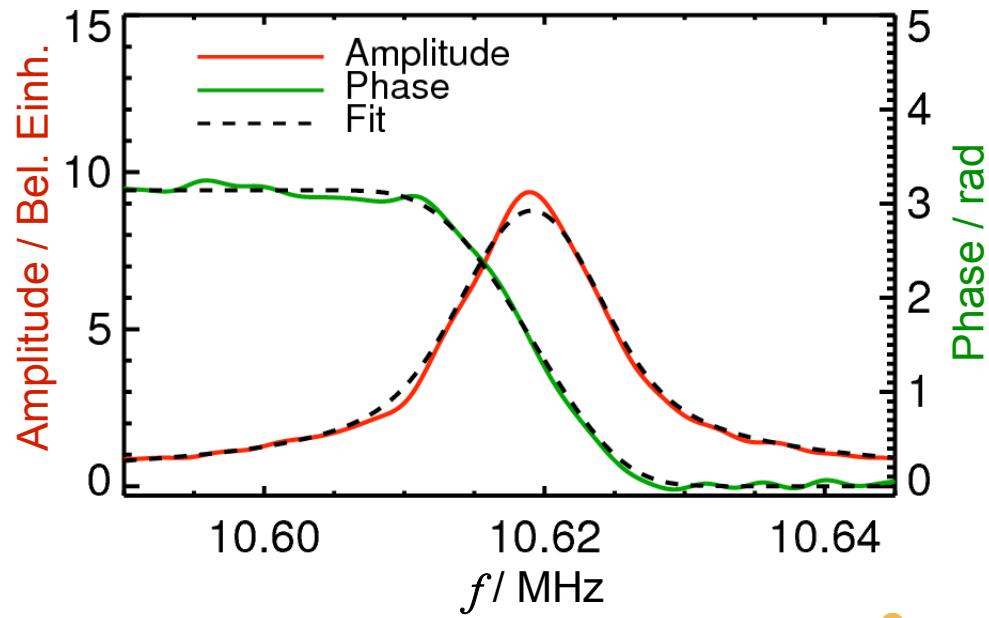
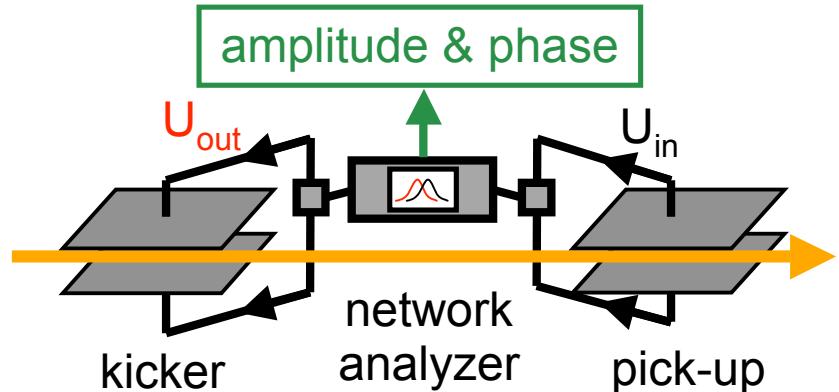
## Requires

- Pick-up
- Sum amplifier for longitudinal spectrum
- Difference amplifier for transverse spectrum
- Spectrum analyzer



# Transverse beam transfer functions (BTFs)

- BTF  $r_0(f)$  defined as ratio of beam response to excitation
- Requires
  - Network analyzer
  - Exciter (kicker)
  - Pick-up
  - Difference amplifier
- Alternative to Schottky diagnosis
- Stability analysis



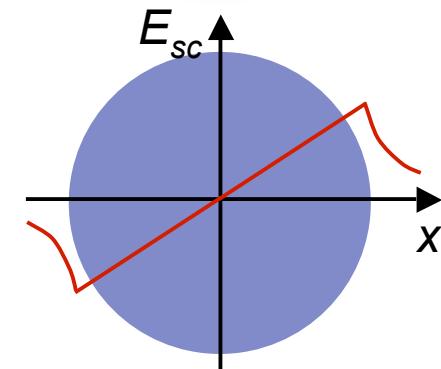
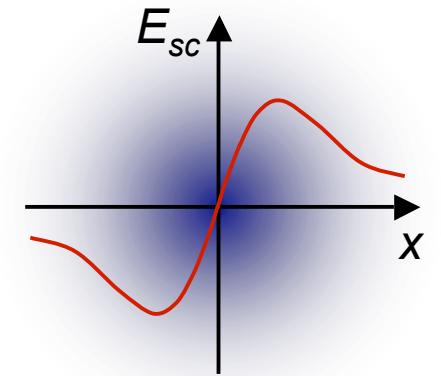
# Impedance and space charge

- Impact of transverse dipolar impedances
  - Coherent tune shift  $\Delta Q_{coh}$
  - Coherent dipolar instability with growth rate  $\tau$ —if not Landau damped
  - Impedance parameters

$$\Delta U_{coh} = \frac{\Delta Q_{coh} f_0}{\sigma_m^\pm} \text{ and } \Delta V = \frac{1}{\tau \sigma_m^\pm}$$

