Partial coherence in undulator beamlines at ultra-low emittance storage rings

Manuel Sanchez del Rio

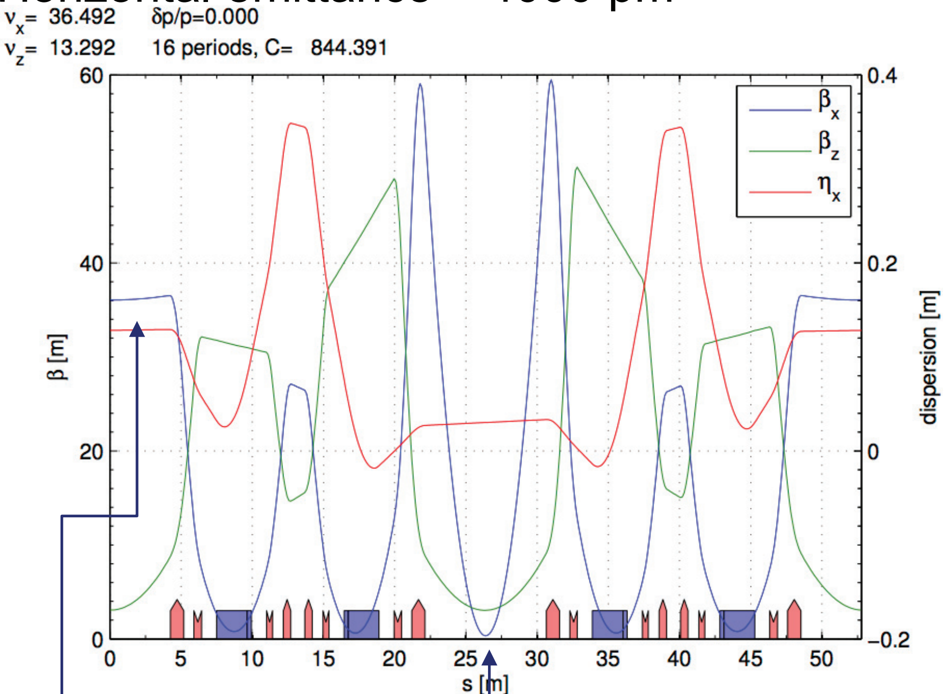
OUTLOOK

MOTIVATION:

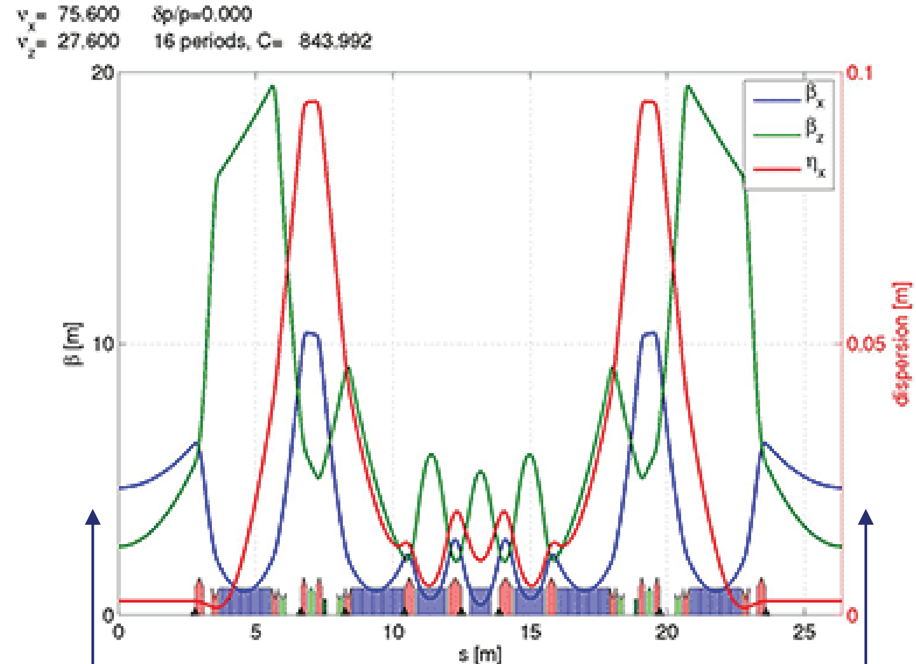
Fully characterize and calculate coherence properties of EBS beamlines.

- Introduction (with a bit of theory)
- Methodology: Coherent mode decomposition.
- Applications
  - ESRF: comparison Hβ-Lβ vs new EBS
  - ID16A

Horizontal emittance = 4000 pm



Horizontal emittance = 147 pm



High Beta:
β_x = 37.59 m
β_y = 2.95 m
σ_x = 415.0 μm
σ_y = 3.43 μ m
$\sigma_{x'}$ = 10.3 μ rad
$\sigma_{z'}$ = 1.16 μ rad

Low Beta:
β_x = 0.35 m
β_y = 3.0 m
σ_x = 50.4 μ m
σ_y = 3.44 μ m
$\sigma_{x'}$ = 107.2 μ rad
$\sigma_{z'}$ = 1.16 μ rad

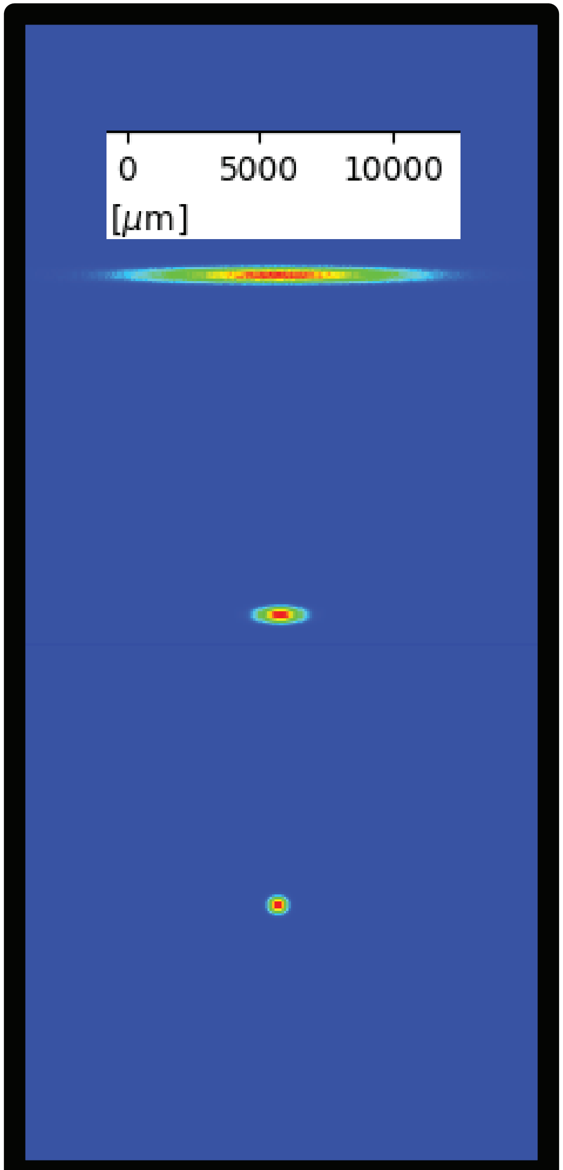
EBS (S28D):
β_x = 6.90 m
β_y = 2.64 m
σ_x = 30.2 μ m
σ_y = 3.64 μ m
$\sigma_{x'}$ = 4.37 μ rad
$\sigma_{z'}$ = 1.37 μ rad

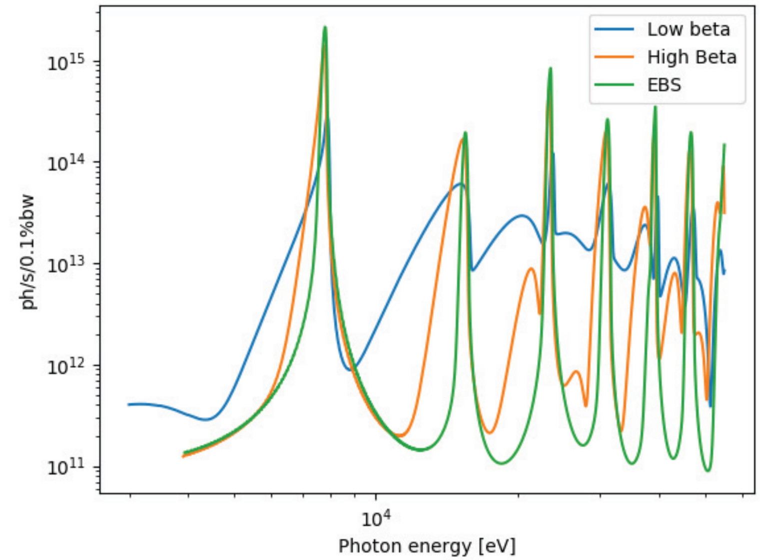
U18 2m @ 8 keV L=2m observed at 30m

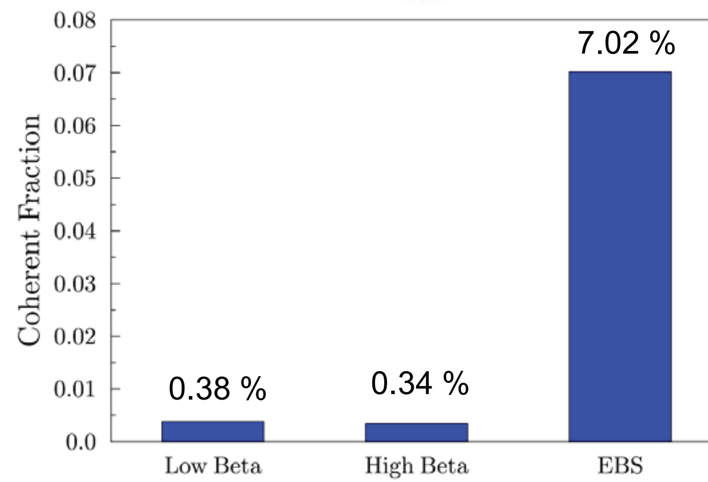
Low Beta U18

High Beta U18

EBS U18







FROM THEORY TO SOFTWARE

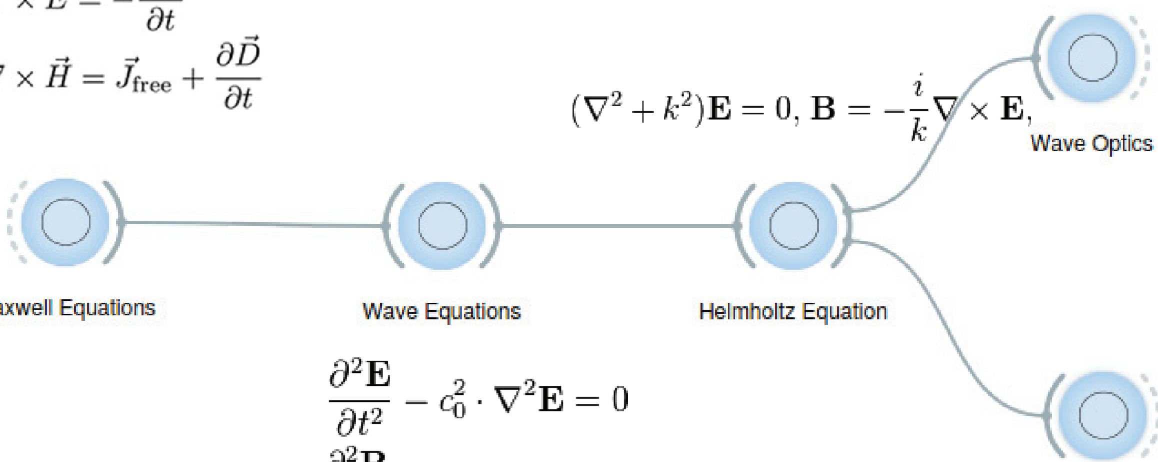
$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

$$(\nabla^2 + k^2)\mathbf{E} = 0,\ \mathbf{B} = -\frac{i}{k}\nabla \times \mathbf{E},$$



$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

$$\vec{E} = \vec{e}\ e^{ik_0 S(r)}$$

$$\vec{H} = \vec{h}\ e^{ik_0 S(r)}$$

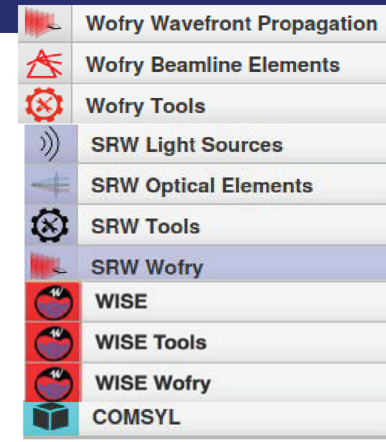
$$(\nabla S)^2 = n^2$$

$$\nabla S = n\,\vec{s}$$

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \nabla n$$

$$\nabla n = 0 \Rightarrow \frac{d\vec{r}}{ds} = 0 \Rightarrow \vec{r} = s\vec{a} + \vec{b}$$

