Coupling Impedance of a Short Insert in the Vacuum Chamber

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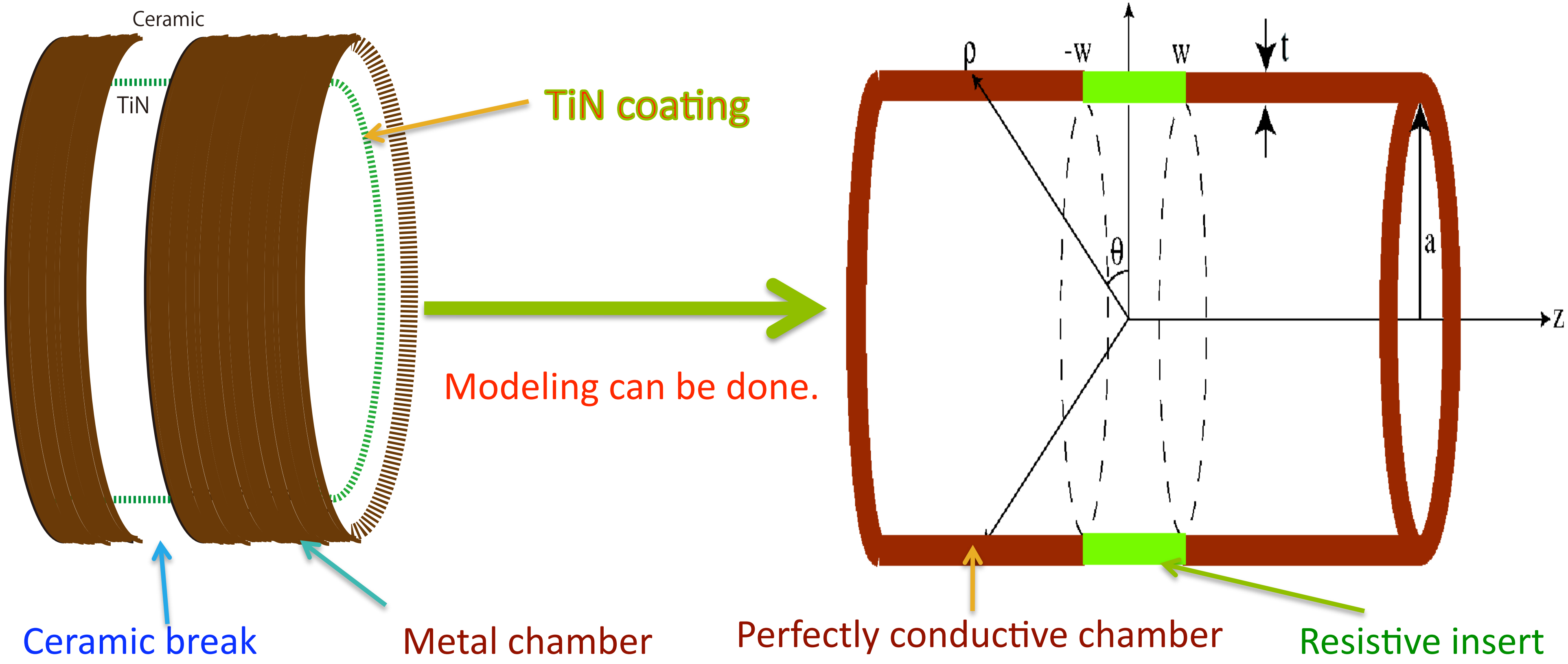
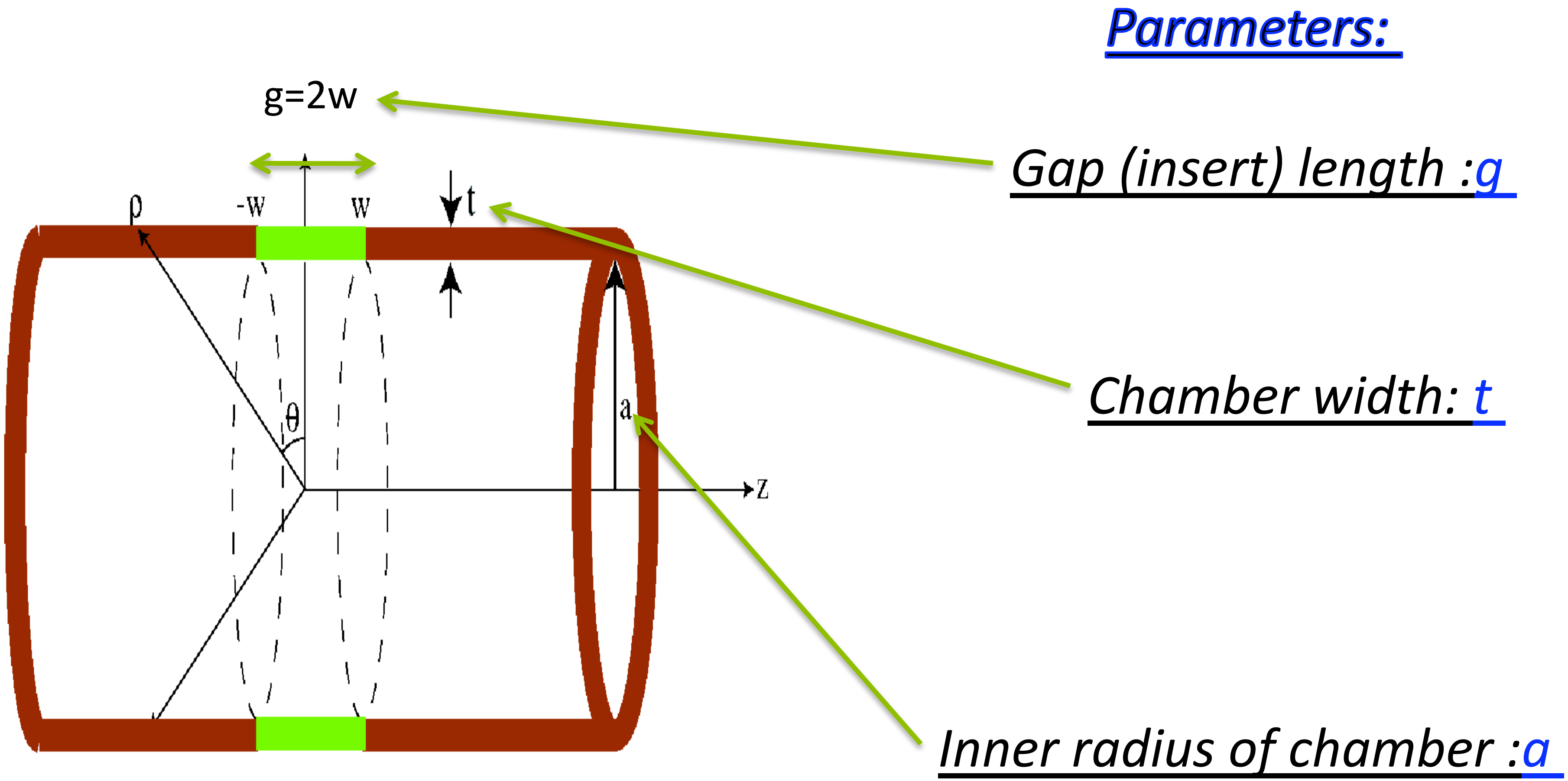
In proton synchrotrons,

- the secondary emission of electrons should be suppressed.
- the inner surface of a short ceramic break is normally coated by a thin Titanium Nitride (TiN) .
- It is typically about ten nm.