- (Direct) space charge
  - Non-linear self-field, very difficult to model  
→ tune spread
  - Linearized self-field (of K-V beam)  
→ incoherent tune shift

$$\Delta Q_{sc} \propto \frac{N}{\epsilon}$$



# Diagnostics with collective effects

High intensity BTF [1] and Schottky band [2]

$$r(f) = \frac{r_0(f_{sc})}{1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})}$$

$$P(f) = \frac{P_0(f_{sc})}{|1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})|^2}$$

with  $\Delta U_{sc} = \frac{\Delta Q_{sc} f_0}{\sigma_m^\pm}$  and  $f_{sc} = f \mp \Delta U_{sc} \sigma_m^\pm$

[1] D. V. Pestrikov, NIM A, **578**, 1, 2007; S. Paret et al., PRST-AB, **13**, 2, 2010

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deformation  
impedance and space charge

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deformation  
impedance and space charge

shift  
space charge only

The diagram illustrates the decomposition of the Schottky term  $r_0(f_{sc})$  into two components: 'deformation' and 'shift'. A red circle highlights  $r_0(f_{sc})$  in the first equation. A blue box encloses the term  $(\Delta U_{coh} + i\Delta V - \Delta U_{sc})$ . Red arrows point from this blue box to both ends of the fraction line. Another red circle highlights  $r_0(f_{sc})$  in the second equation. A blue box encloses the term  $(\Delta U_{coh} + i\Delta V - \Delta U_{sc})$ . Red arrows point from this blue box to both ends of the fraction line. To the left of the first equation, a blue box contains the text 'deformation' above 'impedance and space charge'. To the right of the second equation, a red box contains the text 'shift' above 'space charge only'.

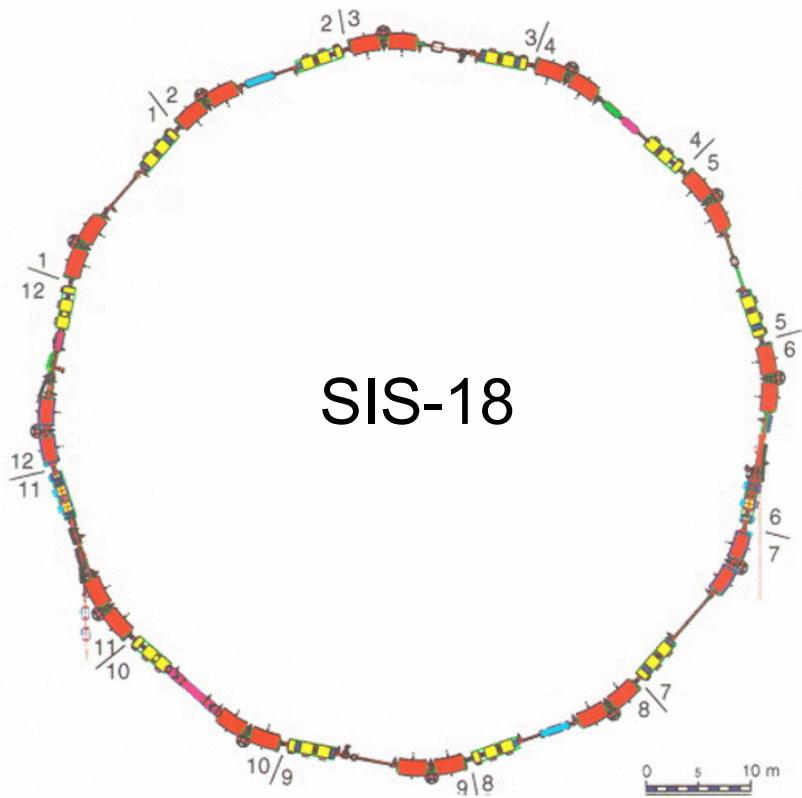
$$\text{with } \Delta U_{sc} = \frac{\Delta Q_{sc} f_0}{\sigma_m^\pm} \text{ and } f_{sc} = f \mp \Delta U_{sc} \sigma_m^\pm$$

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# Experimental setup

- Energy 11.4 MeV/nucleon
- Detection of
  - Ion number  $N$   
varied from  $2.5 \times 10^8$  to  $1.1 \times 10^{10}$  Ar<sup>18+</sup> ions
  - Longitudinal Schottky Spectra  
→ Gaussian **momentum distribution**
  - Beam profiles  
with ionization profile monitor → **emittance**
- $\Delta U_{coh}, \Delta V \ll \Delta U_{sc} \rightarrow$  only  $\Delta U_{sc}$  taken into account