FROM THEORY TO SOFTWARE



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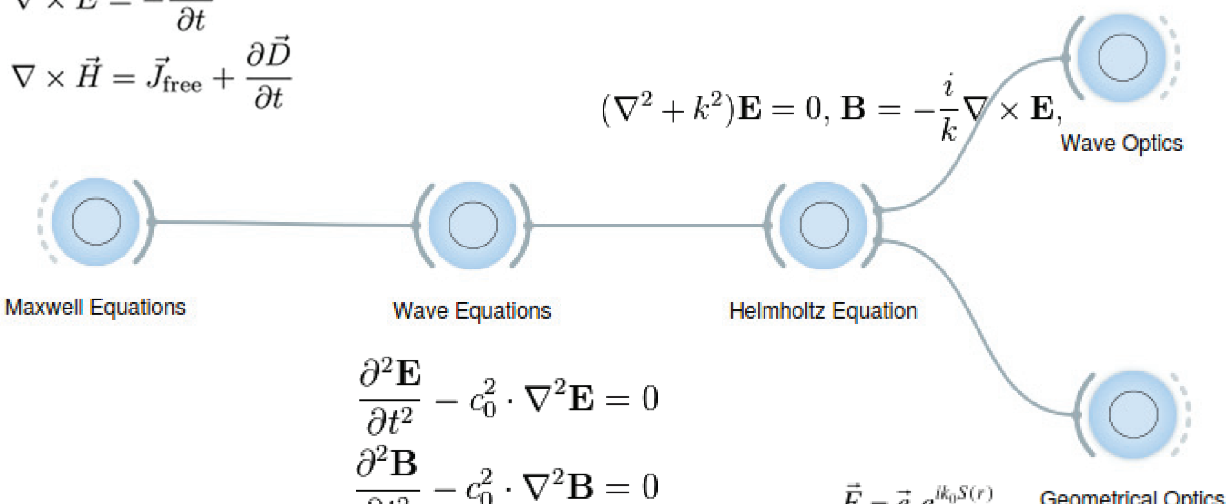

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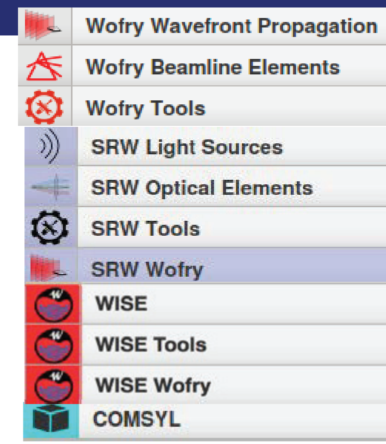

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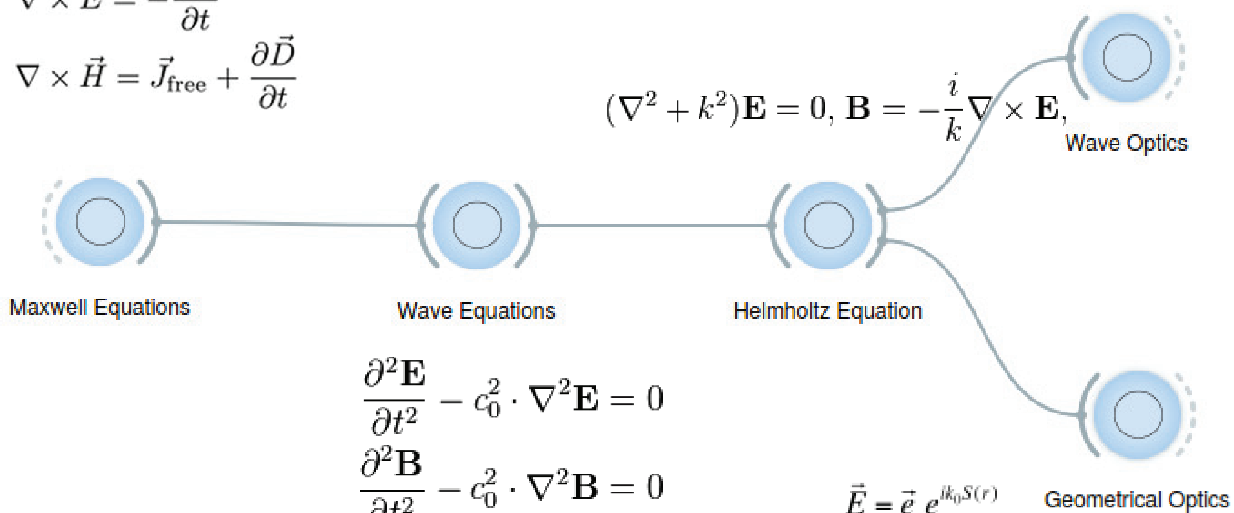

http://www.elettra.eu/oasys.html

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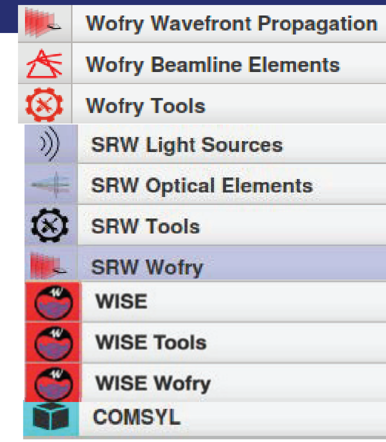

OrAnge SYnchrotron Suite

OASYS

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http://www.elettra.eu/oasys.html

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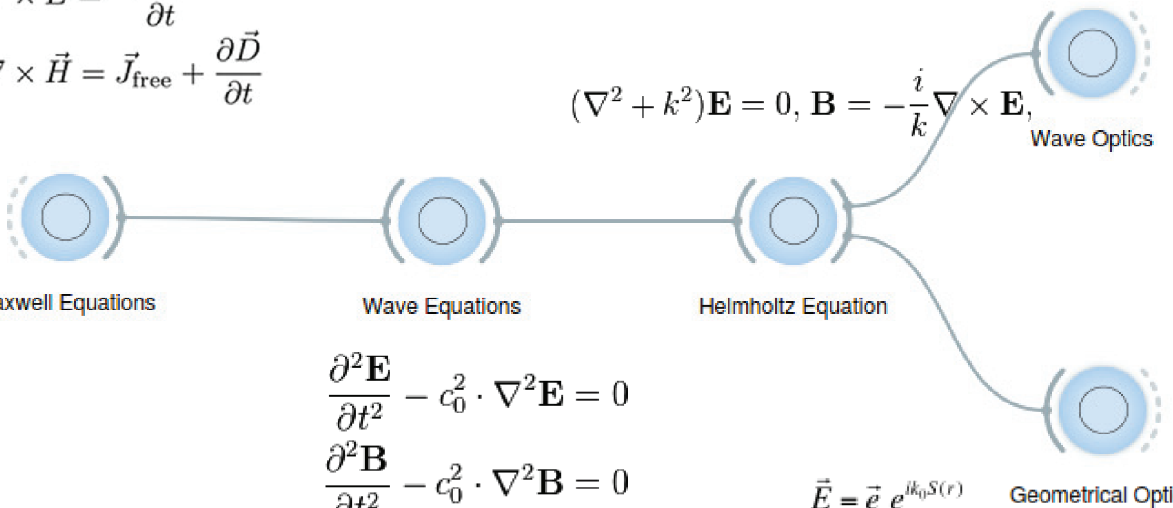

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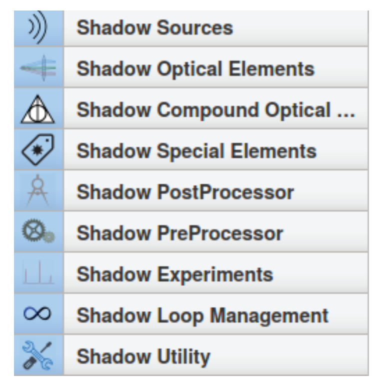
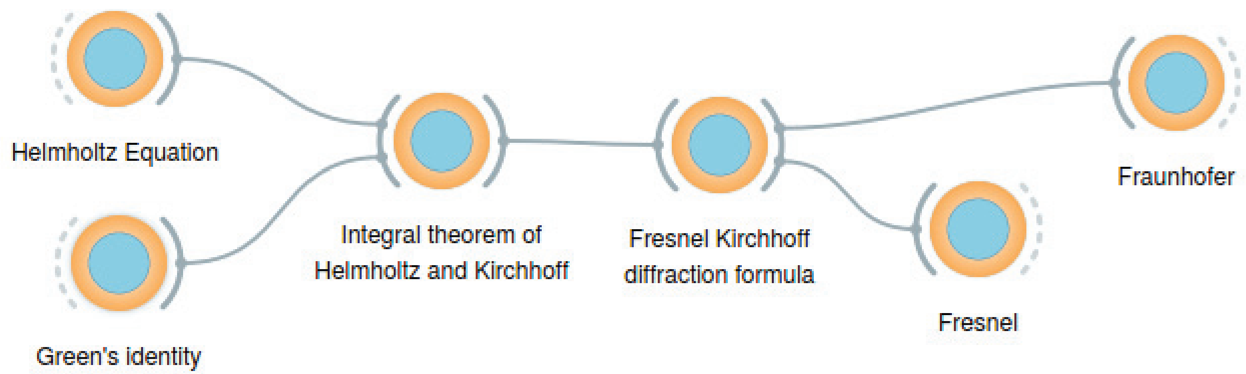
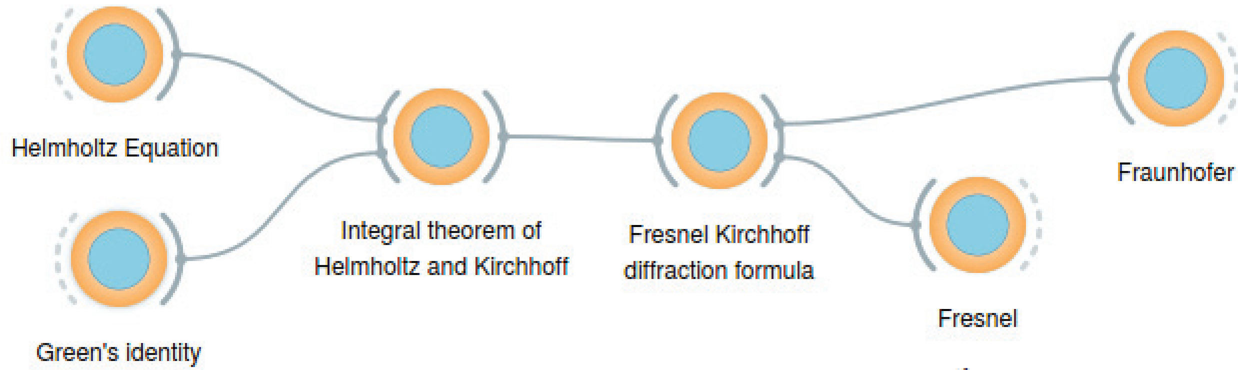
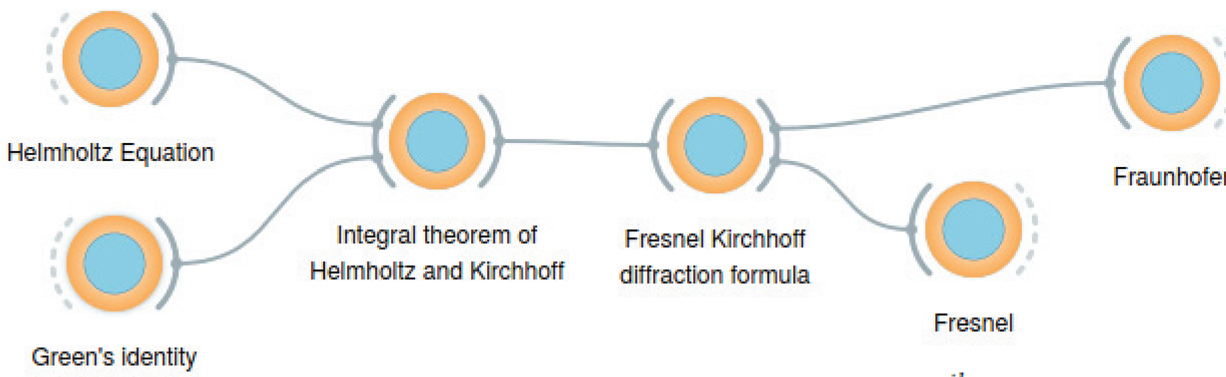


