BEAM LOADING EFFECT SIMULATION IN LINACS

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Abstract

The accurate treatment both self beam space charge and RF field is the main problem for all beam dynamics codes. Traditionally only the Coulomb field is taken into account for low energy beams and the RF part is account for high energy beams. But now the current of accelerated beam enlarges and some radiation effects should be discussed for low energy beam also. The beam loading is being more important. This effect should be studied now not only in electron linacs but for proton one too.

INTRODUCTION

The high-current accelerators has the great perspectives for solving the problems of thermonuclear fusion, safe nuclear reactors, transmutation of radioactive waste and free electron lasers. A large number of low energy particle accelerators are applied in micro- and nanoelectronics, material science, including the study of new construction materials for nuclear industry, in medical physics, in particular for cancer by using the accelerators of protons and light ions, in radiation technology over the past three decades.

The accurate treatments of the beam self field and its influence on the beam dynamics is one of the main problems for developers of high-current RF accelerators. Coulomb field, radiation and beam loading effect are the main factors of the own space charge. Typically, only one of the components is taken into account for different types of accelerators. It is Coulomb field for low energy linacs and radiation and beam loading for higher energies. But both factors should be treated in modern low and high energy high intensity linacs. The mathematical model should be developed for self consistent beam dynamics study taking into account both Coulomb field together with beam loading influence. That is why three-dimensional self-consistent computer simulation of high current beam bunching with transverse and longitudinal motion coupling is very actual.

Let us describe the beam loading effect briefly. The beam dynamics in an accelerator depends not only on the amplitude of the external field but on the beam self field. The RF field induced by the beam in the accelerating structure depends on the beam velocity as well as the current pulse shape and duration in general. The influence of the beam loading can provide irradiation in the wide eigen frequency mode and decrease the external field amplitude. Therefore we should solve the motion equations simultaneously with Maxwell's equations for accurate simulation of beam dynamic.

The most useful methods for self-consistent problem solving are the method of kinetic equation and the method of large particles.

Solving the Maxwell equation can be replaced by solving the Poisson equation if we take into account only the Coulomb part of the own beam field. This equation can be solved by means of the well-known large particles methods as particle in cell (PIC) or cloud in cell (CIC). There is no easy method for dynamics simulation that takes into account the beam loading effect.

Currently there are a large number of commercial programs for electron and ion beam dynamics study. The most famous of them are MAFIA, PARMELA. Unfortunately they are not considering the important aspect associated with the beam loading in the beam bunching for the different cases.

The mathematical model for beam dynamics simulation taking into account the beam loading effect has been developed in MEPhI; the results are described in [1-2]. Now beam intensity in ion and electron linacs has considerably increased and the accounting of beam loading became necessary. New code development has led to necessity for modern computers. The new mathematical model for three-dimensional computer simulation in the Cartesian coordinates system has been developed.

The purpose of the present work is self-consistent high current beam dynamics investigation in uniform and non-uniform traveling wave accelerating structures by means of three-dimensional program BEAMDULAC-BL. The BEAMDULAC code is developing in MEPhI since 1999 [3-4] for high current beam dynamics simulation in linear accelerators and transport channels. Runge-Kutta 4th order method is using for the integration of differential equations of motions. The algorithm of BEAMDULAC-BL code uses any previously defined initial particles distribution in 6D phase space to calculate the Coulomb field distribution and radiation in harmonic form. As a result, the new coordinates, velocities and phases of large particles are determined, and the new values of the self-consistent field is defined. The traditional CIC method is used for Coulomb field calculation.

THE EQUATION OF MOTION IN SELF CONSISTENT FIELD AND SIMULATION METHODS

Let us discuss the methods of beam loading effect simulation used in BEAMDULAC-BL. Usually a longitudinal movement of charged particles is considered only for high current beam dynamics calculation in self-consistent fields. Thus, it is assumed, that the beam is in strong enough focusing field and transverse motion can be neglected. In traditional linear resonant accelerators where longitudinal components of current density $j_z >> j_\perp$ this approach is quite reasonable as the integral of current density and field interaction is defined by an amplitude and a phase of E_z component and the system for the equations of longitudinal dynamics and field excitation will be self-contained. In this approach transverse motion is completely defined by the longitudinal one.

For a long current pulse duration $\tau_u \geq T_f$, considering periodicity on time T_f, it is enough to divide at «large particles» only one bunch. Dimensionless longitudinal field amplitude on an axis is $A_z=e\cdot\lambda\cdot E_{z1}/m\cdot c^2$. Here e – the electron charge; λ – the wave-length; E_z – longitudinal component of electric field; m – the mass of electron; c - velocity of the light.

