RELAXATION, EMITTANCE GROWTH, AND HALO FORMATION IN THE TRANSPORT OF INITIALLY MISMATCHED BEAMS*

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Abstract

In this paper, a simplified theoretical model that allows to predict the final stationary state attained by an initially mismatched beam is presented. The proposed stationary state has a core-halo distribution. Based on the incompressibility of the Vlasov phase-space dynamics, the core behaves as a completely degenerate Fermi gas, where the particles occupy the lowest possible energy states accessible to them. On the other hand, the halo is given by a tenuous uniform distribution that extends up to a maximum energy determined by the core-particle resonance. This leads to a self-consistent model in which the beam density and self-fields can be determined analytically. The theory allows to estimate the emittance growth and the fraction of particles that evaporate to the halo in the relaxation process. Self-consistent N-particle simulations results are also presented and are used to verify the theory.

INTRODUCTION

In experiments that require the transport of intense beams, space charge forces make it virtually impossible to launch a beam with a distribution that corresponds to an exact equilibrium state. As a consequence, as the particles are transported the beam will tend to relax towards a stationary state [1, 2]. Along this process, effects such as emittance growth and halo formation are expected to occur. These effects are very detrimental because they limit beam efficiency and may be responsible for particle losses which can cause wall damage and activation. Therefore, a quantification of the amount of emittance growth and halo formation that can be expected becomes an important issue in the design of such systems. In order to estimate these, a good knowledge of the mechanisms that lead to beam relaxation and, especially, of the *final* stationary state reached by the beam is necessary.

In general, injected beams may deviate from the equilibrium state because of various effects, such as envelope mismatches [3, 4, 5, 6, 7, 8, 9], off-axis motion [10, 11, 12, 8, 13], nonuniformities in the beam distribution [14, 15, 16, 17, 18, 19, 20], and forces due to the surrounding conductors [21, 22, 23]. Among all these effects, the one that has attracted most of attention is the envelope mismatch because it is believed to be a major cause of emittance growth and halo formation. For mismatched beams, an unbalance between the focusing force due to the external applied field and the defocusing forces due to space charge and thermal effects, causes the whole beam to oscillate in a coherent breathing mode. Some single beam particle trajectories resonate with this mode, gaining a lot of energy to form the halo. Based on a low dimensional particle-core model it is possible to observe this resonance process and to determine the maximum range of halo particles [3, 4, 5]. Due to conservation of energy, as the halo is being formed the particles that remain in the core loose energy and the amplitude of the breathing mode decreases. Eventually, halo formation ceases and the stationary state is reached. The whole scenario is analogous to an evaporative cooling process where the core particles cool down via evaporation of hot, energetic halo particles. The thermodynamic equilibrium that corresponds to the Maxwell-Boltzmann distribution [24, 25] is not expected to be attained in this process because the beam dynamics is collisionless [26, 27, 28, 29]. In fact, in the particular case of an initially mismatched high-intensity cold beam, it has been shown that the final stationary state can be very well modeled by a completely cold dense core surrounded by a cloud of energetic particles that carry all the beam emittance [8, 9]. From this model one can successfully determine the total emittance growth and the fraction of particles that form the halo in the stationary state.

In the case of beams with a finite initial emittance, however, the assumption of a completely cold core for the relaxed state is no longer correct. The existence of emittance in the initial distribution indicates that the beam occupies a finite volume in the phase-space. Because the Vlasov dynamics that governs beam evolution is incompressible, this volume has to be preserved. Hence, the occupation of low-energy regions of the phase-space by the particles as the core progressively cools down is limited by the finite density of the initial distribution in phase-space, which is is not compatible with a completely cold core. In other words, although we are dealing with purely classical particles, the conservation of volume in the phase space imposed by the Vlasov equation, leads to a Pauli-like exclusion principle for the beam particles. Taking this into account, here we propose that the stationary state for the core corresponds to a completely degenerate Fermi gas, where the particles occupy the lowest possible energy states accessible to them. This leads to a self-contained model where the beam density and self-fields can be determined analytically as a function of two parameters – the core size and the halo density. This parameters are, in turn, readily obtained by numerically solving two algebraic equations that correspond to the conservation of the total number of particles and the

* This work was supported by CNPq and FAPERGS, Brazil, and by the US-AFOSR under Grant No. FA9550-09-1-0283.
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energy of the system. The results are compared with self-consistent N-particle simulations and a good agreement is found for the density of the stationary state and the emittance growth. In the simulations, the emittance growth is shown to be weakly dependent on the details of the initial beam distribution. The model is also used to estimate the fraction of particles that will evaporate to form the halo. It is worth noting that a more detailed analysis shows that the core distribution is indeed not fully degenerate, but more closely represented by a series of *low temperature* Fermi-Dirac distributions [26, 30]. If on one hand such representation is capable of describing the stationary state in great detail, on the other hand it demands more involved computation and requires an equally detailed knowledge of the initial distribution. In this regard, the model proposed here is a simplification which, however, provides a fair description of the stationary state and that is only based on the knowledge of RMS quantities of the initial distribution.