Outline

- *What was already known about the resistive wall impedance.*
- *A new theory for the impedance of the resistive insert.*
- *Summary.*



We use cylindrical coordinate.

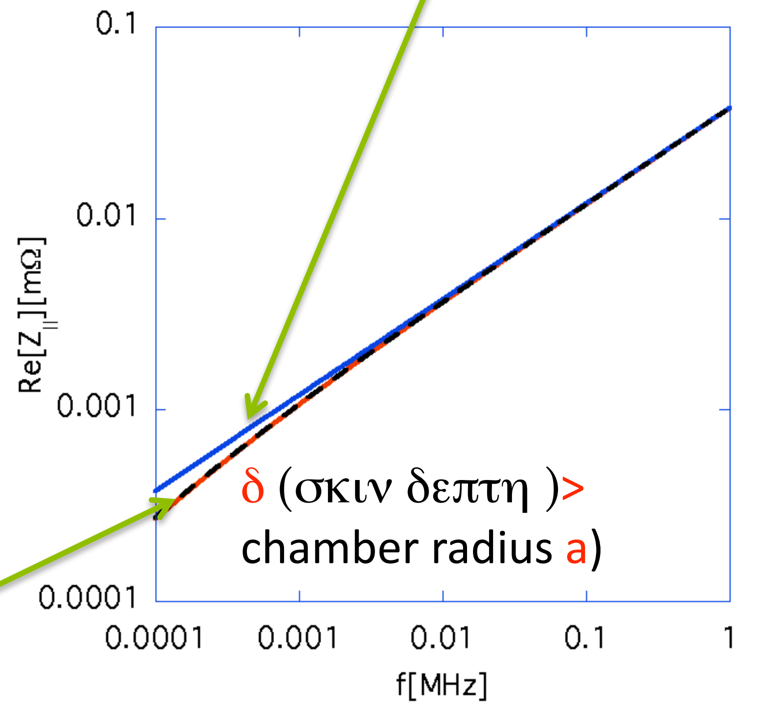
The conventional formula of the resistive wall impedance:

●*the chamber's width t is infinite.*

$$Z_L = gZ_0\sqrt{\frac{2\omega}{cZ_0\sigma_c}}\frac{1+j}{4\pi a}$$

$$\Leftrightarrow \quad Z_{wall} = \frac{g}{\sigma_c \pi((a+\delta)^2 - a^2)}$$

conventional



The real part of resistive wall impedance is equal to the resistance of the wall current.

Z_{wall} or the exact solution.

We borrowed the formula of Metral's work.

●E. Metral *et al*, PAC07(2007)

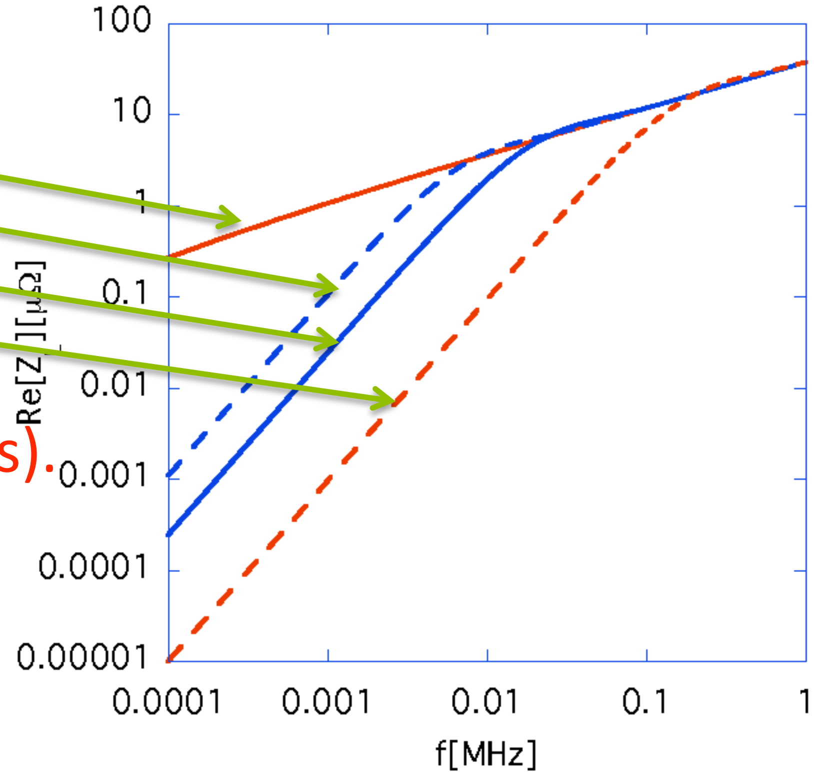
●E. Metral, CERN-AB-2005-084, (2005)

Resistive wall impedance with the finite thickness t
is well approximated as,

$$Z_L = g\frac{e^{\sqrt{2j}t/\delta} - e^{-\sqrt{2j}t/\delta}}{e^{\sqrt{2j}t/\delta} + e^{-\sqrt{2j}t/\delta}} Z_0 \sqrt{\frac{2\omega}{cZ_0\sigma_c}}\frac{1+j}{4\pi a}.$$

a=5cm, σ_c=6*10[6] /Ω m
t=∞
t=5mm
t=3mm
t=1mm

We show four exact solutions.



The wake fields propagate when δ(σκιν δεπτη)>t(chamber thickness).

For the extremely low frequency (δ(σκιν δεπτη)>t(chamber thickness)):