The step-by-step calculation in time domain is used in BEAMDULAC code for beam dynamics simulation as it was noted above. The process is repeated until the particles are not carried the end of accelerator ($\xi= \xi_{end}$), or until the current number of large particles ($N_{now}=N_{ing}-N_r-N_\varphi$) will not lower any minimal value ($N_{now}=0$, here N_{now} – current number of large particles, N_{ing}, N_r and N_φ - number of large-particle injected into the accelerator and out of the acceleration in transverse and phase directions respectively).

The radiation component of own field can be represented in harmonic form as in [1, 2]. The equation of motion can be rewritten in this case as

$$\frac{\mathrm{d}A_z}{\mathrm{d}\xi}+\left(w_1+\frac{1}{2}\cdot\frac{d}{d\xi}\ln B\right)\cdot A_z=\mp\frac{2\cdot B}{N}\times \times\sum_{n=1}^{N}\mathrm{I}_0\left(\frac{2\cdot\pi}{\beta_b}\cdot\sqrt{1-\beta_b^2}\cdot\eta_n\right)\cdot\cos(\psi_n) \quad (1)$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\xi}=2\cdot\pi\cdot\left(\frac{1}{\beta_b}-\frac{1}{\beta_\xi}\right)\pm\frac{2\cdot B}{A_z\cdot N}\times \times\sum_{n=1}^{N}\mathrm{I}_0\left(\frac{2\cdot\pi}{\beta_b}\cdot\sqrt{1-\beta_b^2}\cdot\eta_n\right)\cdot\sin(\psi_n) \quad (2)$$

where N is the number of large particles; β_b is phase velocity, $\eta_n=r_n/\lambda$ and ψ_n are dimensionless particle transverse coordinate and its phase in RF field, $R_p=E^2/2\cdot P$ is the characteristic impedance, P – total external RF power, $w_1=\alpha\cdot\lambda$ is the dimensionless damping factor, $\xi=z/\lambda$ is the dimensionless longitudinal coordinate. The parameter $B=eJ_0\cdot\lambda^2R_p/2\cdot m\cdot c^2$ defines beam and structure coupling on the accelerator axis. The sum in the right part of these expressions is product of all large particles on one period. This part corresponds on own beam field radiation field.

Equivalent RF field amplitude can be written now as:

$$A_{eq}^{\perp}=\frac{\beta_b}{2\cdot\pi\cdot\sqrt{1-\beta_b^2}}\cdot I_1\left(\frac{2\cdot\pi}{\beta_b}\cdot\sqrt{1-\beta_b^2}\cdot\eta\right)\times \times\left\{\left(w_1+\frac{1}{2}\cdot\frac{d}{d\xi}\ln B\right)\cdot A_z\cdot\cos\psi+ +\frac{2\cdot\pi}{\beta_b}\cdot(1-\beta\cdot\beta_b)\cdot A_z\cdot\sin\psi\pm \pm\frac{2\cdot B}{N}\cdot\sum_{n=1}^{N}I_0\left(\frac{2\cdot\pi}{\beta_b}\cdot\sqrt{1-\beta_b^2}\cdot\eta_n\right)\cdot\cos(\psi-\psi_n)\right\} \quad (3)$$

where $A^{\perp}_{eq}$ is the equivalent field amplitude in a non-stationary case when $T_b<<\tau_u<< T_f$ is calculated by the same way, $\eta=r/\lambda$. A_z and $A^{\perp}_{eq}$ now are slow functions of time.

All received equations can be rewritten easily to 3D case.

SIMULATION METHOD EFFICIENCY AND ACCURACY

A number of the test calculations must be done to estimate numerical model accuracy before start beam dynamics simulation for real accelerators. Testing should be done for radiation and Coulomb beam own field. Let us estimate the beam loading effect to the beam bunching for simple examples. The motion of strongly modulated beam consisting of point bunches in traveling wave section with constant β_b=0.9 and injection energy γ_{in}=2.1 was considered. This problem can be solved analytically because the beam self fields influence on bunched particles velocity can be neglected. The bunch in this case can be treated as one large particle. Computer simulation results are obtained in a good agreement with analytical one. The error is less than 2 %. The second test has been carried out to estimate beam current loading. Computer simulation of electron beam motion in traveling wave accelerating structure was considered. Two variants with identical initial conditions were calculated: taking into account beam loading effect in the first case, the second – without (Fig. 1). It is clear from figures that the beam loading must be considered even for short waveguide with current more than 1A. Energy difference in this case equals 12 %.