WAVE OPTICS


Helmholtz Equation
Green's identity
Integral theorem of Helmholtz and Kirchhoff
Fresnel Kirchhoff diffraction formula
Fresnel
Fraunhofer



$$E(\tilde{\mathbf{r}}, z, \omega) = \frac{e^{ikz}}{i\lambda z} \int_A d\mathbf{r}' A(\mathbf{r}') e^{\frac{ik}{2z}(\tilde{\mathbf{r}} - \mathbf{r}')^2}.$$



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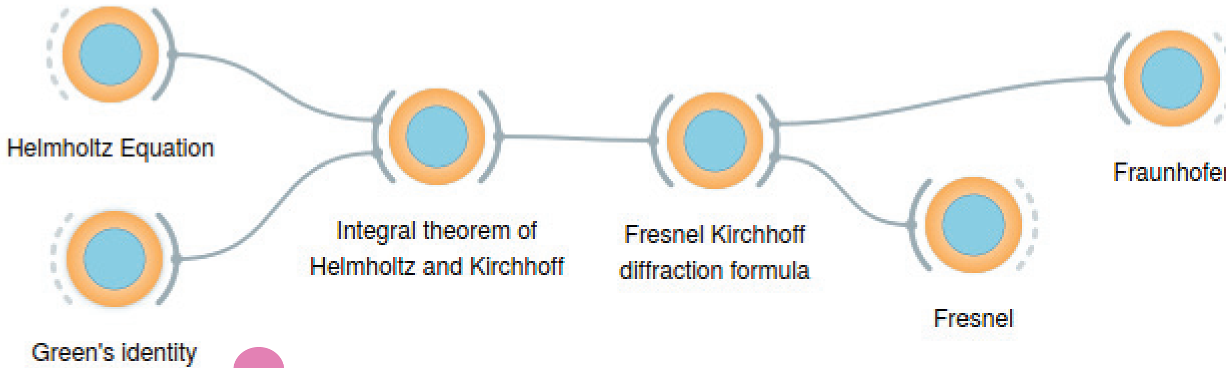
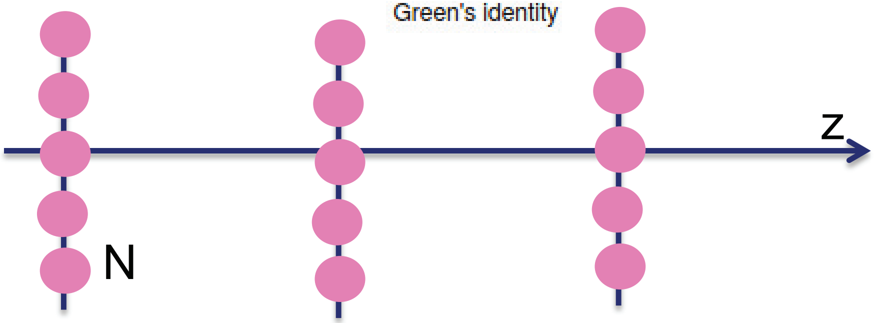
$$E'(\boldsymbol{r}) = \int h(\boldsymbol{r}, \boldsymbol{r}', \omega) E(\boldsymbol{r}', \omega) d\boldsymbol{r}'$$

WAVE OPTICS





$$E(\tilde{\mathbf{r}}, z, \omega) = \frac{e^{ikz}}{i\lambda z} \int_A d\mathbf{r}' A(\mathbf{r}') e^{\frac{ik}{2z}(\tilde{\mathbf{r}} - \mathbf{r}')^2}.$$

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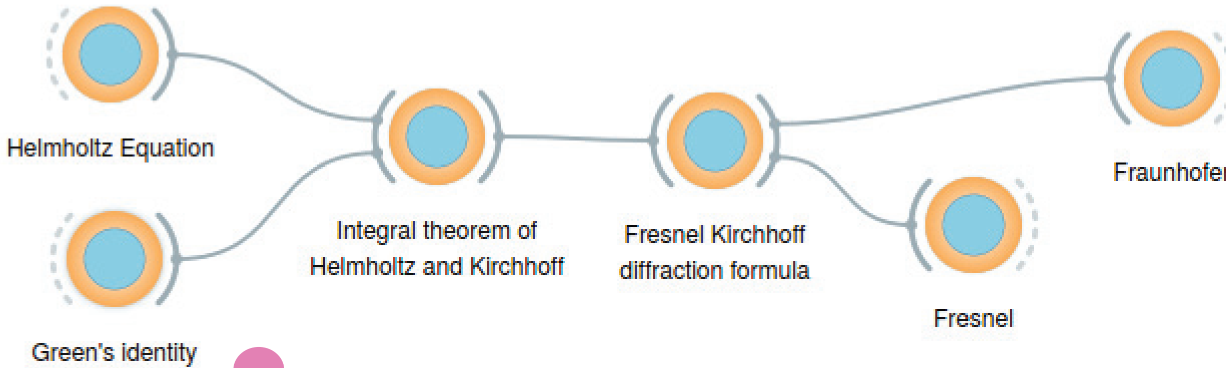

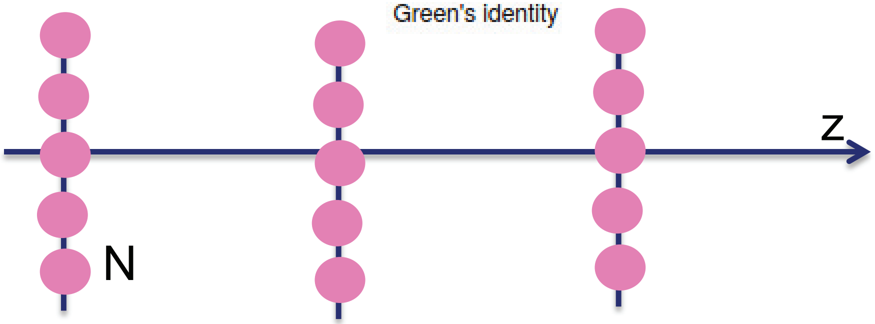

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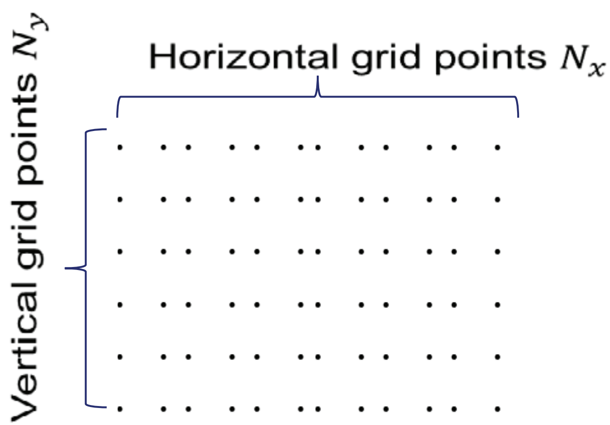

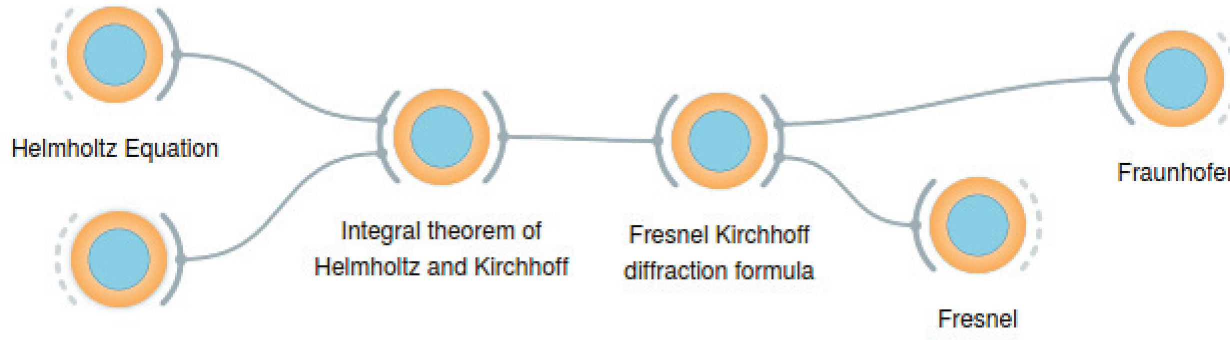

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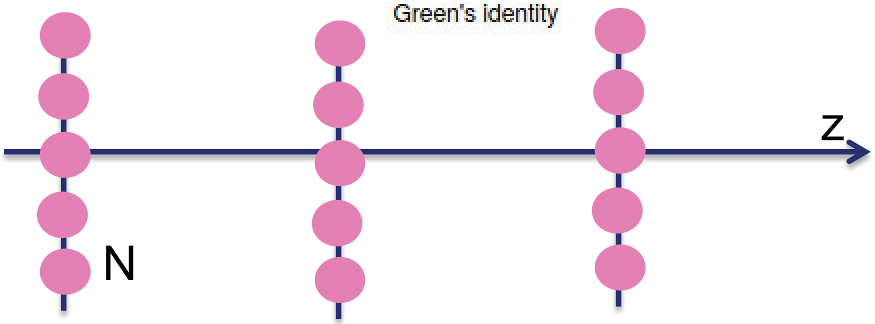

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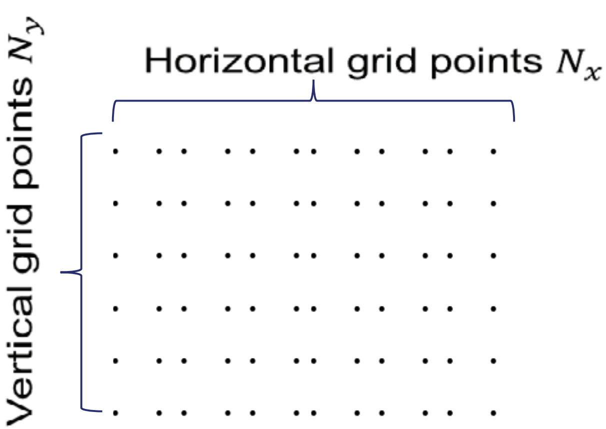
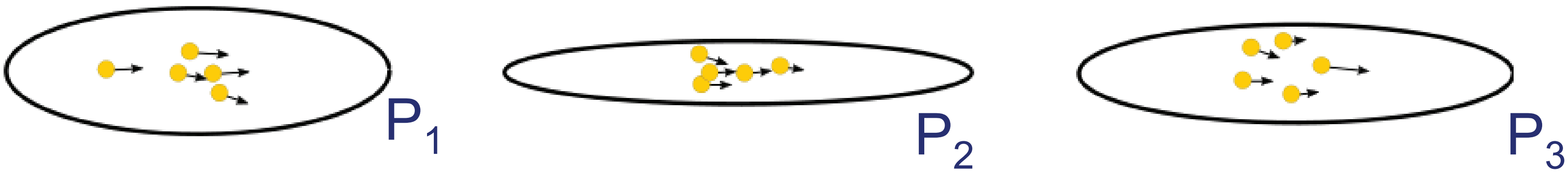


Estimation of operations:
- $(N_x \times N_y)^2$ operations: N_x N_y integrals
- $N_x \times N_y$ with FTT (Fourier Optics)

for $N \sim 10^3$

- Both integral and Fourier methods OK in 1D
- Only Fourier methods in 2D

Statistically distributed bunches



Order of 10^9 electrons per bunch

$$\boxed{f(\mathbf{r},\boldsymbol{\theta},\gamma,z) = C \cdot \exp\left(-\mathbf{u}^{\mathrm{T}}\Sigma^{-1}\boldsymbol{u}\right)}$$

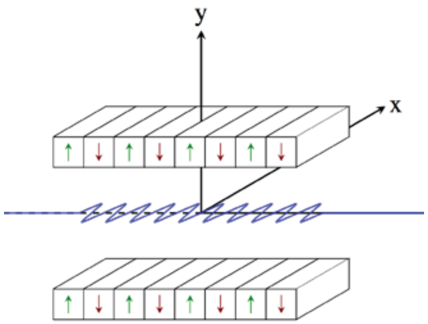
Phase space vector $\boldsymbol{u} = (x, \theta_x, y, \theta_y, \gamma, z)$

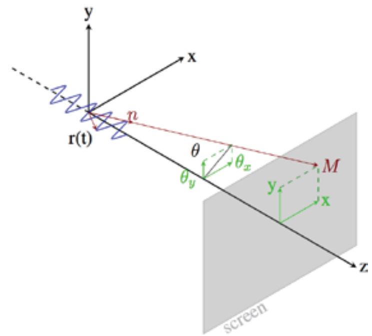
6x6 covariance matrix Σ:

$$\begin{bmatrix} \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \end{bmatrix} \quad \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \\ & & & & & \blacksquare \end{bmatrix} \quad \begin{bmatrix} \blacksquare & \blacksquare & & & & \\ \blacksquare & \blacksquare & & & & \\ & & \blacksquare & \blacksquare & & \\ & & \blacksquare & \blacksquare & & \\ & & & & \blacksquare & \\ & & & & & \blacksquare \end{bmatrix} \quad \begin{bmatrix} \blacksquare & & & & & \\ & \blacksquare & & & & \\ & & \blacksquare & & & \\ & & & \blacksquare & & \\ & & & & \blacksquare & \\ & & & & & \blacksquare \end{bmatrix}$$