$$\Re[Z_L] = \frac{2g\pi Z_0^2\sigma_c f^2 t^3}{3ac^2}.$$

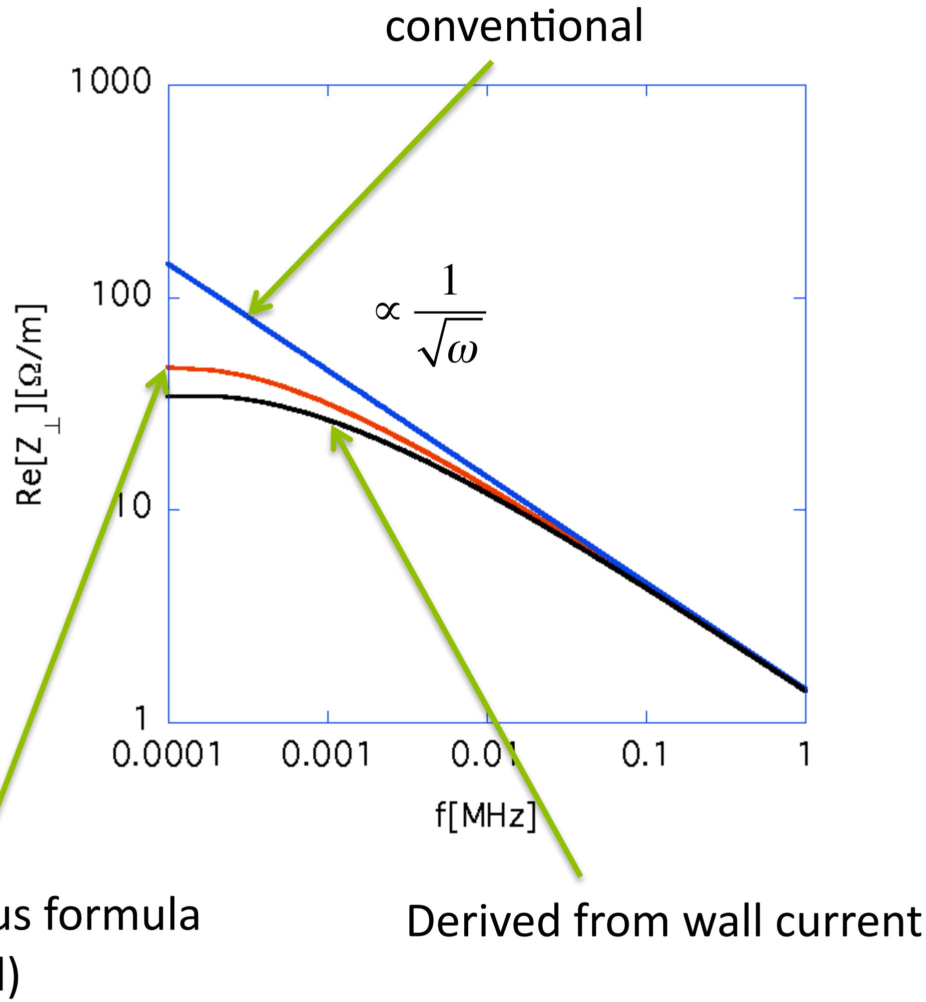
What about the transverse impedance?

Conventional formula **for the infinite thickness**:

$$Z_T = gc\sqrt{\frac{Z_0\omega\sigma_c}{2c}}\,\frac{1+j}{\pi\sigma_c\omega a^3}.$$

Fields are stored inside the circular area given by the radius a+δ.
Ιτ ισ ωελλ αππροξιματεδ ασ

$$\Re[Z_{wall,T}] = \frac{2\beta c}{\omega(a+\delta)^2} Z_{wall}.$$



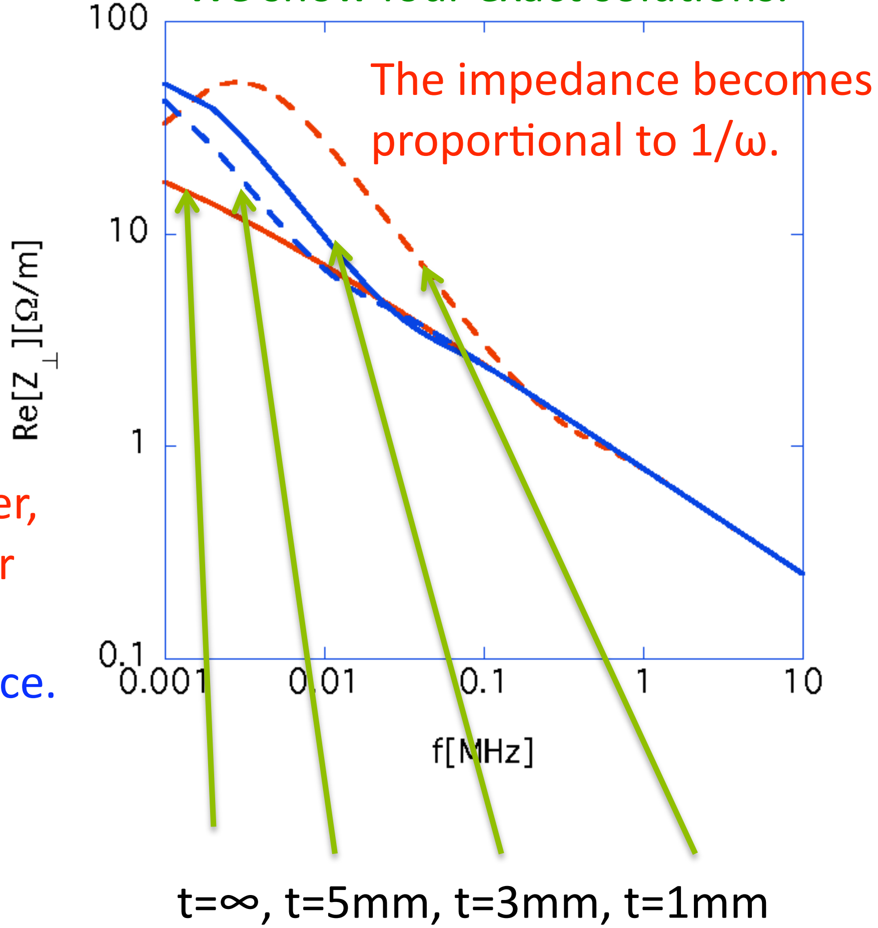
For the transverse impedance,
the real part of resistive wall impedance is approximately equal to the resistance of the wall current, as well.

Transverse impedance *with the finite thickness*

In the low frequency, the approximate formula can be obtained by replacing the skin depth δ by the chamber thickness t in the conventional formula.

$$Z_T = \begin{cases} \dfrac{gc(1+j)}{\omega\sigma_c \pi t a^3} & \text{for } \delta > t \\ \dfrac{gc(1+j)}{\omega\sigma_c \pi \delta a^3} & \text{for } \delta < t \end{cases}$$

We show four exact solutions.



The entire wall current runs in the thin chamber, while the skin depth is larger than the chamber thickness t.

This is different from the longitudinal impedance.

In the quite low frequency region:

$$f < f_L \equiv \frac{3c}{4\pi Z_0 \sigma_c t a},$$

the wake fields propagate out, again.

Is this picture applicable for the resistive insert ?

- Fortunately, we have a theory of the gap impedance.

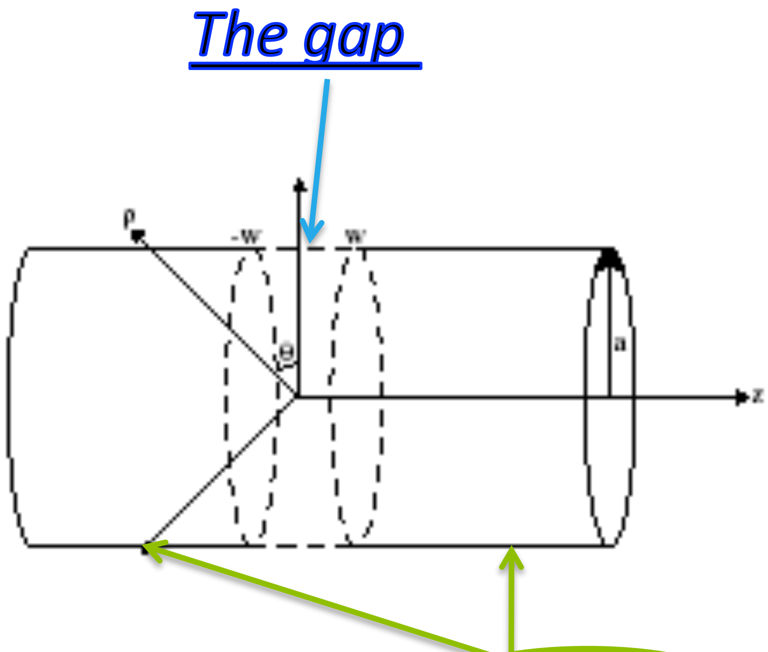


FIG. 1: The gap between metal chambers.

