SELF-CONSISTENT BEAM DYNAMICS IN RF LINACS WITH NON-SYNCHRONOUS HARMONICS FOCUSING*

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Abstract

It was done the studies on high intensity ion beam dynamics in axisymmetric rf linacs both analytically, in terms of the so-called smooth approximation, and numerically in [1-3] rather carefully. For all that, effects of beam self-space-charge field were not taken into consideration under analytical investigations of the focusing by means of non-synchronous harmonics up to date. These effects are said to affect a focusing parameters choice deeply. A "beam-wave" Hamiltonian is derived under assumption that a bunch has an ellipsoidal form. Analytical results specify that given in [4] and it is verified numerically.

INTRODUCTION

Linac design is of interest to many fields of science, industry and medicine (e.g. nuclear physics, surface hardening, ion implantation, hadron therapy). The number of linacs is increased steadily. The most significant problem for low-energy high-current beams of charged particles is the question of its stability because of the influence of Coulomb's repelling forces. Beam motion stability can be realized by means of the following focusing types: alternating phase focusing, radio frequency quadrupoles, focusing by means of the nonsynchronous wave field as well as the undulator one.

However, system with alternating phase focusing is not suitable for low-energy high-current beam acceleration, because it requires small values of synchronous phase. Radio frequency quadrupoles showed itself as initial linac sections well, but careful beam dynamics study gives, that considerable part of input rf power is spent on transverse focusing. Due to such rf power disproportion between degrees of freedom radio frequency quadrupoles have a small acceleration rate usually. Acceleration and focusing can be realized by means of the electromagnetic waves which are nonsynchronous with a beam (the so-called undulator focusing). Unfortunately, systems without the synchronous wave are effective only for light-ion beams. For low-energy heavy-ion beams to be accelerated it is necessary to have the synchronous wave with particles.

Linac sections with rf focusing by the nonsynchronous harmonics can be adequate alternative to that with alternating phase focusing and radio frequency quadrupoles, joining its advantages. Zero-intensity beam dynamics analysis in linac sections with rf focusing by the nonsynchronous harmonics was done previously [1-3]. For intense, high-brightness beams from rf linacs, it is important to have analytical results together with numerical ones which help do a linac parameters choice to ensure total beam stability. In this paper analytical results specify that given in [4] and it is verified numerically.

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BASIC RELATIONS

Self-consistent beam dynamics is described by the 2nd Newton's law together with Poisson's equation as

$$\begin{cases} \dfrac{d}{dt}\left(m\dfrac{d\mathbf{R}}{dt}\right) = q\left(\mathbf{E} - \nabla_{\mathbf{R}}\Phi_c\right); \\ \nabla^2_{\mathbf{R}}\Phi_c = -\rho/\varepsilon_0 , \end{cases} \tag{1}$$

where m is a beam mass, $\mathbf{R}$ is a beam radius-vector, q is a beam charge, $\mathbf{E}$ is an external rf field, Φ_c is the self-space-charge field potential, ρ is a beam charge density, ε_0 is the free space permittivity.

Let us express rf field in axisymmetric periodic resonant structure as an expansion by the standing wave spatial harmonics assuming that a structure period is a slowly varying function of the longitudinal co-ordinate z

$$\begin{cases} E_z = \sum\limits_n E_n I_0(k_n r)\cos\left(\int k_n dz\right)\cos \omega t; \\ E_r = \sum\limits_n E_n I_1(k_n r)\sin\left(\int k_n dz\right)\cos \omega t, \end{cases} \tag{2}$$

where E_n is the nth harmonic amplitude of RF fields the axis; $k_n = (\theta + 2\pi n)/D$ is the propagation wave number for the nth RF field spatial harmonic; D is the geometric periods of the resonant structure; θ is the phase advances per period D; ω is the circular frequency; I_0, I_1 are modified Bessel functions of the 1st kind of orders 0 & 1.

One assumes the beam velocity does not equal one of the spatial harmonic phase-velocities except the synchronous harmonic of rf field, the geometric period of rf structure being defined as $D = \beta_s\lambda(s + \theta/2\pi)$, where s is the synchronous harmonic number, β_s is the relative velocity of the synchronous particle, λ denotes rf wavelength.

The analytical investigation of the beam dynamics in a polyharmonic field (2) is a difficult problem. Rapid longitudinal and transverse oscillations as well as a strong dependence of field components on transverse coordinates does not allow us to use the linear approximation in the paraxial region for a field series. Nevertheless the self-consistient analytical beam dynamics investigation can be carried out by means of the so-called smooth approximation [5].