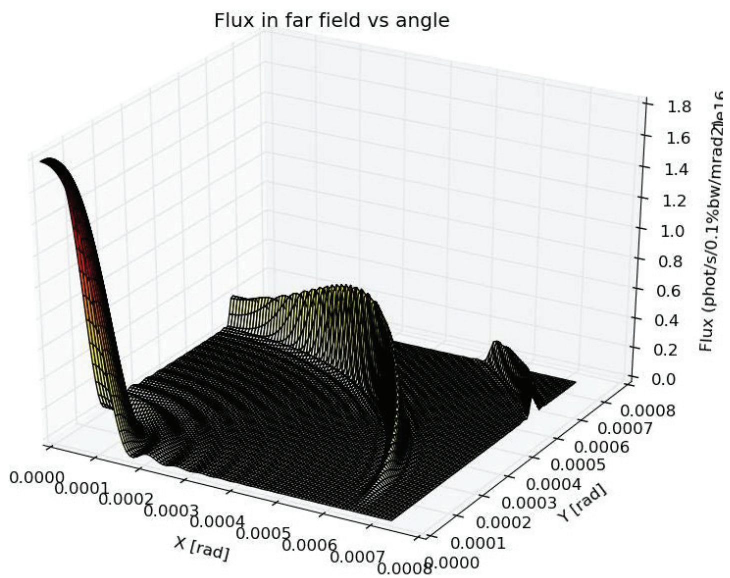
General | No z-coupling | Finite alpha | Symmetry point

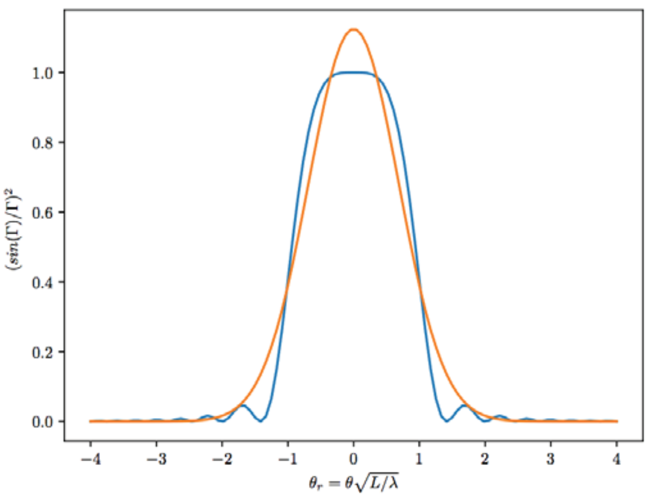
SINGLE ELECTRON PHOTON EMISSION (ZERO EMITTANCE)



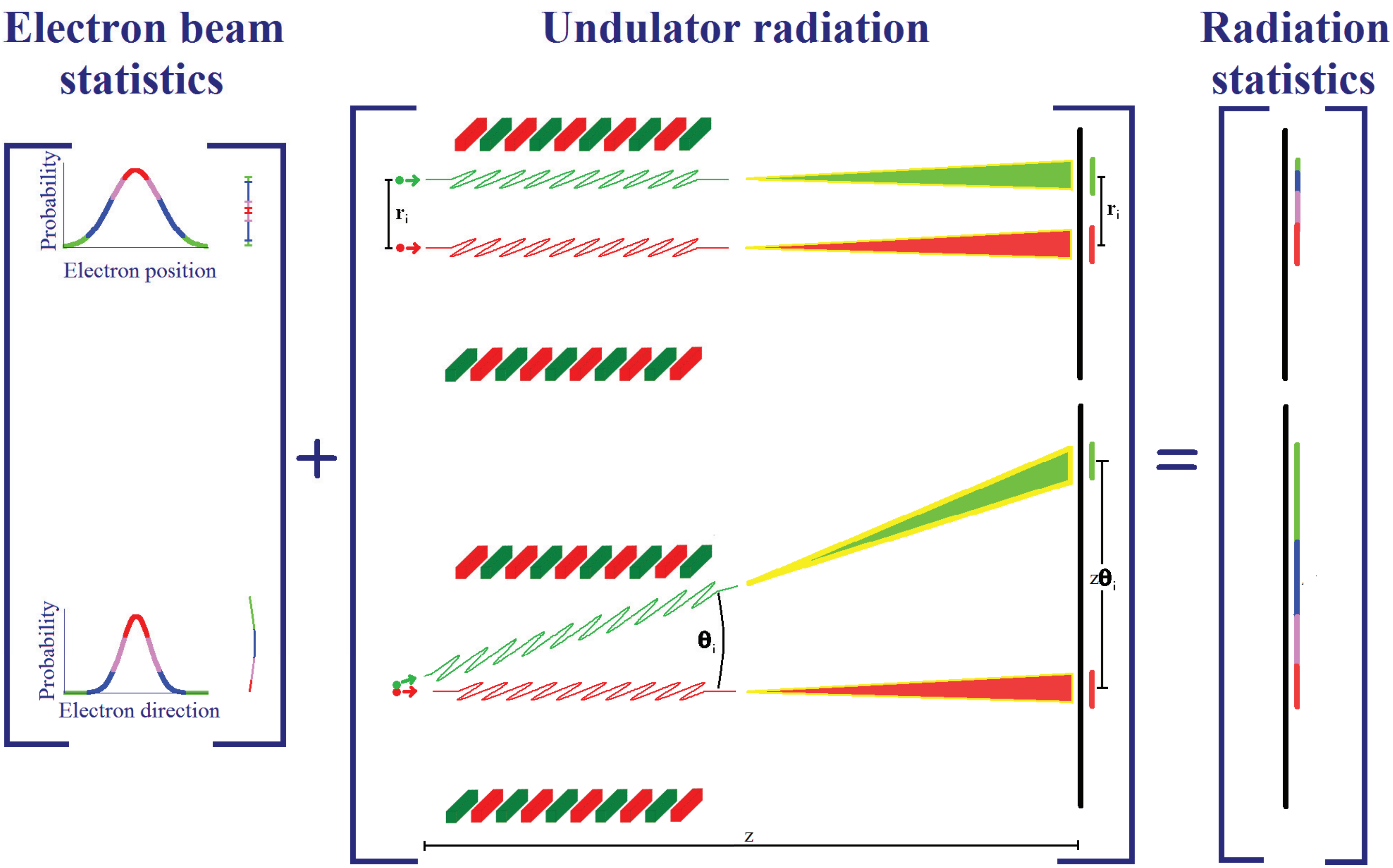


$$\frac{d^2I}{d\omega d\Omega} = \frac{eI}{8\pi^2 c\epsilon_0 h}10^{-9}\left|\int_{-\infty}^{\infty}\left[\frac{n\times\left[(n-\beta)\times\dot{\beta}\right]}{(1-\beta\cdot n)^2} + \frac{c}{\gamma^2 R}\frac{(n-\beta)}{(1-\beta\cdot n)^2}\right]e^{i\omega(t'+R(t')/c)}dt'\right|^2$$





beam statistics ➡ statistics of the emission



Kim, K.-J. Proc. SPIE 0582 (1986)

Mutual coherence function $\Gamma(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2)\rangle_e$

$$\left(\Delta_{\boldsymbol{r}_1} - \frac{1}{c^2}\frac{\partial^2}{\partial t_1^2}\right)\left(\Delta_{\boldsymbol{r}_2} - \frac{1}{c^2}\frac{\partial^2}{\partial t_2^2}\right)\Gamma(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = (4\pi)^2 \Gamma_Q(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2)$$

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Wide-sense stationary:

Mutual coherence function $\Gamma(\boldsymbol{r}_1,t_1,\boldsymbol{r}_2,t_2) = \langle E^*(\boldsymbol{r}_1,t_1)E(\boldsymbol{r}_2,t_2)\rangle_e$

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Wide-sense stationary:

$$\langle E^*(\boldsymbol{r}_1,t_1)E(\boldsymbol{r}_2,t_2)\rangle_e = \langle E^*(\boldsymbol{r}_1,0)E(\boldsymbol{r}_2,t_2-t_1)\rangle_e$$

Mutual coherence function $\Gamma(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2)\rangle_e$

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Wide-sense stationary:

$$\langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2)\rangle_e = \langle E^*(\boldsymbol{r}_1, 0) E(\boldsymbol{r}_2, t_2 - t_1)\rangle_e$$

Storage ring emission is *wide-sense stationary* if

- the bunch length is long enough
- the radiation frequency is large enough
- the monochromator resolution is not too high

Geloni, G., et al. Nucl. Inst. and Meth. in Physics 588 463-493 (2008)

Frequency representation:

$$\langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2) \rangle_e \xleftrightarrow[\text{FT}]{} \langle E^*(\boldsymbol{r}_1, \omega_1) E(\boldsymbol{r}_2, \omega_2) \rangle_e$$

CROSS SPECTRAL DENSITY

Frequency representation:

$$\langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2) \rangle_e \overset{\text{FT}}{\longleftrightarrow} \langle E^*(\boldsymbol{r}_1, \omega_1) E(\boldsymbol{r}_2, \omega_2) \rangle_e$$

In consequence:

CROSS SPECTRAL DENSITY

Frequency representation:

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In consequence:

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Frequency representation:

$$\langle E^*(\boldsymbol{r}_1, t_1) E(\boldsymbol{r}_2, t_2) \rangle_e \xleftrightarrow[\text{FT}]{} \langle E^*(\boldsymbol{r}_1, \omega_1) E(\boldsymbol{r}_2, \omega_2) \rangle_e$$

In consequence:

$$\langle E^*(\boldsymbol{r}_1, \omega_1) E(\boldsymbol{r}_2, \omega_2) \rangle_e \Rightarrow$$

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$$\langle E^*(\boldsymbol{r}_1, t_1)E(\boldsymbol{r}_2, t_2)\rangle_e \xleftrightarrow{\text{FT}} \langle E^*(\boldsymbol{r}_1, \omega_1)E(\boldsymbol{r}_2, \omega_2)\rangle_e$$

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much simpler

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- **Cross spectral density** (CSD) **[everything]**

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \langle E_1^*(\boldsymbol{r}_1, \omega) E_2(\boldsymbol{r}_2, \omega) \rangle_e$$

- **Spectral density** (kind of “intensity” / “energy”)

$$S(\boldsymbol{r}, \omega) = W(\boldsymbol{r}, \boldsymbol{r}, \omega)$$

- **Spectral degree of coherence**

$$\mu(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \frac{W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)}{\sqrt{S(\boldsymbol{r}_1, \omega) S(\boldsymbol{r}_2, \omega)}}$$

(incoherent) $0 \le |\mu(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)| \le 1$ (comp coherent)

CROSS SPECTRAL DENSITY

$W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$ **four-dimensional** for fixed frequency at a distance z.

Propagation:

$$W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \int W(\boldsymbol{r}_1', \boldsymbol{r}_2', \omega) h^*(\boldsymbol{r}_1, \boldsymbol{r}_1', \omega) h(\boldsymbol{r}_2, \boldsymbol{r}_2', \omega) d\boldsymbol{r}_1' \, d\boldsymbol{r}_2'$$

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$N_x, N_y \in [100{,}1000]$.

Memory size ~ $N_x^2 N_y^2$

$100^4 = 10^8$ **to** $1000^4 = 10^{12}$

complex numbers (16 bytes), i.e. at least Gb to Tb.

Computation of W **takes a lot of time, i.e. calculation of** 10^8 **to** 10^{12} **elements.**

Propagation of W **takes a lot of time for calculating** 10^8 **to** 10^{12} **4d integrals.**

COHERENT MODE DECOMPOSITION

The cross spectral density function W can be represented in **coherent modes**:

$$\boxed{\mathrm{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n^{\infty} \lambda_n(\omega) \phi_n^*(\boldsymbol{r}_1, \omega) \phi_n(\boldsymbol{r}_2, \omega)}$$

Trade **4d** spatial dependencies to **sum of 2d** at fixed frequency.

$\phi_n(\boldsymbol{r}, \boldsymbol{\omega})$ coherent mode

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Each mode propagate like a wavefront so one can build the CSD at any point by propagating the modes to that point.