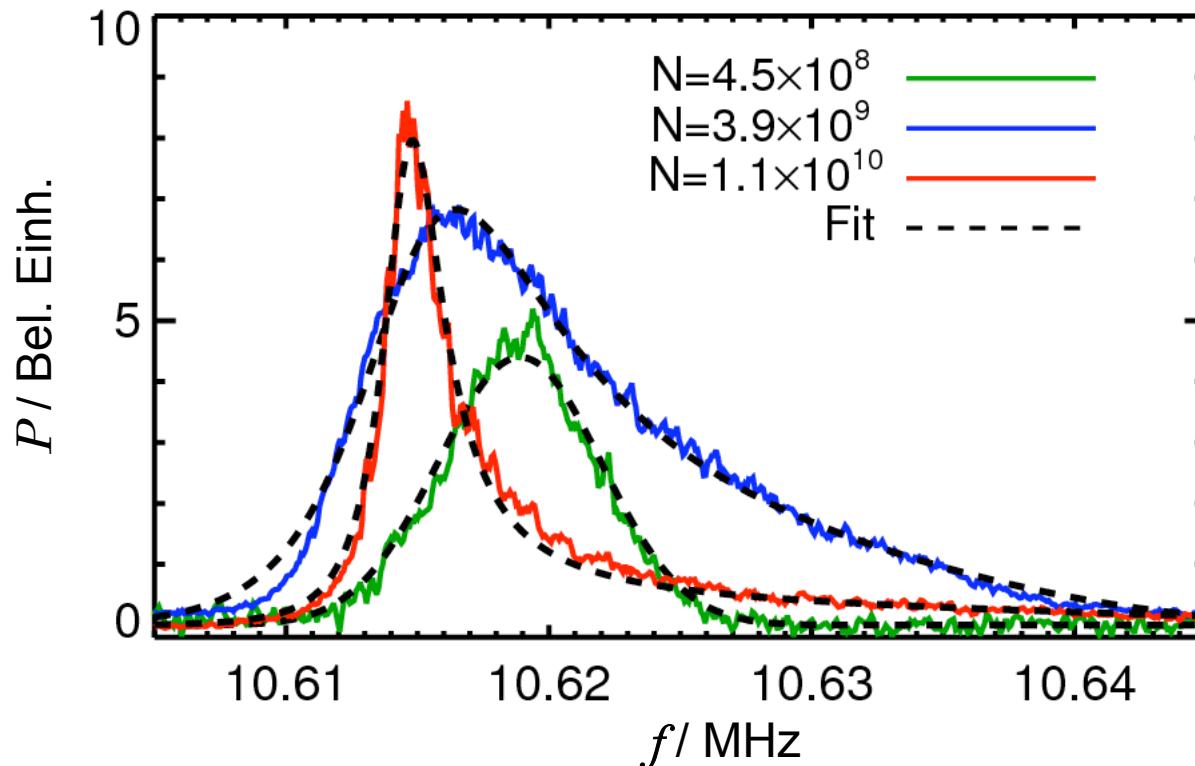
SIS-18

# Measured Schottky bands

- Fit of

