COUPLING IMPEDANCES OF A SHORT INSERT IN THE VACUUM CHAMBER

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Abstract

We have developed a theory to calculate both longitudinal and transverse impedances of a resistive short (typically shorter than the chamber radius) insert with cylindrical symmetry, sandwiched by perfectly conductive chambers on both sides. It is found that unless the insert becomes extremely thin (typically a few nm for a metallic insert) the entire image current runs on the thin insert, even in the frequency range where the skin depth exceeds the insert thickness, and therefore the impedance increases drastically from the conventional resistive-wall impedance. In other words, the wake fields do not leak out of the insert unless it is extremely thin.

INTRODUCTION

In proton synchrotrons, the inner surface of a short ceramic break is normally coated by a thin (typically about ten nm) Titanium Nitride (TiN) to suppress the secondary emission of electrons. The skin depth can be larger than the thickness of the TiN coating in low frequency, and the wake fields may interact with the outside world through the coating. It is thus important to construct a theory of resistive insert taking into account its thickness effects.

We have developed a theory to describe the impedance of a short insert by generalizing a theory of a gap, where the respective components are sandwiched by perfectly conductive chambers[1, 2]. The main difference between the gap and the insert is that the insert has a finite skin depth, and this skin depth effect will modulate how wake fields propagate in the chamber. Main objective of this paper is to study how the impedance of the insert will change from that of the conventional resistive-wall theory to that of a gap, when the thickness of the insert is changed compared to the skin depth.

In numerical examples shown in figures, unless specified otherwise, we consider a beam pipe radius $a = 5$ cm with an insert of length $g = 8$ mm, and conductivity $\sigma_c = 6 \times 10^6/\Omega$ m. This can be a model for a short ceramic break with TiN coating in a copper beam pipe.

LONGITUDINAL IMPEDANCE

The longitudinal coupling impedance of the resistive short insert is expressed as

$$Z_{L,insert} = \frac{Z_0}{j\beta ak I_0^2(\bar{k}a)}/[Y_{pole} + Y_{cut} - \frac{2\pi\sqrt{jk\beta Z_0\left(\sigma_c + j\frac{k\beta\epsilon'}{Z_0}\right)}\tanh\sqrt{jk\beta Z_0\left(\sigma_c + j\frac{k\beta\epsilon'}{Z_0}\right)}t}{k^2\beta^2 g}], \tag{1}$$

where

$$Y_{pole} = -\sum_{s=1}^{\infty}\frac{4\pi a(1 - e^{-j\frac{b_s g}{2a}})}{g b_s^2}, \tag{2}$$

$$Y_{cut} = -\frac{\int_0^\infty d\zeta \frac{4(1-e^{-jw\sqrt{k^2\beta^2+\frac{\zeta}{(a+t)^2}}})}{\zeta\left(k^2\beta^2+\frac{\zeta}{(a+t)^2}\right)H_0^{(1)}(e^{j\frac{\pi}{2}}\sqrt{\zeta})H_0^{(2)}(e^{j\frac{\pi}{2}}\sqrt{\zeta})}}{g\pi(a+t)}$$
$$\simeq \frac{2\sqrt{2}(1-j)}{\sqrt{k\beta g}}, \tag{3}$$

$Z_0 = 120\pi$, $k = \omega/c\beta$, $\bar{k} = k/\gamma$, ϵ' is the relative dielectric constant of the insert. Here, $b_s^2 = k^2\beta^2a^2 - j_{0,s}^2 = -\beta_s^2$, $j_{0,s}$ are s-th zeros of $J_0(z)$ and $H_m^{(1)}(z)$ is the Hankel function of the first kind. We should notice that b_s approaches $-j\beta_s$ for $j_{0,s} > k\beta a$.

At first, let us check the accuracy of the formula Eq.(1) by comparing with ABCI results [3]. Recently, ABCI has been upgraded and can now handle a resistive material inside a cavity. We choose the chamber thickness $t = 2$mm, the relative dielectric constant of the insert $\epsilon' = 10$ and its conductivity $\sigma_c = 50[/\Omega\text{m}]$. In order to simulate correctly, the mesh size should be sufficiently smaller than the chamber thickness. In our case, it is divided into ten meshes. At high frequency where the skin depth becomes smaller than the mesh size, ABCI cannot accurately simulate field behavior. That is about 1GHz for the present choice of mesh size. At higher frequency where the skin depth is smaller than the mesh size, the theory predicts the insert impedance better than the ABCI [1]. In ABCI, we put a huge cavity in the outside of the insert to simulate open space. Figure 1 shows the comparison results of the real (left) and the imaginary (right) parts of the impedance, respectively. Quite good agreements can be seen between the two results.

