LONGITUDINAL PEAK DETECTED SCHOTTKY SPECTRUM

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Abstract

The peak detected Schottky spectrum is used for beam observation in the CERN SPS and now also in the LHC. This tool was always believed, however without proof, to give a good picture of the particle distribution in synchrotron frequencies similar to the longitudinal Schottky spectrum of unbunched beam for revolution frequencies. The analysis shows that for an optimised experimental set-up the quadrupole line from the spectrum of the peak detected signal is very close to the synchrotron frequency distribution inside the bunch - much closer than that given by the traditional longitudinal bunched-beam Schottky spectrum. The analysis of limitations introduced by a realistic experimental set-up is based on its realisation in the SPS.

INTRODUCTION

The so called "peak detected Schottky" (PD Schottky) signal is a beam diagnostics tool developed and used extensively in the SPS [1, 2] since the late seventies, especially during $p\bar{p}$ operation. This technique has already been used in the LHC.

The theory of Schottky signals for unbunched and bunched beams both in the longitudinal and transverse plane is well developed (e.g. [3]-[5]). In the case of an unbunched beam the longitudinal Schottky spectra gives the particle distribution in revolution frequencies and therefore in particle momentum. For the bunched beam, information about the momentum spread (dispersion) can also be extracted in most cases [6].

The PD Schottky is a special case of the bunched beam longitudinal Schottky signal, different from the usual technique since it uses only one selected piece of information from the beam current - its (average) peak amplitude. This method is in fact closer to the unbunched beam Schottky spectra in that it also provides almost direct information about the particle distribution in oscillation frequency, which for an unbunched beam is the revolution frequency and for a bunched - the synchrotron frequency [7]. The deviation of the PD Schottky spectrum from the synchrotron frequency distribution is mainly defined by the experimental set-up.

PEAK DETECTED SIGNAL

The peak detected signal is used as a beam diagnostics tool to control beam lifetime and stability and can also be used as input for Schottky diagnostics. In the SPS and LHC a simple circuit, Fig. 1, consisting of fast switching diode and capacitor detects the peak of the bunch current signal from the wide-band pick-up. The spectrum is obtained using the dynamic spectrum analyser.

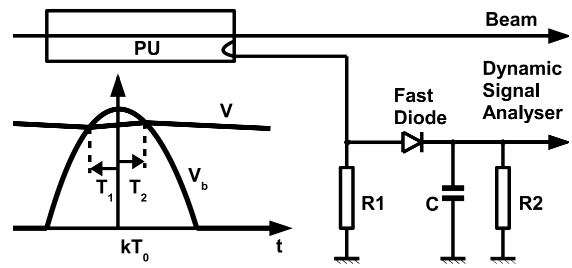

Figure 1: The simplified scheme of the bunch peak detection used for longitudinal Schottky signal in the SPS.

The parameters relevant to the Schottky measurements in the SPS and LHC and used in different examples below are presented in Table 1.

Table 1: The PD Schottky Parameters in the SPS and LHC

Parameter			SPS	LHC
revol. period	T_0	μs	23.0	88.9
RF harmonic	h		4620	35640
resistance	R_1	Ω	50	50
resistance	R_2	MΩ	1.0	1.0
capacitance	C	pF	240	920
PD decay time	$1/\mu$	μs	240	920
PD growth time	$1/\alpha$	ns	12	12
acquisition time	T_a	s	1.6	3.2

The fast diode is open during the bunch passage with current I_b, when $V_b = I_b R_1 \geq V$. The voltage V measured at resistance R_2 during this time interval $(-T_1, T_2)$ can be found from the following equation (valid for $R_2 \gg R_1$)

$$\frac{dV}{dt} = \alpha(V_b - V), \tag{1}$$

where $\alpha C = 1/R_1 + 1/R_2$. The solution of eq. (1), valid for $-T_1 < t < T_2$, is

$$V(t) = \alpha \int_{-T_1}^{t} V_b(t')\, e^{-\alpha(t-t')}\, dt' + V(-T_1) e^{-\alpha(t+T_1)}. \tag{2}$$

with additional conditions

$$V(-T_1) = V_b(-T_1), \tag{3}$$

$$V(T_2) = V_b(T_2). \tag{4}$$

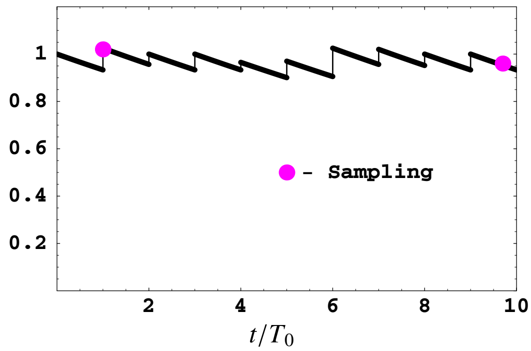

Figure 2: Illustration of the PD signal in time domain used for Schottky spectrum measurements.