- The impedance due to the gap filled with resistive materials can be calculated by generalizing the theory of the gap impedance.

The formula of the longitudinal impedance of the resistive insert

$$Z_{L,\text{insert}} = \frac{4Z_0 I_1^2(\bar{k}\sigma)e^{-jkz}}{j\beta\gamma\sigma^2 a\bar{k}^3 I_0^2(\bar{k}a)[Y_{\text{pole}} + Y_{\text{cut}} - \frac{\pi\sqrt{jk\beta Z_0(\sigma_c + j\frac{k\beta\epsilon'}{Z_0})}}{\bar{k}^2\beta^2 w}\tanh\sqrt{jk\beta Z_0(\sigma_c + j\frac{k\beta\epsilon'}{Z_0})}t]},$$

$$Y_{\text{pole}} = -\sum_{s=1}^{\infty}\frac{\pi a(2 - e^{-j(b_s/a)(z+w)} - e^{j(b_s/a)(z-w)})}{wb_s^2},$$

$$Y_{\text{cut}} = -\frac{1}{w\pi(a+t)}\int_0^{\infty} d\zeta \frac{2 - e^{-j(z+w)\sqrt{k^2\beta^2+\frac{\zeta}{(a+t)^2}}} - e^{j(z-w)\sqrt{k^2\beta^2+\frac{\zeta}{(a+t)^2}}}}{\zeta(k^2\beta^2 + \frac{\zeta}{(a+t)^2})H_0^{(1)}(e^{j(\pi/2)}\sqrt{\zeta})H_0^{(2)}(e^{j(\pi/2)}\sqrt{\zeta})} \simeq \frac{2(1-j)}{\sqrt{k\beta w}}.$$

Radiation term

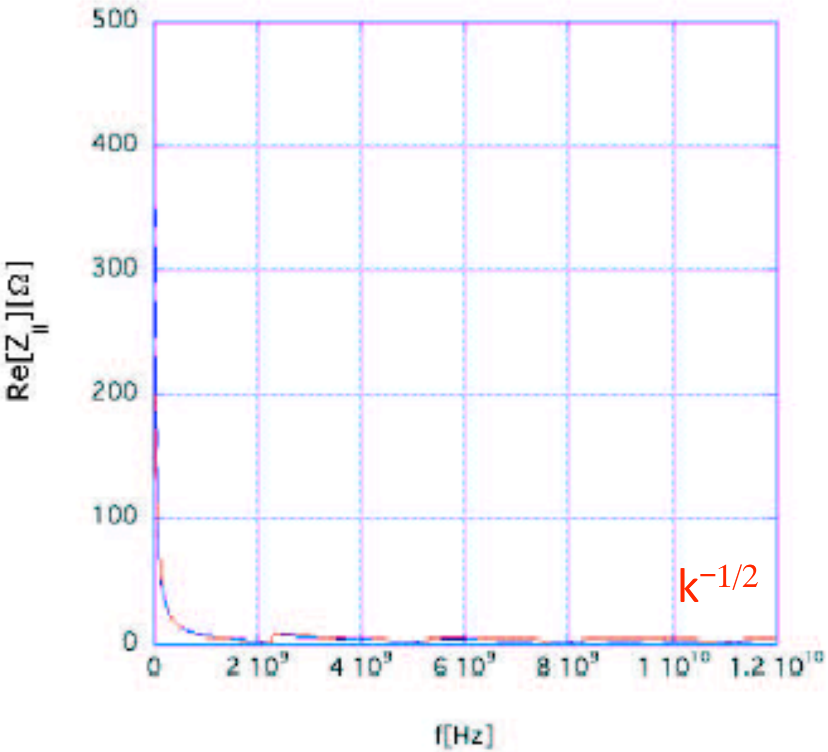
Resistive wall term

For the extremely small t, we reproduce the gap impedance.

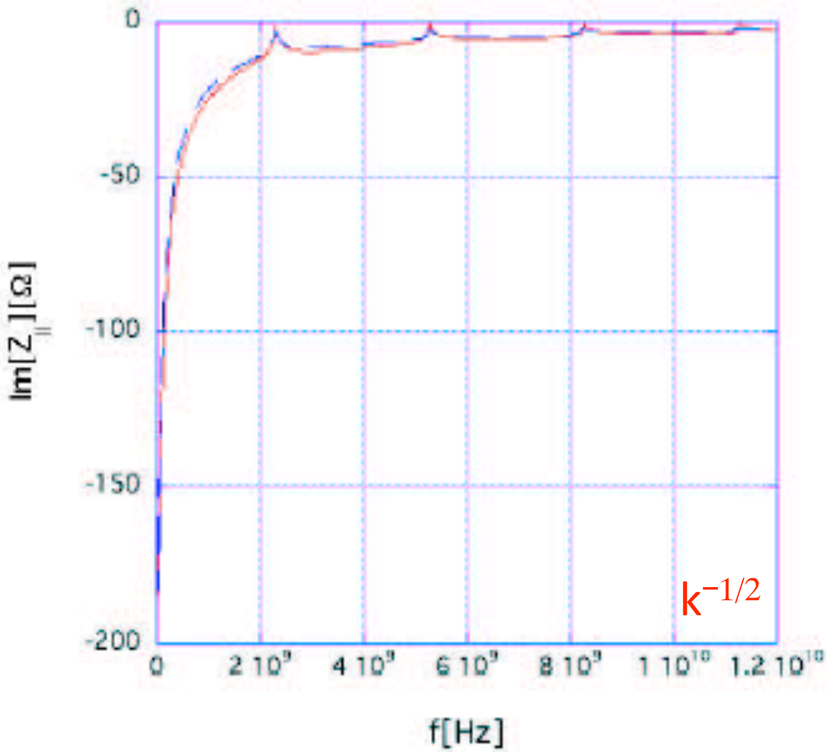
The feature of the longitudinal impedance of the gap

Parameters: a=5cm,g=8mm

- Real part of the impedance is huge.
- The radiation effect causes the energy loss.



- The impedance is capacitive.
- The gap can be assumed to be a capacitor.



- The impedance satisfies the diffraction theorem.
- Dips in the figure correspond to the cutoff frequency.

