Dynamics of Intense Inhomogeneous Charged Particle Beams*

Renato Pakter, F.B. Rizzato, Y. Levin,
E.G. Souza, A. Endler, and R.P. Nunes

Instituto de Física
Universidade Federal do Rio Grande do Sul

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Introduction

- In the transport of intense beams, space charge forces make it virtually impossible to launch beams in an exact equilibrium state.
- As a consequence, the beam tends to relax towards a stationary state, leading to emittance growth and halo formation.
- E.g., initially mismatched beams (particle-core resonance).
- This is also the case for beams with nonequilibrium density profiles.
- Here we investigate the mechanism that leads to beam relaxations in this case, concentrating in very intense beams where thermal effects are negligible.

Model and beam equations

- We consider a continuous, axisymmetric beam propagating with constant axial velocity v_z, focused by a uniform magnetic field B_z
- Neglecting thermal effects, beam evolution is dictated by the single particle transverse dynamical equation (normalized):

$$\frac{d^2 r}{dz^2} = -\kappa_z r + \frac{Q(r)}{r},$$

where $Q(r) = \frac{K}{N_b}\int_0^r n(r')r'dr'$ is the charge inside the circle of radius r

- As long as the fluid description (laminar flow) is valid $Q(r)=Q(r_0)$ is invariant and the beam evolution is completely determined by the knowledge of $r=r\,(r_0,z)$
- In particular, the density is given by

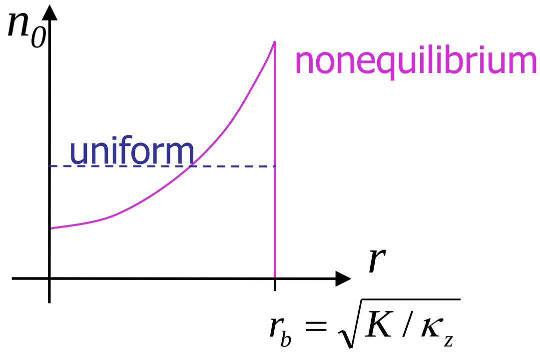
$$n(r,z) = n_0(r_0)\left(\frac{r_0}{r}\right) \Big/ \left(\frac{\partial r}{\partial r_0}\right)\Bigg|_{r_0 = r_0(r,z)}$$

Equilibrium beam

- Equating the force to zero, we obtain the equilibrium beam profile

$$\frac{d^2 r}{dz^2} = -\kappa_z r + \frac{Q(r)}{r} = 0 \quad \Rightarrow \quad Q(r) = k_z r^2,$$

- Which corresponds to a uniform density profile up to the matched radius $r_b = \sqrt{K / \kappa_z}$
- For any other distribution the particles are not in equilibrium and will start to move -> What is the main mechanism for beam relaxation?



Fast wave breaking

- Anderson (1998) identified the occurrence of a wave breaking for nonuniformities above a certain threshold.

- Using perturbation theory to first order we obtain for each particle trajectory

$$r(r_0, z) = r_{eq} + A\cos(wz)$$

where $A(r_0) = r_0 - r_{eq}$, $r_{eq}(r_0) = \sqrt{Q(r_0)/\kappa_z}$, and $w = \sqrt{2\kappa_z}$

- Therefore

$$\frac{\partial r}{\partial r_0} = \frac{dr_{eq}}{dr_0} + \frac{dA}{dr_0}\cos(wz)$$

- And a wave breaking occurs within one plasma cycle when

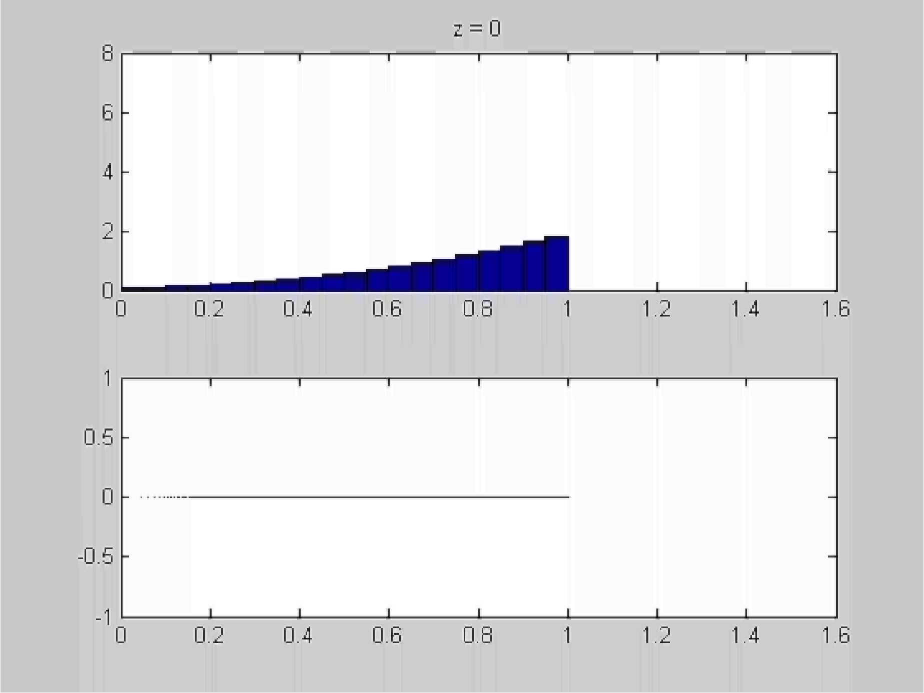
$$\frac{dA}{dr_0} > \frac{dr_{eq}}{dr_0} \quad \Rightarrow \quad \sqrt{Q(r_0)} > \frac{dQ}{dr_0}$$

