Longitudinal peak detected Schottky spectrum

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Acknowledgements: T. Bohl, T. Linnecar, U.Wehrle

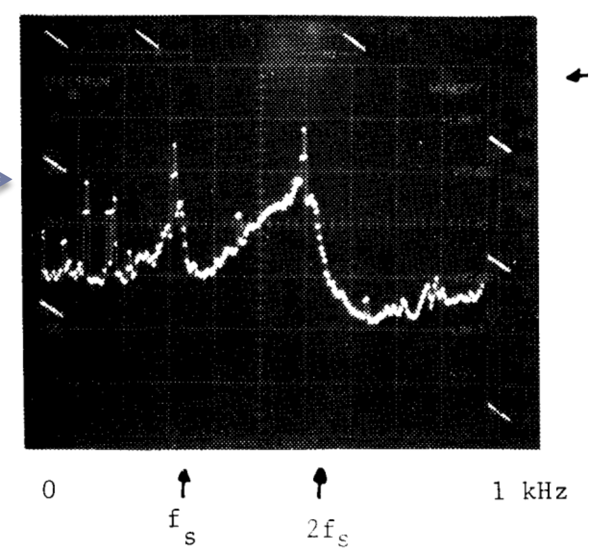
Outline

- History and motivation
- Peak detection
  - experimental set-up
  - peak detected signal
- Peak detected Schottky spectrum
  - theory
  - measurements
  - comparison with traditional bunched beam Schottky

History and motivation

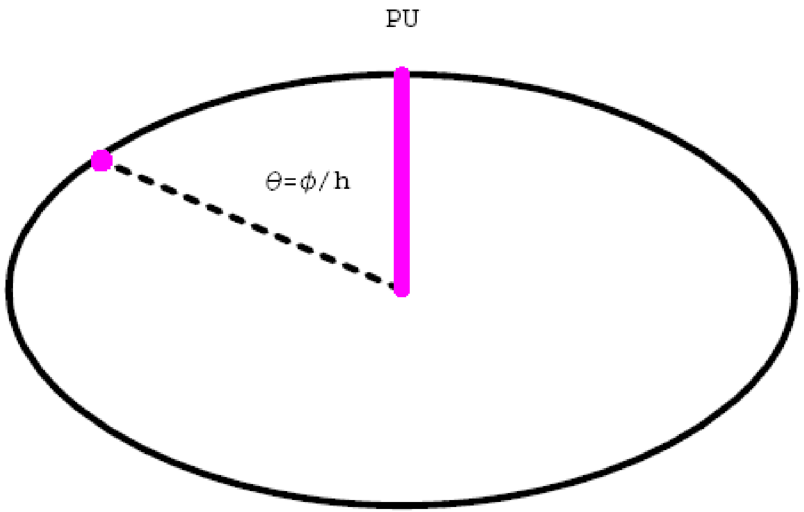
- The "peak detected (PD) Schottky" is a beam diagnostics tool developed by D. Boussard and T. Linnecar and extensively used in the SPS since the late 70s
- The quadrupole line was always believed to represent well the particle distribution in synchrotron frequencies, similar to longitudinal Schottky spectrum of unbunched beam for revolution frequencies
- "Traditional" Schottky system (high sensitivity PU at ~1 GHz) was not used anymore (at least in longitudinal plane)
- Installed for beam observation in the LHC

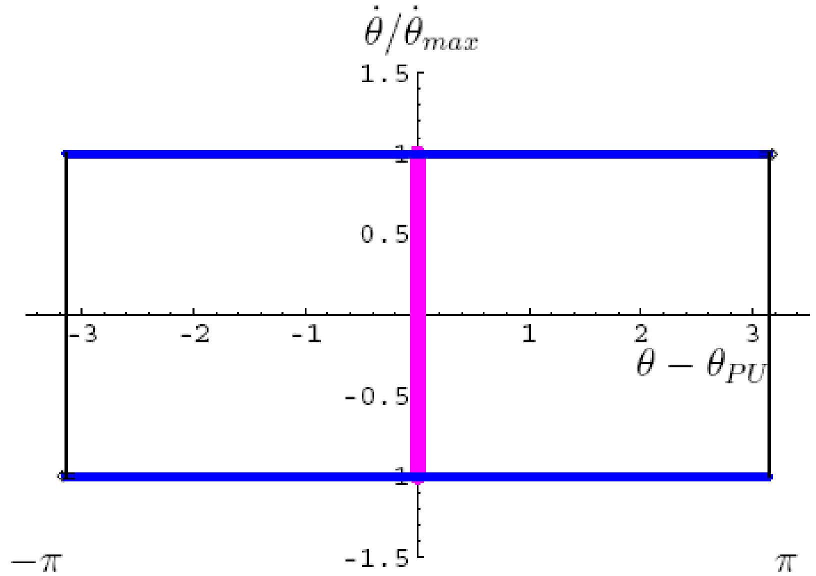
First measurements in the SPS,
coast at 120 GeV/c, 1978
SPS Improvement Reports 154, 162, 167



References for this work:
[1] E. S., CERN-BE-2009-010 RF, 2009
[2] E. S., T. Bohl, T. Linnecar, PAC'09

Longitudinal Schottky spectrum - unbunched beam (1/2)





Particles contributing to the Schottky signal at any given moment at the pick-up - once per revolution period.

Longitudinal phase space

Unbunched beam - theory (2/2)

For a single (n-th) particle circulating in the ring with a revolution frequency ω_n the time-domain signal at the pick-up

$$I_n(t) = e\omega_n \sum_{k=-\infty}^{\infty} \delta(\omega_n t + \theta_n - 2\pi k) = \frac{e\omega_n}{2\pi} \sum_{k=-\infty}^{\infty} \mathrm{e}^{-ik(\omega_n t + \theta_n)},$$

where θ_n is the particle azimuthal position at moment t=0 relative to the pick-up. The total beam current is a sum over all N particles

$$I(t) = \sum_{n=1}^{N} I_n(t) = \frac{e}{2\pi} \sum_{n=1}^{N} \sum_{k=-\infty}^{\infty} \omega_n \,\mathrm{e}^{-ik(\omega_n t + \theta_n)}.$$

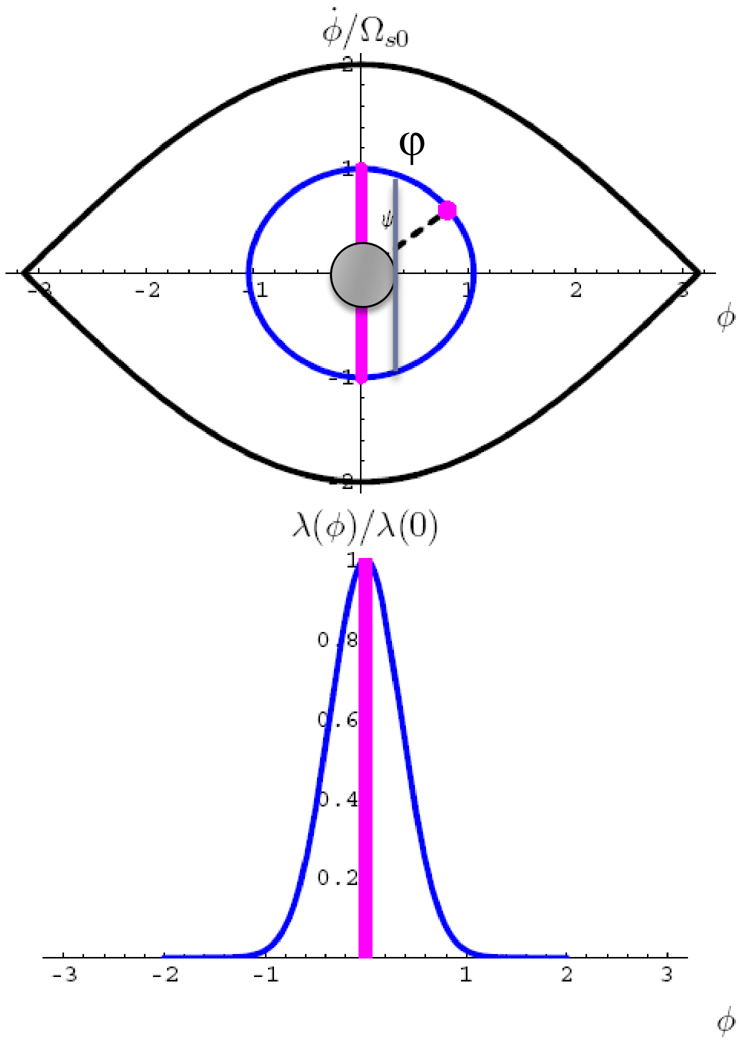
At positive frequencies

$$I(t) = \frac{e}{2\pi} \sum_{n=1}^{N} \omega_n + \frac{e}{\pi} \sum_{n=1}^{N} \sum_{k=1}^{\infty} \omega_n \cos k(\omega_n t + \theta_n).$$

The r.m.s. beam current per band (Schottky current) has lines at all harmonics k with increasing width $k\Delta\omega_0$. The power spectral density ~1/k.