Thus, the solution of the motion equation (the particle path) in the rapidly oscillating field we shall search as a sum of a slowly varying beam radius-vector component and a rapidly oscillating one. After some manipulations (as it was done in Ref. [1-5]) one can readily obtain the motion equation, in the synchronous particle frame, in the well-known form

$$\frac{d^2\mathbf{Q}}{d\tau^2} = -\nabla_{\mathbf{Q}} U_{\mathrm{ef}}\,, \tag{3}$$

where $\mathbf{Q} = \{\zeta,\ \eta\}$, $\mathbf{Q} = 2\pi(\overline{\mathbf{R}} - \mathbf{R}_s)/\beta_s\lambda$, $\overline{\mathbf{R}}$ is the mean value of $\mathbf{R}$ over rapid oscillation period, $\mathbf{R}_s$ is the synchronous particle radius-vector, $\tau = \omega t$, U_{ef} is the effective potential function (EPF) which is defined as

$$U_{\mathrm{ef}} = U_{\mathrm{ext}} + U_{\mathrm{c}}\,. \tag{4}$$

Here U_{ext} is the external rf field potential which consists of three terms:

$$\begin{aligned} U_0 &= -\frac{1}{2} e_s \left[I_0(\eta)\sin(\zeta + \varphi_s) - \zeta\cos\varphi_s - \sin\varphi_s \right]; \\ U_1 &= \frac{1}{16}\sum_{n\neq s}\frac{e_n^2}{\nu_{n,s}^2} w_{n,s}^{(0)}(\eta) + \frac{1}{16}\sum_{n}\frac{e_n^2}{\mu_{n,s}^2} w_{n,s}^{(0)}(\eta); \\ U_2 &= \frac{1}{16}\sum_{\substack{n\neq s\\ k_n+k_p=2k_s}}\frac{e_n e_p}{\nu_{n,s}^2}\left[w_{n,s,p}^{(1)}(\eta)\cos(2\zeta + 2\varphi_s) \right. \\ &\quad \left. + 2\zeta\sin 2\varphi_s - \cos 2\varphi_s \right] + \frac{1}{8}\sum_{\substack{n\neq s\\ k_n-k_p=2k_s}}\frac{e_n e_p}{\nu_{n,s}^2} \\ &\quad \times\left[w_{n,s,p}^{(2)}(\eta)\cos(2\zeta + 2\varphi_s) + 2\zeta\sin 2\varphi_s \right. \\ &\quad \left. - \cos 2\varphi_s \right]. \end{aligned} \tag{5}$$

$e_i = qE_i\lambda/2\pi\beta_s mc^2$, c is the light velocity, $s, n, p \in \mathrm{N}\cup\{0\}$, $\nu_{i,j} = (k_i - k_j)/k_j$, $\mu_{i,j} = (k_i + k_j)/k_j$, $\iota_{i,j} = 0.5(\nu_{i,j} + \mu_{i,j})$ and the functions of the dimensionless transverse coordinate are defined as

$$\begin{aligned} w_{i,j}^{(0)}(\eta) &= I_0^2(\iota_{i,j}\eta) + I_1^2(\iota_{i,j}\eta) - 1; \\ w_{i,j,l}^{(1)}(\eta) &= I_0(\iota_{i,j}\eta) I_0(\iota_{l,j}\eta) - I_1(\iota_{i,j}\eta) I_1(\iota_{l,j}\eta); \\ w_{i,j,l}^{(2)}(\eta) &= I_0(\iota_{i,j}\eta) I_0(\iota_{l,j}\eta) + I_1(\iota_{i,j}\eta) I_1(\iota_{l,j}\eta). \end{aligned} \tag{6}$$

Charge density is considered as a constant due to the fact that U_{ext} does not depend on time variable explicitly. Therefore one can write the self-space-charge field potential for the biaxial beam shape in a form

$$U_{\mathrm{c}} = -\frac{3qI_{\mathrm{b}}\lambda^3}{64\pi^3 W_0 \varepsilon_0 c}\left(A\zeta^2 + B\eta^2\right), \tag{7}$$

where $W_0 = mc^2$, I_{b} is the beam current and coefficients are

$$A = \begin{cases} \dfrac{2}{\ell(\rho^2 - \ell^2)} - 2\dfrac{\arctan\left(\sqrt{\rho^2 - \ell^2}/\ell\right)}{(\rho^2 - \ell^2)^{3/2}}, & \text{if } \ell < \rho; \\[2ex] \dfrac{1}{(\ell^2 - \rho^2)^{3/2}}\ln\left(\dfrac{\ell + \sqrt{\ell^2 - \rho^2}}{\ell - \sqrt{\ell^2 - \rho^2}}\right) - \dfrac{2}{\ell(\ell^2 - \rho^2)}, & \text{if } \ell > \rho; \\[2ex] 2/3\ell^3, & \text{if } \ell = \rho. \end{cases}$$

