HEAD-TAIL BUNCH DYNAMICS WITH SPACE CHARGE

Vladimir Kornilov and Oliver Boine-Frankenheim,
GSI, Planckstr. 1, 64291 Darmstadt, Germany

Abstract

Significant progress has been made recently in the understanding of the effects of direct space charge on the transverse head-tail bunch dynamics. Different analytic approaches for head-tail modes in bunches for different space-charge parameter regimes have been suggested. Besides head-tail eigenmode characteristics, Landau damping in a bunch exclusively due to space charge has been predicted. In this contribution we compare results of particle tracking simulations with theoretical predictions for the eigenfrequencies and eigenfunctions of head-tail modes in a Gaussian bunch. We demonstrate the space-charge induced Landau damping in a bunch and quantify damping rates for different modes and space-charge tune shifts. Under conditions below the mode coupling threshold we study the head-tail instability with space charge. Our results show that the space-charge induced damping can suppress the instability for moderately strong space charge. For strong space charge the instability growth rates asymptotically reach constant vales, in agreement with theoretical predictions.

INTRODUCTION

The standard head-tail theory, i.e. the model of Sacherer [2, 3], does not include the effect of an incoherent tune spread on head-tail modes. A model for the head-tail instability with arbitrary space charge has been suggested in Ref. [4], for a bunch in a square-well potential and an airbag bunch distribution in the longitudinal phase space. Only in recent works [5, 6] analytical treatments of head-tail modes with space charge for realistic bunche distributions (as e.g. Gaussian) have been proposed. However, numerical simulations appear to be indispensable for a comprehensive stability analysis in different beam parameter regimes and with various collective effects taken into account. Here, we present particle tracking simulations for head-tail modes in a Gaussian bunch with space charge. We use two different particle tracking codes, PATRIC [7] and HEADTAIL [8], in order to compare different numerical implementations. As an exemplary instability driving source, the resistive-wall impedance is considered. In this work we consider the single-bunch head-tail instability for the parameters well below the threshold for mode coupling.

An important phenomenon, discussed in Refs. [5, 6], is Landau damping in a bunch exclusively due to space charge. In a coasting beam space charge can not provide Landau damping of its own, even if the coherent frequency overlaps the tune spread induced by nonlinear space charge [9]. In the case of a bunch, the synchrotron motion plays an important role and the space-charge tune spread due to the longitudinal density profile provides Landau damping. Here, we demonstrate this Landau damping in particle tracking simulations and examine its role for the stability of head-tail modes at moderate and stronger space charge.

BUNCH SPECTRUM WITH SPACE CHARGE

There is no simple analytical answer for the space-charge effect on head-tail modes in bunches with an arbitrary bunch profile $\lambda(\tau)$. However, such a theory could be very useful for code validation and for the interpretation of simulation results. An analytical solution for head-tail modes in bunches with arbitrary space charge has been derived in Ref. [4]. The model assumes an airbag distribution in the longitudinal phase space and a square-well (or barrier) potential and thus a constant line density, which means a constant $\Delta Q_{\rm sc}$. The longitudinal momentum distribution has two opposing flows of particles $\Big[\delta(v_0{-}v_{\rm b}){+}\delta(v_0{+}v_{\rm b})\Big]$, the synchrotron tune in this bunch is $Q_{\rm s} = v_{\rm b}/(2\tau_{\rm b}Rf_0)$, where $\tau_{\rm b}$ is the full bunch length and f_0 is the revolution frequency. The model considers "rigid flows", i.e. only dipole oscillations without variation in the transverse distribution of the flows are included. It also assumes that all betatron tune shifts are small compared to the bare tune $|\Delta Q| \ll Q_0$. The resulting tune shift due to space charge (without impedances) is given by

$$\Delta Q = -\frac{\Delta Q_{\rm sc}}{2} \pm \sqrt{\frac{\Delta Q_{\rm sc}^2}{4} + k^2 Q_{\rm s}^2}\,, \qquad (1)$$

where "+" is for modes $k \geq 0$.