References: J. Borer et al. (1974), D. Boussard (1987) …

Peak amplitude signal



- Longitudinal phase space: some particles contribute each revolution period to Peak Amplitude (PA) signal

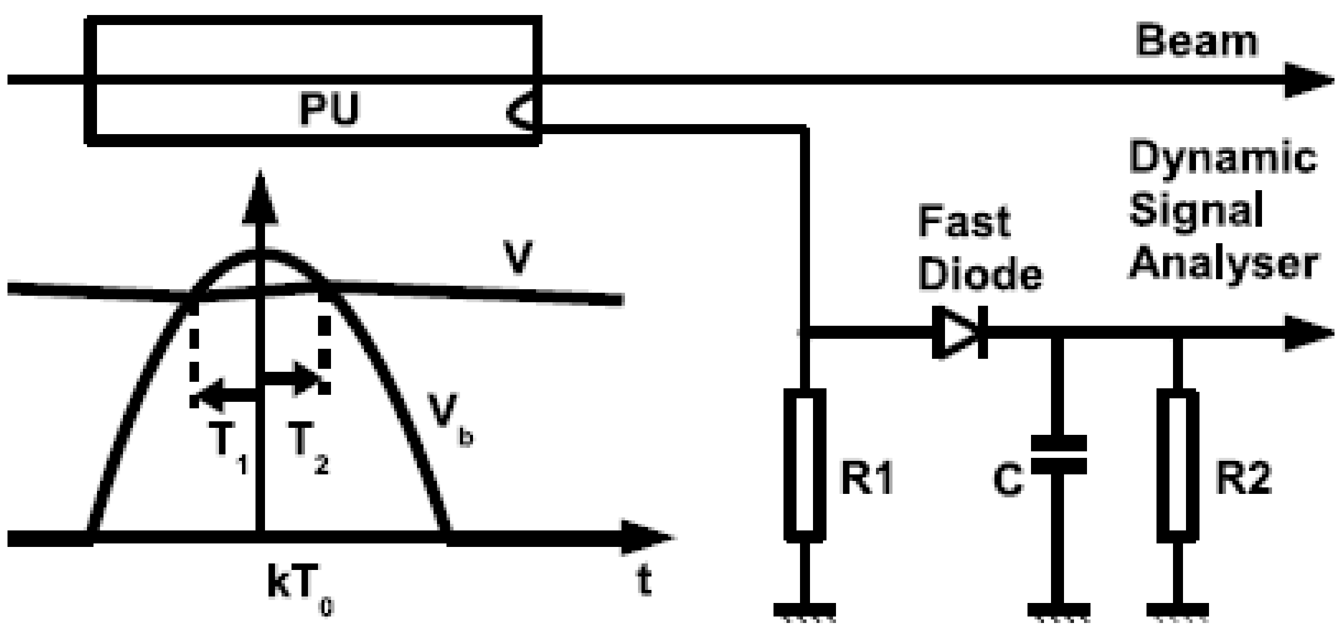
- Only particles with oscillation amplitude $\varphi_\alpha \geq \varphi$ contribute to the PA signal at position φ twice per synchrotron period – core is under-represented
  $\rightarrow$ how much?

- Measured peak amplitude signal:

$$I_{av}^{pd}(t_k) = \frac{1}{2\Phi}\int_{-\Phi}^{\Phi} I(t_k,\phi)d\phi$$

Peak Detected (PD) signal (1/5)

The simplified PD scheme
(used for longitudinal Schottky in the SPS and LHC)

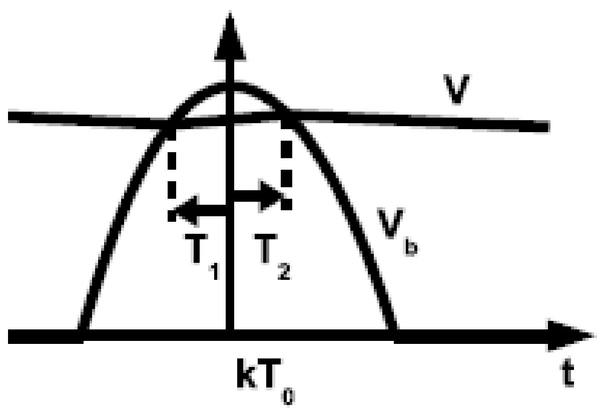


PD signal (2/5)

SPS and LHC parameters relevant to the PD Schottky measurements

Parameter			SPS	LHC
revol. period	T_0	μs	23.0	88.9
RF harmonic	h		4620	35640
resistance	R_1	Ω	50	50
resistance	R_2	MΩ	1.0	1.0
capacitance	C	pF	240	920
PD decay time	$1/\mu$	μs	240	920
PD growth time	$1/\alpha$	ns	12	12
acquisition time	T_a	s	1.6	3.2

PD signal (3/5)



The fast diode is open during the bunch passage (current I_b), when $V_b = I_b R > V$.

The voltage V measured at resistance R_2 during this time interval $(-T_1, T_2)$ can be found from the equation (valid for $R_2 >> R_1$)

$$\frac{dV}{dt} = \alpha(V_b - V);$$

where $\alpha = 1/(R_1C) + 1/(R_2C)$

The solution

$$V(t) = \alpha \int_{-T_1}^{t} V_b(t')\, e^{-\alpha(t-t')}\, dt' + V(-T_1) e^{-\alpha(t+T_1)}$$

The diode is off for the rest of the revolution period and we have

$$\frac{dV}{dt} = -\mu V, \qquad V(t) = V(T_2)\, e^{-\mu(t-T_2)}$$, where $\mu = 1/(R_2C)$.

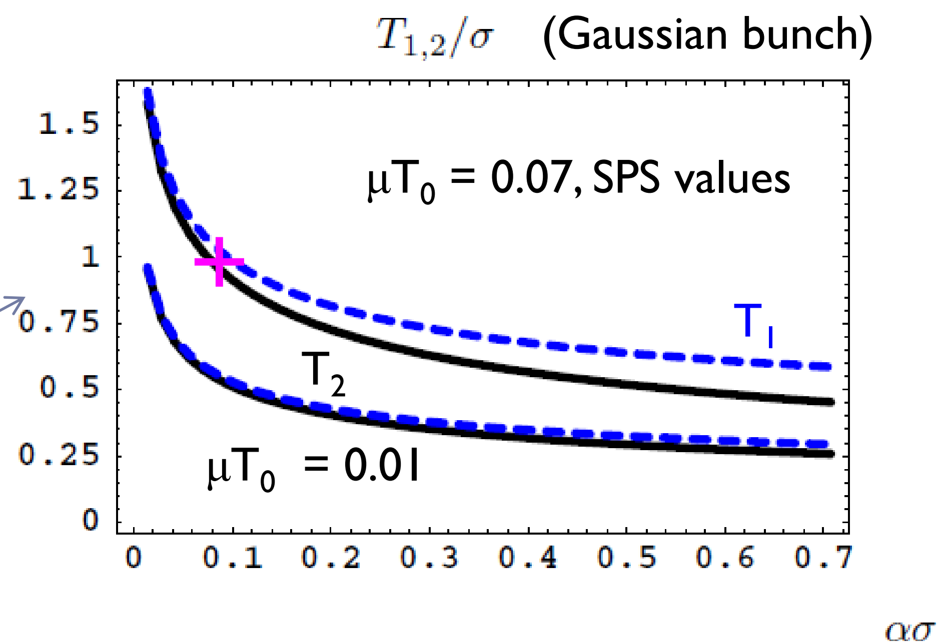
PD signal (4/5)