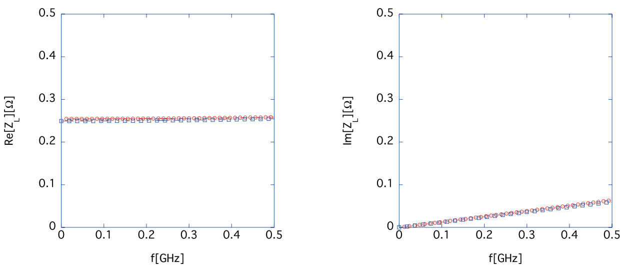

Figure 1: Comparison of the real (left) and the imaginary (right) parts of the longitudinal impedance, calculated by the present theory (red curves) and ABCI (blue curves), respectively.

Frequency-dependence and Length-dependence of the Impedance

In this subsection, we assume that the chamber thickness t satisfies the condition :

$$t > t_{min} \equiv (\frac{4g}{\pi^2 Z_0^3 \sigma_c^3})^{\frac{1}{4}}. \tag{4}$$

We exclude an extremely thin insert case here. This assumption allows us to neglect the effect from "radiation terms" such as Y_{pole} and Y_{cut} in Eq.(1) in the low frequency region where the skin depth exceeds the insert thickness t. The thickness t_{min} is typically a few ten nm for a metallic insert.

Krinsky *et al* and Stupakov [4, 5] studied the impedance of a short insert. Their results indicate that when $g \ll (Z_0 \sigma a^4/4\pi)^{1/3}$ and

$$f \gg f_D \equiv \frac{c}{2\pi}\sqrt{\frac{2Z_0\sigma_c}{g}}, \tag{5}$$

(the frequency f_D is typically of the order of THz in our short insert in MKS unit), then

$$Z_L \simeq \frac{(1-j)2Z_0\sqrt{g}}{2\pi a\sqrt{\pi k}}, \tag{6}$$

and is proportional to $\sqrt{g}$.

Let us consider the case that the thickness of the insert is larger than $2^{1/2}\pi^{3/4}t_{min}$ and see if our theory can reproduce the formula Eq.(6) in the extremely high frequency region $f \gg f_D$. In this frequency region, we may take a limit of t to infinity in Eq.(1), and the following inequality can be applied to Eq.(1):

$$|Y_{cut}| \gg \left|\frac{\pi\sqrt{jk\beta Z_0\sigma_c}}{k^2\beta^2 w} + Y_{pole}\right|. \tag{7}$$

Then, Eq. (1) becomes

$$Z_{L,insert} \simeq \frac{(1-j)2Z_0\sqrt{g}}{2\pi a I_0^2(\bar{k}a)\sqrt{\beta\pi k}}. \tag{8}$$

Specifically for a relativistic beam, Eq.(8) reproduces Eq.(6). These results show that the impedance decreases in proportional to $k^{-1/2}$ in the extremely high frequency, as predicted by the diffraction theory [6].

In the intermediate region of $f \ll f_D$ where the skin depth δ is still smaller than the insert thickness t, we can apply the following inequality to Eq.(1):

$$|Y_{pole} + Y_{cut}| \ll \left|\frac{\pi\sqrt{jk\beta Z_0\sigma_c}}{k^2\beta^2 w}\right|. \tag{9}$$

We then obtain the conventional formula of the resistive-wall impedance for a relativistic beam [7]:

$$Z_{L,insert} \simeq gZ_0\sqrt{\frac{2\omega}{cZ_0\sigma_c}}\frac{1+j}{4\pi a}, \tag{10}$$

which is proportional to the length of the insert g.

In the low frequency region where the skin depth exceeds the insert thickness t:

$$f < f_\delta \equiv \frac{c}{\pi Z_0 \sigma_c t^2}, \tag{11}$$

but the effect from radiation terms such as Y_{pole} and Y_{cut} are still negligible in Eq.(1), we obtain

$$\Re[Z_{L,insert}] \simeq \frac{g}{2\pi a\sigma_c t}. \tag{12}$$

When the thickness of the insert is smaller than $2^{1/2}\pi^{3/4}t_{min}$ but larger than t_{min}, Eq.(12) becomes valid all the way up to f_D.