The diode is off for the rest of the revolution period and

$$\frac{dV}{dt} = -\mu V, \qquad V(t) = V(T_2)\, e^{-\mu(t-T_2)}, \tag{5}$$

where $\mu = 1/(R_2 C)$. The voltage is sampled during this period (typically 2048 points), see Fig. 2.

After a transient period, in the quasi-stationary situation variations of T_1 and T_2 from turn to turn are small and defined only by statistical fluctuations (Schottky noise). Then in the first approximation (and for $T_1 \ll T_0$)

$$V(-T_1) \simeq V(T_2)\, e^{-\mu T_0}.$$

Taking into account solution (2) together with (3-4) allows the stationary values of T_1 and T_2 to be found as functions of beam (bunch length for a given particle distribution) and experimental set-up (α and μ) parameters. They are shown in Fig. 3 for a Gaussian line density with rms bunch length σ. One can see that in this model for the SPS set-up $T_1 \simeq T_2 \simeq \sigma$.

The signal detected at the moment t, after the k-th bunch passage, is $V_k e^{-\mu(t-t_k)}$, where $t_k = kT_0$ and

$$V_k = \Delta V_k + V_{k-1} e^{-\delta} = \sum_{q=0}^{k} \Delta V_{k-q}\, e^{-q\delta}. \tag{6}$$

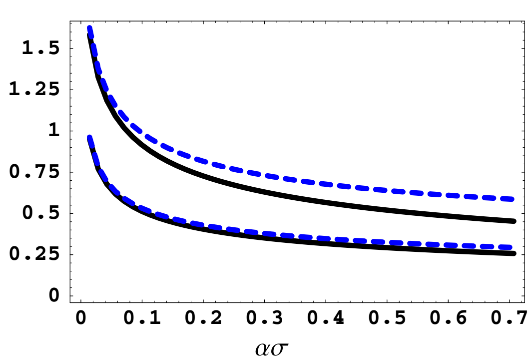

Figure 3: T_1/σ (dashed line) and T_2/σ (solid line) for $\mu T_0 = 0.07$ (two upper curves, SPS values) and $\mu T_0 = 0.01$ (lower curves) found for a Gaussian line density.

Here $\delta = \alpha(T_2 + T_1) + \mu T_0$ and the increase in voltage at each revolution turn, as follows from (2),

$$\Delta V_k = R_1 \alpha \int_{t_k - T_1}^{t_k + T_2} I_b(t_k - t')\, e^{-\alpha(t_k + T_2 - t')} dt'. \tag{7}$$

is proportional to the average bunch peak amplitude.

In the SPS set-up $\alpha T_2 = 0.083$, $\mu T_0 = 0.077$ and $\delta \simeq 0.25$ for $T_2 = T_1 = 1$ ns.

PEAK DETECTED SCHOTTKY SPECTRUM

A particle with phase $\psi_n = \Omega_n t + \psi_{n0}$ will be detected at the azimuthal position ϕ (RF phase) twice per synchrotron period $2\pi/\Omega_n$ at time t_1 and t_2, when

$$\psi_n = \Omega_n t_\phi + 2\pi m$$
$$\psi_n = \pi - \Omega_n t_\phi + 2\pi m,$$

where $m = \pm 0, 1, ..., \infty$, and

$$t_\phi = t_\phi(\mathcal{E}_n, \phi) = \int_0^\phi \frac{d\phi'}{\sqrt{2[\mathcal{E}_n - W(\phi')]}}.$$