BEAM MODEL AND EQUATIONS

We consider an unbunched beam propagating with a constant axial velocity $\beta_b c$ along the inner channel of a circular grounded conducting pipe of radius r_w; the beam is focused by a uniform solenoidal magnetic field of magnitude B_z. Both the pipe and the focusing field are aligned with the z axis. Given the uniform motion along z, we define a longitudinal coordinate $s = \beta_b ct$ that plays the role of time in the system. It is convenient to work in the Larmor frame of reference [31], which rotates with respect to the laboratory frame with the angular velocity $\Omega_L = qB_o/2\gamma_b mc$, where q, m and $\gamma_b = (1-\beta_b 2)^{-1/2}$ are, respectively, the charge, mass and relativistic factor of the beam particles. In the paraxial approximation, the beam distribution function $f(\mathbf{r},\mathbf{v},s)$ evolves according to the Vlasov-Maxwell system [31]

$$\frac{\partial f}{\partial s} + \mathbf{v}\cdot\nabla f + (-\sigma_0^2\mathbf{r} - \nabla\psi)\cdot\nabla_{\mathbf{v}} f = 0, \quad (1)$$

$$\nabla^2\psi = -\frac{2\pi K}{N}\, n(\mathbf{r},s), \quad (2)$$

where $n(\mathbf{r},s) = \int f\, d\mathbf{v}$ is the beam density profile, $\sigma_0 = qB_z/2\gamma_b\beta_b mc^2$ is the vacuum phase advance per unit axial length which determines the focusing field strength, $K = 2q^2N/\gamma_b^3\beta_b^2 mc^2$ is the beam perveance that is a measure of the beam intensity, $N = \int f\, d\mathbf{r}d\mathbf{v}$ =const. is the conserved number of particles per unit axial length, $\mathbf{r}$ is position vector in the transverse plane, and $\mathbf{v} \equiv d\mathbf{r}/ds$. As discussed in the Introduction, it is exactly because the beam evolves according to the Vlasov Equation (1), that the total phase-space volume occupied by the particles has to be conserved. In Eqs. (1) and (2), ψ is a normalized potential that incorporates both self-electric and self-magnetic field interactions. Due to the presence of the pipe surrounding the beam, the self-field potential satisfies the boundary condition $\psi(r = r_w) = 0$. In view of the axisymmetry of the external focusing field, we assume that the beam distribution has no θ dependence, so that $f = f(r, v_r; v_\theta; s)$, where the angular velocity v_θ is a constant of motion for the beam particles. For the Vlasov dynamics, if the distribution function only depends on the phase-space variables through the single particle energy, i.e., $f(\mathbf{r},\mathbf{v}) = f(\varepsilon)$, where

$$\varepsilon(\mathbf{r},\mathbf{v}) = (v^2/2) + (\sigma_0^2 r^2/2) + \psi(r), \quad (3)$$

it will be stationary. When that is not the case, the distribution will vary as function of s, tending to relax to a stationary state.

The beam envelope $r_b = [2\langle r^2\rangle]^{1/2}$ is a measure of the transverse size of the beam and evolve according to [31]

$$r_b'' + \sigma_0^2 r_b - (K/r_b) - (\epsilon^2/r_b^3) = 0, \quad (4)$$

where the emittance of the beam is defined as

$$\epsilon = 2[\langle r^2\rangle\,\langle v^2\rangle - \langle r v_r\rangle^2]^{1/2}, \quad (5)$$

the prime denotes derivative with respect to s, the angled brackets represent the average over the beam distribution, and $v = (v_r^2 + v_\theta^2)^{1/2}$. While for equilibrium beam distributions the emittance is a conserved quantity, for a non-stationary beam the emittance $\epsilon = \epsilon(s)$ generally grows as the beam relaxes towards the stationary state. It is clear from Eq. (4) that there is a competition between the focusing force imposed by the external magnetic field and the defocusing forces due to space charge and emittance. For matched beams these forces are balanced in such a way that the beam envelope remains mostly constant along the transport. Equating $r_b'' = 0$ in Eq. (4) we obtain the matched beam envelope

$$r_b^* = \left[\frac{K + \left(K^2 + 4\sigma_0^2\epsilon^2\right)^{1/2}}{2\sigma_0^2}\right]^{1/2}. \quad (6)$$

More generally, however, the initial distribution will have a mismatched envelope. In this case, the envelope will start to oscillate due to the unbalanced focusing and defocusing forces, and will start to induce halo formation as described by the particle-core model [3, 4, 5]. In order to quantify the initial beam envelope mismatch, we define a mismatch parameter as given by $\mu \equiv r_b(0)/r_b^*(0)$.