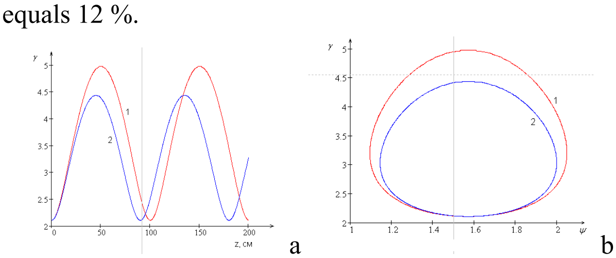

Figure 1: Energy dependence on longitudinal coordinate (a) and phase portrait (b). Curve 1 – without beam loading effect; 2 – taking into account beam loading.

PROTON AND ELECTRON BEAM DYNAMICS SIMULATION

The results of beam dynamics simulation were compared with the measurement data obtained on the traveling wave electron linac U-28 of Radiation-accelerating centre in NRNU MEPhI. The main U-28 characteristics are given at Table 1. Three-dimensional code BEAMDULAC-BL was used for beam dynamics simulation in U-28.

The results of simulation are presented in the Table 2. It is shown, that beam loading effect is too small for beam current $I \leq 0.2$ A. Results of numerical simulation are in a good agreement with experimental one.

Table 1: Parameters of U-28 Linac

Parameterer	Value
Average output energy, MeV	10
Range output energy, MeV	2 - 12
Max pulse beam current, mA	440
Max average beam current, μA	170
Normalized energy spectrum $(\Delta W/W)_{min}$, %	3
Pulse duration, μs	0.5 – 2.5
Pulse repetition, 1/s	400

Table 2: Results of the Electron Beam Dynamic Simulation

Parameter	Injection	Output
Velocity, β	0.5681	0.999
Average energy, MeV	0.6219	9.525
Beam current, mA	200	103
Current transmission, %	51.7	
Phase losses, %	45.4	
Transverse losses, %	3.0	

The computer simulation in a wide range input beam current has been carried out to study the beam loading effect on beam output energy (Fig. 2). Initial current variation leads to the beam output energy and current transmission coefficient decreasing. In particular, at the high initial currents more than half injected particles were lost, that lead to beam loading effect attenuation and deep beam energy adjustment impossibility.

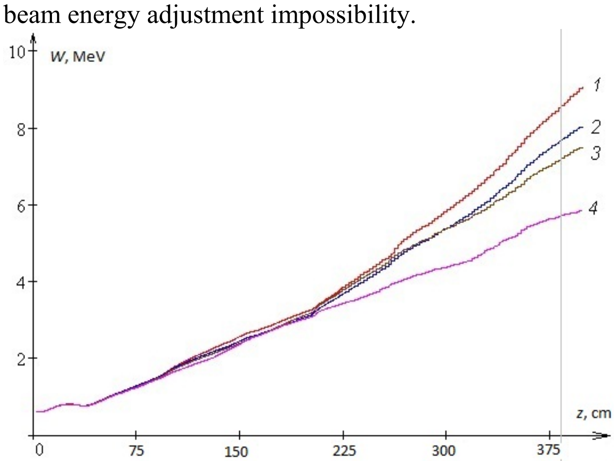

Figure 2: Electron beam energy dependence on accelerator length for different beams current: 1: I=0.2 A, 2: I=1 A, 3: I=2 A, 4: I=5 A.

The especial version of computer code has also been developed to study the beam loading effect in proton and ion linacs. Due to non relativistic particles velocities beam static self field becomes very essential. Computer simulation results for proton beam are presented in Table 3. It was shown that the Coulomb field has the main influence to particle dynamics. For low injection current (I < 0.24 A in our example) the beam loading effect has a weak influence to the proton beam bunching. It can be explained by low output beam energy in comparison with input RF power and small value of the parameter $E{\cdot}\lambda/\sqrt{P}$ defining beam and structure coupling. But some interest nonlinear effects were observed when beam loading was taken into account.

It is interesting to consider the possibility of average output protons energy variation for high beam currents. It can be possible to change output beam energy by injection beam current variation. Computer simulation results for proton linac with different input beam currents shows that it is not obviously possible to change the output energy by input current varying unlike electron linac.

Table 3: Results of the Proton Beam Dynamics Simulation

Parameter	Injection	Output
Velocity, β	0.015	0.057
Average energy, MeV	0.1055	1.553
Beam current, mA	200	154
Current transmission, %	77	
Phase losses, %	23	

CONCLUSIONS

The high current electron beam dynamics study in the linear accelerator is carried out for stationary beam loading. The mathematical model of self-consistent three-dimensional high current beams simulation in linacs has been described. Using this model the algorithm and computer code was done. The analysis of an electron beams dynamic in the traveling wave linacs let us make the conclusion, that even for low beam current (less than I<1 A) beam loading effect should be taken into account. Computer simulation of beam loading effect for high current electron and proton beam linacs was carried out. The developed methods can be used to solve the wide range of accelerator and RF electronics problems.

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