Dependence of the Insert Impedance on its Thickness

Before studying the thickness dependence of the insert impedance, let us study the thickness dependence of the resistive-wall impedance in order to compare them with our results afterwards. We numerically calculate the resistive-wall impedance for different thicknesses of the chamber by borrowing the general formulae of the resistive-wall impedance with finite thickness from Metral *et al*'s recent work [8, 9]. The results for a relativistic beam are shown in Fig.2. The red, the blue and the black lines show the cases that the insert thickness t is equal to infinity, 10μm and 1μm, respectively. The impedance starts to deviate from that for the infinitely thick chamber when the skin depth exceeds the chamber thickness. Apparently, the wake fields leak out at low frequency. The dependence of Z_L on the conductivity σ_c, the frequency f and the chamber thickness t, for the case that the skin depth exceeds t, can be approximately written as

$$\Re[Z_L] \simeq \frac{2g\pi Z_0^2\sigma_c f^2 t^3}{3ac^2}. \tag{13}$$

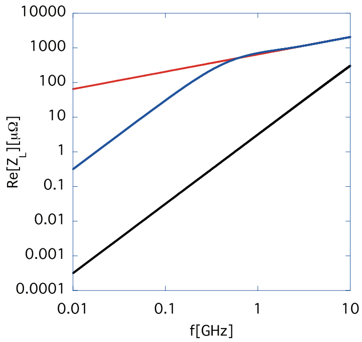

Figure 2: The dependence of the real part of the longitudinal resistive-wall impedance of a uniform beam pipe (no insert) on the thickness of the chamber.

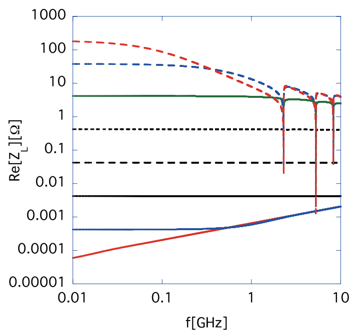

Figure 3: The thickness dependence of the longitudinal impedance of the insert in the relativistic beam case.

Contrary to our intuition, the impedance becomes larger as the conductivity of the material σ_c increases.

Now, let us discuss the properties of the impedance of the insert by changing the thickness of it. The thickness dependence of the real part of the insert impedance obtained by Eq.(1) is shown in Fig.3. The red, the blue, the black, the black dashed, the black dot, the green and the blue dashed and the red dashed lines represent the cases that the thickness t is equal to 100μm, 10μm, 1μm, 100 nm, 10 nm, 1 nm, 100 pm and 10 pm, respectively.

When the skin depth is smaller than the thickness of the insert but the frequency f is lower than f_D, the impedance of the insert is identical to the resistive-wall impedance given by Eq.(10) (see the results for the case that t is equal to 100μm).

As we find from the result of $t = 10\mu$m in Fig.3, if the skin depth exceeds the insert thickness ($f_\delta \simeq 0.42$ GHz in this case. See Eq.(11)), the real part of the impedance becomes independent of the frequency. The imaginary part is still inductive for the insert with this thickness. This indicates that the whole wall current runs in the thin insert, despite of the fact that the skin depth exceeds the insert thickness in most of frequencies. In other words, the beam current is completely shielded by the wall current in the insert, and the wake fields do not propagate out of the chamber. If this picture is correct, the real part of impedance should be equal to the resistance of the wall current Z_{wall}. Actually, the results of $t = 1\mu$m to $t = 100$ nm (even including the result of 10 nm that is smaller than t_{min}) described in Fig.3 are equal to the resistance of the wall current Z_{wall}:

$$Z_{wall} = \frac{g}{\sigma_c \pi((a+t)^2 - a^2)} \simeq \frac{g}{2\pi a \sigma_c t}. \qquad (14)$$

This behavior of the insert impedance is quite different from that of the resistive-wall impedance of the chamber with finite thickness for a relativistic beam, which was discussed in the first paragraph of this subsection.