COMSYL

The coherent modes are the solution of the homogenous Fredholm equation of second kind:

$$A_W[\phi_n] = \lambda_n \phi_n$$

i.e. an eigenvalue problem for:

$$A_W[f](\boldsymbol{r}_2) = \int W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) f(\boldsymbol{r}_1) d\boldsymbol{r}_1$$

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Coherent modes of X-ray beams emitted by undulators in new storage rings
Mark Glass and Manuel Sanchez del Rio
EPL, 119 3 (2017) 34004

DOI: https://doi.org/10.1209/0295-5075/119/34004
Free preprint: https://arxiv.org/abs/1706.04393

See the COMSYL Wiki pages https://github.com/mark-glass/comsyl/wiki for more information including the full thesis manuscript.

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Solved by COMSYL (Coherent modes for synchrotrons)

Open-source at:

https://github.com/mark-glass/comsyl

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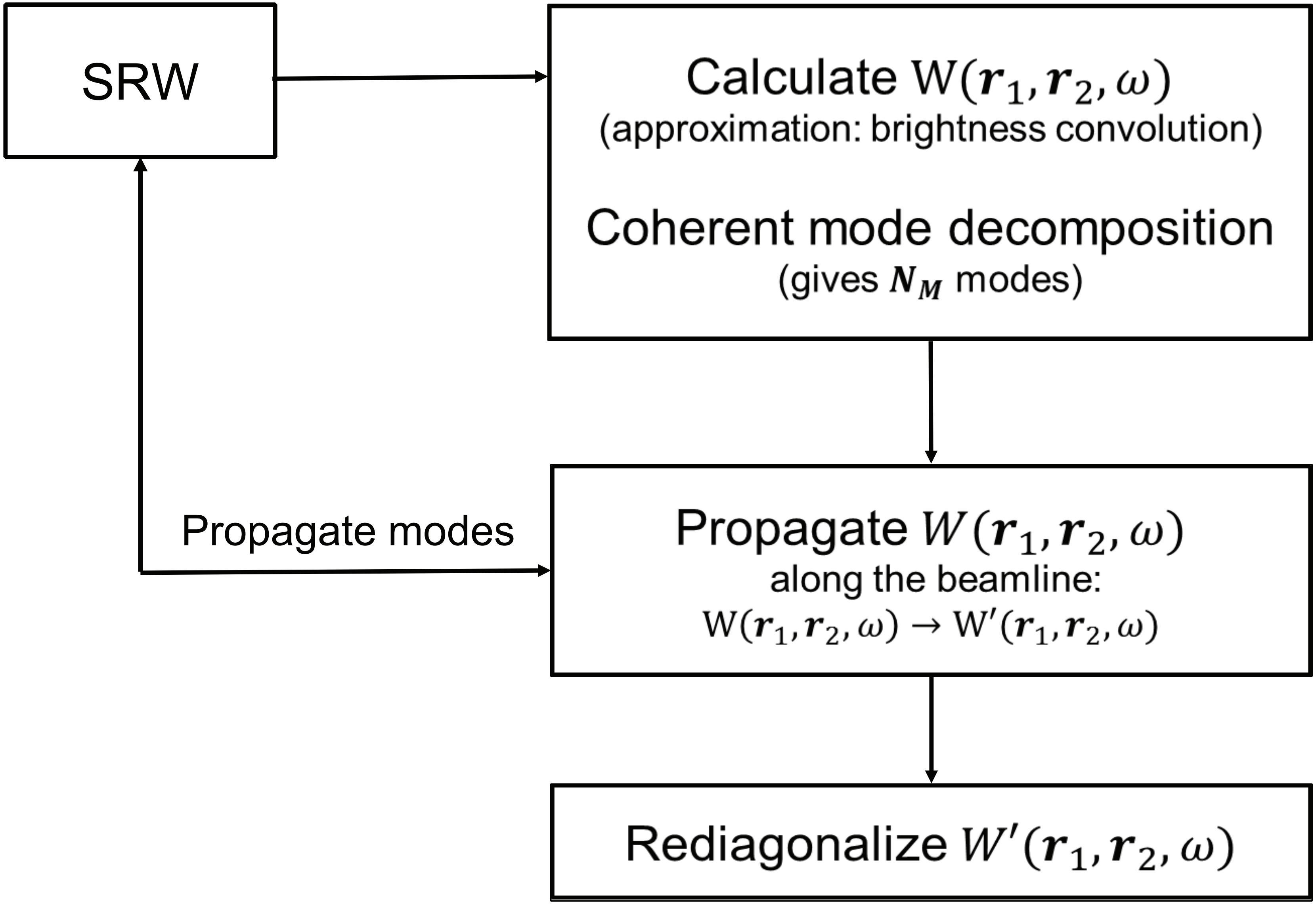
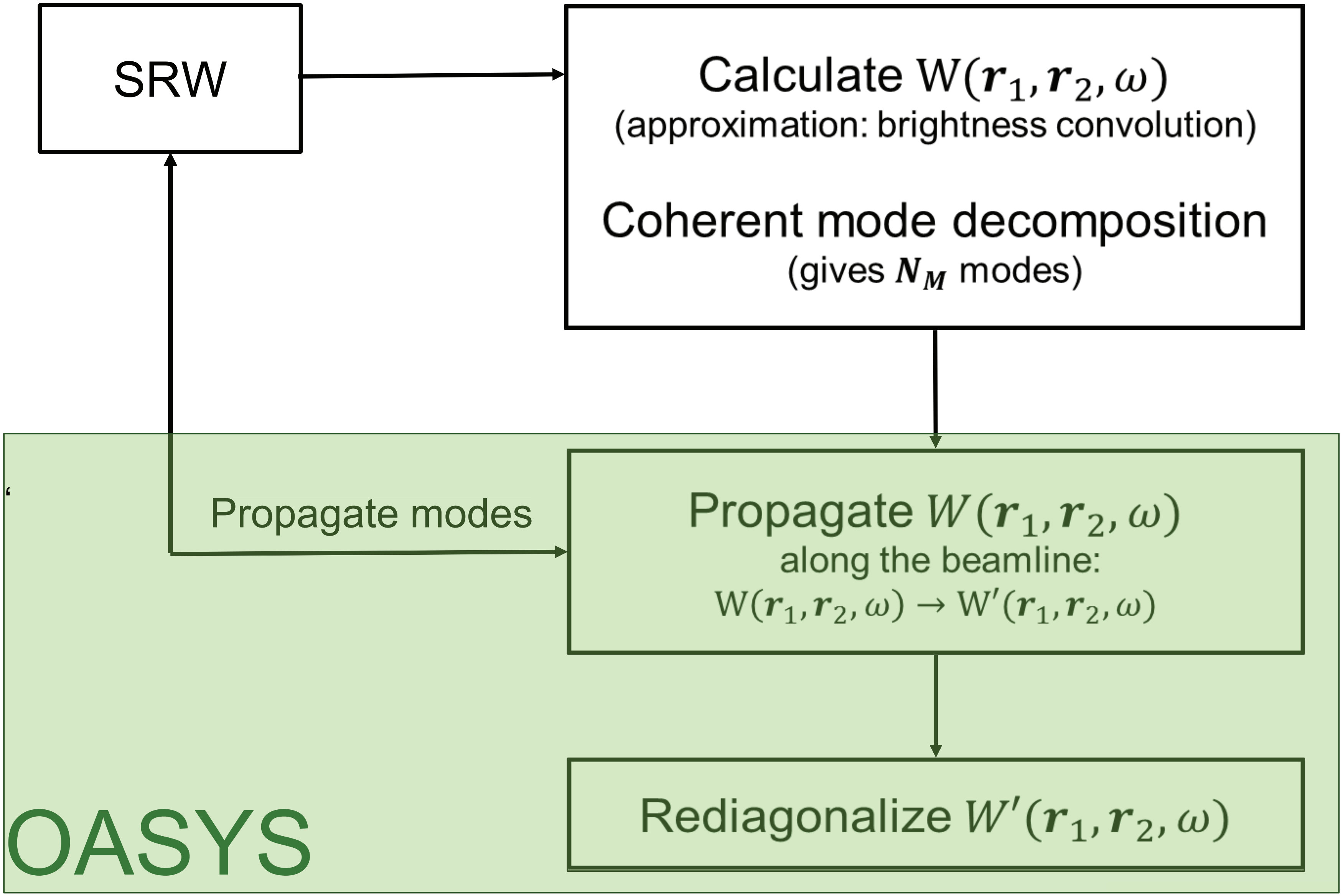
STRATEGY


SRW
Calculate $\mathrm{W}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
(approximation: brightness convolution)
Coherent mode decomposition
(gives $\boldsymbol{N_M}$ modes)
Propagate modes
Propagate $W(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
along the beamline:
$\mathrm{W}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) \rightarrow \mathrm{W}'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
Rediagonalize $W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$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(approximation: brightness convolution)
Coherent mode decomposition
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Propagate modes
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Rediagonalize $W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$
OASYS

STRATEGY

COMSYL

SRW

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(approximation: brightness convolution)

Coherent mode decomposition
(gives N_M modes)

OASYS

Propagate modes

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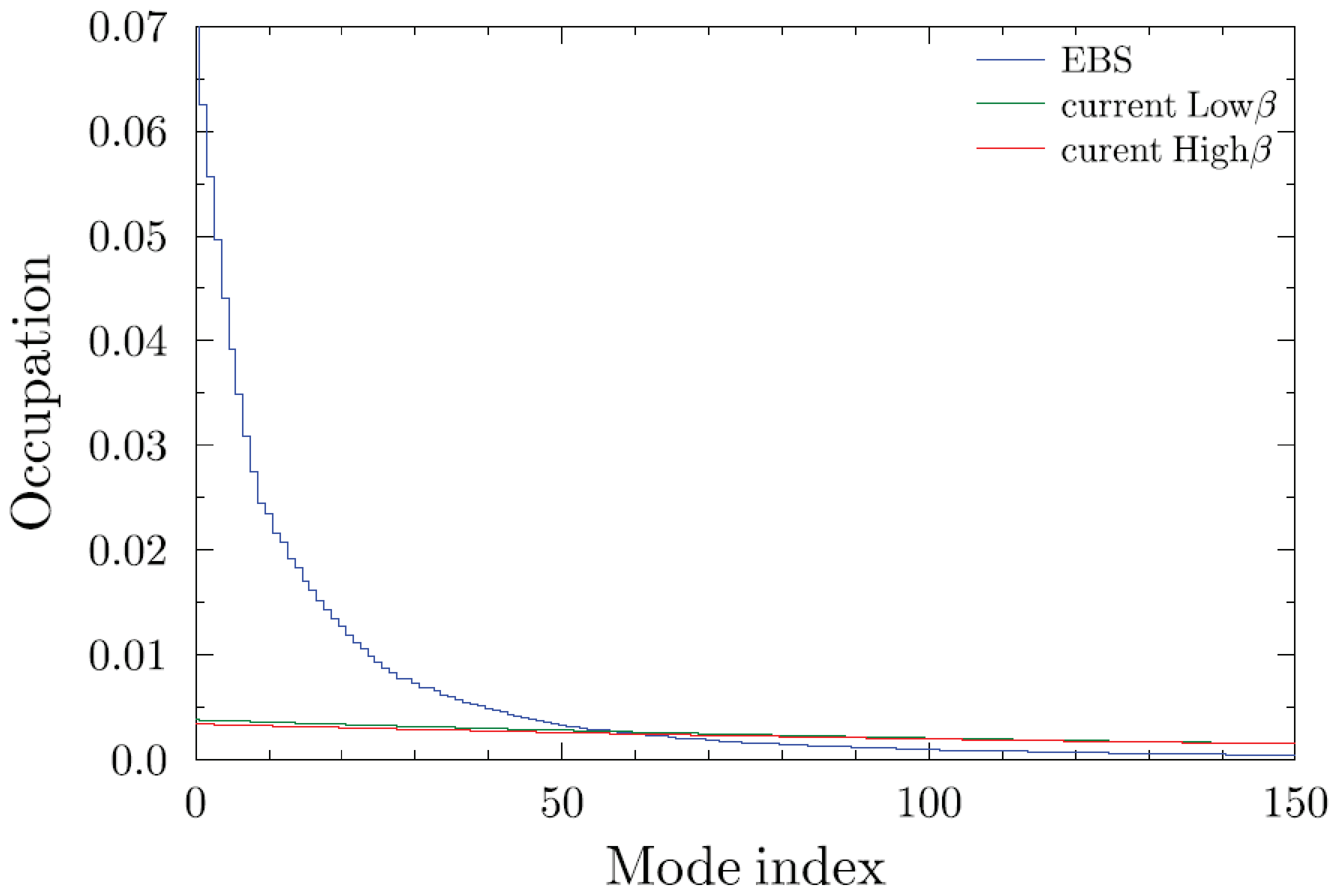
Rediagonalize $W'(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$

COMPARISON EBS VS CURRENT LATTICE: MODE SPECTRUM

$\lambda_n(\omega)$ eigenvalue (*mode intensities*)