We consider below a single RF system with potential well $W(\phi) = \Omega_{s0}^2(1 - \cos\phi)$, where $\Omega_{s0} = 2\pi f_{s0} = 2\pi/T_{s0}$ is a linear synchrotron frequency and for a particle with phase oscillation amplitude ϕ_a the synchrotron energy $\mathcal{E} = W(\phi_a)$. The particle contribution to a bunch current at ϕ is

$$I_n(t,\phi) = \frac{e}{2} \sum_m [\delta(t - t_1) + \delta(t - t_2)] =$$

$$= \frac{e\Omega_n}{4\pi} \sum_m [e^{im\Omega_n t_\phi} + e^{im(\pi - \Omega_n t_\phi)}]\, e^{-im(\Omega_n t + \psi_{n0})}. \tag{8}$$

Collecting contributions at ϕ from all particles the increase in voltage (7) can be written in the form

$$\Delta V_k = \frac{e}{2\pi} B \sum_n \sum_{m=-\infty}^{\infty} \Omega_n A_m(\mathcal{E}_n)\, e^{-im(\Omega_n t_k + \psi_{n0})}, \tag{9}$$

where $B = 2R_1 \alpha T e^{-\alpha T}$ and $A_m = A_m(\mathcal{E}_n)$ is

$$A_m = \frac{1}{2\Phi} \int_{-\Phi_{max}}^{\Phi_{max}} e^{\frac{\alpha\phi}{h\omega_0}} [e^{im\Omega_n t_\phi} + e^{im(\pi - \Omega_n t_\phi)}] d\phi.$$

Here $\Phi = h\omega_0 T_2$ and the limit of integration Φ_{max} is a function of $\mathcal{E}_n$

$$\Phi_{max} = \Phi \quad \text{for } \mathcal{E}_n \geq W(\Phi), \tag{10}$$
$$\Phi_{max} = \phi_a(\mathcal{E}_n) \quad \text{for } \mathcal{E}_n \leq W(\Phi), \tag{11}$$

since particles with synchrotron energy $\mathcal{E}_n > W(\Phi)$ contribute to the whole range of measurement $0 \leq \phi \leq \Phi$ while for particles with $\mathcal{E}_n < W(\Phi)$ the contribution is restricted to the range $0 \leq \phi \leq \phi_a(\mathcal{E}_n)$ with $\phi_a(\mathcal{E}_n)$ determined by equation $\mathcal{E}_n = W(\phi_a)$. As it will be shown below the shape of the Schottky signal is mainly affected by these functions.

Examples of A_m as a function of the synchrotron oscillation amplitude ϕ_a for different multipoles m and values of Φ in a single RF system are shown in Fig. 4. For a single RF system and $\Phi \ll \pi$ functions A_m can be calculated analytically:

$$\begin{array}{lll} A_1 = & \frac{\alpha_\phi \phi_a}{3}\left(\frac{\phi_a}{\Phi}\right) & \text{for } \phi_a \le \Phi \\ A_1 = & \frac{\alpha_\phi \Phi}{3}\left(\frac{\Phi}{\phi_a}\right) & \text{for } \phi_a \ge \Phi \\ A_2 = & \frac{1}{3}\left(\frac{\phi_a}{\phi}\right) & \text{for } \phi_a \le \Phi \\ A_2 = & 1 - \frac{2}{3}\left(\frac{\Phi}{\phi_a}\right)^2 & \text{for } \phi_a \ge \Phi \end{array}$$

where $\alpha_\phi = \alpha/(h\omega_0)$. Maximum value of A_1, $\alpha_\phi \Phi/3$, is usually much less than 1, maximum of A_2, and it is achieved at synchrotron oscillation amplitude $\phi_a = \Phi$.