When the thickness of the insert is extremely thin like $t \ll t_{min}$, the situation is quite different from the above case. The results of $t = 1$nm to $t = 10$pm in Fig.3 correspond to this case. The frequency f_D and the skin depth δ are no longer dominant parameters. The new parameter:

$$f_c \equiv \frac{\sigma_c^2 Z_0^2 t^2 c}{4\pi g}, \qquad (15)$$

plays a more important role in the impedance. In the frequency region $f \ll f_c$, the contribution from the wall current dominates in the impedance. In the rest of the frequency, the radiation effects become dominant contributions. The dips for these cases in Fig.3 correspond to the cut-off frequencies of the chamber. The imaginary part of impedance becomes capacitive, which is opposite to the result of $t > t_{min}$.

The physical reason of why the whole wall current tends to run on the thin insert except for the extremely thin insert case is that the nature tries to minimize the energy loss of a beam, which is smaller when the wall current runs on the thin insert with large resistance than it converts to the radiation out to free space(= gap impedance). When $t \ll t_{min}$, the real part of the correct impedances using the present theory is smaller than the hypothetical impedances calculated by extending the simple formula (14) to these extreme thicknesses. The impedance i.e. the energy loss of a beam becomes small by the wall current converting to outer radiation than staying in the extremely thin insert.

TRANSVERSE IMPEDANCE

The expression for the transverse impedance $Z_{T,insert}$ is given as

$$Z_{T,insert} \simeq -\frac{jZ_0}{2\beta\gamma^2 a I_1^2(\bar{k}a)} /$$

$$\left(-\frac{2\pi\sqrt{jk\beta Z_0 \sigma_c}}{k^2\beta^2 g} \tanh\sqrt{jk\beta Z_0 \sigma_c} t + Y'_{pole} + Y'_{cut} \right), \qquad (16)$$

where

$$
\begin{aligned}
Y'_{pole} = & \sum_{s=1}^{\infty}\left[-\frac{4\pi a(1-e^{-j\frac{b_{1,s}g}{2a}})}{g b_{1,s}^2}\right. \\
& \left.+\frac{4\pi a J_1(j'_{1,s})(1-e^{-j\frac{b'_{1,s}g}{2a}})}{k^2\beta^2 a^2 g j'^2_{1,s} J''_1(j'_{1,s})}\right] \\
& -\frac{4\pi H_1^{(2)}(h'_{1,0})(1-e^{-j\frac{d'_{1,0}g}{2(a+t)}})}{k^2\beta^2(a+t)g h'^2_{1,0} H''^{(2)}_1(h'_{1,0})} \\
& +\frac{4\pi(1-e^{-j\frac{k\beta g}{2}})}{g k^2\beta^2}\left(\frac{H_1^{(2)}(h'_{1,0})}{(a+t)h'^2_{1,0}H''^{(2)}_1(h'_{1,0})}-\frac{1}{2a}\right),
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
Y'_{cut} = & -\frac{\int_0^\infty d\zeta \frac{4(1-e^{-j\frac{g}{2}\sqrt{k^2\beta^2+\frac{\zeta}{(a+t)^2}}})}{\zeta\left(k^2\beta^2+\frac{\zeta}{(a+t)^2}\right)H_1^{(1)}(e^{j\frac{\pi}{2}}\sqrt{\zeta})H_1^{(2)}(e^{j\frac{\pi}{2}}\sqrt{\zeta})}}{\pi(a+t)g} \\
& +\int_0^\infty d\zeta \frac{\frac{4(e^{-j\frac{k\beta g}{2}}-e^{-j\frac{g}{2}\sqrt{k^2\beta^2+\frac{\zeta}{(a+t)^2}}})}{\zeta^2 H'^{(1)}_1(e^{j\frac{\pi}{2}}\sqrt{\zeta})H'^{(2)}_1(e^{j\frac{\pi}{2}}\sqrt{\zeta})}}{k^2\beta^2(a+t)\pi g} \\
\simeq & \; 4\tan^{-1}\frac{1}{\sqrt{jkg}} \\
& +\frac{-2+4\sqrt{1+jk\beta g}\sinh^{-1}\frac{e^{-j\frac{\pi}{4}}}{\sqrt{k\beta g}}+2e^{-j\frac{k\beta g}{2}}}{k^2\beta^2(a+t)^2\sqrt{1+jk\beta g}},
\end{aligned}
\tag{18}
$$

$b_{1,s}=\sqrt{k^2\beta^2a^2-j_{1,s}^2}$, $b'_{1,s}=\sqrt{k^2\beta^2a^2-j'^2_{1,s}}$, $d'_{1,0}=\sqrt{k^2\beta^2a^2-h'^2_{1,0}}$, $j_{n,s}$ are the s-th zeros of $J_n(z)$, $j'_{1,s}$ are the s-th zeros of $J'_1(z)$ and $h'_{1,0}=0.501184+j0.643545$: the 0-th zero of $H'^{(2)}_1(z)$(the differential of the Hankel function of the second kind). We should notice that $b'_{1,s}$ approaches $-j\sqrt{j'^2_{1,s}-k^2\beta^2a^2}$ for $j'^2_{1,s}>k^2\beta^2a^2$.