In order to verify the space-charge implementation for long-time simulations with a particle tracking code, we have introduced the barrier-airbag bunch distribution in both PATRIC and HEADTAIL codes. For the transverse space charge force, the "frozen" electric field model was used, i.e. a fixed potential configuration which follows the mass center for each single slice. This approach is justified for the "rigid-slice" regime and can be considered as a reasonable approach for moderate and strong space charge [10, 5]. A round transverse cross-section and a homogeneous transverse beam profile were used in the simulations in this work. An excellent agreement between the airbag theory [Eq. (1)] and simulations has been achieved, a detailed description of the code validation was presented in

Ref. [11].

For a realistic case, we consider a Gaussian bunch, i.e. a Gaussian line density profile and a Gaussian momentum distribution. Coherent oscillation spectra for bunches with $q = 5$ and $q = 20$ are shown in Fig. 1, where we introduce a space charge parameter $q = \Delta Q_{\rm sc}/Q_{\rm s}$. The space charge parameter q is calculated for the peak value of $\Delta Q_{\rm sc}$ in the bunch center. Head-tail eigenfrequencies from the airbag theory are given in Fig. 1 with red dashed lines. The differences in the tune shifts between the Gaussian bunch and the airbag bunch are below ≈12% for $q = 5$, and below ≈5% in the case of $q = 20$. Especially for strong space charge the airbag theory Eq. (1) gives a surprisingly good prediction for the bunch eigenfrequencies, even in the case of a Gaussian bunch, which can also be seen using results of Refs. [5, 6].

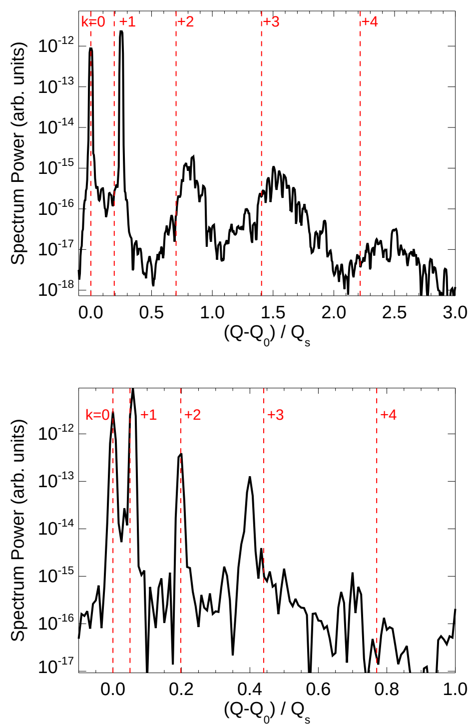

Figure 1: Transverse bunch spectrum from simulations for a Gaussian bunch with space charge: top plot $q = 5$, bottom plot $q = 20$. Red dashed lines are head-tail modes for the airbag bunch, Eq. (1). To clarify the notation we note that the eigenfrequencies without space charge are $(Q - Q_0)/Q_{\rm s} = 1$ for $k = 1$, $(Q - Q_0)/Q_{\rm s} = 2$ for $k = 2$, etc.

LANDAU DAMPING DUE TO SPACE CHARGE

In simulations for a Gaussian bunch we have observed Landau damping due to the effect of space charge. This kind of damping is provided by the variation of the space-charge tune shift along the bunch which causes a tune spread. Note that this is in opposite to a coasting beam, where space charge can not produce Landau damping of its own. A regular exponential decrease of the mode amplitude in time has been observed. Results of a space charge scan for the modes $k = 1$ and $k = 2$ are presented in Fig. 2.