$d_n(\omega) = \frac{\lambda_n(\omega)}{\sum_j \lambda_j(\omega)}$ occupation (mode distribution)

d_0 is the Coherent fraction



2m U18 @ current or EBS, with energy spread, 1.harmonic (8keV)

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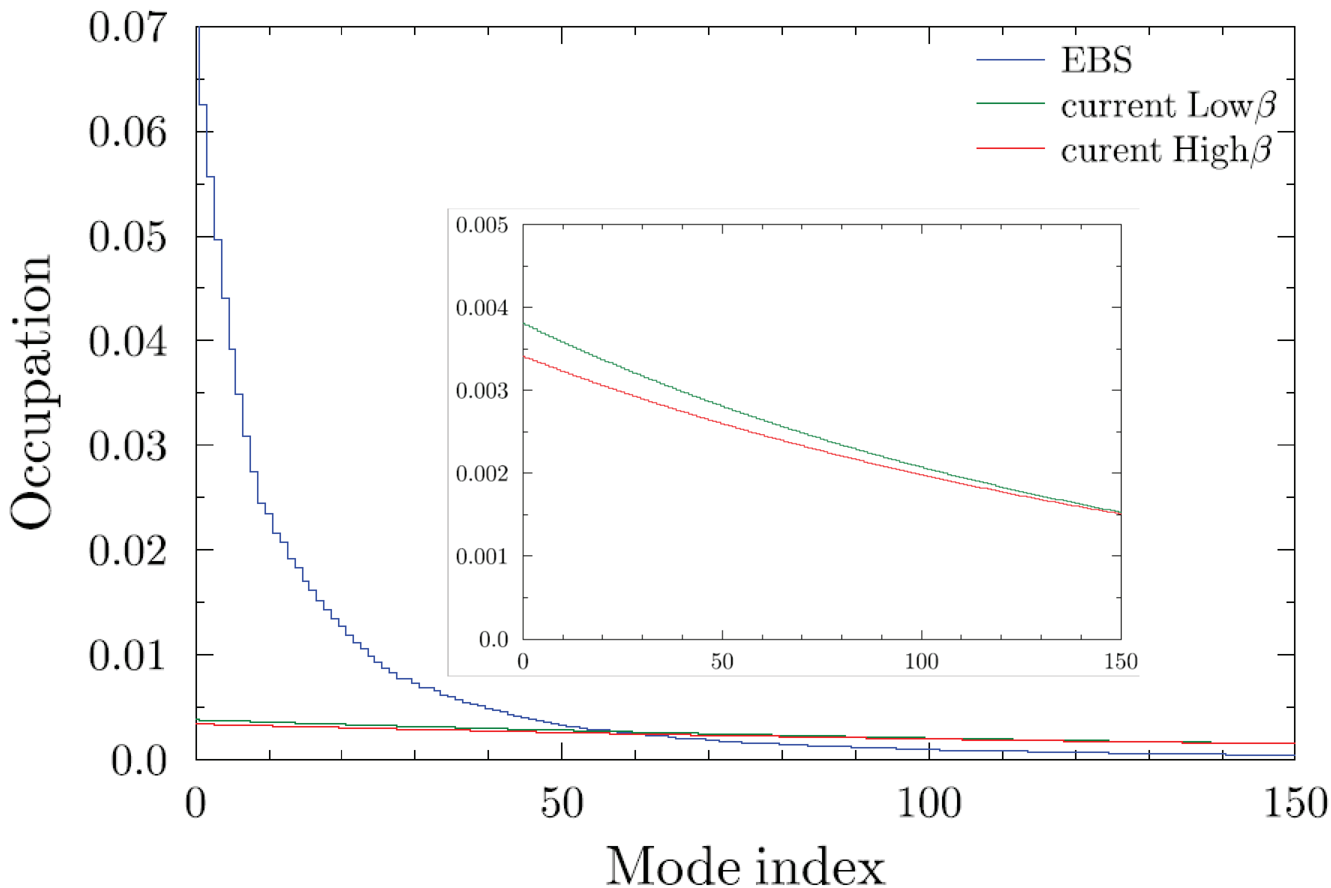

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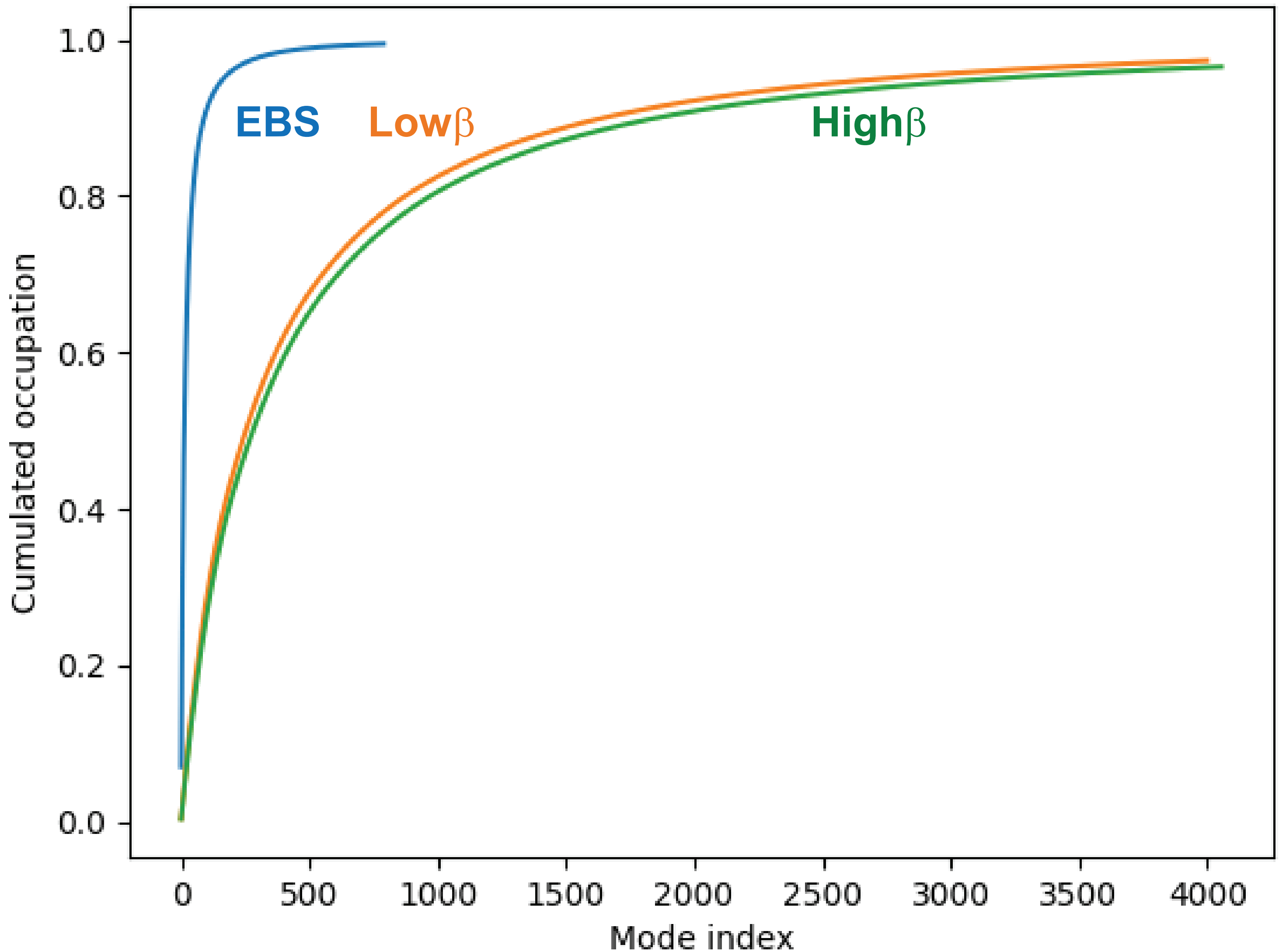
CUMULATED OCCUPATION



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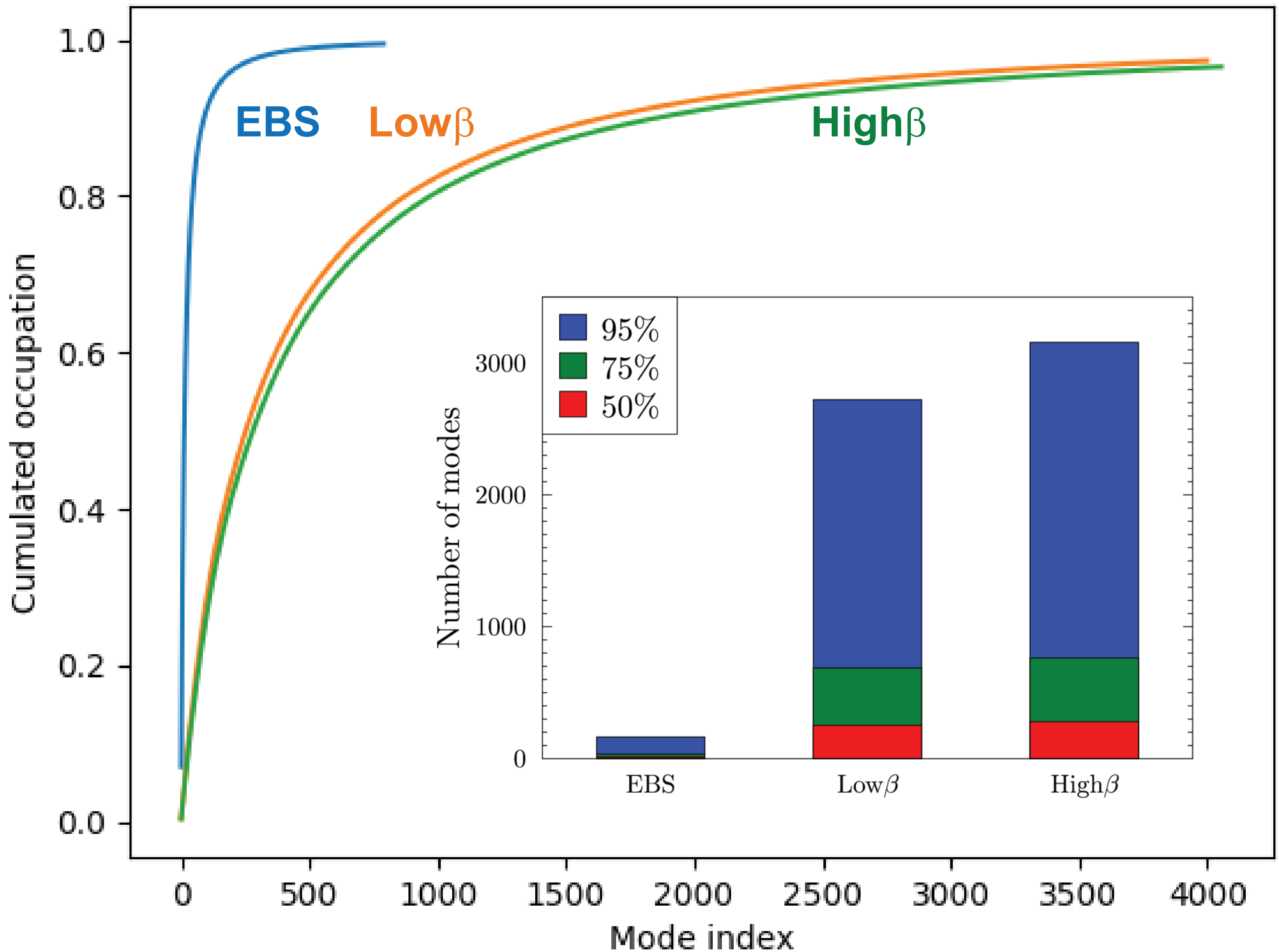
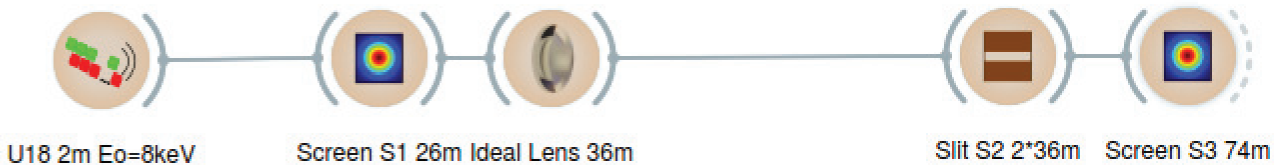
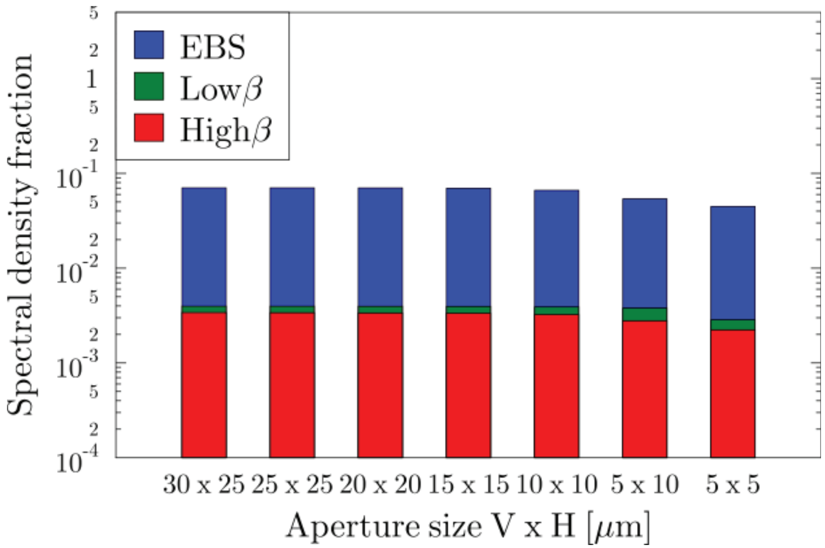