After a transient period variations of T_1 and T_2 from turn to turn are small and defined only by statistical fluctuations (Schottky noise). Then

$$V(-T_1) = V(T_2)\, e^{-\mu T0}.$$

The stationary values of $T_{1,2}$ can be found from conditions

$$V(-T_1) = V_b(-T_1),\ \ V(T_2) = V_b(T_2)$$



$\rightarrow$ For the SPS max $\alpha\sigma = 1/12$ and $T_1 \approx T_2 \approx \sigma$

PD signal (5/5)

The signal detected at the moment t, after the k-th bunch passage, is

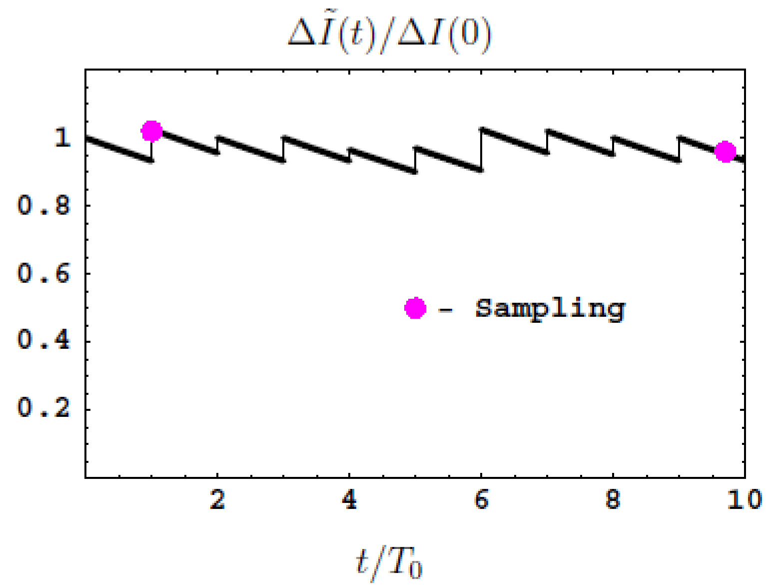
$V_k e^{-\mu(t-t_k)}$, where $t_k = kT_0$

with voltage

$$V_k = \Delta V_k + V_{k-1} e^{-\delta} = \sum_{q=0}^{k} \Delta V_{k-q}\ \mathrm{e}^{-q\delta}$$

Here $\delta = \alpha(T_2 + T_1) + \mu T_0.$

$\delta \simeq 0.25$ for $T_2 = T_1 = 1$ ns



The change in voltage at each revolution turn is proportional to the average bunch peak amplitude:

$$\Delta V_k = R_1 \alpha \int_{t_k - T_1}^{t_k + T_2} I_b(t_k - t')\ \mathrm{e}^{-\alpha(t_k + T_2 - t')} dt'.$$

PD Schottky spectrum (1/5)

A particle will be at position φ when its synchrotron angle ψ_n is equal to

$$\begin{aligned}\psi_n &= \Omega_n t_\phi + 2\pi m,\\ \psi_n &= \pi - \Omega_n t_\phi + 2\pi m,\end{aligned} \qquad t_\phi = t_\phi(\mathcal{E}_n, \phi) = \int_0^\phi \frac{d\phi'}{\sqrt{2[\mathcal{E}_n - W(\phi')]}}.$$

This will happen at times t_1 and t_2. Then similar to the "ideal" (PA) case a single particle contribution to a bunch current is

$$I_n(t,\phi) = \frac{e}{2}\sum_m [\delta(t-t_1) + \delta(t-t_2)] = \frac{e\Omega_n}{4\pi}\sum_m [\mathrm{e}^{im\Omega_n t_\phi} + \mathrm{e}^{im(\pi-\Omega_n t_\phi)}]\,\mathrm{e}^{-im(\Omega_n t + \psi_{n0})}.$$

Collecting possible contributions at φ from all particles

$$I(t,\phi) = \sum_n I_n(t,\phi).$$

PD Schottky spectrum (2/5)

The increase in voltage during bunch passage ~ average bunch peak amplitude

$$\boxed{\Delta V_k = R_1 \alpha_\phi \int_{-\Phi}^{\Phi} I(t_k, \phi) e^{-\alpha_\phi(\Phi - \phi)} d\phi.}$$

$$\alpha_\phi = \alpha/(h\omega_0)$$

or

$$\Delta V_k = \frac{e}{2\pi} B \sum_n \sum_m \Omega_n A_m(\mathcal{E}_n) e^{-im(\Omega_n t_k + \psi_{n0})},$$

$$B = 2R_1 \alpha_\phi \Phi e^{-\alpha_\phi \Phi}.$$

where

$$\boxed{A_m(\mathcal{E}_n) = \frac{1}{4\Phi} \int_{-\Phi_{max}}^{\Phi_{max}} e^{\alpha_\phi \phi} [e^{im\Omega_n t_\phi} + e^{im(\pi - \Omega_n t_\phi)}] d\phi.}$$

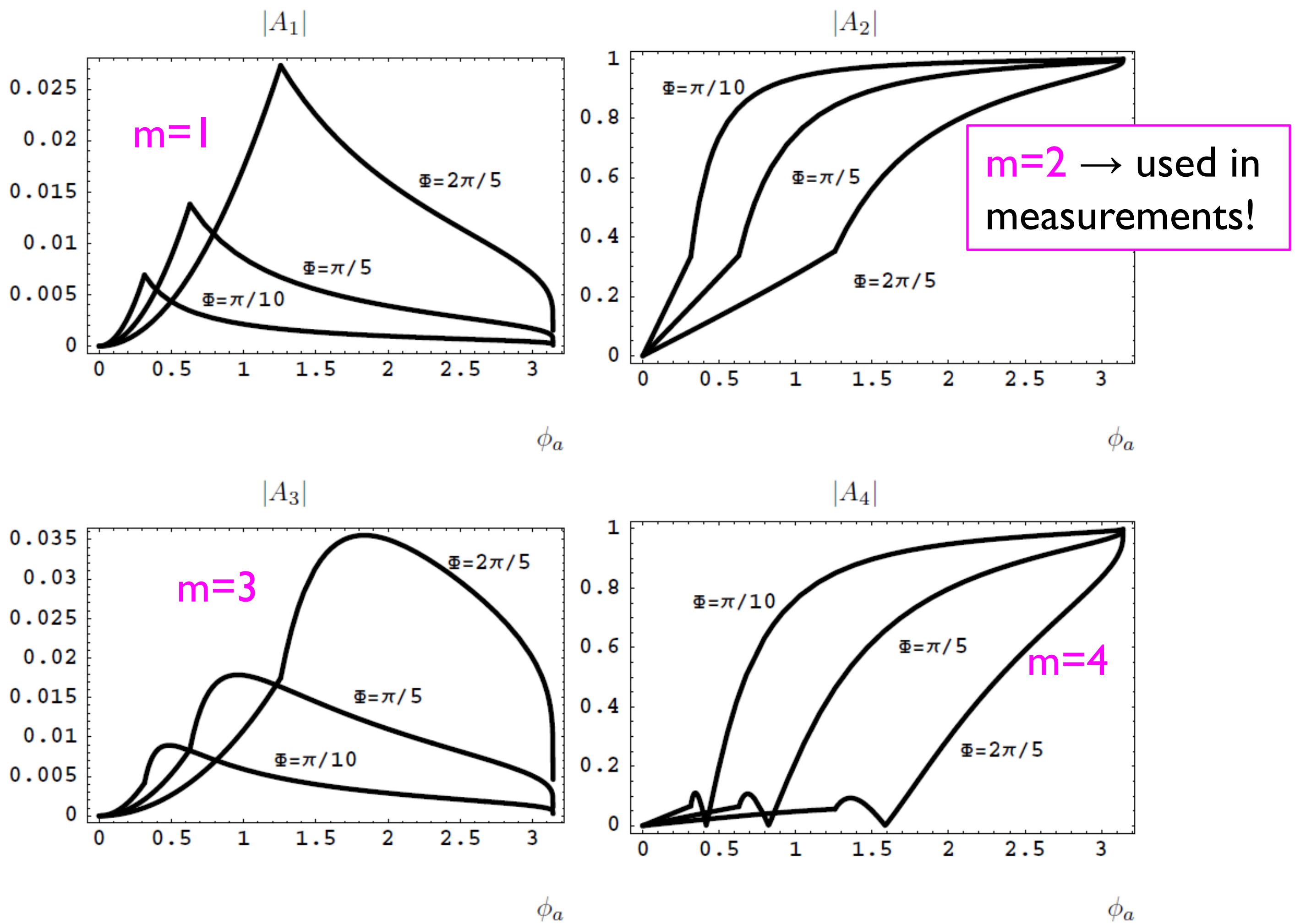
$$\begin{aligned} \Phi_{max} &= \Phi & \text{for } \mathcal{E}_n > W(\Phi), \\ \Phi_{max} &= \phi_a(\mathcal{E}_n) & \text{for } \mathcal{E}_n < W(\Phi), \end{aligned}$$