In order to characterize bunch Landau damping for different head-tail modes and bunch parameters we consider an initial perturbation with an eigenmode. As a reasonable approximation, we trigger a k–mode of the airbag bunch [4] $\overline{x}_k(\tau) = A_0 \exp(-i\zeta\tau)\cos(k\pi\tau/\tau_{\rm b})$ and follow the time evolution of the perturbation. Here $\zeta = \xi Q_0/\eta$ is the normalized chromaticity, $\Delta Q_\xi/Q = \xi\Delta p/p$, η is the slip factor. Using our simulations it is possible to demonstrate that, on the one hand, the eigenmodes in a Gaussian bunch are very close to the airbag modes, and on the other hand, to compare these eigenmodes with the eigenfunctions obtained in Refs. [5]. For this, we start with the airbag $\overline{x}_k(\tau)$ for $k = 1$ and $k = 2$ with $q = 6$, $\xi = 0$ and observe the bunch dipole traces after approximately two damping times of $k = 2$. A comparison of these numerical traces (red lines) with the airbag eigenmodes (blue lines) and with the eigenfunctions from [5] (green lines) is presented in Fig. 3. For the airbag modes $\tau_{\rm b} = 4\sigma_z/R$ was chosen, where σ_z is the rms bunch length of the Gaussian bunch. Starting with the airbag $k = 1$ mode, there is no contribution from $k = 0$ and $k = 2$ because it is an odd function. For $k = 2$, the $k = 1$ mode is excluded because it is an even function, and we exclude the $k = 0$ mode by making the integrated bunch offset zero. Additionally we note that in the case of a large difference between the true eigenmode and an approximation, this difference is given by modes of higher k, which are Landau damped much faster than the mode considered.

Landau damping examples are shown in Fig. 4, where the momentum $M_k = \int \overline{x}_{\rm code}\cos(k\pi\tau/\tau_{\rm b}){\rm d}\tau$ is plotted turn-by-turn for two cases; here $\overline{x}_{\rm code}(\tau)$ is the simulation output. Simulations for bunch truncations between $2\sigma_z$ and $3\sigma_z$ of the half-length did not provide significant differ-

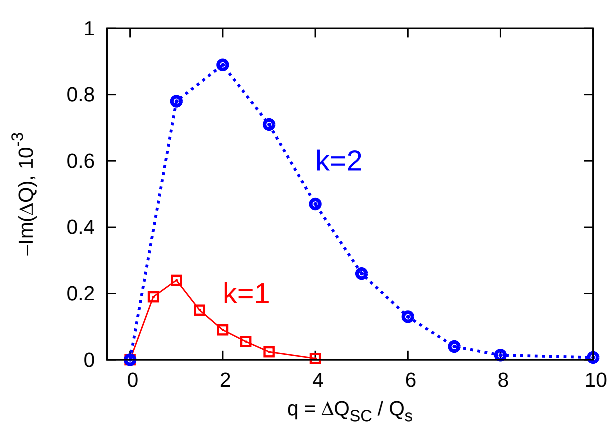

Figure 2: Damping decrement of the $k = 1$ and $k = 2$ modes obtained from simulations for a Gaussian bunch, $Q_{\rm s} = 0.01$.

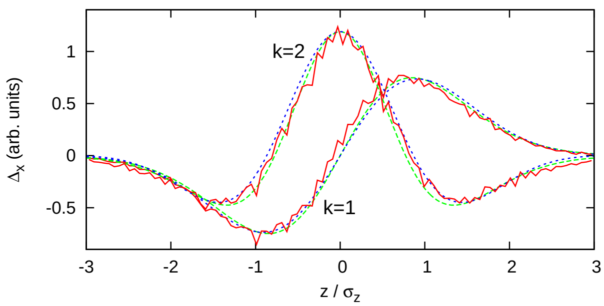

Figure 3: Dipole moments from simulations for a Gaussian bunch (red lines), theory eigenfunctions for a Gaussian bunch from Refs. [5] (green lines) and dipole moments with the beam offset for analytical eigenmodes of an airbag bunch (blue lines).