Comparison with ABCI results

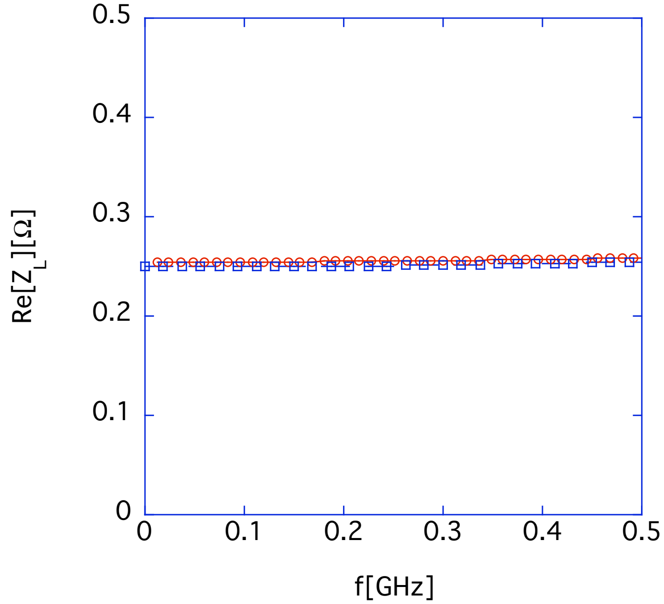
Parameters: a=5cm, g=8mm, τ=2mm, ε'=10, σ_c =50/Ωm.

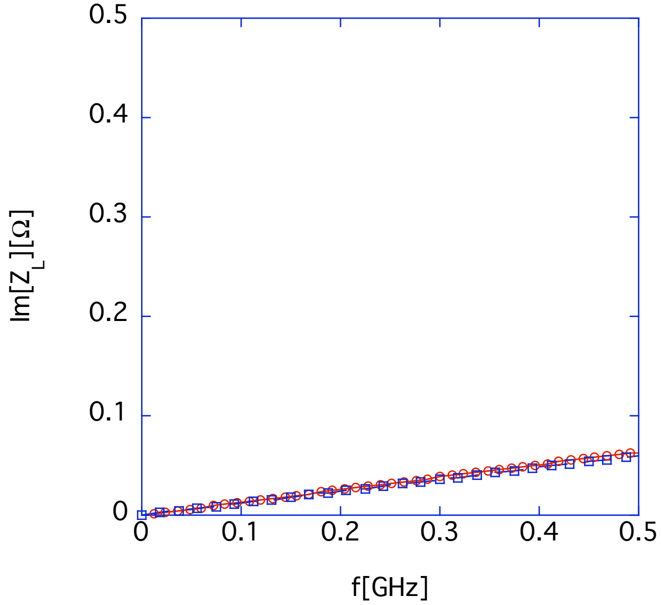
■In order to simulate correctly, the mesh size should be sufficiently smaller than the chamber thickness.

■The chamber thickness is divided into ten meshes.

■At high frequency (where the skin depth becomes smaller than the mesh size), ABCI cannot accurately simulate field behavior.

■That is about 1GHz for the present choice of mesh size.





The theoretical and simulation results are in good agreement.

The feature of the longitudinal impedance of the insert.

Here, we introduce the parameter of the insert thickness t_{min}:

$$t_{\min} \equiv \left(\frac{4g}{\pi^2 Z_0^3 \sigma_c^3} \right)^{\frac{1}{4}} \sim$$

Typically a few ten nm

In the case that the insert's thickness t is larger than $2^{1/2} \pi^{3/4} t_{min}$ **(this is typical case),**

For the extremely high frequency region,

$$f >> f_D \equiv \frac{c}{2\pi} \sqrt{\frac{2Z_0\sigma_c}{g}}$$

~typically the order of THz,

the impedance is approximated to

$$Z_L \approx \frac{(1-j)2Z_0\sqrt{g}}{2\pi a\sqrt{\pi k}},$$

which satisfies the diffraction theorem reproduces Krinsky and Stupakov's result (PRST**7**,114401(2004), PRST**8**,44401(2005)).

For the region $f<<f_D$, the impedance becomes the conventional resistive wall impedance

For the region $\delta>t$,

$$\Re[Z_L] \approx \frac{g}{2\pi a\sigma_c t}.$$

The real part of the impedance is identical to the resistance of the wall current.

In the case that the insert's thickness is smaller than $2^{1/2}\pi^{3/4} t_{min}$, but larger than t_{min},

For the extremely high frequency region f>> f_D,
the impedance is approximated to

$$Z_L \approx \frac{(1-j)2Z_0\sqrt{g}}{2\pi a\sqrt{\pi k}},$$ which satisfies the diffraction theorem.

For the region f< f_D,

$$\Re[Z_L] \approx \frac{g}{2\pi a \sigma_c t}$$

This is identical to the resistance of the wall current.

Whether the impedance is the radiation dominant or the resistive wall dominant, the insert length g dependence changes in the impedance.

In the region where the radiation is dominant, the impedance is proportional to √g.
In the region where the resistive wall (or wall current) is dominant,
the impedance is proportional to g.

For the insert with **$t < t_{min}$** , f_D and δ are no longer dominant parameters.

■The new parameter :

$$f_c \equiv \frac{\sigma_c^2 Z_0^2 t^2 c}{4\pi g}$$

plays an important role.

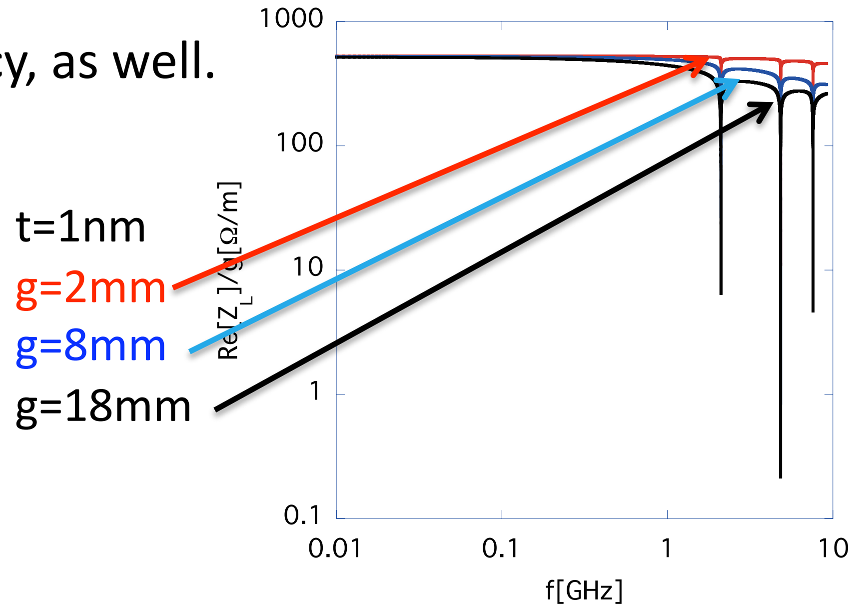
■As the insert becomes thinner,
this upper limit f_c moves to a lower frequency.

■ As the insert becomes longer,
this upper limit f_c moves to a lower frequency, as well.

For the low frequency region $f \ll f_c$,
<u>the contribution from the wall current</u>
<u>is dominant</u>.