Fast Breaking - Parabolic beam

- If we consider an initial parabolic beam profile

$$n_0(r) = \frac{N_b}{\pi r_b^2}\left[1+\chi\left(2\frac{r^2}{r_b^2}-1\right)\right], \quad r \le r_b$$

fast wave breaking occur for χ>0.8 or χ<-0.5.



Slow wave breaking*

- However, if we go to higher order perturbation theory, we note that frequency depends on r_0

$$r(r_0, z) = r_{eq} + A\cos(wz), \quad w(r_0) = \sqrt{2\kappa_z}\left[1 + \frac{1}{12}\left(\frac{A(r_0)}{r_{eq}(r_0)}\right)^2\right]$$

- And

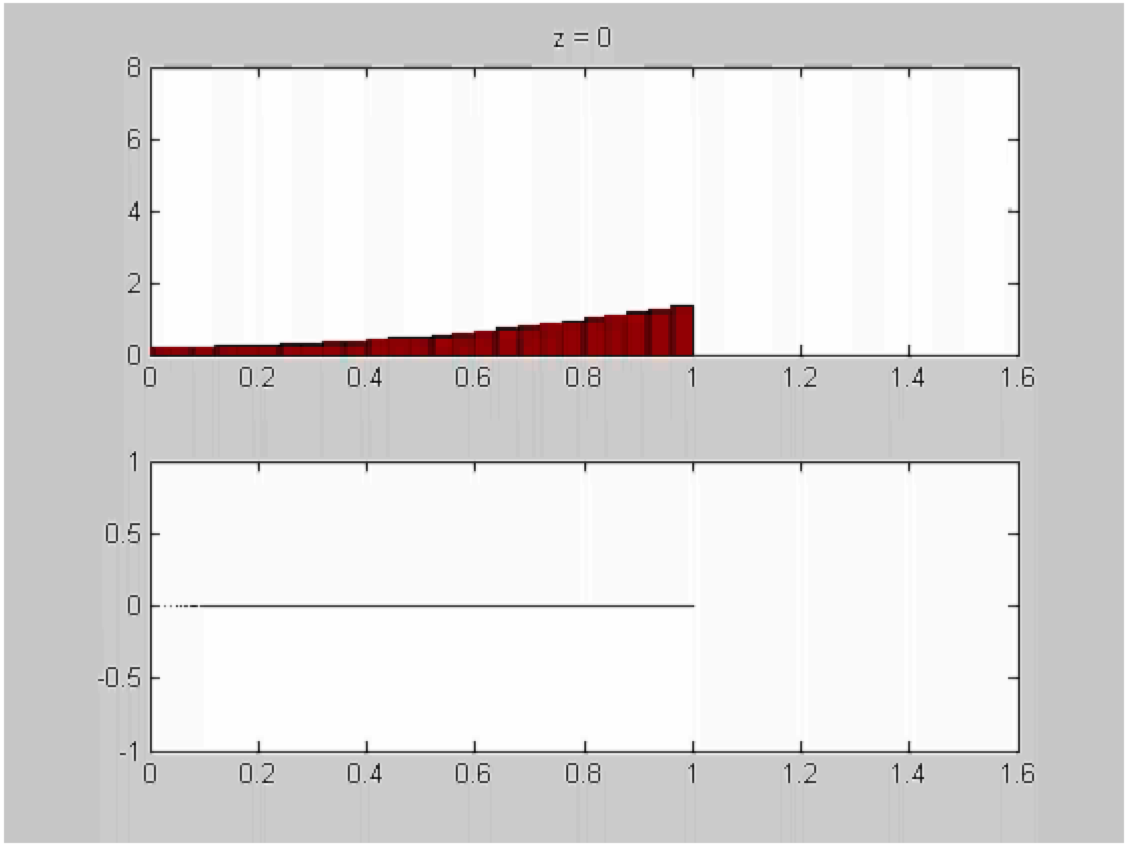
$$\frac{\partial r}{\partial r_0} = \frac{dr_{eq}}{dr_0} + \frac{dA}{dr_0}\cos(wz) - A\sin(wz)\frac{dw}{dr_0}z$$