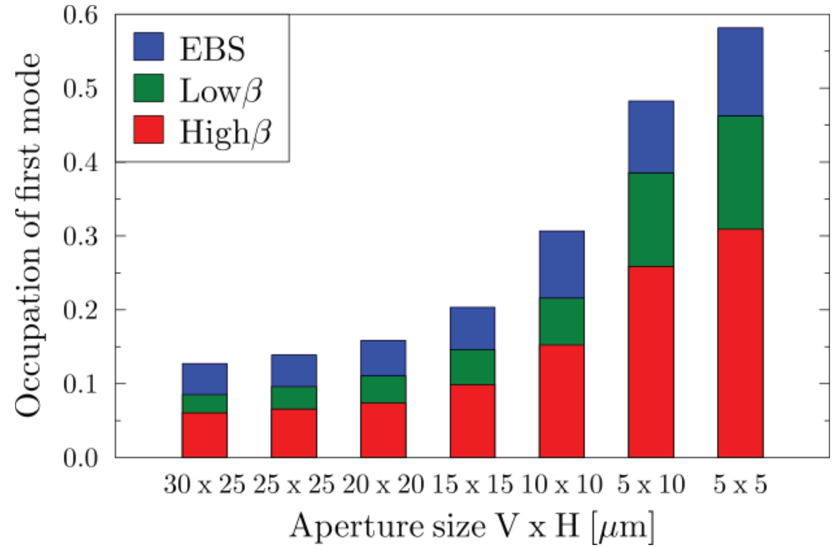
COMPARISON EBS VS CURRENT LATTICE (IMAGING BEAMLINE)

A typical "coherence beamline": 2m U18 E_0=8keV



At S_3 after second diagonalization:





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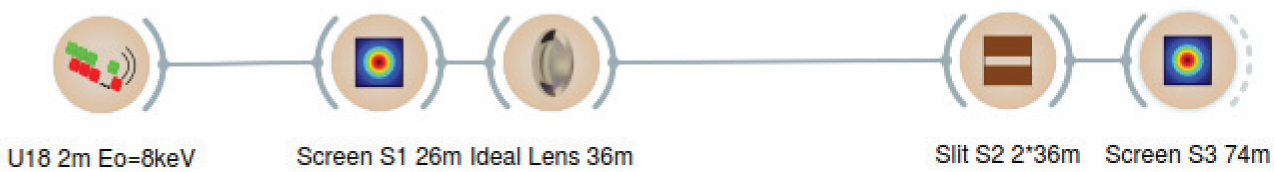

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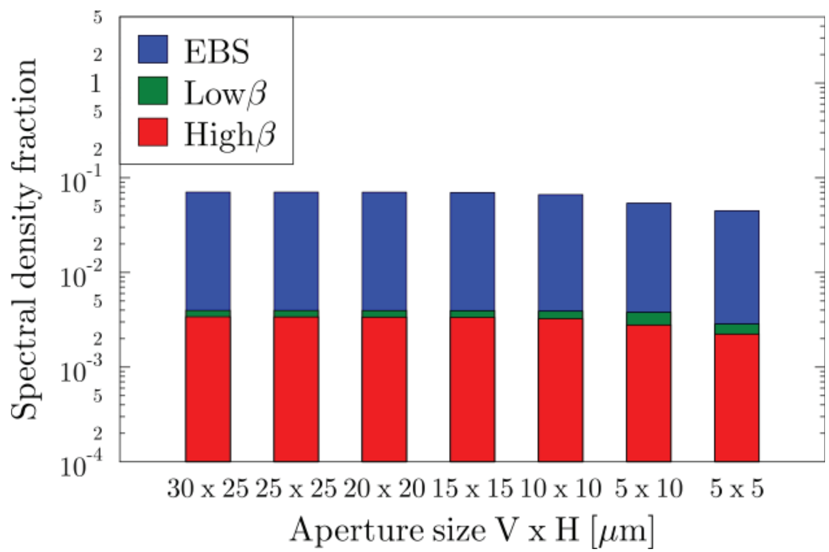

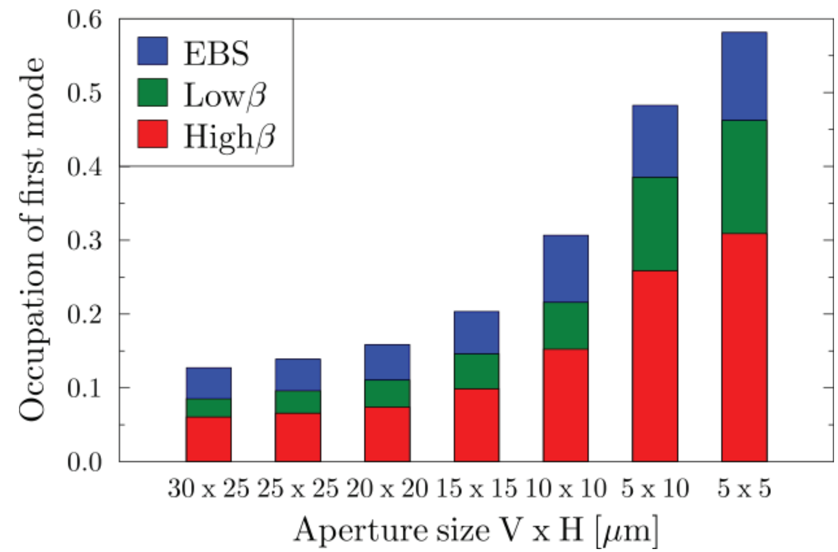

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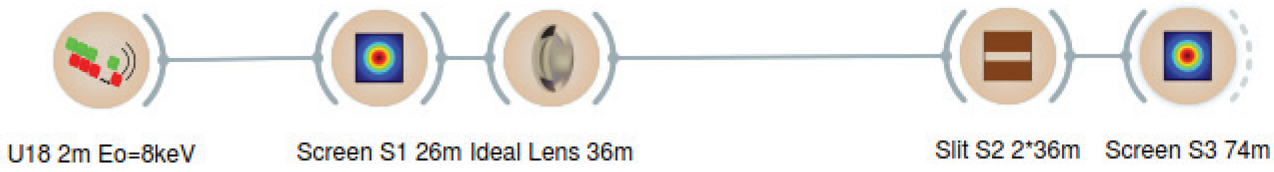

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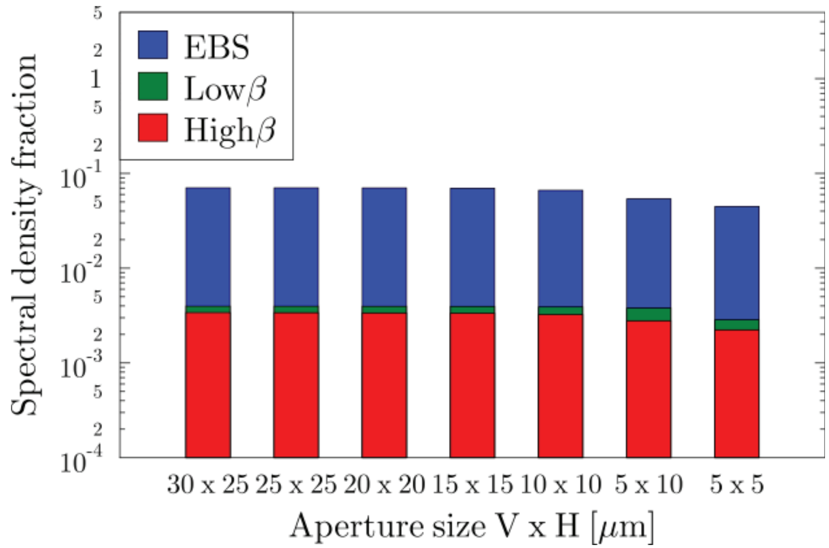
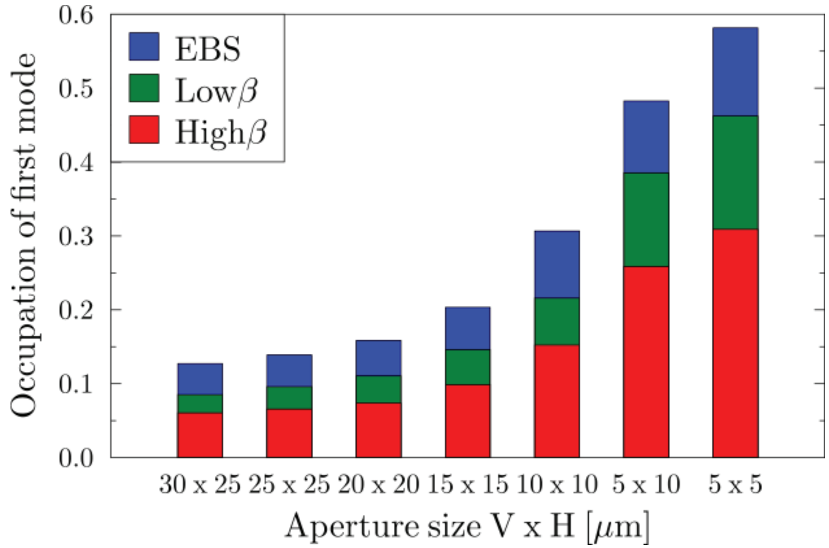
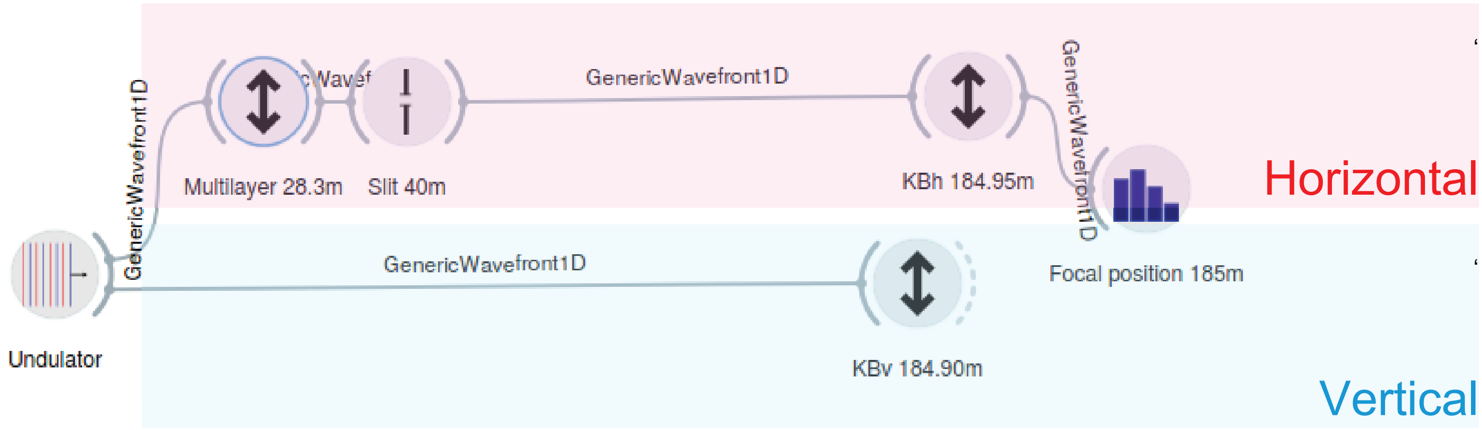




Quantifies "less flux" but "more coherent"