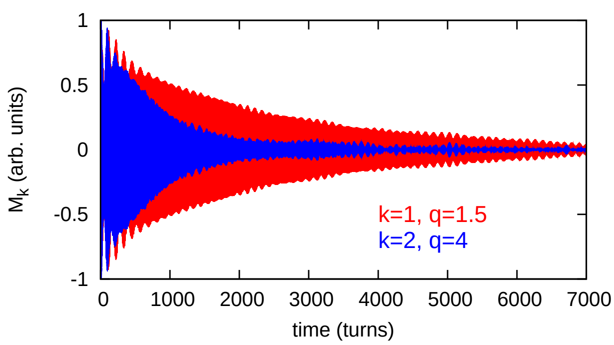

Figure 4: Simulations with a Gaussian bunch: examples for Landau damping due to the space-charge effect in a bunch for $k = 1$ (red line) and $k = 2$ (blue line), $Q_s = 0.004$.

ences. For stronger space charge, the longitudinal bunch tails above 2σ should be important for Landau damping.

The effect of Landau damping can be also observed on the bunch spectra in Fig. 1. The modes $k = 2$ and $k = 3$ are strongly damped at $q = 5$ (the top plot), which is not the case at $q = 20$ (see the bottom plot in Fig. 1).

We discuss now a physical interpretation of Landau damping observed in the simulations. The explanation is illustrated in Fig. 5. Due to the line density variation along the bunch, particles with different synchrotron amplitudes have different space-charge tune shifts. Thus we consider the incoherent spectrum related to a chosen k-mode. The upper boundary of the effective spectrum, which is given by particles with large synchrotron amplitudes, can be roughly estimated from the longitudinal average of the space-charge intensity. Assuming $\pm 2\sigma_z$ as the relevant area for the efficient space-charge tune spread, and taking into account the modulation by the synchrotron motion,

$$\Delta Q_{\max} \approx -0.23 Q_s q + k Q_s \ , \qquad (2)$$

see Fig. 5. This part of the incoherent spectrum is relevant for the resonant interaction with the coherent oscillation since it is close to the coherent frequency, see Fig. 5.

The lower boundary of the incoherent spectrum $\Delta Q_{\min}$, which is located well below the coherent line, corresponds to the strongest space charge tune shift and is represented by the particles with small synchrotron amplitudes. As we demonstrate in Fig. 1, the airbag theory is a good approximation for the space-charge frequency shifts of head-tail eigenmodes even for a Gaussian bunch. If we suggest that transverse Landau damping due to space charge should be active when the coherent head-tail mode lies within the effective spectrum, we would expect the area of Landau damping to be as illustrated in the bottom plot of Fig. 5. Note that the dependencies of the damping rates on q in Fig. 2 are qualitatively similar to the curves in the bottom plot of Fig. 5.

From this interpretation of bunch Landau damping it is easy to see that the resonant interaction between the coherent mode and individual particles should happen in the bunch tails, as it is also discussed in [5]. Indeed, particles in the effective spectrum close to $\Delta Q_{\max}$ have large synchrotron amplitudes and this part of the effective spectrum is the closest to the coherent frequency, see Fig. 5. In order to support this argumentations, we consider the trasverse rms beam size along the bunch in a damping simulation. The energy transfer from the coherent motion to the incoherent oscillations should lead to an increase of the individual betatron amplitudes, which, in turn, can increase the local rms beam size in bunch tails. Figure 6 shows the rms beam size distribution along the bunch at the simulation start, as the eigenmode $k = 2$ has been excited, and