A quantity that plays a key role in the determination of the final stationary state of the beam is its average energy per particle. This is given by

$$\mathcal{E} = \frac{\langle v^2\rangle}{2} + \frac{\sigma_0^2\langle r^2\rangle}{2} + \mathcal{E}_\psi, \quad (7)$$

and is conserved along the transport. In Eq. (7), $\mathcal{E}_\psi$ is the beam self-field energy per particle given by [31]

$$\mathcal{E}_\psi = \frac{1}{4\pi K}\int |\nabla\psi|^2 d\mathbf{r} = \frac{1}{2K}\int_0^{r_w}\left(\frac{\partial\psi}{\partial r}\right)^2 r dr. \quad (8)$$

The aim in the next Sections is to determine the final stationary state achieved by a beam of known initial distribution.

DETERMINING THE FINAL STATIONARY STATE

We start our analysis by considering a beam whose initial distribution corresponds to a phase-space waterbag. That is, the particles are uniformly distributed up to a maximum radius r_m and a maximum speed v_m,

$$f_0(\mathbf{r}, \mathbf{v}) = \frac{N}{\pi^2 \epsilon_0^2} \Theta(r_m - r)\, \Theta(v_m - v), \quad (9)$$

where $\Theta(x)$ is the Heaviside step function and $\epsilon_0 = \epsilon(0) = r_m v_m$ is the initial beam emittance. Its energy per particle can be readily computed by solving the Poisson equation and using Eqs. (7) and (8) to give

$$\mathcal{E}_0 = \frac{v_m^2}{4} + \frac{\sigma_0^2 r_m^2}{4} + \frac{K}{8} - \frac{K}{2} \log\left(\frac{r_m}{r_w}\right). \quad (10)$$

The waterbag distribution given by Eq. (9) is quite convenient for our discussion because it has the property that all the occupied regions in phase-space have the same density $N/\pi^2\epsilon_0^2$. Hence, as the beam relaxes, the incompressibility of the Vlasov dynamics will limit the occupation of the lower energy states available to the progressively colder core to this density value. In the final stationary state, therefore, the core will resemble a degenerate Fermi gas of density $N/\pi^2\epsilon_0^2$ that extends up to a Fermi energy ε_F in the phase-space. The value of ε_F is yet unknown, but will be determined self-consistently. As for the halo, the particle-core model allows us to determine the maximum radius that the halo particles can attain, r_h [3, 5]. Since the particle located at r_h represents the outermost one, we can easily determine its energy as $\varepsilon_h = \sigma_0^2 r_h^2/2 - K \log(r_h/r_w)$. While for initially cold beams it was found that the halo particles tend to stay along the separatrix of the particle-core model resonance [9], for finite emittance beams it was observed that they typically spread uniformly in phase-space up to the energy ε_h [26, 30]. Putting all this information together, we write the final stationary distribution as

$$f_s(\mathbf{r}, \mathbf{v}) = \frac{N}{\pi^2 \epsilon_0^2} \left[\Theta(\varepsilon_F - \varepsilon) + \chi \Theta(\varepsilon_h - \varepsilon) \Theta(\varepsilon - \varepsilon_F)\right], \quad (11)$$

where χ is the ratio between halo and core density in phase-space. The distribution is represented in Fig. 1(b). Note that f_s only depends on the phase-space coordinates through the single particle energy ε, defined in Eq. (3) and is consequently an equilibrium distribution. It depends on two still unknown parameter, ε_F and χ. By integrating $f_s(\mathbf{r}, \mathbf{v})$ over the velocity space we can determine the beam density profile and self-consistently solve Poisson equation (2). Both the density $n_s(r)$ and the self-field $\psi_s(r)$ of the stationary state can be writen analytically in terms of modified Bessel functions. By imposing the conservation of particles and conservation of energy, namely,

$$\int f_s(\mathbf{r}, \mathbf{v}) d\mathbf{r} d\mathbf{v} = N, \quad (12)$$

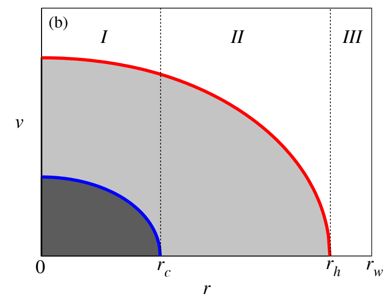

Figure 1: Proposed final stationary distribution in phase space formed by a dense core (dark gray) and a tenuous halo (light gray).