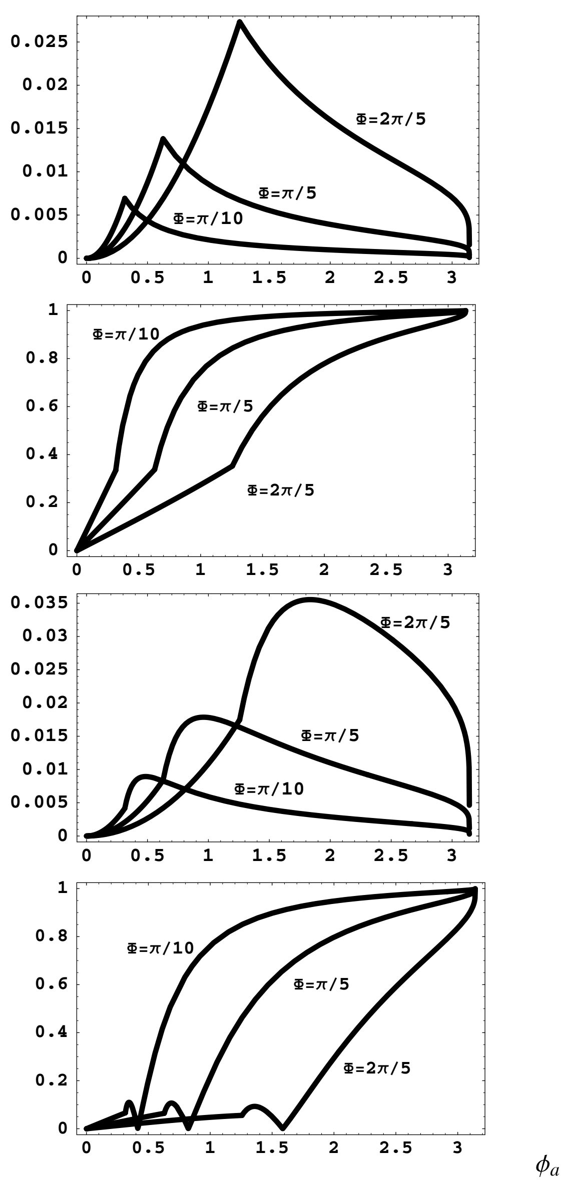

Figure 4: Function $|A_m(\phi_a)|$ for m = 1, 2, 3 and 4 (from top to bottom) calculated for $\alpha_\phi = 0.07$ and $\Phi = 2\pi/5$, $\Phi = \pi/5$ and $\Phi = \pi/10$.

Using expression (9) the summation over q in (6) can be performed and finally for the PD signal we obtain

$$V_k = \frac{eB}{2\pi} \sum_n \sum_m \Omega_n \, A_m(\mathcal{E}_n) \, Q_m(\Omega_n) \, \mathrm{e}^{-im(\Omega_n t_k + \psi_{n0})}. \quad (12)$$

For large enough k, so that $k\delta \gg 1$, the function Q_m is

$$Q_m(\Omega_n) = \sum_{q=0}^{k} \mathrm{e}^{im\Omega_n T_0 q - q\delta} \simeq \frac{1}{1 - \mathrm{e}^{im\Omega_n T_0 - \delta}}.$$

Averaging over initial phase ψ_{n0} (similar to that for unbunched beam Schottky, see e.g. [4]) and replacing the sum over all particles by the integral over the distribution function $F(\Omega) = dN/d\Omega$ (normalised to unity) the power spectral density of the PD signal can be written in the form [7]

$$P(\omega) = \frac{P_0}{\Omega_{s0}^2} \sum_{m=1}^{\infty} \int \Omega^2 F(\Omega) \, |A_m(\Omega)|^2 |Q_m(\Omega)|^2 \, S^2 d\Omega, \quad (13)$$

where $P_0 = e^2 N f_{s0} B^2$. Function S depends on the acquisition time T_a with

$$S^2 = |S(\omega - m\Omega)|^2 = \frac{2T_a}{T_{s0}} \frac{\sin^2[(\omega - m\Omega)T_a/2]}{[(\omega - m\Omega)T_a/2]^2}.$$

Taking into account the structure of the PD signal, Fig. 2, and the fact that measurements are done at some sampling rate t_s which is different from T_0, function S becomes

$$|S|^2 = \mathrm{e}^{-2\mu t_0} \frac{2t_s^2}{T_{s0} T_a} \frac{\sin^2[(\omega - m\Omega)T_a/2]}{\sin^2[(\omega - m\Omega)t_s/2]}, \quad (14)$$

where t_0 is a time of signal acquisition (sampling) after the bunch passage and discrete frequencies are replaced by a continuous spectrum. Since $t_0 < T_0$ we have $\mathrm{e}^{-2\mu t_0} \simeq 1$. The signal has some additional noise if sampling is not at a multiple of the revolution period T_0 [7].