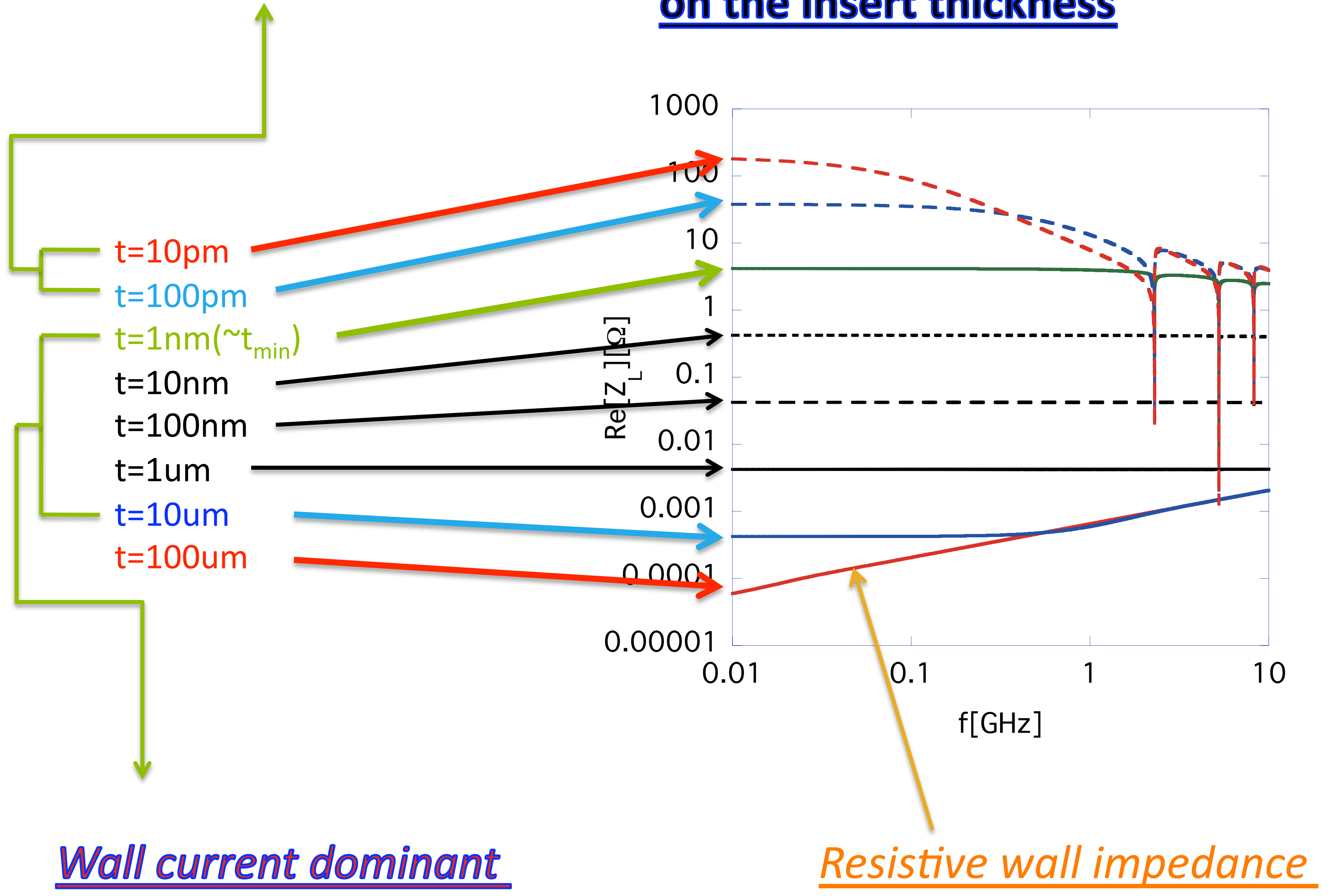
In the rest of the frequency region,
<u>the radiation effects dominates to</u>
<u>the impedance.</u>

<u>The dependence on the length of the insert.</u>



This figure shows the impedance per unit length.

The dependence of the impedance on the insert thickness



The expression of the transverse impedance

$$Z_{T,\text{insert}} \simeq -\frac{jZ_0 I_1(\bar{k}r_b)e^{-jkz}}{\beta\gamma r_b a k I_1^2(\bar{k}a)\left(-\frac{\pi\sqrt{jk\beta Z_0\sigma_c}}{k^2\beta^2 w}\tanh\sqrt{jk\beta Z_0\sigma_c}\,t + Y'_{\text{pole}} + Y'_{\text{cut}}\right)},$$

$$Y'_{\text{pole}} = \sum_{s=1}^{\infty}\left[-\frac{\pi a(2 - e^{-j(b_{1,s}/a)(z+w)} - e^{j(b_{1,s}/a)(z-w)})}{wb_{1,s}^2} + \frac{\pi a J_1(j'_{1,s})(2 - e^{-j(b'_{1,s}/a)(z+w)} - e^{j(b'_{1,s}/a)(z-w)})}{k^2\beta^2 a^2 w j'^2_{1,s} J''_1(j'_{1,s})}\right]$$
$$-\frac{\pi H_1^{(2)}(h'_{1,0})(2 - e^{-j[d'_{1,0}/(a+t)](z+w)} - e^{j[d'_{1,0}/(a+t)](z-w)})}{k^2\beta^2(a+t)wh'^2_{1,0}H''^{(2)}_1(h'_{1,0})} + \frac{\pi(2 - e^{-jk\beta(z+w)} - e^{jk\beta(z-w)})}{wk^2\beta^2}$$
$$\times\left(\frac{H_1^{(2)}(h'_{1,0})}{(a+t)h'^2_{1,0}H''^{(2)}_1(h'_{1,0})} - \frac{1}{2a}\right),$$