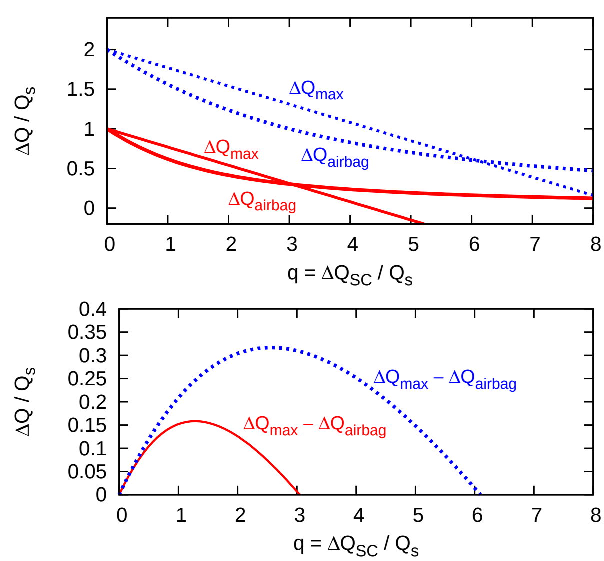

Figure 5: Illustration for the active area of the bunch Landau damping with the betatron tune shifts as functions of the space-charge parameter. The red lines correspond to the $k = 1$ mode, the blue dashed lines show the $k = 2$ mode. The parameter $\Delta Q_{\max}$ is the upper boundary of the effective spectrum and $\Delta Q_{\rm airbag}$ is the eigenfrequency of the head-tail modes from the airbag theory Eq. (1).

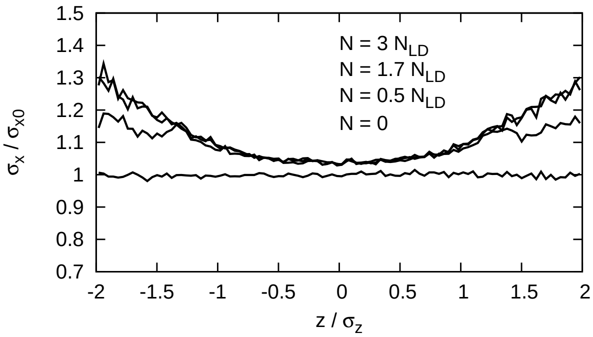

Figure 6: Time developement of the trasverse rms beam size along the bunch during Landau damping of the $k = 2$ mode at $q = 3$. Lines from bottom to top: $N/N_{\mathrm{LD}} = 0, 0.5, 1.7$ and 3; N_{LD} is the inverse damping decrement for this mode.

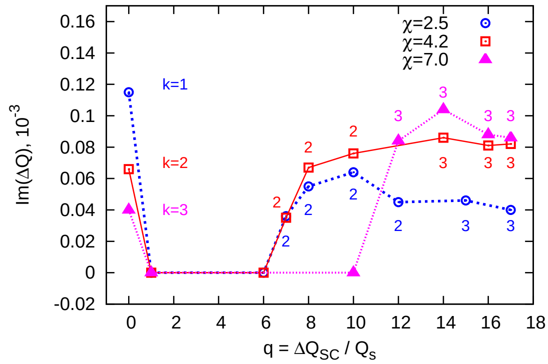

Figure 7: Growth rates of the most unstable head-tail modes obtained in simulations for a Gaussian bunch for three different head-tail phases $\chi = \zeta\tau_{\mathrm{b}}$ in a dependence on the space-charge parameter. The mode index k is given for each data point with the corresponding color.

three curves during the damping phase. The rms beam size increase in the bunch tails clearly indicates the resonant energy transfer.

An estimation for the Landau damping rate $\mathrm{Im}(\Delta Q) \sim -k^4 Q_{\mathrm{s}}/q^3$ has been obtained in Refs. [5] for strong space charge, which was defined as $q \gg 2k$. In this sense Landau damping demonstrated in our simulations relates to moderate space charge. Nevertheless, it is still interesting to compare some ultimate points of Fig. 2 with this estimation. This comparison shows a reasonable agreement for the dependence of the damping rate on the mode number and on the space charge parameter. However, the absolute values for $\mathrm{Im}(\Delta Q)$ from our simulations are smaller by approximately an order of magnitude. Further simulations for stronger space charge should shed more light on this issue.

HEAD-TAIL INSTABILITY WITH SPACE CHARGE

The code verification in the case of head-tail modes with space charge has been done using the airbag theory [4] with a short range wake, for details see Ref. [11].