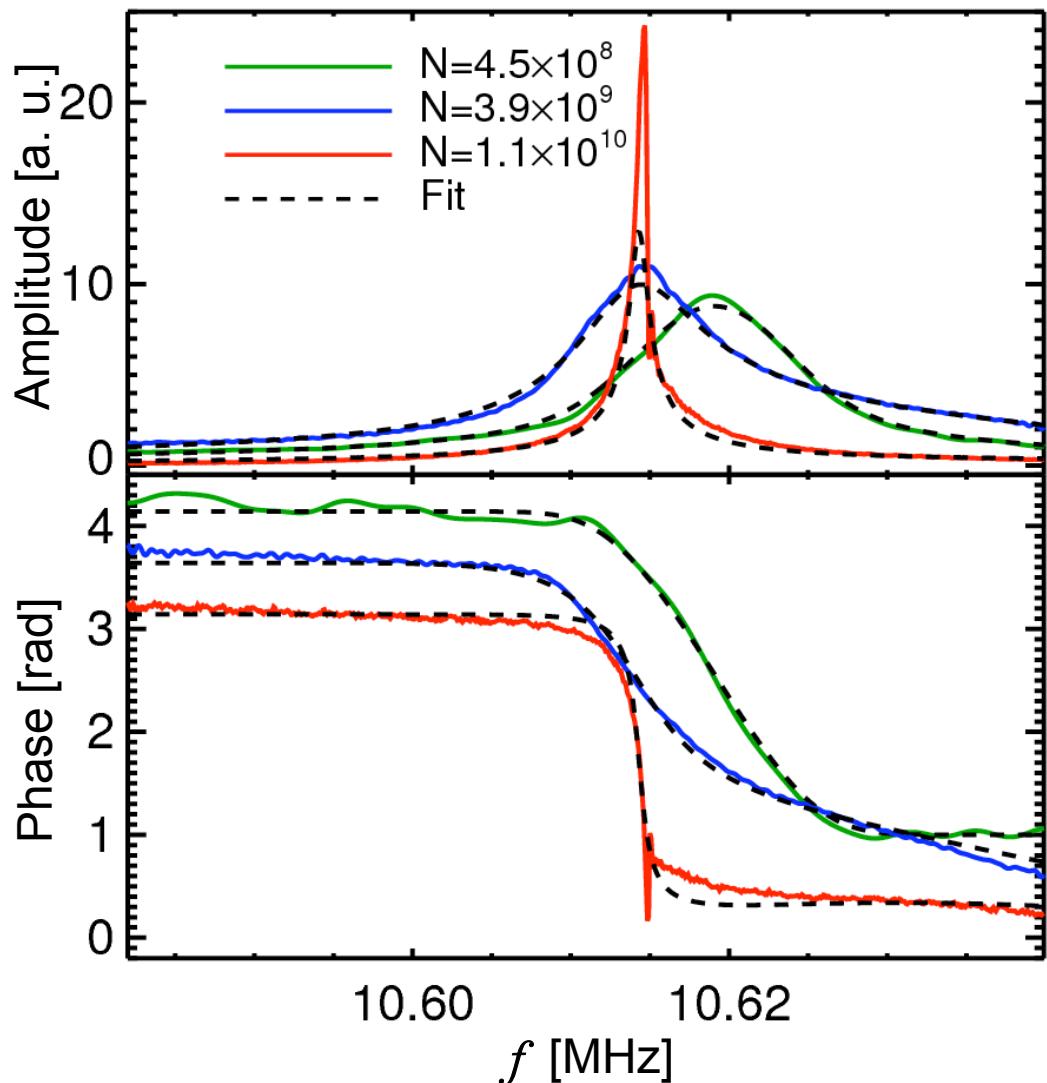
$$P(f) = \frac{P_0(f_{sc})}{|1 + \Delta U_{sc} r_0(f_{sc})|^2}$$