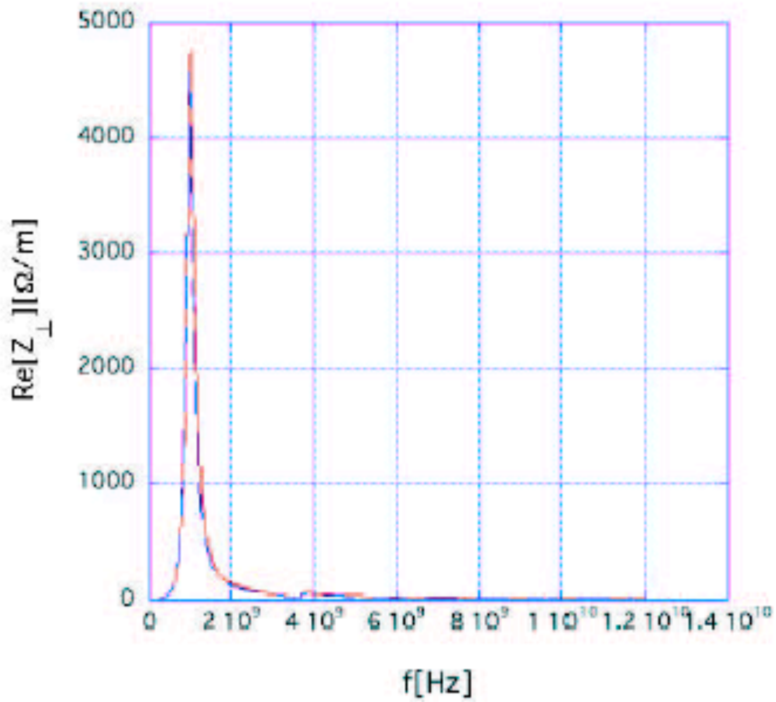
$$Y'_{\text{cut}} = -\frac{1}{\pi(a+t)w}\int_0^{\infty} d\zeta \frac{(2 - e^{-j(z+w)\sqrt{k^2\beta^2+\zeta/(a+t)^2}} - e^{j(z-w)\sqrt{k^2\beta^2+\zeta/(a+t)^2}})}{\zeta(k^2\beta^2 + \frac{\zeta}{(a+t)^2})H_1^{(1)}(e^{j(\pi/2)}\sqrt{\zeta})H_1^{(2)}(e^{j(\pi/2)}\sqrt{\zeta})}$$
$$+\int_0^{\infty} d\zeta \frac{(e^{-j(z+w)k\beta} + e^{j(z-w)k\beta} - e^{-j(z+w)\sqrt{k^2\beta^2+\zeta/(a+t)^2}} - e^{j(z-w)\sqrt{k^2\beta^2+\zeta/(a+t)^2}})}{k^2\beta^2(a+t)\pi w\zeta^2 H'^{(1)}_1(e^{j(\pi/2)}\sqrt{\zeta})H'^{(2)}_1(e^{j(\pi/2)}\sqrt{\zeta})}$$
$$\simeq 4\tan^{-1}\frac{1}{\sqrt{2jkw}} + \frac{-2 + 4\sqrt{1+2jk\beta w}\sinh^{-1}\frac{e^{-j(\pi/4)}}{\sqrt{2k\beta w}} + e^{-j(z+w)k\beta} + e^{j(z-w)k\beta}}{k^2\beta^2(a+t)^2\sqrt{1+2jk\beta w}},$$