$$B = \begin{cases} \dfrac{\arctan\left(\sqrt{\rho^2 - \ell^2}/\ell\right)}{(\rho^2 - \ell^2)^{3/2}} - \dfrac{\ell}{\rho^2(\rho^2 - \ell^2)}, & \text{if } \ell < \rho; \\[2ex] \dfrac{\ell}{\rho^2(\ell^2 - \rho^2)} + \dfrac{1}{2(\ell^2 - \rho^2)^{3/2}}\ln\left(\dfrac{\ell - \sqrt{\ell^2 - \rho^2}}{\ell + \sqrt{\ell^2 - \rho^2}}\right), & \text{if } \ell > \rho; \\[2ex] 2/3\ell^3, & \text{if } \ell = \rho. \end{cases}$$

Here ℓ and ρ [m] are RMS beam length & radius. Using Eq. (3) one can obtain a "beam-wave" Hamiltonian.

Maclaurin series of the EPF is

$$U_{\mathrm{ef}} = \Omega_{0\zeta}^2\zeta^2/2 + \Omega_{0\eta}^2\eta^2/2 + o\left(\|\mathbf{Q}\|^3\right), \tag{8}$$

and the expansion coefficients are given by

$$\begin{aligned} \Omega_{0\zeta}^2 &= \frac{1}{2} e_s\sin\varphi_s - \frac{1}{4}\sum_{\substack{n\neq s\\ k_n+k_p=2k_s}}\frac{e_n e_p}{\nu_{n,s}^2}\cos 2\varphi_s \\ &\quad - \frac{1}{2}\sum_{\substack{n\neq s\\ k_n-k_p=2k_s}}\frac{e_n e_p}{\nu_{n,s}^2}\cos 2\varphi_s - \frac{3qI_{\mathrm{b}}\lambda^3 A}{32\pi^3 W_0\varepsilon_0 c}; \\ \Omega_{0\eta}^2 &= -\frac{1}{4} e_s\sin\varphi_s - \frac{3qI_{\mathrm{b}}\lambda^3 B}{32\pi^3 W_0\varepsilon_0 c} \\ &\quad + \frac{3}{32}\sum_{n}\frac{e_n^2}{\mu_{n,s}^2}\iota_{n,s}^2 + \frac{3}{32}\sum_{n\neq s}\frac{e_n^2}{\nu_{n,s}^2}\iota_{n,s}^2 \\ &\quad + \frac{1}{32}\sum_{\substack{n\neq s\\ k_n+k_p=2k_s}}\frac{e_n e_p}{\nu_{n,s}^2}\left(\iota_{n,s}^2 + \iota_{p,s}^2 - \iota_{n,s}\iota_{p,s}\right)\cos 2\varphi_s \\ &\quad + \frac{1}{16}\sum_{\substack{n\neq s\\ k_n-k_p=2k_s}}\frac{e_n e_p}{\nu_{n,s}^2}\left(\iota_{n,s}^2 + \iota_{p,s}^2 + \iota_{n,s}\iota_{p,s}\right)\cos 2\varphi_s\,. \end{aligned} \tag{9}$$

It is necessary that the parameters of the channel will be chosen in terms of the conditions $\Omega_{0\zeta}^2 > 0, \Omega_{0\eta}^2 > 0$ (for the simultaneous transverse and longitudinal focusing).

In terms of Eq. (9) one can readily write threshold current values for longitudinal and transverse beam motion:

$$I_{\mathrm{lg}} = \frac{32\pi^3 W_0 \varepsilon_0 c}{3qA\lambda^3}\left(\frac{1}{2}e_s \sin\varphi_s - \frac{1}{4}\sum_{\substack{n\neq s\\ k_n+k_p=2k_s}} \frac{e_n e_p}{\nu_{n,s}^2}\cos 2\varphi_s \right.$$

$$\left. - \frac{1}{2}\sum_{\substack{n\neq s\\ k_n-k_p=2k_s}} \frac{e_n e_p}{\nu_{n,s}^2}\cos 2\varphi_s \right);$$

$$I_{\mathrm{tr}} = \frac{32\pi^3 W_0 \varepsilon_0 c}{3qB\lambda^3}\left(-\frac{1}{4}e_s \sin\varphi_s + \frac{3}{32}\sum_{n\neq s} \frac{e_n^2}{\nu_{n,s}^2}\iota_{n,s}^2 \right.$$