For the SPS experimental set-up the distortion of the PD Schottky spectra due to function $|Q_m(\Omega)|^2$, which can be also written in the form

$$|Q_m(\Omega)|^2 = \frac{\mathrm{e}^{\delta}/2}{\cosh\delta - \cos(m\Omega T_0)},$$

is very small. A few examples of this function for different δ are given in Fig. 5.

The Schottky power spectrum is mainly affected by the form-factor $|A_m(\Omega)|^2$. It is obvious from Fig. 4 that only a quadrupole line $m = 2$ can represent well the synchrotron frequency distribution inside the bunch. All odd multipoles are also significantly suppressed in amplitude, indeed $(A_1/A_2)^2 \sim (\alpha_\phi \Phi)^2$ and in the SPS set-up $\alpha_\phi \Phi \le 0.1$. Examples of A_1 and A_2 as functions of the synchrotron oscillation frequency Ω are shown in Fig. 6 for different Φ. The smaller the integration time Φ the closer the shape of the measured quadrupole band is to the synchrotron frequency distribution.

Examples of the quadrupole line in the PD Schottky spectrum for Gaussian distribution function with $\sigma_\phi =$

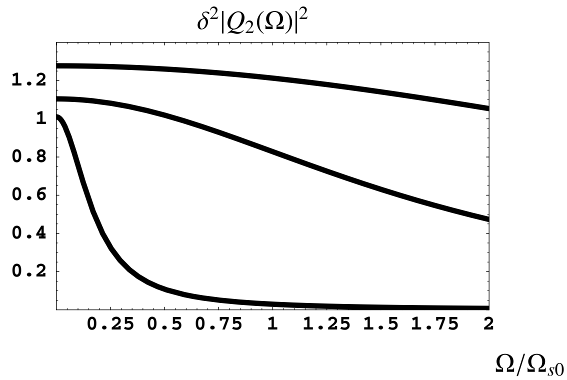

Figure 5: Function $\delta^2|Q_2(\Omega)|^2$ for $\delta = 0.25$ (top), $\delta = 0.1$ (middle), $\delta = 0.01$ (bottom) with $T_0\Omega_{s0} = 0.03$.

$h\omega_0\sigma = \pi/4$ and $\Phi = \pi/8$ (top) and $\Phi = \pi/4$ (bottom) are shown in Fig. 7. These spectra can be compared with the corresponding distribution function in synchrotron frequency as well as with Schottky spectra of the "ideal" case ($A_2 = 1$ in (13), only taking the finite acquisition time T_a into account). As Q_m is a fairly flat function of Ω for not too small δ, the measured PD Schottky spectrum deviates from $F(\omega/m)$, mainly due to $A(\Omega)$. The distortion is smaller for smaller Φ and in the limit of $t_\phi = 0$, $A_m = (1 + (-1)^m)/2$, so that only even multipoles (reducing as $1/m$ in amplitude) are present in the spectrum. The quadrupole line gives the best reproduction of particle distribution.

For comparison, the dipole sideband at revolution harmonic ph in the traditional Schottky spectra (e.g. [4]), calculated by replacing the function $(Q_mA_m)^2$ in $P(\omega)$ by the

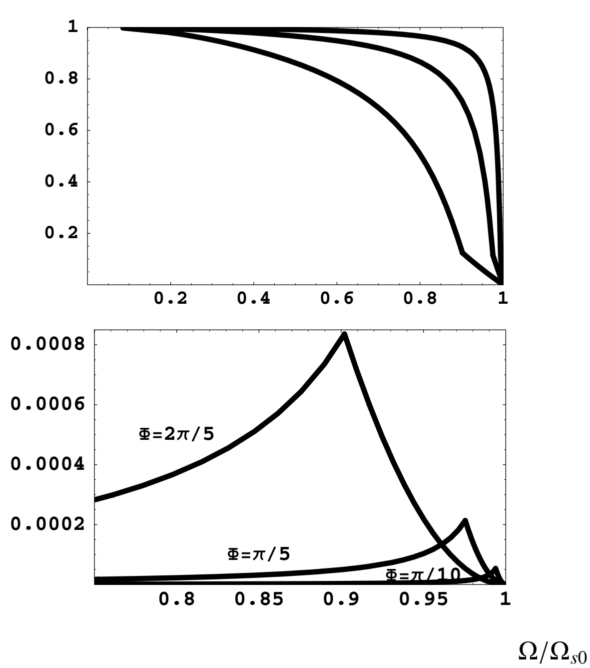

Figure 6: Top: functions $|A_2(\Omega)|^2$ for $\Phi = \pi/10$ (top curve), $\Phi = \pi/5$ (middle) and $\Phi = 2\pi/5$ (bottom). Bottom: $|A_1(\Omega)|^2$ for the same Φ and $\alpha_\phi = 0.07$.