$$\mathcal{E}_n = W(\phi_a)$$

Terms with odd m are suppressed as ~ $\alpha_\varphi \Phi$

$$\begin{aligned} A_1 &= \frac{\alpha_\phi \phi_a}{3} \left(\frac{\phi_a}{\Phi}\right) & \text{for } \phi_a \leq \Phi \\ A_1 &= \frac{\alpha_\phi \Phi}{3} \left(\frac{\Phi}{\phi_a}\right) & \text{for } \phi_a \geq \Phi \\ A_2 &= \frac{1}{3} \left(\frac{\phi_a}{\phi}\right) & \text{for } \phi_a \leq \Phi \\ A_2 &= 1 - \frac{2}{3} \left(\frac{\Phi}{\phi_a}\right)^2 & \text{for } \phi_a \geq \Phi \end{aligned}$$

PD Schottky spectrum (3/5)



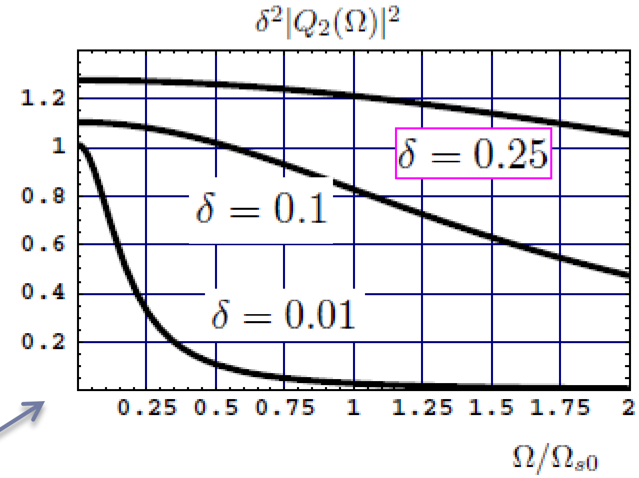
PD Schottky spectrum (4/5)

Finally the PD signal

$$V_k = \frac{eB}{2\pi} \sum_n \sum_m \Omega_n \, A_m(\mathcal{E}_n) \, Q_m(\Omega_n) \, e^{-im(\Omega_n t_k + \psi_{n0})}$$

The sum Q_m is due to the previous turns

$$Q_m(\Omega_n) = \sum_{q=0}^{k} e^{im\Omega_n T_0 q - q\delta} \simeq \frac{1}{1 - e^{im\Omega_n T_0 - \delta}}$$



- Q_m is a fairly flat function (for not too small δ, in SPS δ=0.25)

The power spectral density of the PD signal

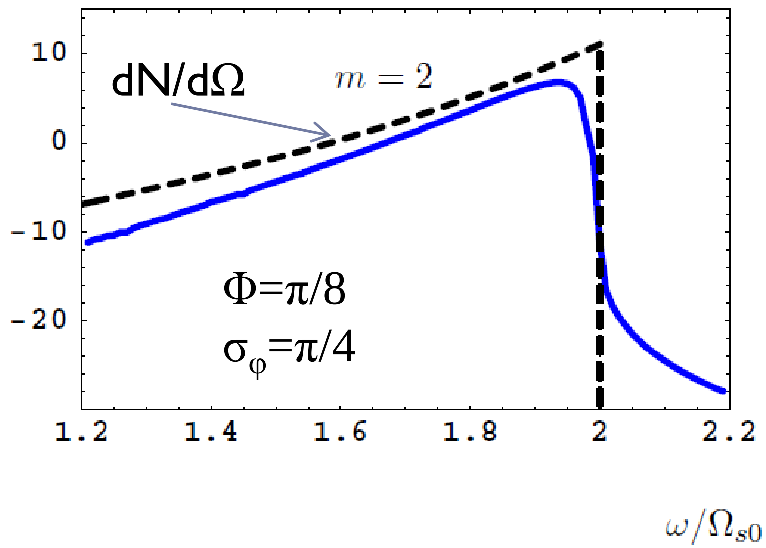
$$\boxed{P(\omega) = \frac{P_0}{\Omega_{s0}^2} \sum_{m=1}^{\infty} \int \Omega^2 F(\Omega) |A_m(\Omega)|^2 |Q_m(\Omega)|^2 \, S^2 d\Omega,}$$

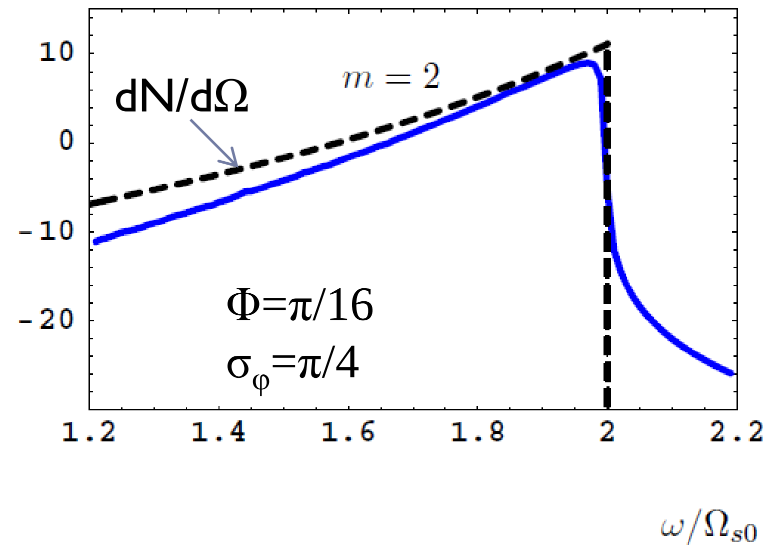
where $P_0=e^2Nf_{so}B^2$ and for acquisition time T_a

$$S^2 = |S(\omega - m\Omega)|^2 = \frac{2T_a}{T_{s0}} \frac{\sin^2[(\omega - m\Omega)T_a/2]}{[(\omega - m\Omega)T_a/2]^2}.$$

PD Schottky spectrum (5/5)