- Good agreement at low, medium and maximal intensity



# Measured BTFs

- Noise suppression via time gating
- Fit of  $r(f)$ 
  - Good agreement at low intensity
  - Deviations at high intensity

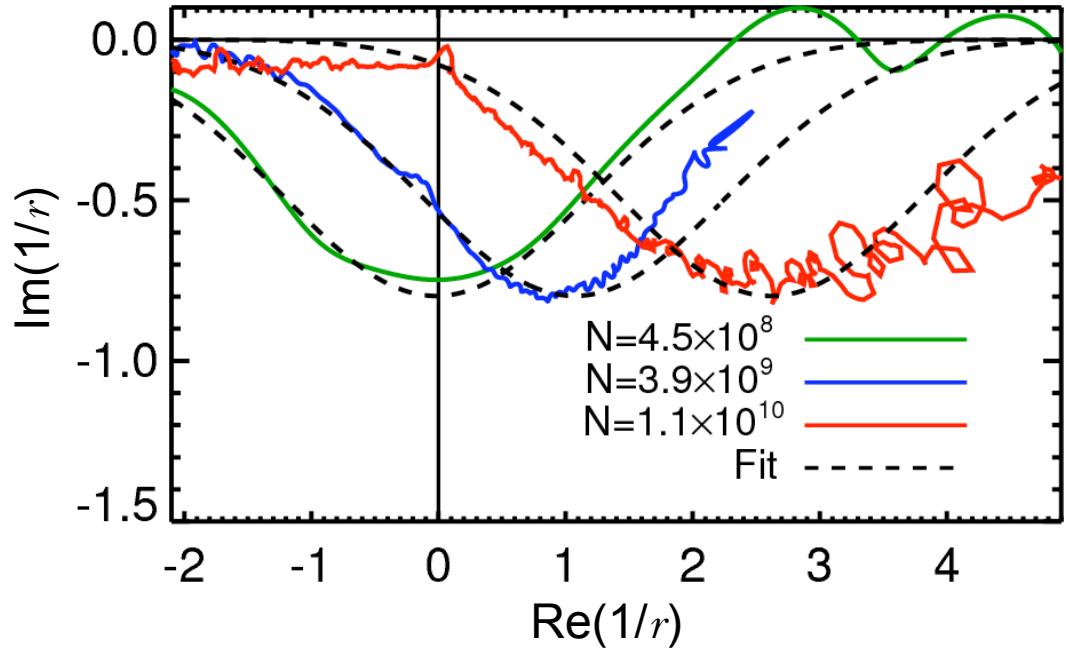


# Measured stability diagrams

- Stability diagram with space charge

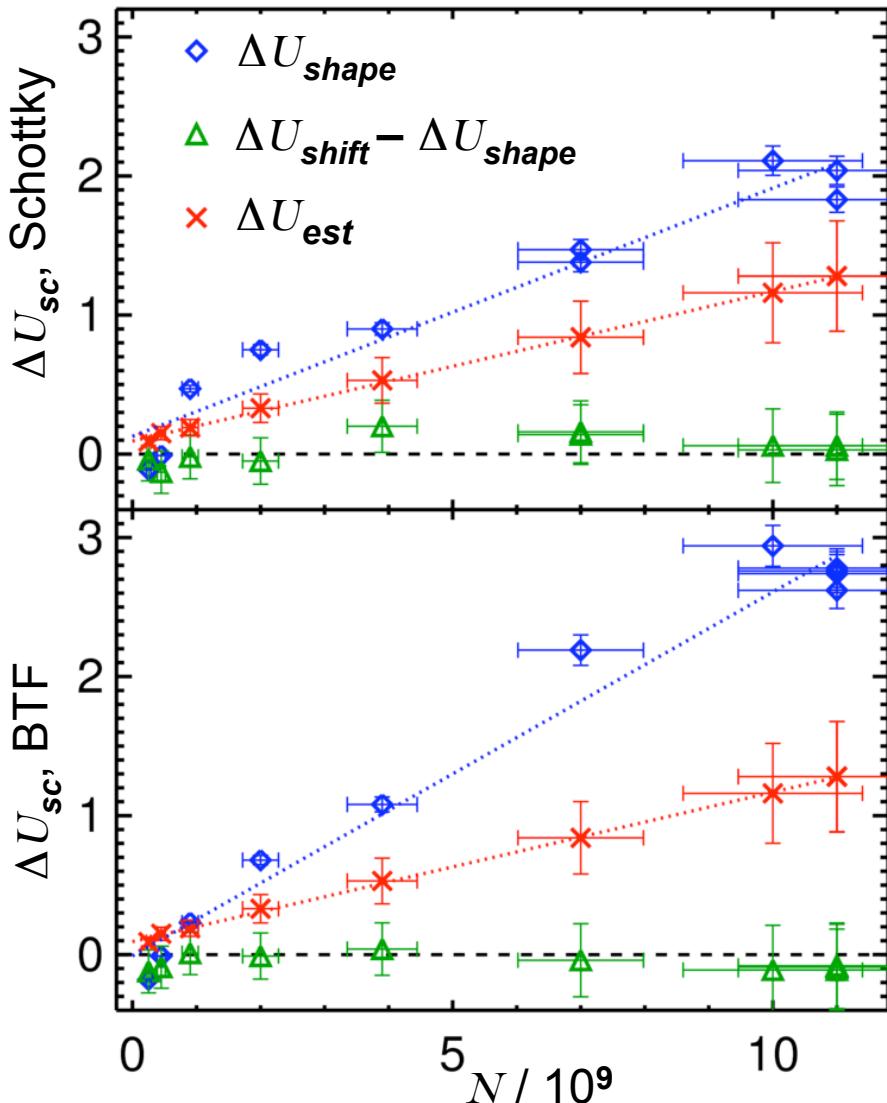
$$\frac{1}{r(f)} = \frac{1}{r_0(f_{sc})} + \Delta U_{sc}$$

- Shifted as expected
- Approximately shaped as expected
- Disturbed by noise at high intensity



# Measured space-charge parameter

- Estimation with beam parameters →  $\Delta U_{\text{est}}$
- Deformation of signal →  $\Delta U_{\text{shape}}$
- Position of signal ( $f_{sc}$ ) →  $\Delta U_{\text{shift}}$   
Consistency  
→  $\Delta U_{\text{shift}} - \Delta U_{\text{shape}} = 0$
- $\Delta U_{sc}$  grows linearly with  $N$
- Measured  $\Delta U_{sc}$  larger than estimation
- Larger  $\Delta U_{sc}$  for BTF



# Possible error sources

## Beam parameters

- Uncertainty of beta function at profile monitor
- Degradation of detector components