$$\frac{\sigma_0^2 r_{bs}^2}{2} - \frac{K}{4} + \mathcal{E}_{\psi s} = \mathcal{E}_0, \quad (13)$$

we determine the unknown parameter ε_F and χ. In Eq. (13), $r_{bs}^2 = 2\langle r^2 \rangle = 2\int r^2 n_s(r) r dr$, $\mathcal{E}_{\psi s}$ is obtained by subtitutin $\psi_s(r)$ in Eq. (8), and use has been made of $2\langle v^2 \rangle = \sigma_0^2 r_b^2 - K$, valid for any equilibrium beam distribution.

Once the final stationary state has been determined, we can compute the total emittance growth that occurs in the beam relaxation process, that is given by $\epsilon_s = r_{bs}\sqrt{\sigma_0^2 r_{bs}^2 - K}$, as well as the fraction of particles that evaporate to the halo, $\mathcal{F}_h = (N\chi/\pi^2\epsilon_0^2) \int \Theta(\varepsilon_h - \varepsilon)\Theta(\varepsilon - \varepsilon_F) d\mathbf{r} d\mathbf{v}$.

So far, we have only considered the relaxation of beams with an initial distribution given by the waterbag distribution, in Eq. (9). In general, however, we may expect initial distributions that present a nonuniform density in phase-space. In order to handle such cases, we can discretize the nonuniform distribution into p levels [30]. While this procedure allows for a very detailed description of the beam, it demands an equally detailed knowledge of the beam initial distribution. In many practical situations, however, there is no such knowledge and all that is known from the initial beam are the RMS quantities, like the envelope and the emittance. Taking this into consideration, we take the lowest order $p = 1$ and approximate any given initial distribution by Eq. (9) with the envelope r_m and emittance ϵ_0 corresponding to the actual beam. With this, we can estimate the final stationary state, the emittance growth, and the halo fraction for any beam, just based on its initial envelope and emittance. Self-consistent simulations are presented in the next section to verify the validity of this approximation.

NUMERICAL RESULTS

In order to test the theory presented, we perform N-particle self-consistent simulations. The simulations are based on Gauss's law where the field at a certain radial coordinate r depends on the total number of particles with coordinates smaller than r [4]. This method precludes the effects of collisions between individual particles and is convenient because instabilities and profile distortions around the round shape are not expected here [32, 33]. In the simulations we launch $N = 5000$ macroparticles according to a prescribed distribution and evolve them until a stationary state is reached. We consider three different initial beam distributions, namely, a waterbag given by Eq. (9), a semigaussian distribution and a full gaussian distribution both in space and velocity. The analysis is simplified if we measure longitudinal and transverse coordinates in units of σ_0^{-1} and $(\epsilon_0/\sigma_0)^{1/2}$, respectively. Then, the initial beam is characterized by two parameters only: $K/\sigma_0\epsilon_0$ and the mismatch parameter μ. In the results presented below, the halo size used in the theory is not directly obtained by the particle-core model, but rather, the one approximated by the empirical formula proposed by Ref. [5], namely, $r_h = 2r_b^*(1 + \log\mu)$, where r_b^* is the matched beam envelope of Eq. (6).

In Fig. 2, we compare the final stationary particle distribution obtained from the theory (solid lines) and the N-particle simulation (dots) for three different cases. In panel (a) we present the results for an initial waterbag distribution with $K/\sigma_0\epsilon_0 = 0.1$ and $\mu = 1.5$. This parameter set corresponds to a mildly space-charge dominated beam that is comparable to that found in the experiments of Ref. [7]. Despite the *small* space-charge forces a large halo is apparent. Clearly the model agrees very well with the simulation results, describing very closely both the core and the halo particle distributions. In panel (b), we consider a beam with the same initial distribution as in (a), but with larger space-charge forces corresponding to $K/\sigma_0\epsilon_0 = 1.0$. Again, a very good agreement is found. In panel (c), we present an example with a different initial distribution. In particular, we consider the same parameters as in panel (b), namely $K/\sigma_0\epsilon_0 = 1.0$ and $\mu = 1.5$, but now for a fully gaussian distribution. As expected, because the initial distribution is nonuniform both in the configuration and the velocity space, the final agreement between the final stationary state reached in the simulation and the theory is not as impressive as in the previous cases. Nevertheless, taking into consideration the crudeness and simplicity of the model, the results are still quite satisfactory. Particularly concerning the halo distribution which is reasonably close to the actual one.