Here, we consider a Gaussian bunch and with the wake function of the thick resistive wall, $W_{\mathrm{rw}}(z) \propto 1/\sqrt{z}$. The effect of the wake is taken into account in the single-bunch regime, multi-turn effects are not included. We consider a beam below transition, thus three exemplar negative chromaticities were considered. A synchrotron tune of $Q_{\mathrm{s}} = 0.01$ was chosen. Figure 7 summarizes results of our particle tracking simulations. Without space charge, at $q = 0$, we obtain the head-tail modes $k = 1$, $k = 2$ and $k = 3$ as the most unstable modes for the respective chromaticity, and examine the effect of increasing space charge for a constant impedance. A simulation is started with a non-disturbed bunch and the instability development is observed, thus this method provides only positive $\mathrm{Im}(\Delta Q)$, the simulation points inside of the stable areas are not shown for simplicity. The first important observations is

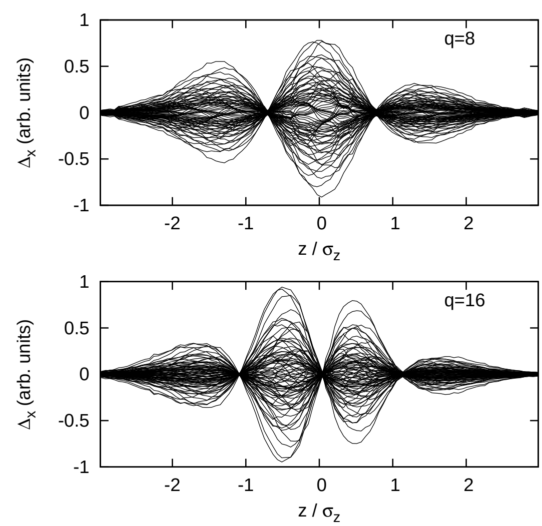

Figure 8: Examples of bunch dipole traces of head-tail instabilities from Fig. 7 for the head-tail phase $\chi = 4.2$. The upper plot for $q = 8$ demonstrates the $k = 2$ mode, while the lower plot for $q = 16$ shows the $k = 3$ mode.

that for moderate space charge Landau damping suppresses all the head-tail modes, in agreement with the results of the previous section. Secondly, for large ΔQ_{sc}, above the tune shift range for strong Landau damping, the growth rates do not experience significant changes with increasing space charge. At the same time, lowest-order modes ($k = 1$ and $k = 2$) leave the role of the strongest head-tail instability to higher-order modes. An example of the most prominent modes for different space charge strengths is shown in Fig. 8.

The observation of the growth rate saturation for strong space charge can be made in the analytic airbag theory [4] for the short-range exponential wake. However, Landau damping does not appear in this case due to a constant line density. On the other hand, the behavior of head-tail modes at strong space charge can be understood in terms of the calculations in Refs. [5], where the author argues that treating a wake as a perturbation provides the related tune shift in the form of a diagonal element of the wake operator,

$$\Delta Q = \frac{\kappa}{N_{\rm ion}\lambda_0 R} \int_0^{z_{\rm b}} {\rm d}z \int_z^{z_{\rm b}} {\rm d}s\, W(s-z) d_k(s) d_k^*(z)\ , \quad (3)$$

where $d_k(s) = \lambda(s)\overline{x}_k(s)$, $\int \lambda(s){\rm d}s = N_{\rm ion}$, $\kappa = \lambda_0 q_{\rm ion}^2/(4\pi\gamma m\omega_0^2 Q_0)$. In the case of the airbag bunch, where the eigenfunctions do not depend on space charge, this means that space charge has no effect on ΔQ. Also for an arbitrary bunch profile, e.g. Gaussian, the space-charge induced deformation of the eigenfunctions is small at strong space charge [5, 6]. Hence, the tune shift should saturate with increasing space charge and Eq. (3) should give an estimation for the mode growth rate at saturation. Using this expression for the beam parameters in our simulations, we obtain for the most unstable modes $\mathrm{Im}(\Delta Q) = 0.06 \times 10^{-3}$ ($k = 2$) for $\chi = 4.2$ and $\mathrm{Im}(\Delta Q) = 0.055 \times 10^{-3}$ ($k = 3$) for $\chi = 7.0$. Hence, we find a reasonable agreement between our simulations and the ansatz Eq. (3) for estimations of the head-tail instability growth rates at strong space charge.