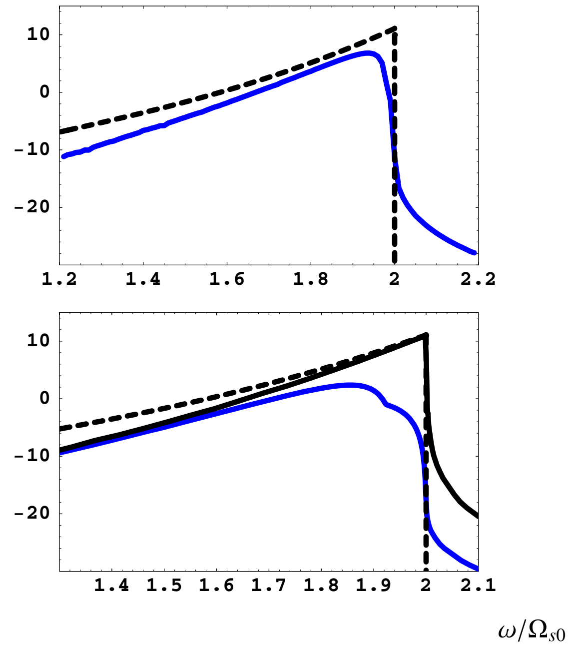

Figure 7: Quadrupole PD Schottky band P/P_0 (blue) for $\Phi = \pi/8$ (top) and $\Phi = \pi/4$ (bottom), $\sigma_\phi = \pi/4$, measurement time $T_a = 320\,T_{s0}$. In all figures logarithmic scale (dB), $2\Omega_{s0}/mF(\omega/m)$ as dashed line, solid line (bottom figure) - ideal case with $A_2 = 1$.

Bessel function $J_m^2(p\phi_a)$, with $p = 5$ in the SPS and $p = 12$ in LHC [9], is shown in Fig. 8. One can see that this Schottky line would give a very good measurement of a zero-amplitude synchrotron frequency, but has a very perturbed (modulated) presentation of a synchrotron frequency distribution, at least for $p \gg h$, which is usually the case due to the wish to have Schottky measurements at frequencies significantly higher than bunch spectrum.

The calculated spectrum finally can be compared with the measured PD Schottky spectrum, Fig. 9. For low intensity beam (top figure), the amplitude of the dipole and sextupole lines is always much smaller than of a quadrupole line as should be expected for a small integration distance Φ and parameter α_ϕ. This distance is further reduced if the finite reaction time of the fast diode is taken into account. The shape of the quadrupole line is close to calculated and the octupole line even has the double hump as functions A_4 in Fig. 4. The example of measurements done for high intensity bunches is shown in Fig. 9 (bottom). The first measurements of Schottky spectrum in LHC at 450 GeV with $\sigma_\phi = \pi/7$ can be found in ([8]).

The PD Schottky spectrum has been used as a powerful beam diagnostic in many different studies, such as measurements of the quadrupole frequency shift with intensity (for evaluation of the low-frequency inductive impedance of the SPS), beam dynamics in a double RF system and beam loss studies. From example in Fig. 10 one is able to see how an external excitation (at 790 Hz) can depopulate and even creates holes in certain areas of the bunch