$$+ \frac{1}{32}\sum_{\substack{n\neq s\\ k_n+k_p=2k_s}} \frac{e_n e_p}{\nu_{n,s}^2}\left(\iota_{n,s}^2 + \iota_{p,s}^2 - \iota_{n,s}\iota_{p,s}\right)\cos 2\varphi_s$$

$$+ \frac{1}{16}\sum_{\substack{n\neq s\\ k_n-k_p=2k_s}} \frac{e_n e_p}{\nu_{n,s}^2}\left(\iota_{n,s}^2 + \iota_{p,s}^2 + \iota_{n,s}\iota_{p,s}\right)\cos 2\varphi_s$$

$$\left. + \frac{3}{32}\sum_{n} \frac{e_n^2}{\mu_{n,s}^2}\iota_{n,s}^2 \right).$$

COMPUTER SIMULATION RESULTS

The analytical results obtained above were used to estimate the beam threshold current at the linac with $\theta = \pi$. The beam was the unbunched 2.5 keV/u lead ions Pb^{25+} with charge-to-mass ratio 0.12. We consider there are two spatial harmonics at the linac. One of it is the synchronous harmonic with $s = 0$, and another one is the nonsynchronous (focusing) with $n = 1$. In the beginning, beam dynamics simulation was conducted to calculate threshold beam current values under next conditions: system length – 2.44 m; bunching length and field increasing one were the same and the former being equal to 1.75 m; channel aperture – 5 mm; input/output value of the equilibrium particle phase φ_s were $\pi/2$ and $\pi/6$; synchronous harmonic maximal value at the axis was equal to 16.1 kV/cm; the ratio of the harmonic amplitudes e_1/e_0 was equal to 9. The equilibrium particle phase linearly increases at the bunching length and plateaus further. Note that the variation of the synchronous harmonic amplitude against longitudinal coordinate (at field increasing length) was calculated by using the technique described in [3]. Initial beam radius was 1 mm. Threshold beam currents behavior is shown in Fig. 1. One can see that threshold values increase at the linac length. It can seem strange but it should not forget that harmonic amplitudes are increasing functions too. Thus one can see that total threshold current value is defined by the longitudinal one and the beam current value should not be grater than that at the linac input.

Further, the results obtained above were verified by means of a modified version of the specialized computer code BEAMDULAC–ARF3 [1] under different beam current values. Results obtained above agree within a few percent with the numerical simulation ones. Threshold current, which ensure high particle transmission, is equal to 6 mA for the chosen parameter set. There are longitudinal (a) and transverse (b) beam phase space projections together with separatrix and RMS ellipse under 5 μA beam current in Fig. 2 (GC – beam gravity center). The output beam energy and current transmission coefficient are 103 keV/u and 85% respectively.

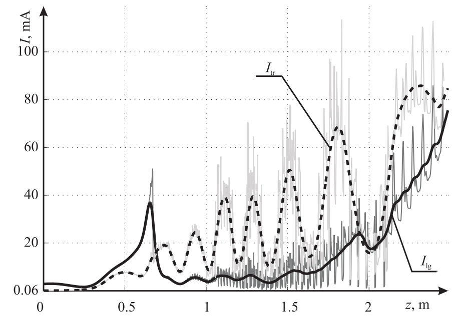

Figure 1: Threshold beam currents.

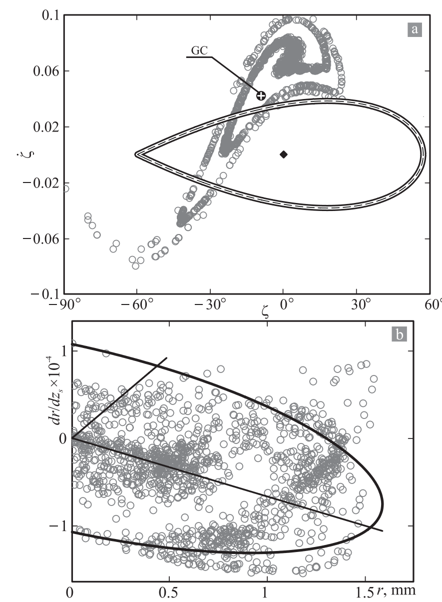

Figure 2: Beam phase space projections.

SUMMARY

Beam dynamics model with regard for particles interactions was made. Threshold beam currents were evaluated in terms of this model. The necessary restrictions on the linac parameters were imposed. The numerical simulations of the self-consistent low-velocity high-brightness heavy-ion beam dynamics confirmed the analytical results obtained in toto.

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