We compare the emittance growth calculated from the model and obtained from the simulations with different initial conditions. These results are presented in Fig. 3(a) as a function of the mismatch parameter for $K/\sigma_0\epsilon_0 = 0.1$. The theoretical results are found to be in good agreement with the numerical results. We also apply the theory to estimate the fraction of particles that evaporate from the core to form the halo. In Fig. 3(b), we show $\mathcal{F}_h$ as a function of the mismatch parameter μ obtained from the theory. A nearly linear dependence of the halo fraction with the mismatch parameter is observed. The figure also indicates that the halo fraction decreases as the space-charge is increased. In fact, this trend is verified by computing $\mathcal{F}_h$ as a function of $K/\sigma_0\epsilon_0$ for fixed μ (not shown). We note that although the halo fraction is a decreasing function of $K/\sigma_0\epsilon_0$, the total charge in the halo, given by $K\mathcal{F}_h$, grows as the beam becomes more intense.

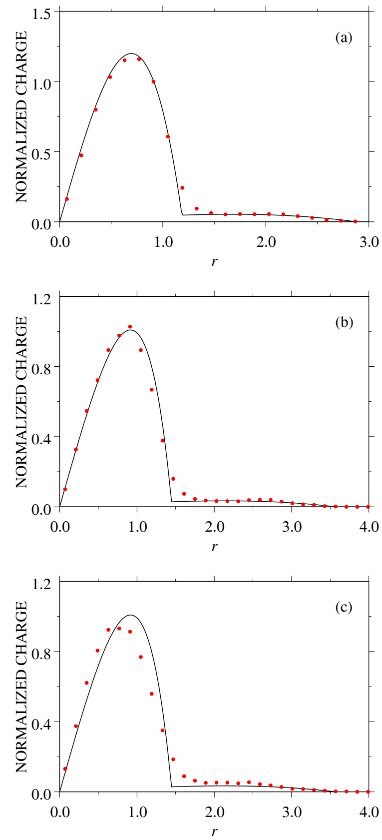

Figure 2: Comparison of the normalized charge as a function of radius obtained from the theory (solid curve) and the N-particle simulation (red dots).

As far as emittance growth is concerned, the results presented in Fig. 3(a) for a low space-charge beam are very similar to those obtained from the *free-energy model* described in Ref. [1]. Thus, our emittance growth estimates should also agree very well with the experimental results presented in Ref. [7]. Nevertheless, in contrast to the free-energy model [1], the theory derived here not only allows for emittance growth estimates, but also provides o good description of the final stationary distribution attained by the beam, including halo density and fraction. Therefore, it would be interesting to validate the model against experimental results of halo formation in mismatched space-charge dominated beams [7, 34, 35, 36].

CONCLUSION

A simplified theoretical model that allows to predict the final stationary state attained by an initially mismatched beam is presented. The theory allows to estimate important quantities, such as, the emittance growth and the fraction of particles that evaporate to the halo in the relaxation process. In particular, regarding the halo fraction, the model fore-

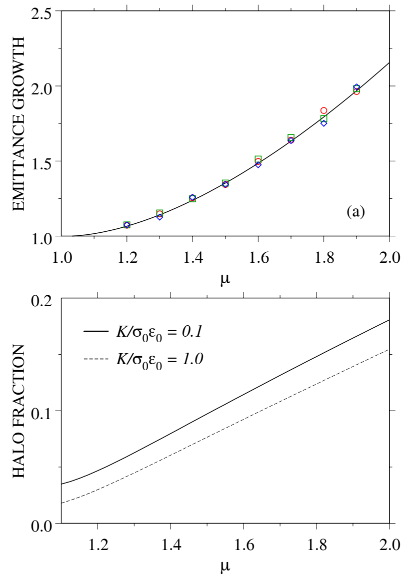

Figure 3: In (a), the emittance growth ϵ_s/ϵ_0 as a function of the mismatch parameter μ obtained from the theory (solid curve) and the N-particle simulations (symbols) for $K/\sigma_0\epsilon_0 = 0.1$. The symbols correspond to the different initial distribution in the simulation: waterbag (circle), semi-gaussian (square), and gaussian beam (diamond). In (b), the halo fraction $\mathcal{F}_h$ as a function of the mismatch parameter μ obtained from the theory.

sees a nearly linear increase with the mismatch amplitude, as well as an inverse dependence with the space-charge parameter $K/\sigma_0\epsilon_0$. Self-consistent N-particle simulations were performed to verify the predictions of the theory.

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