secular term that continuously growths and will always lead to wave breaking

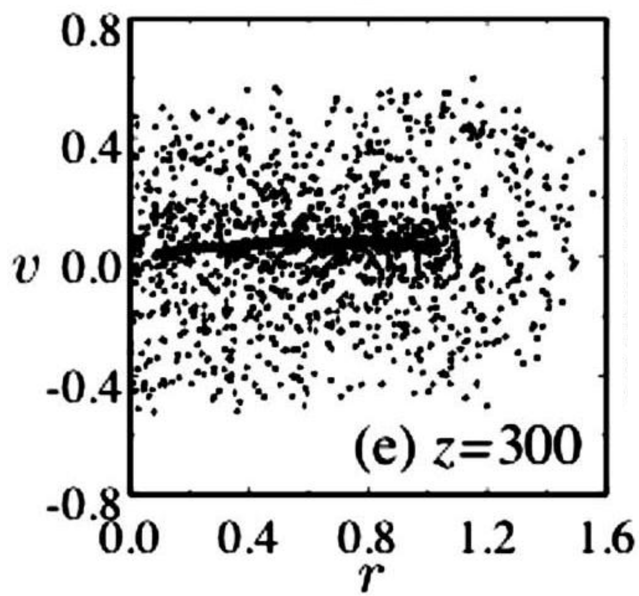
*F.B. Rizzato, R. Pakter, and Y. Levin, Phys. Plasmas (2007)