Similarly to the longitudinal case, let us examine the frequency-dependence of the transverse impedance. We start to study from the extremely high frequency and then will gradually lower the frequency. When the thickness of the insert is larger than $2^{1/2}\pi^{3/4}t_{min}$ (see Eq.(4)), the wake field leaks out of the insert in the high frequency region specified by $f \gg f_D$ (see Eq.(5)). In this frequency range, the transverse impedance is approximately given by

$$
Z_T \simeq \frac{(1-j)Z_0\sqrt{kg}}{8\sqrt{2}\beta\gamma^2 a I_1^2(\bar{k}a)}, \tag{19}
$$

which becomes for a relativistic beam

$$
Z_T \simeq \frac{(1-j)Z_0\sqrt{g}}{2\sqrt{2}k^{3/2}a^3}. \tag{20}
$$

In the frequency region where $f \ll f_D$ but still the skin depth is smaller than the insert thickness, the impedance is approximately written as

$$
Z_{T,insert} \simeq \frac{\sqrt{jk\beta Z_0}gI_1(\bar{k}r_b)}{2\gamma\pi r_b a\sqrt{\sigma_c}I_1^2(\bar{k}a)}, \tag{21}
$$

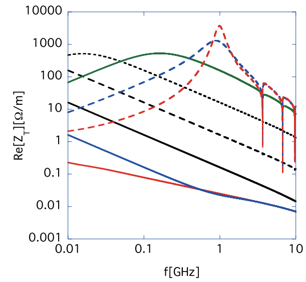

Figure 4: The thickness dependence of the transverse impedance of the insert for the relativistic beam case.

which reproduces the conventional resistive-wall impedance for a relativistic beam: [7]

$$
Z_{T,insert} = gc\sqrt{\frac{Z_0\omega\sigma_c}{2c}}\frac{1+j}{\pi\sigma_c\omega a^3}. \tag{22}
$$

In a lower frequency where the skin depth exceeds the thickness of the insert, the impedance becomes

$$
\Re[Z_{T,insert}] = \frac{gc}{2\pi^2 f a^3\sigma_c t}. \tag{23}
$$

When the thickness of the insert is smaller than $2^{1/2}\pi^{3/4}t_{min}$ but larger than t_{min}, Eq.(23) gives correct impedance all the way up to f_D.

Finally, in the region of

$$
f \ll f_L \equiv \frac{3c}{4\pi Z_0\sigma_c t a}, \tag{24}
$$

the wake fields leak out of the insert again. It is almost identical to the gap impedance $Z_{gap,\perp}$ described in reference [2] and goes down toward zero as the frequency approaches to zero.

Now, we consider the thickness dependence of the insert. The dependence of the real part of the insert impedance on the insert thickness is shown in Fig.4. The red, the blue, the black, the black dashed, the black dot, the green, the blue dashed and the red dashed lines represent the cases for the thickness t equal to 100μm, 10μm, 1μm, 100 nm, 10 nm, 1 nm, 100 pm and 10 pm, respectively.

Similar to the longitudinal case, we at first consider the case that the thickness of the insert t is larger than t_{min}. The result of $t=100\mu$m in Fig.4 corresponds to the case that the skin depth δ is smaller than the thickness of the insert t, which reproduces Eq.(22). The results of $t=10\mu$m to $t=100$nm in Fig.4 represent the case that the skin depth δ exceeds the thickness of the chamber t except at the low frequency extreme $f \ll f_L$ (See Eq.(24)). These impedances (even including the result for 10 nm which is smaller than t_{min}) agree very well with those obtained from simple formula

$$
\Re[Z_T] \simeq \frac{2\beta c}{a^2\omega}Z_{wall} = \frac{\beta c g}{2\pi^2 f a^3\sigma_c t}, \tag{25}
$$

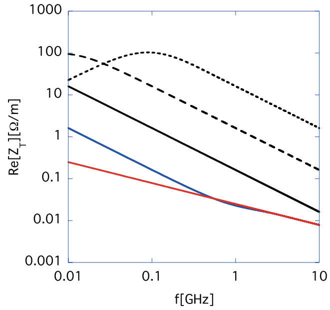

Figure 5: The dependence of the real part of the transverse resistive-wall impedance of a uniform beam pipe (no insert) on the thickness of the chamber.

where Z_{wall} is identical to Eq.(14).