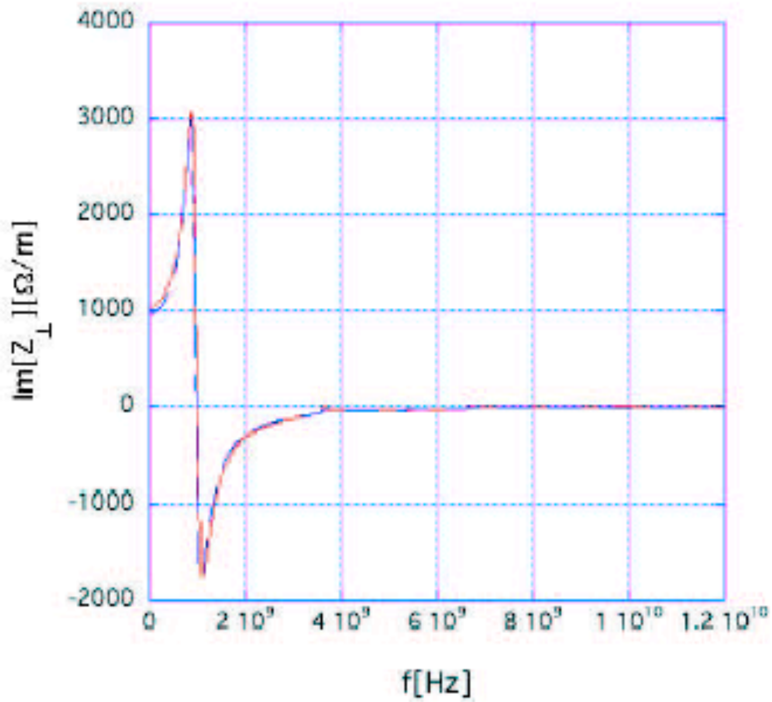
Resistive wall term

Radiation term

The feature of the transverse impedance of the gap

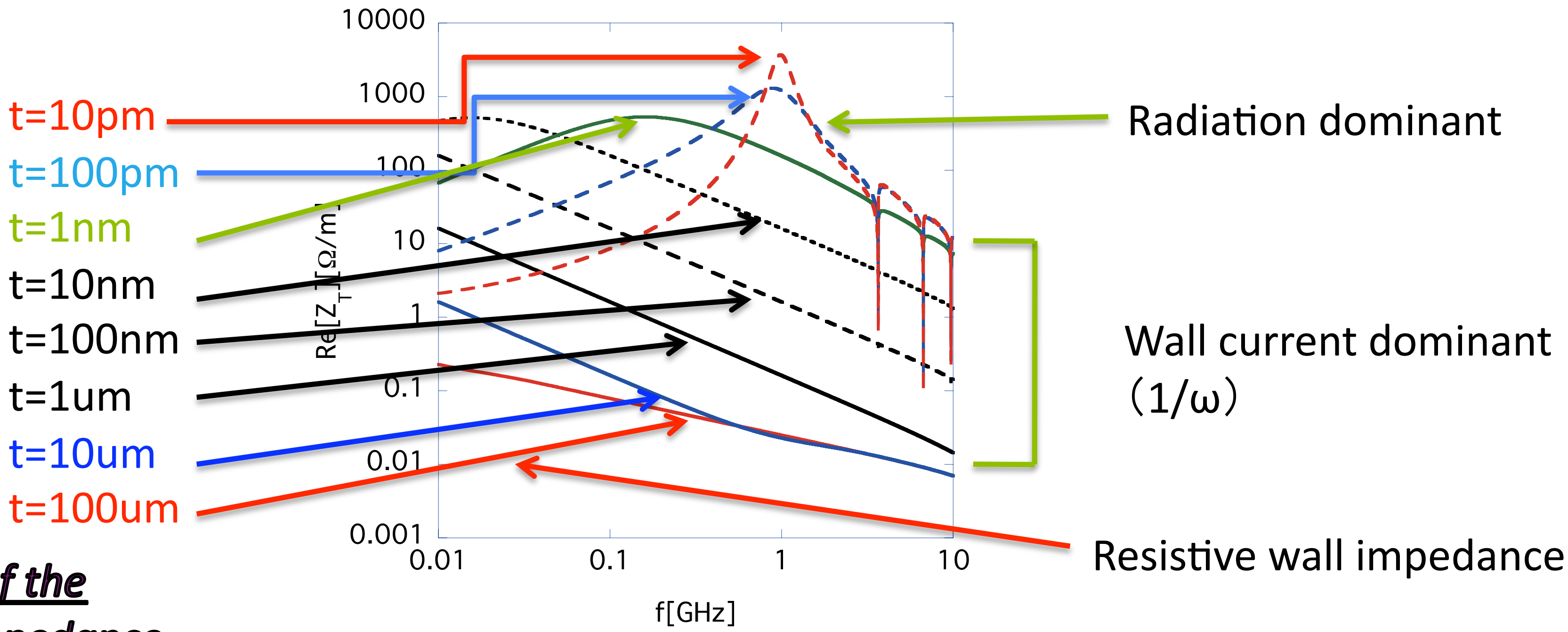
Parameters: a=5cm,g=8mm

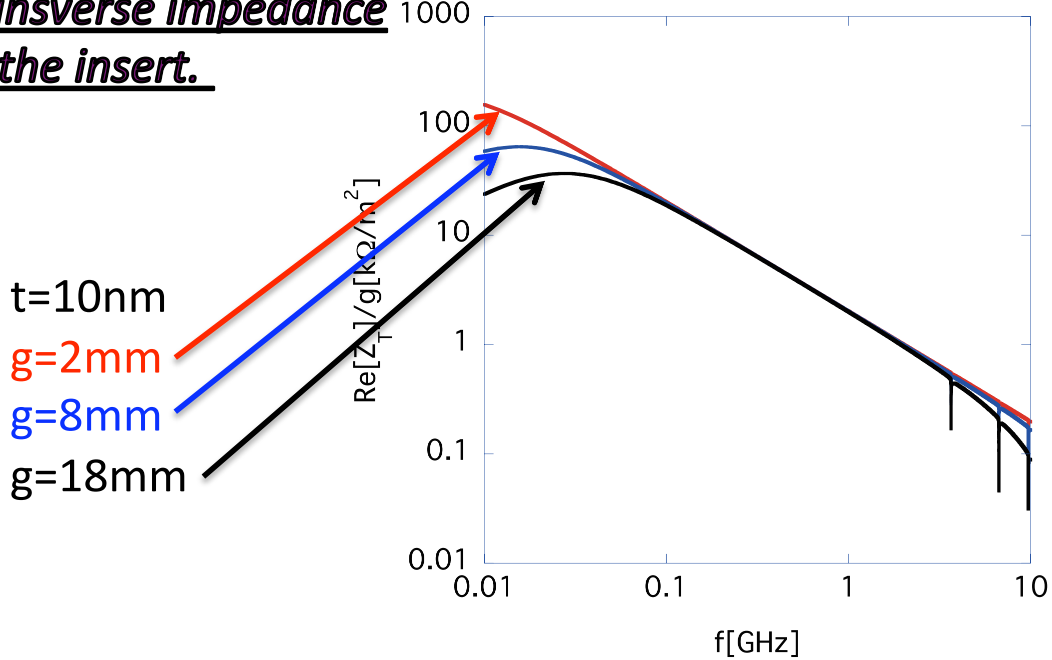




- Resonance occurs when the wavelength is equal to the circumference of the chamber.
- The impedance satisfies the diffraction theorem, as well.

The feature of the transverse impedance of the insert.





■As the insert becomes longer,

●the upper limit f_c **(= $\sigma_c^2 Z_0^2 t^2 c/4\pi g$)** moves to a lower frequency.

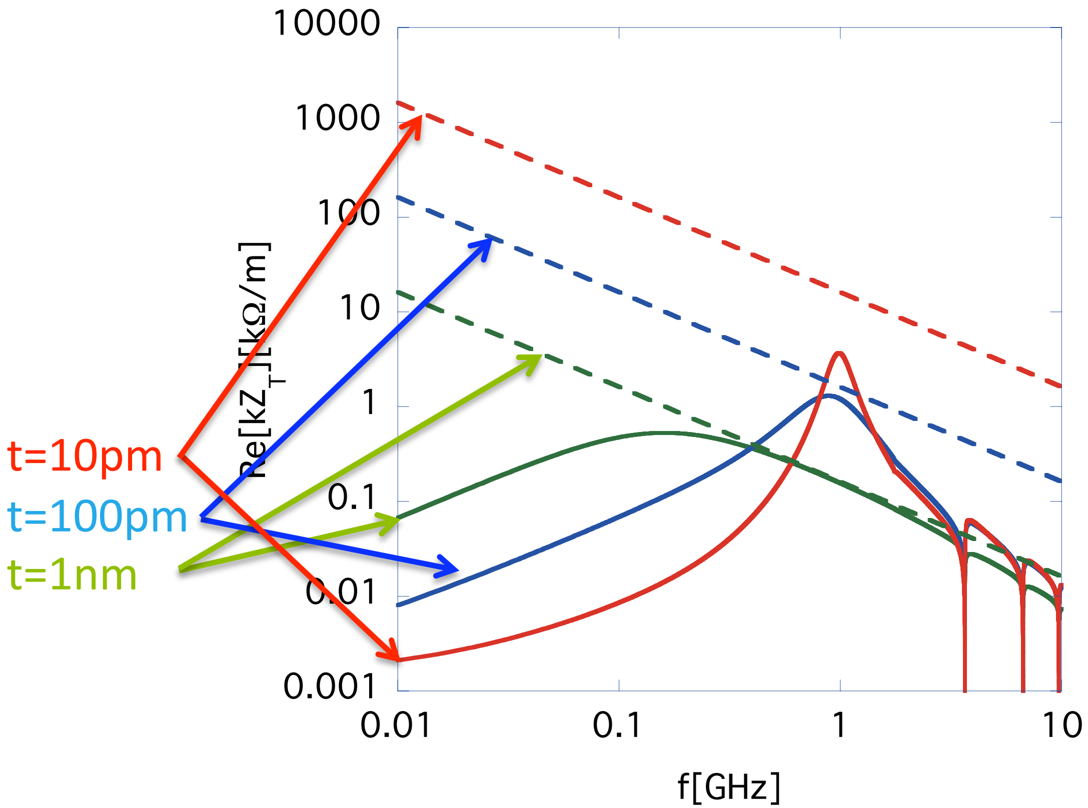
●the lower limit f_r **(=$(gc^3/4\pi^3 Z_0^2 \sigma_c^2 a^4 t^2)^{1/3}$)** moves to a higher frequency.

The wall current dominant region becomes narrower.

The impedance per a unit length → Gap impedance

The physical reason of why the whole wall current tends to run on the thin insert (except for the extremely thin insert case) is

that the nature tries to minimize the energy loss of a beam.



This picture shows the extremely thin insert case.

The dashed lines show the results,
in the case that
the impedance is hypothetically wall current dominant.

The lines show the real impedance.

The impedance is bigger when the wall current runs on the thin insert with large resistance, than the wall current converts to the radiation out to free space (= gap impedance) <u>only in this extremely thin insert.</u>

Summary

- ■The theory to describe the impedances of a short insert was developed.
- ■The theory is consistent with the resistive wall impedance and the gap impedance.
- ■ Nature tries to minimize the energy loss of a beam.

Concretely,

◆In the case that **$t > t_{min}$** (typically a few ten nm), the entire wall current runs in the thin insert, even when the skin depth exceeds the thickness of the insert

● the impedances increase drastically from the conventional resistive-wall impedance.

◆In the case that **$t < t_{min}$** , the contribution from the wall current starts to diminish.

●For the longitudinal impedance, the parameter **$f_c (= \sigma_c^2 Z_0^2 t^2 c / 4\pi g)$** specifies **the upper limit** of the frequency **where the wall current effects are dominant** .

•As the insert becomes thinner, this upper limit moves to a lower frequency.

●For the transverse impedance, the another parameter **$f_r (=(g c^3 / 4\pi^3 Z_0^2 \sigma_c^2 a^4 t^2)^{1/3})$** specifies **the lower limit** of the frequency region where **the wall current effects are dominant**.

•As the insert becomes thinner, the lower limit moves to a higher frequency.

•The frequency region where the wall current effects dominate in the impedance becomes narrower from the higher and the lower sides.