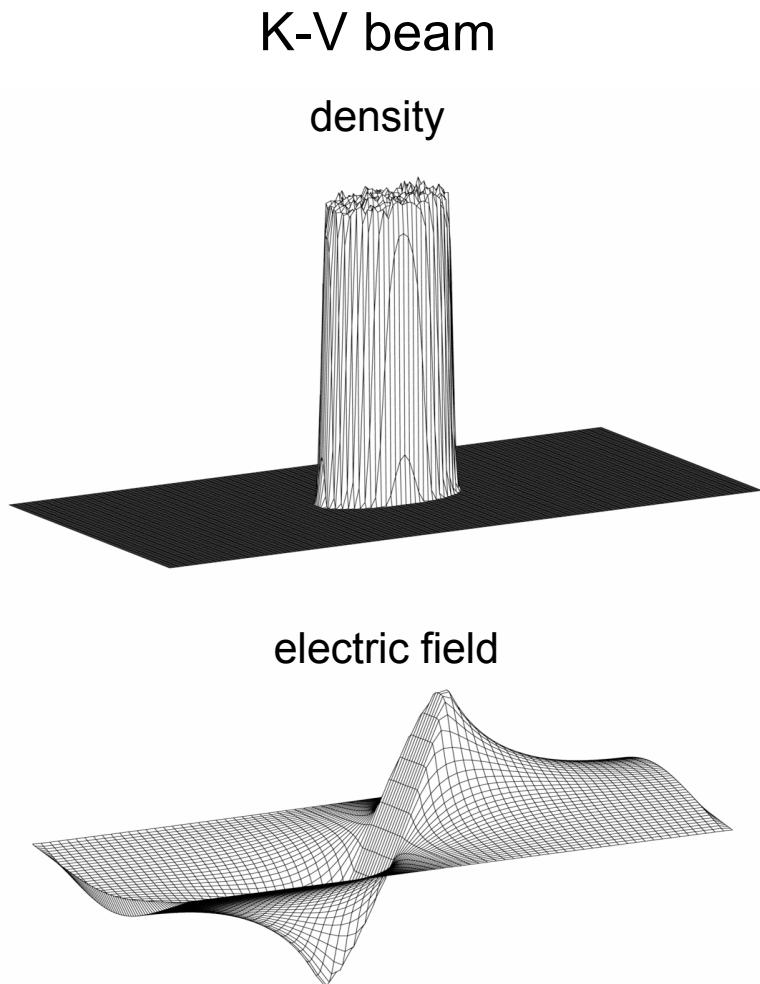
## BTFs

Beam of high intensity close to coherent instability

- Nonlinear response to excitation?
- Perturbation by resonance?

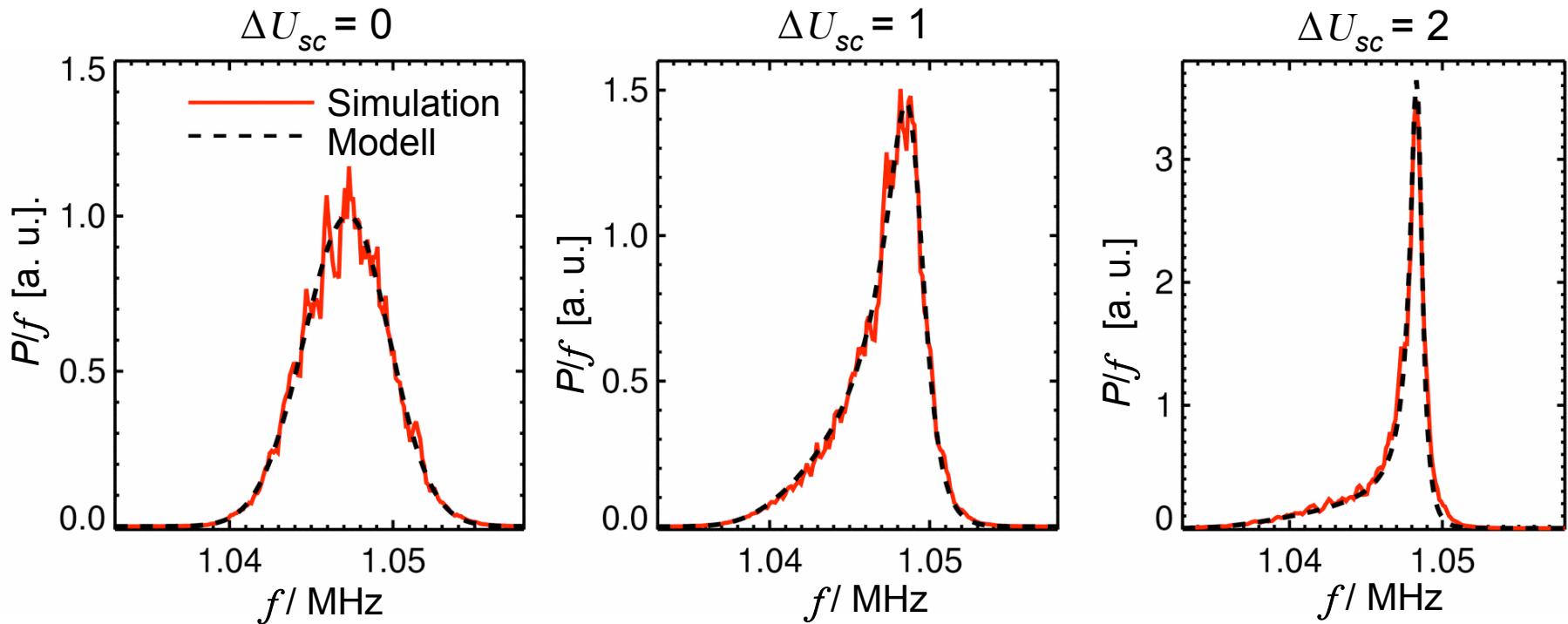
# PIC simulations

- Random macro particle distribution in phase space
  - Fluctuation of dipole moment → transverse Schottky spectrum
- Self-consistent field computation in 2D
- Options:
  - Excitation with noise for BTF
  - Impedance kicks
- Transverse profiles: K-V beam or Gaussian
- Maximal  $\Delta U_{sc} = 2$