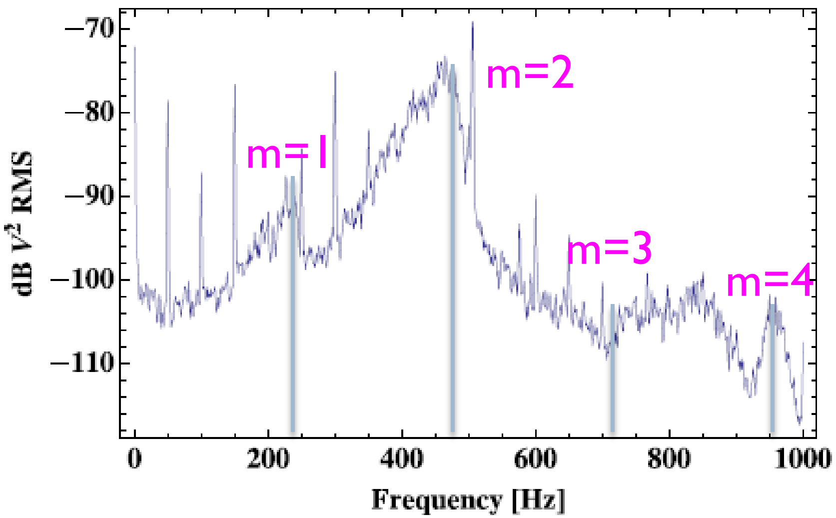
Peak Detected Schottky: quadrupole bands P_2/P_0

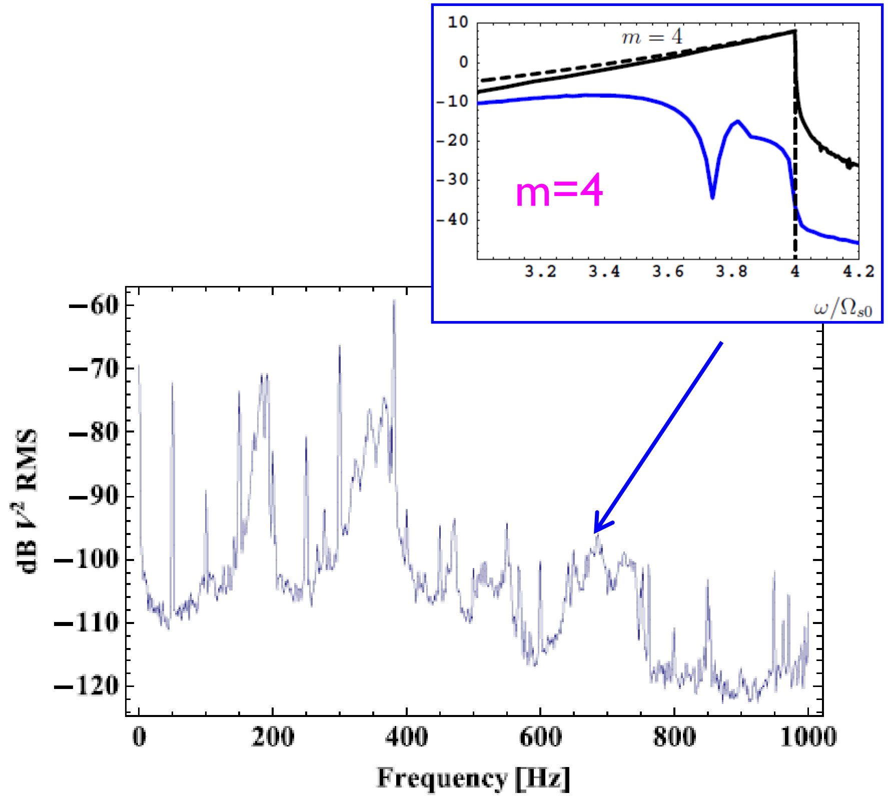




- The measured PD Schottky spectrum deviates from distribution function dN/dΩ mainly due to $|A(\Omega)|^2$
- The distortion is smaller for smaller averaging distance Φ

Measured PD Schottky spectrum in SPS





Low intensity single bunch at 26 GeV/c, f_{s0}=240 Hz, $\sigma_\varphi=\pi/4$

High intensity bunch in coast at 270 GeV/c, f_{s0}=192 Hz, $\sigma_\varphi=\pi/12$

Courtesy T. Bohl

Bunched-beam ("traditional") longitudinal Schottky spectrum

The spectral density of current fluctuations for bunched beam.
(e.g. S. Chattopadhyay, Some fundamental aspects of fluctuations and coherence in charged-particle beams in storage rings, CERN-84-11, 1984)

$$P_L(\omega) = \frac{e^2 N \omega_0^2}{2\pi} \sum_{p=-\infty}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{1}{m} F\left(\frac{\omega - p\omega_0}{m}\right) |I_{mp}(\mathcal{E})|^2,$$

where $\mathcal{E}$ corresponds to the synchrotron frequency $\Omega=\omega-p\omega_0$
and the "distortion" factor is

$$\boxed{I_{mp}(\mathcal{E}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{p\phi(\mathcal{E},\psi)/h - im\psi}\, d\psi.}$$

For short bunches in a single RF system

$$I_{mp}(\mathcal{E}) \simeq i^m J_m(p\phi_a/h_1)$$

Traditional Schottky spectrum: dipole sidebands

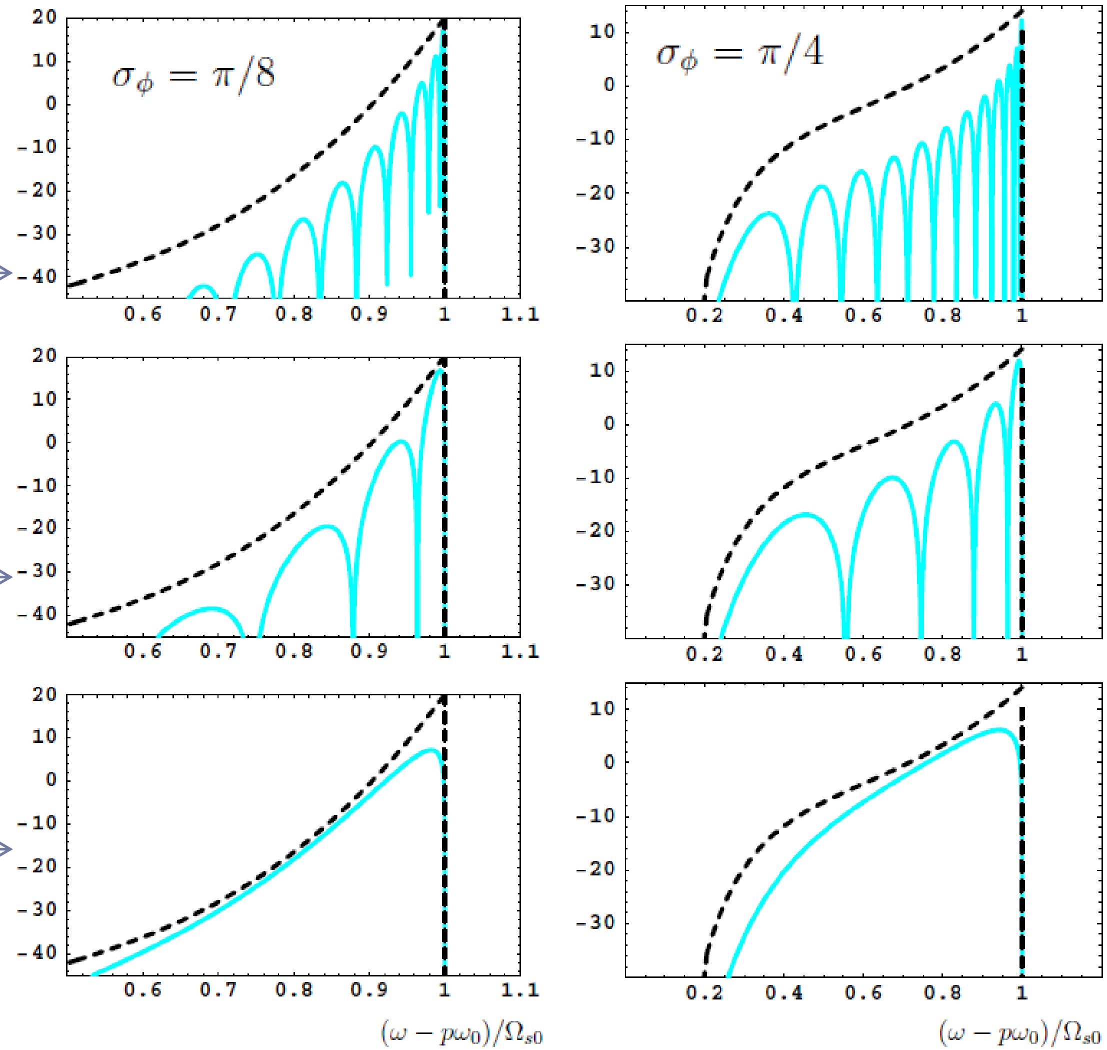
P_{pm}/P_0 [dB]