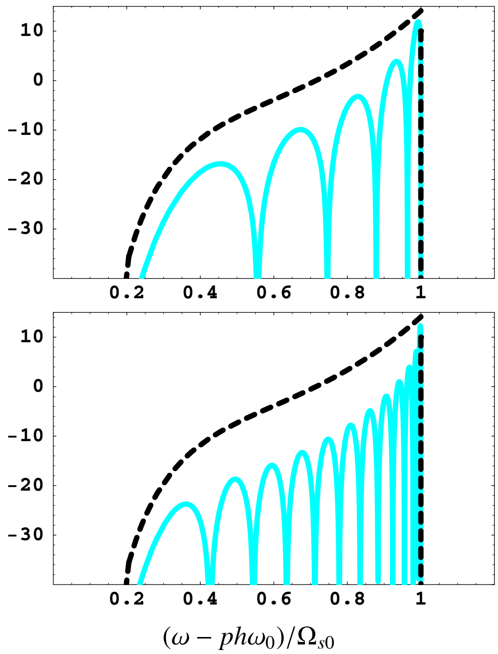

Figure 8: Dipole sidebands (a.u.) in the traditional Schottky spectrum (cyan) for $T_a = \infty$, $p = 5$ (top, SPS 1 GHz system) and $p = 12$ (bottom, LHC 4.8 GHz system). In all figures logarithmic scale (dB), $2\Omega_{s0}/mF(\omega/m)$ as dashed line, $\sigma_\phi = \pi/4$.

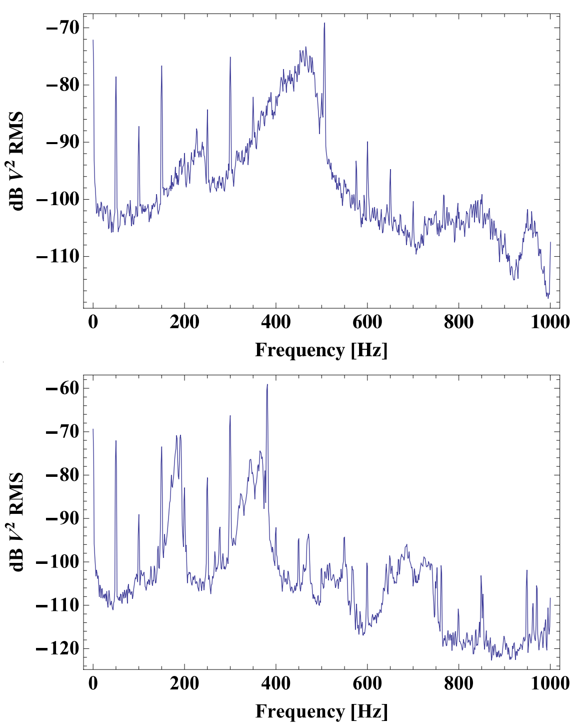

Figure 9: Measured PD Schottky spectrum in the SPS at 26 GeV/c. Top: low intensity bunch with $\sigma_\phi = \pi/4$, $f_{s0} = 240$ Hz. Bottom: one of four bunches spaced by 525 ns with average intensity of $\sim 8 \times 10^{10}$ during the coast at 270 GeV/c, $f_{s0} = 192$ Hz, $\sigma_\phi = \pi/12$.

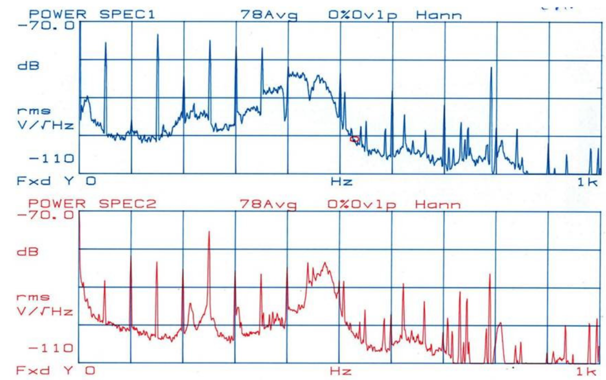

Figure 10: Measured PD Schottky spectrum in the SPS at 26 GeV/c for the nominal LHC batch (72 bunches) in the ring, 17 min after the beginning of the store, at the head (top) and the tail of the batch (bottom), $f_{s0} = 257$ Hz [2].

[2]. Removal of this source from the feedback electronics, improved beam transmission. Lifetime of bunches at the tail of the batch was less than at the head, the difference in synchrotron frequency distribution is also visible from Schottky spectrum.

SUMMARY

The quadrupole line of the PD Schottky spectrum represents the particle distribution in synchrotron frequency modified by nonlinearity of the synchrotron frequency (factor Ω^2) and experimental set-up (function A_2). The deviation introduced by the latter is mainly defined by the distance (phase Φ) over which the bunch peak amplitude averaging is performed. The connection between Φ and bunch parameters obtained allows the existing Schottky measurements to be understood and possible improvements to be foreseen.

ACKNOWLEDGMENTS

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