# Schottky simulations

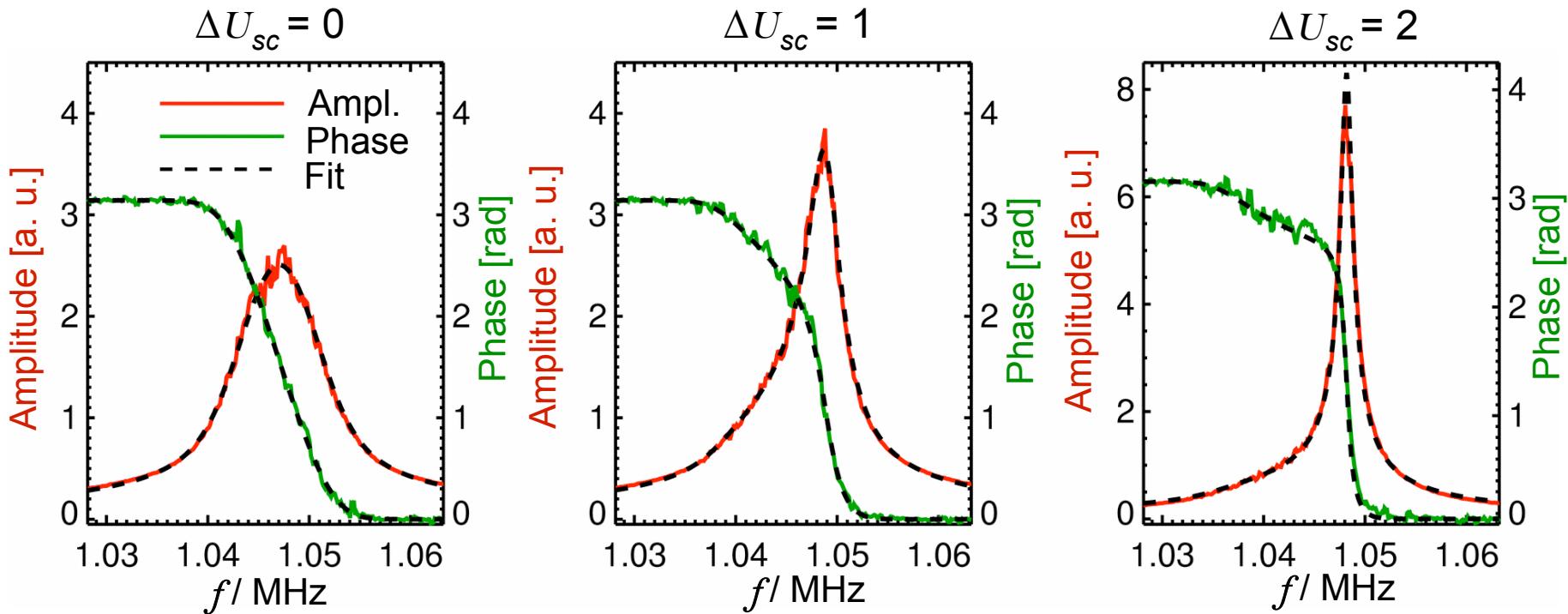
Results for beam with Gaussian transverse profile



- $\Delta U_{sc}$  fitted to data
- Excellent agreement with data and expected  $\Delta U_{sc}$
- Similar results for K-V und Gaussian profiles