$\Omega_{s0}F_{dist}(\omega)$

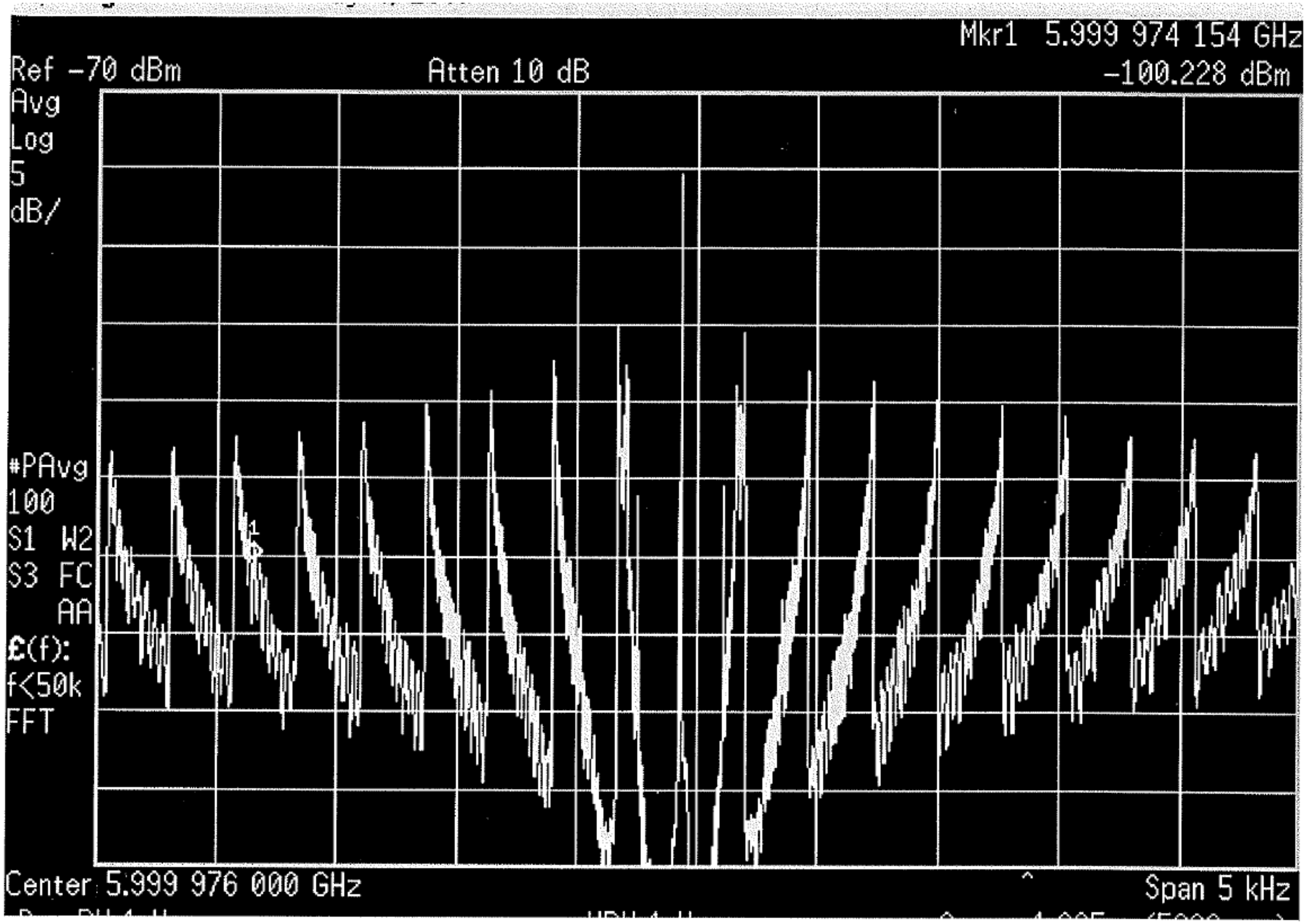
LHC: 4.8 GHz, p=12h

SPS: 1.0 GHz, p=5h

p=h

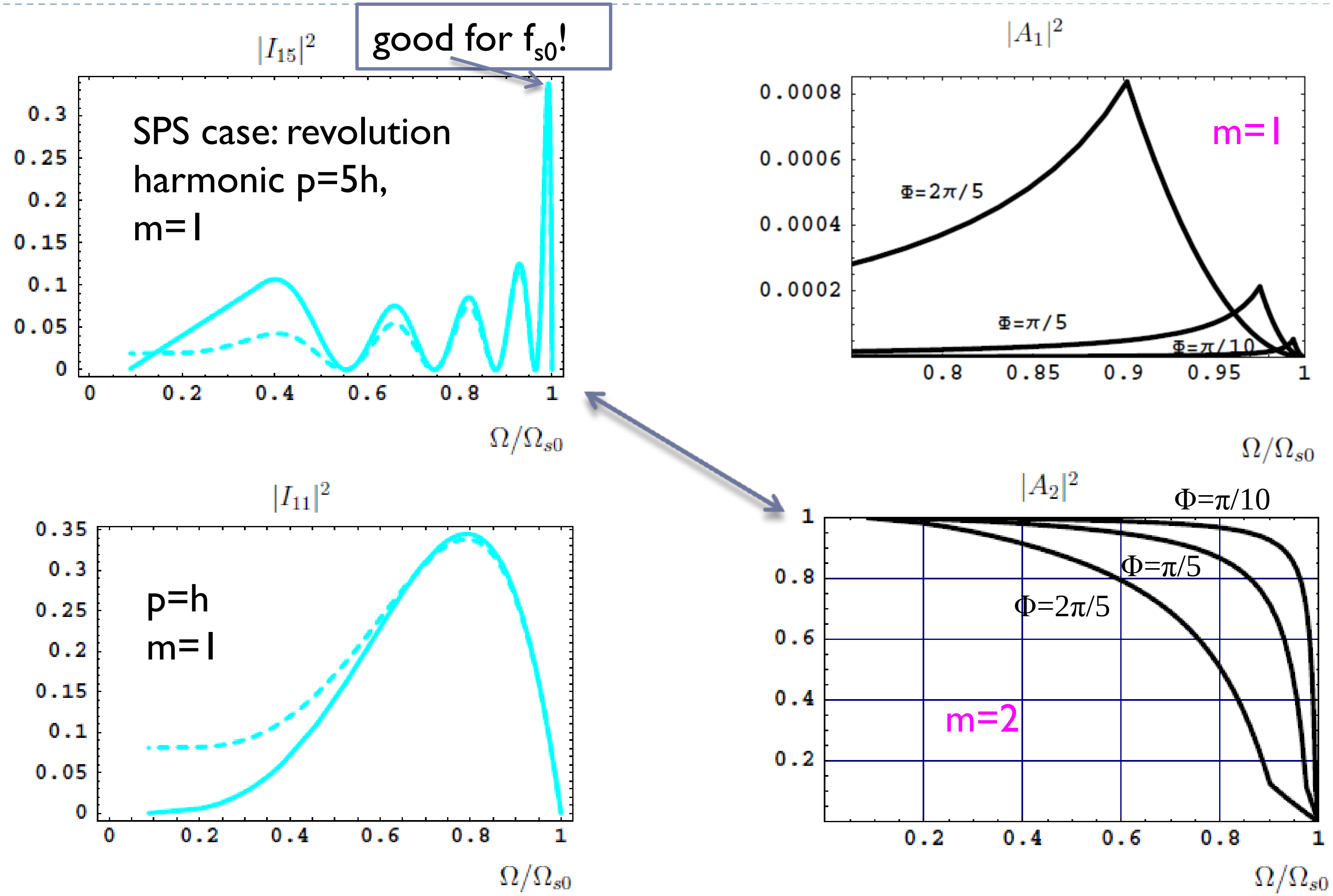


Traditional longitudinal Schottky spectrum



measurements in RHIC, M. Brennan

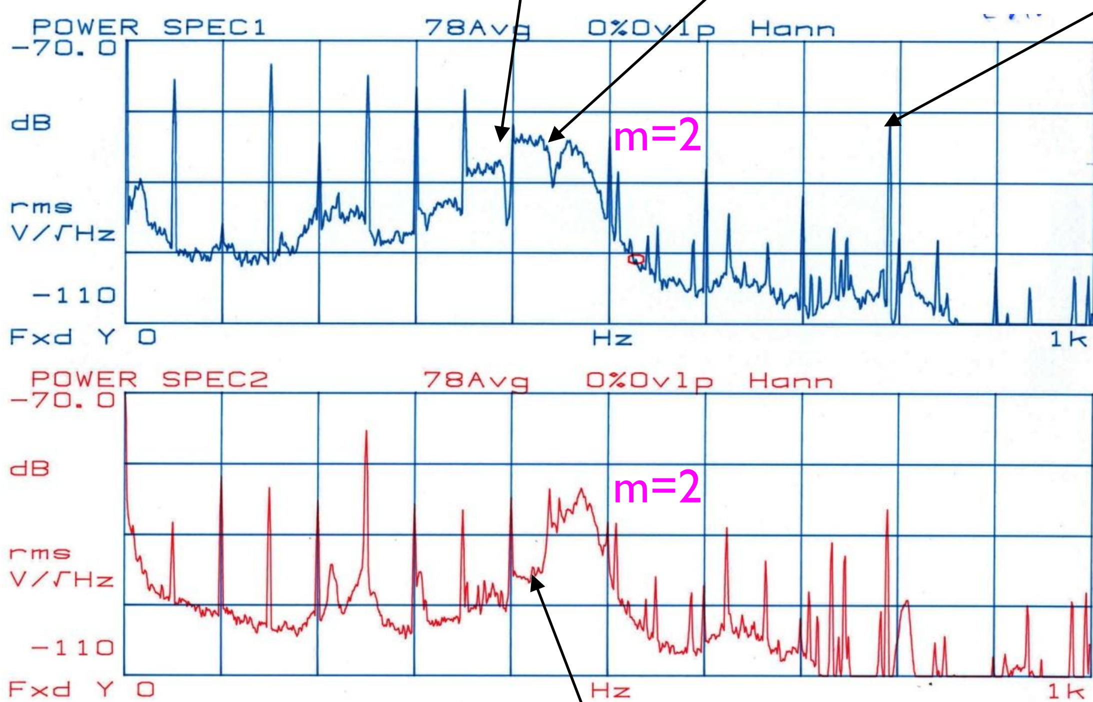
Comparison: traditional and PD Schottky



Examples
Losses along the injection plateau – noise

Peak detected Schottky
after 17 min store

395 Hz hole caused by coherent line at 790 Hz.
The other?



Head of batch