The case of $t = 10\mu$m especially helps us to understand the behavior of the real part of the impedance, which starts to deviate from Eq.(22) and becomes proportional to f^{-1} when the skin depth exceeds the insert thickness ($f < f_\delta$. See Eq.(11)).

In the frequency region specified by $f_L < f < f_\delta$, the whole wall current runs on the thin insert, and wake fields are still confined inside the chamber. Contrary to the longitudinal impedance, this picture of the insert impedance is applicable to that of the resistive-wall impedance for the transverse impedance.

In order to compare the resistive-wall impedance with the insert impedance, we numerically calculate the resistive-wall impedance for different thicknesses of the chamber by borrowing the general formula of the resistive-wall impedance with finite thickness from Metral *et al*'s recent work [8, 9]. The results for a relativistic beam are shown in Fig.5. The red, the blue, the black, the black dashed and the black dot lines show the cases that the insert thickness t is equal to infinity, 10μm, 1μm,100nm and 10nm, respectively. The entire wall current runs on the chamber for the resistive-wall impedance as well, after the skin depth exceeds the chamber thickness. But at the region $f < f_L$ (but not quite lower as in the short insert) where the skin depth is much larger than the chamber thickness, the resistive-wall impedance starts to fall off.

In the case that the thickness of the insert t is extremely thin like $t \ll t_{min}$, the situation becomes significantly different. The results of $t = 1$nm to 10 pm in Fig.4 correspond to this case. The parameter f_L should be replaced by a new parameter:

$$f_r \equiv \frac{1}{2\pi}(\frac{2gc^3}{Z_0^2\sigma_c^2 a^4 t^2})^{\frac{1}{3}}. \qquad (26)$$

Contrary to the longitudinal case, f_r as well as f_c (see Eq.(15)) are used to classify the property of the impedance along the frequency axis. In the frequency region $f_r < f \ll f_c$ the contribution from the wall current dominates in the impedance, while the radiation effects dominate in the rest of the frequency. Since the wall current effect dominates in the impedance in the frequency region $f_r < f \ll f_c$, the impedance is proportional to the length

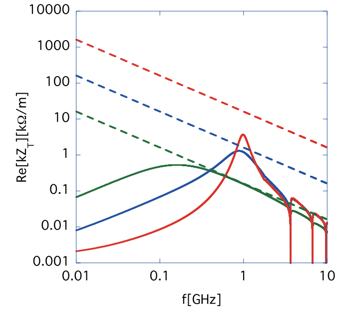

Figure 6: The transverse impedances for the case of $t \ll t_{min}$. The green solid and dashed, the blue solid and dashed, and the red solid and dashed lines show the cases that the thickness of the insert t is equal to 1nm, 100 pm and 10 pm, respectively. The solid lines are based on the present theory, while the dashed lines are calculated hypothetically by extending the simple formula (25) to these extreme thicknesses.

of the insert g. The impedance is proportional to $\sqrt{g}$ in the higher frequency region, as the contributions from the radiation dominate in the impedance. The wake field makes dips in the impedance curve for the frequency that is larger than the cut-off frequency of the chamber. Especially, in the case of the infinitesimally thin insert, the impedance is identical to the gap impedance, in the entire frequency.

Like the longitudinal case, the transition thickness of the insert at which the wall current starts converting to the outer radiation from running on the thin insert is determined by which case minimizes the impedance and thus the energy loss of a beam. Figure 6 demonstrates this fact by comparing the correct impedances with the hypothetical ones obtained by extending the simple formula (25) to these extreme thicknesses.

SUMMARY

The theory to describe the impedances of a short insert has been developed. The theory is consistent with the conventional resistive-wall impedance and the gap impedance.

Even in the thin insert the entire image current runs on the insert, and therefore the impedances increase drastically from the conventional resistive-wall impedance [7].

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