# BTF simulations

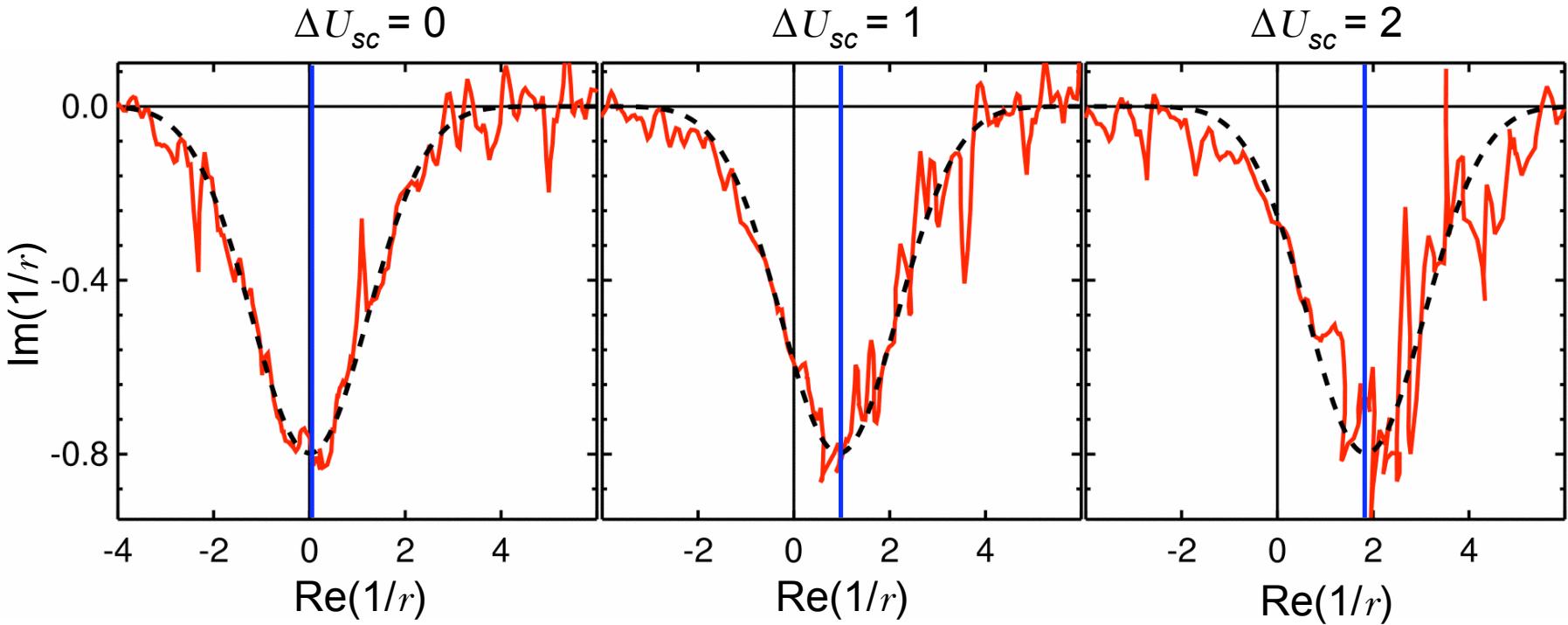
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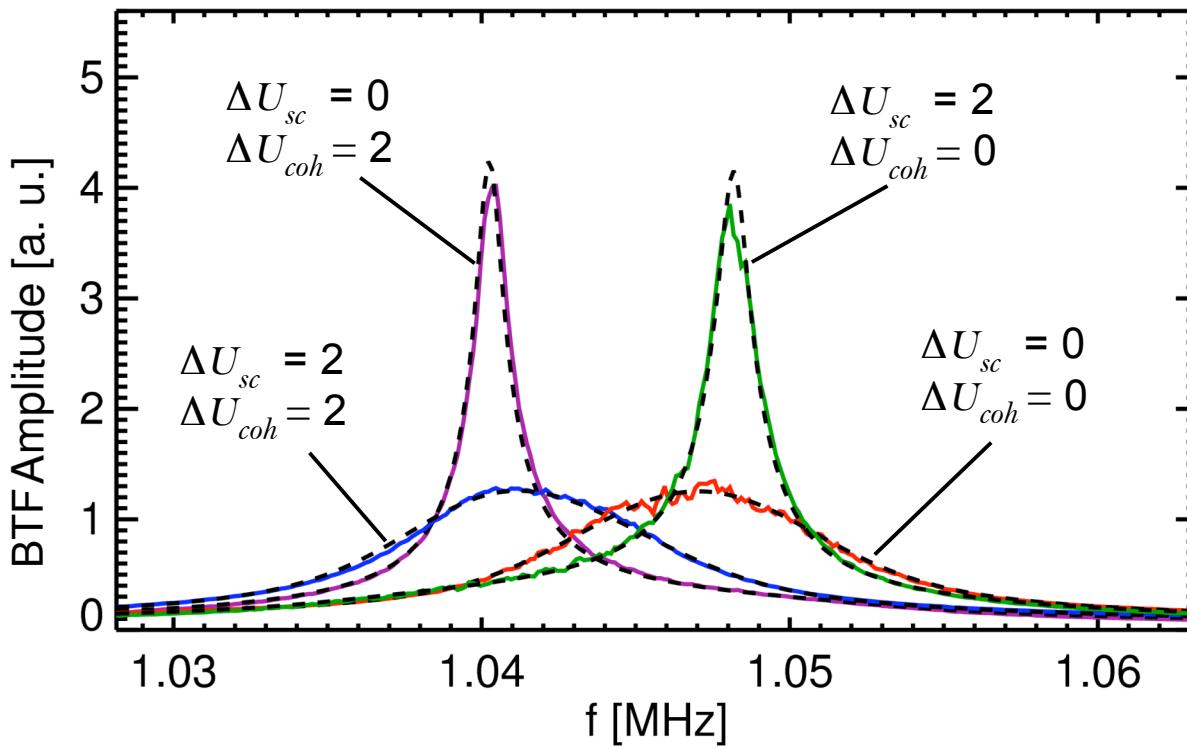
# Simulated stability diagrams

- Good agreement with model
- More noise at high intensity



# Simulation with impedance

Variation of  $\Delta U_{coh}$  and  $\Delta U_{sc}$  for direct comparison



Shift and deformation agree with model

# Summary

## Analytic linear space-charge model

- Different from dipolar impedance

## Experiment

- Measurement of transverse Schottky spectra and BTFs
- Verification of model despite deviations in some parts
- Direct measurement of  $Q$ ,  $\Delta Q_{sc}$  und  $\Delta U_{sc}$

## Simulation

- Transverse Schottky spectra and BTFs with space charge and imaginary impedances
- Excellent agreement with model



# **Thank you for your attention**

# Measured $\Delta Q$