Tail of batch

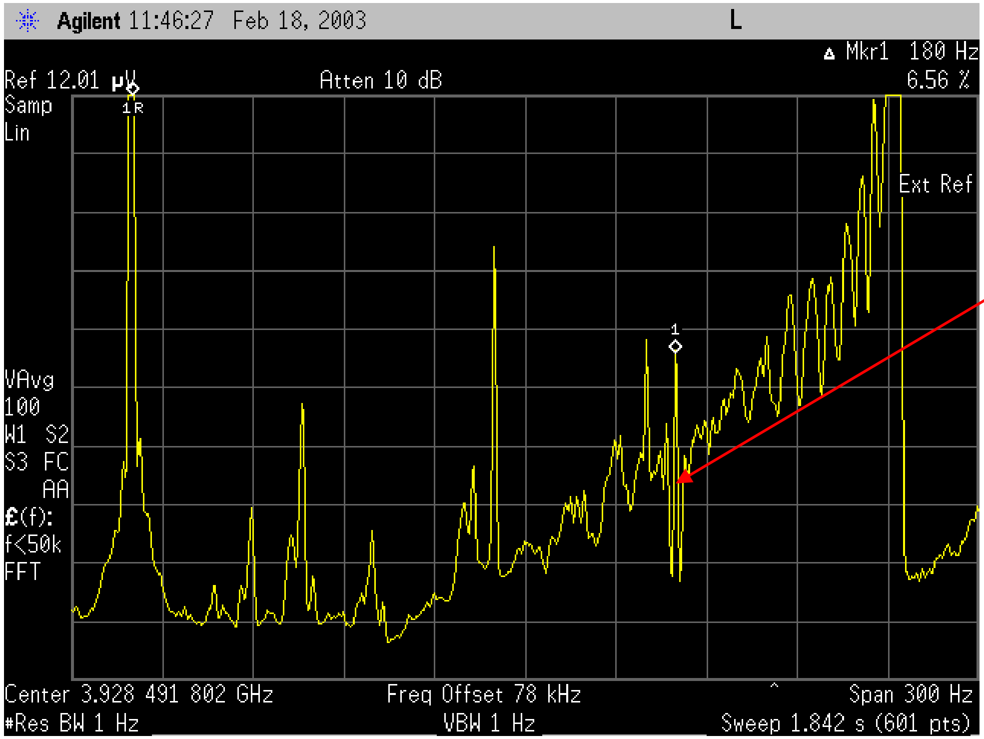
V_{RF} = 2MV, f_s = 257 Hz

Depopulated area – impedance, RF,
electron cloud …?

talk of T. Linnecar, LTC, 2005

Examples
Losses along the injection plateau - noise

- Sometimes particles are redistributed within the bunch. Often however they are lost completely. (Resonant islands, bridges due to white noise?)
- These effects also seen in ppbar and recently in RHIC



RHIC

Traditional Schottky at 4 GHz.

Hole appears during coast.