Slow wave breaking

$$\chi = 0.75$$



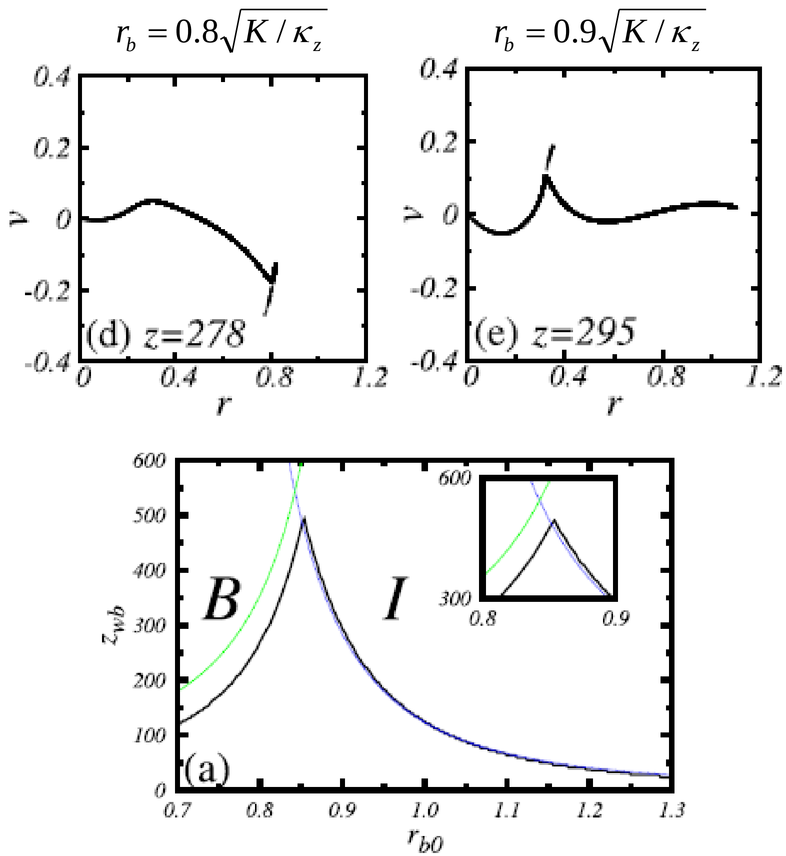
Stationary State



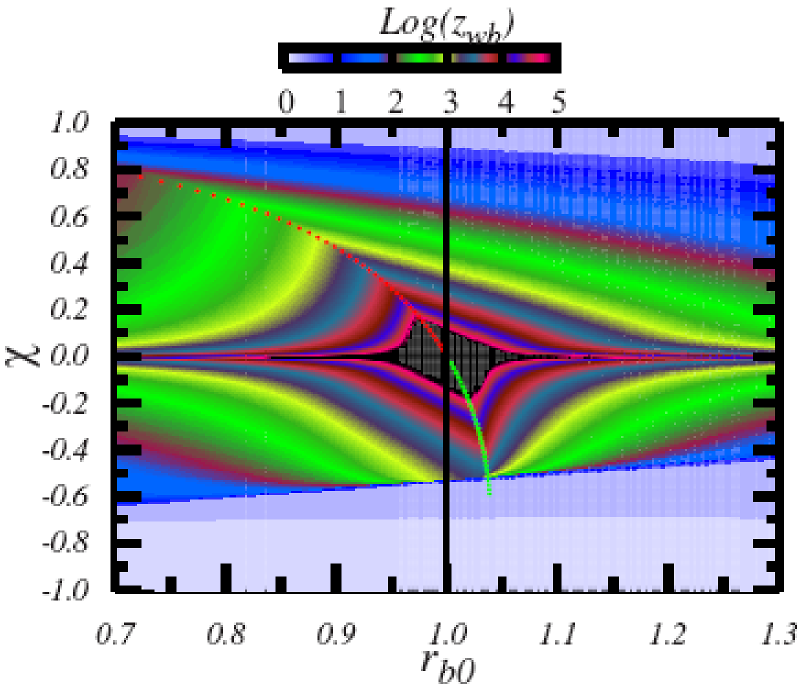
- Wave breaking leads to halo formation and occurs for any nonuniformity.
- Is it possible to avoid it?

Effect of beam mismatch

- mismatching the beam changes the location of the break and the axial distance where it occurs



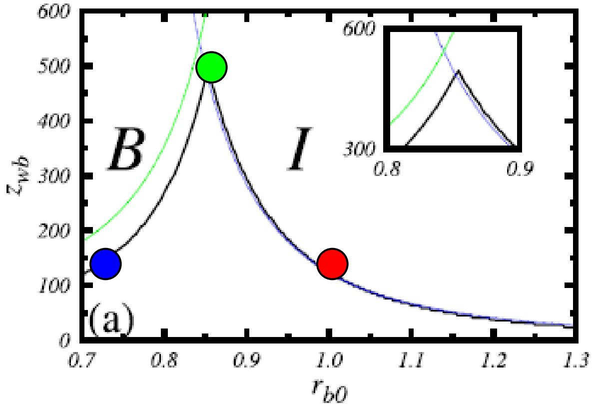
Effect of beam mismatch*

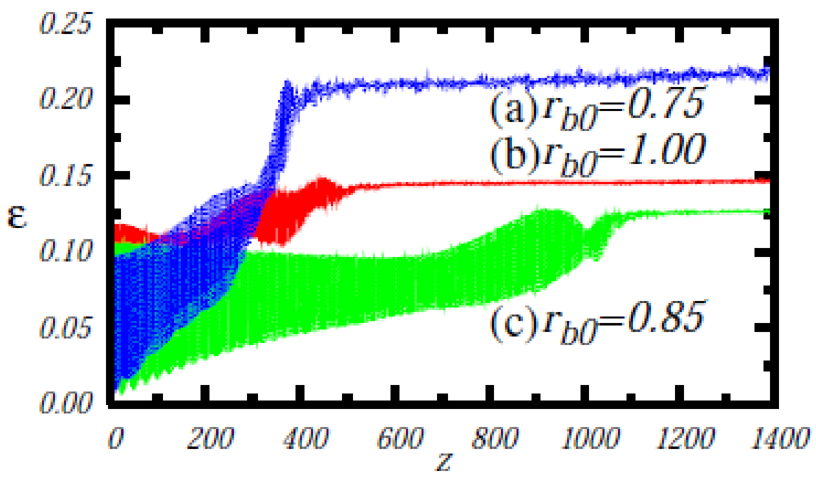


- Although we cannot completely avoid the break, it is possible to postpone it by suitably mismatching the beam

*E.G. Souza, A. Endler, R. Pakter, F.B. Rizzato, R.P. Nunes, Appl. Phys. Lett (2010)

Beams with Finite temperature





Conclusions

- We have analyzed the transport of intense inhomogeneous beams.
- Two types of wave breaking cases were identified: a fast breaking commanded by amplitude gradients and a slow breaking commanded by frequency gradients.
- In particular, the slow breaking has no threshold and is bound to happen no matter how small is the beam nonuniformity.
- It was shown that a judicious choice of an envelope mismatch can significantly extend the beam life time before the breaking occurs.