CONCLUSIONS

The effect of space charge on the weak head-tail instability has been studied using particle tracking simulations. An analytical theory [4] for an airbag bunch with a short-range wake was employed for code validation and for the interpretation of realistic simulation results. The airbag theory gives a good prediction of the bunch eigenfrequencies, even in the case of a realistic Gaussian bunch. It has been demonstrated that the transverse eigenfunctions in a Gaussian bunch with space charge correspond to eigenmodes obtained in Refs. [5], which, interestingly, are also very close to the airbag [4] eigenmodes.

Landau damping of head-tail modes exclusively due to transverse space charge was demonstrated in simulations for a Gaussian bunch. The range of the space charge strength where Landau damping is mostly prominent depends on the mode index k and can be understood using a simple argumentation in terms of the mode eigenfrequency and the band of incoherent frequencies. The time develepment of the rms beam size confirmes that the resonant energy transfer of Landau damping happens in the bunch tails.

Simulations of the head-tail instability with space charge and the resistive-wall impedance showed that Landau damping can effectively stabilize the bunch at moderate space charge. In agreement with the airbag theory and with the results of Refs. [5], the instability growth rates saturate at strong space charge. Absolute values of growth rates are in a good agreement with the method [5] to estimate the head-tail instability growth rates at strong space charge using a diagonal element of the wake operator Eq. (3).

Applying our results to experimental observations in existing machines and to future machines we discuss two interesting examples. The head-tail instabilities observed in CERN PS correspond rather well to the Sacherer theory [2], as reported in [13]. Indeed, the space charge parameter for bunches in CERN PS is of order of $q \approx 150$, which is far above the range of Landau damping for observed modes. On the other hand, at such a strong space charge the growth rates of head-tail modes should be saturated and should not be very different from the no-space-charge estimation. Another example concerns the nominal parameters for uranium bunches in SIS100 [1, 14]. Here the space charge parameter lies in the range of $q \approx 20$. Landau damping might then give a significant contribution to the stability of the head-tail modes.

ACKNOWLEDGEMENTS

We are grateful to Alexey Burov (FNAL) for useful discussions. We thank Giovanni Rumolo (CERN) for our fruitful collaboration.

REFERENCES

[1] FAIR Baseline Tech. Report 2006: http://www.gsi.de/fair/reports/btr.html

[2] F. Sacherer, Proc. First Int. School of Particle Accelerators, Erice, p. 198 (1976)

[3] F. Sacherer, CERN Report CERN/SI-BR/72-5 (1972)

[4] M. Blaskiewicz, Phys. Rev. ST Accel. Beams **1**, 044201 (1998)

[5] A. Burov, Phys. Rev. ST Accel. Beams **12**, 044202 (2009); A. Burov, Phys. Rev. ST Accel. Beams **12**, 109901(E) (2009)

[6] V. Balbekov, Phys. Rev. ST Accel. Beams **12**, 124402 (2009)

[7] O. Boine-Frankenheim, V. Kornilov, Proc. of ICAP2006, 2-6 Oct., Chamonix Mont-Blanc, (2006)

[8] G. Rumolo and F. Zimmermann, Phys. Rev. ST Accel. Beams **5**, 121002 (2002)

[9] V. Kornilov, O. Boine-Frankenheim, and I. Hofmann, Phys. Rev. ST Accel. Beams **11**, 014201 (2008)

[10] A. Burov and V. Lebedev, Phys. Rev. ST Accel. Beams **12**, 034201 (2009)

[11] V. Kornilov and O. Boine-Frankenheim, Proceedings of ICAP2009, San Francisco (2009)

[12] O. Boine-Frankenheim and V. Kornilov, Phys. Rev. ST Accel. Beams **12**, 114201 (2009)

[13] E. Métral, G. Rumolo, R. Steerenberg and B. Salvant, Proceedings of PAC07, Albuquerque, New Mexico, USA, p. 4210 (2007)

[14] V. Kornilov, O. Boine-Frankenheim, I. Hofmann, GSI Technical Report GSI-Acc-Note-2008-006, GSI Darmstadt (2008)