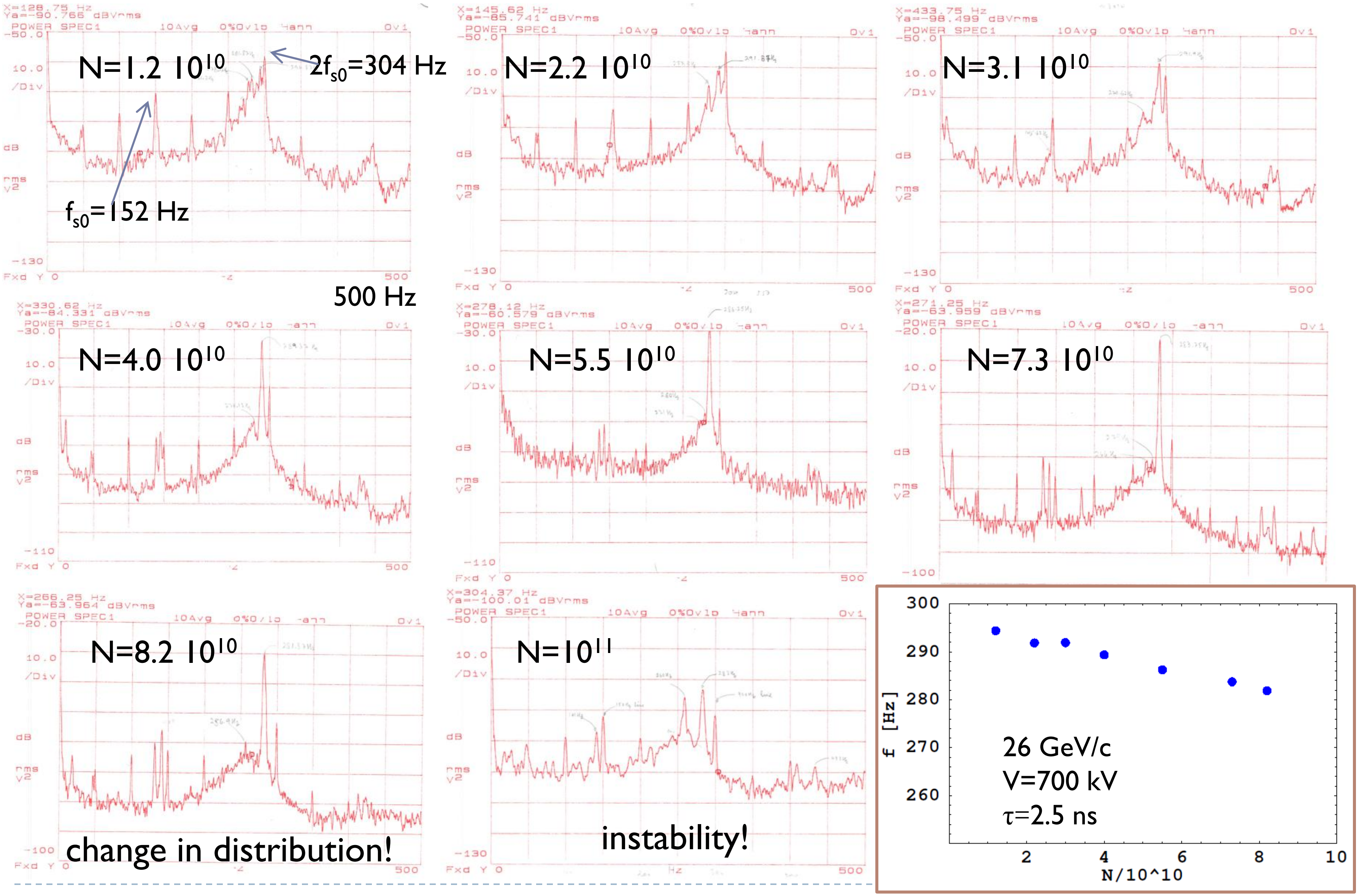
In this case, particles are <u>lost</u> from the bunch, not redistributed.

Cured by removing source in RF.

M. Brennan (BNL)

Talk of T. Linnecar, LTC, 2005

Examples: quadrupole frequency shift with intensity in SPS

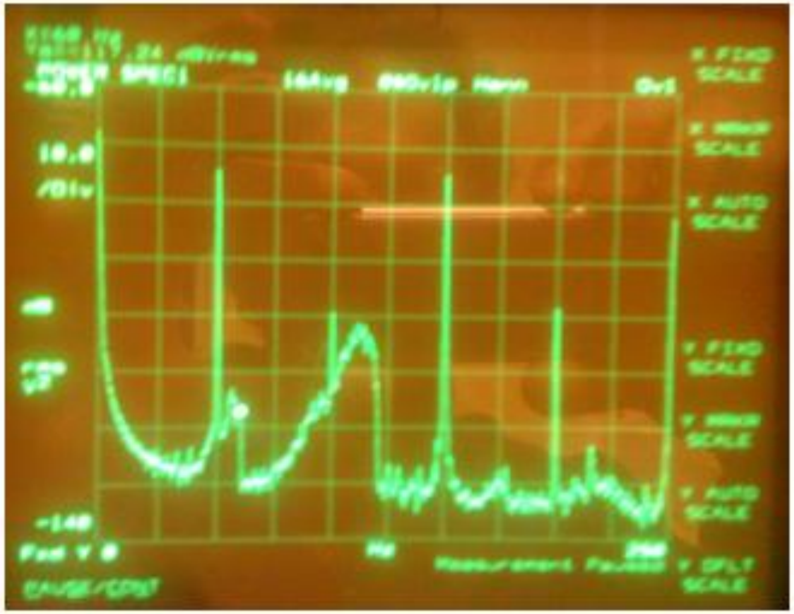


Summary

- The PD Schottky spectrum have been used at CERN for a long time
- The quadrupole line of the PD Schottky spectrum represents the particle distribution in synchrotron frequency modified by the synchrotron frequency nonlinearity (Ω^2) and experimental set-up, function $(A_2Q_2)^2$
- The measured PD signal is proportional to an average over the bunch current around its peak value. The distortion mainly depends on the distance Φ over which the averaging is performed.
- In optimised experimental set-up, for the PD Schottky spectrum the deviation from synchrotron frequency distribution function is much less than for the traditional Schottky (which nevertheless gives good measurement of a zero-amplitude synchrotron frequency f_{s0})

Spare slides

Measured PD Schottky spectrum in LHC



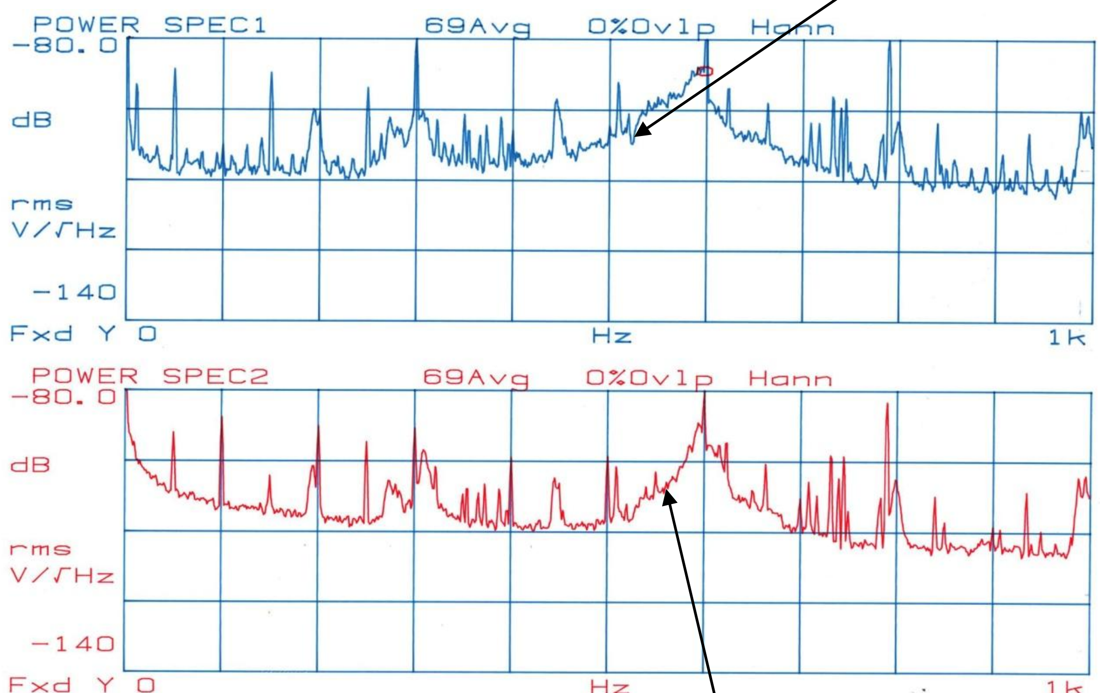
LHC coast at 450 GeV/c, September 2008,
f_{s0}=66 Hz, $\sigma_\phi = \pi/7$. Courtesy T. Bohl.
Scales: vertical 10dBV^2_{rms}/div, horizontal 25 Hz/div

Examples of application
Losses along the injection plateau - noise

Peak detected Schottky after 15 min store

Holes in spectrum are less for this voltage

527 Hz = 790/3 x 2!



Head of batch

Tail of batch

Shorter bunch, concave distribution

V_{RF} = 3 MV, f_s = 315 Hz

T. Linnecar et al. HB2004