EBS will deliver more flux and more coherence

ID16A U18 L=1.4m @17.225 keV



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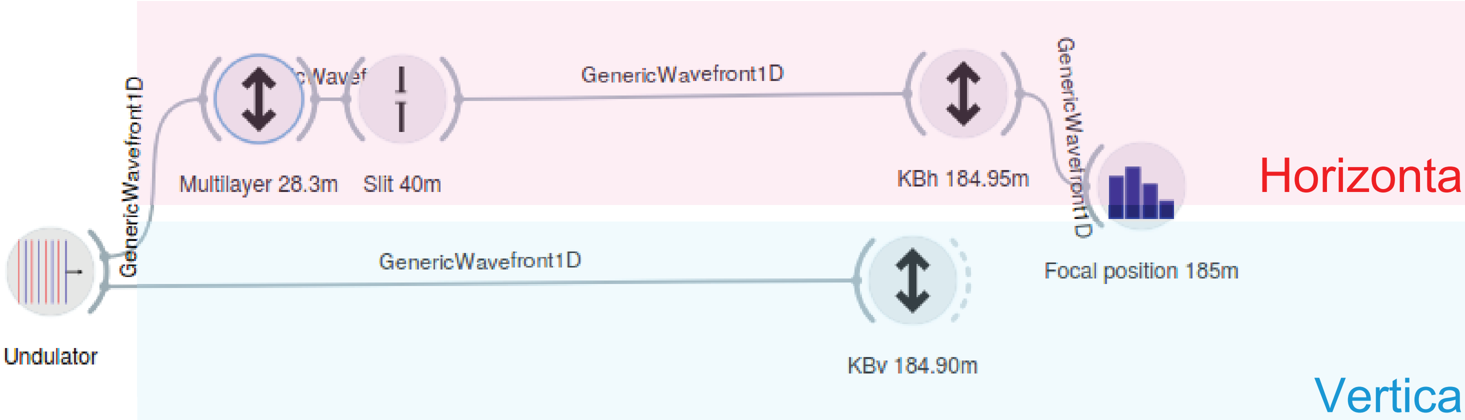

1) Extreme demagnification

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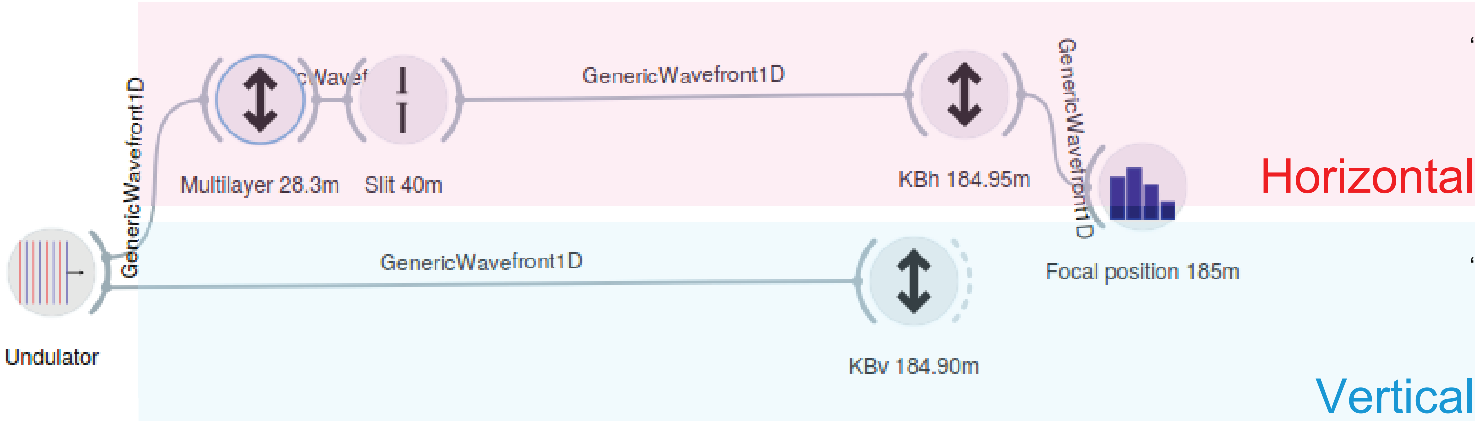

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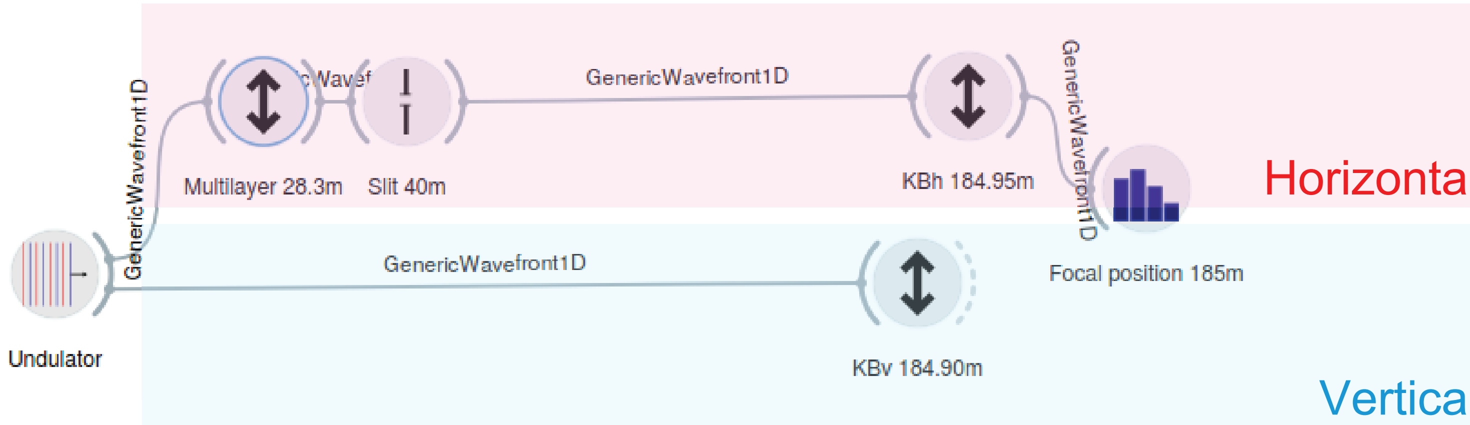
Plane	Source	Multilayer	Slit	KB(V)	KB(H)	focal plane
D	0.0	28.3	40.0	184.90	184.95	185.0
H		2.42:1			2899:1	
V				1849:1		



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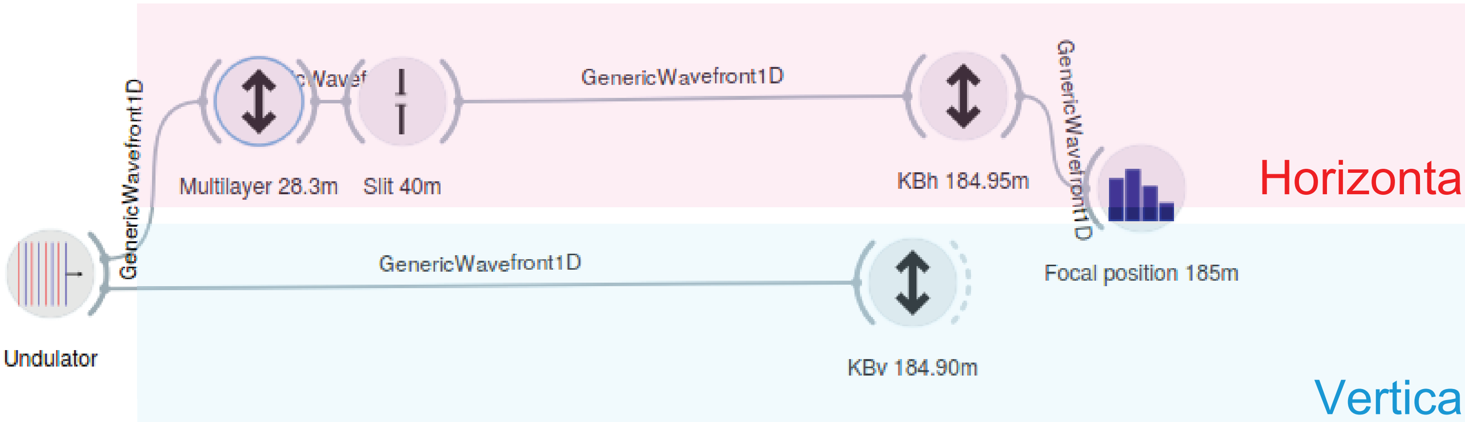

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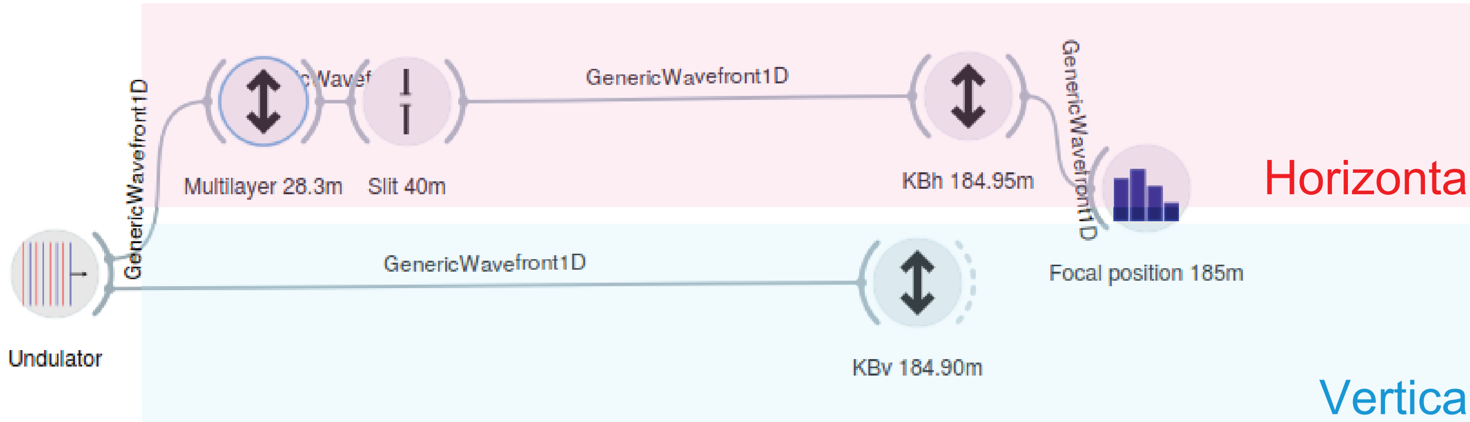

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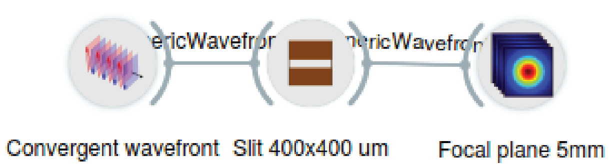

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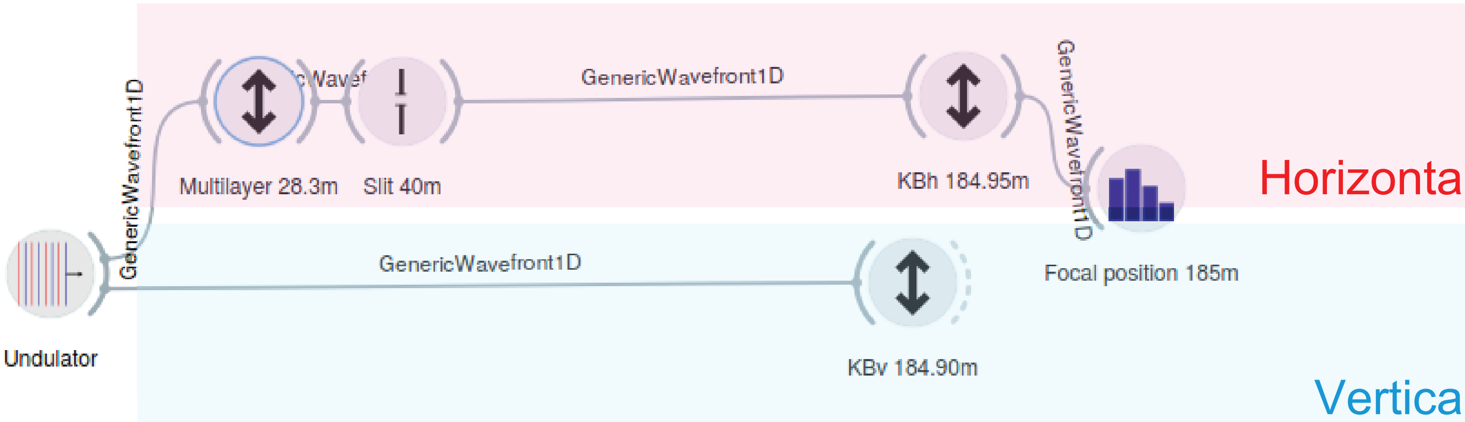

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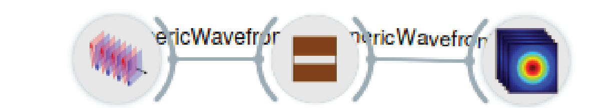

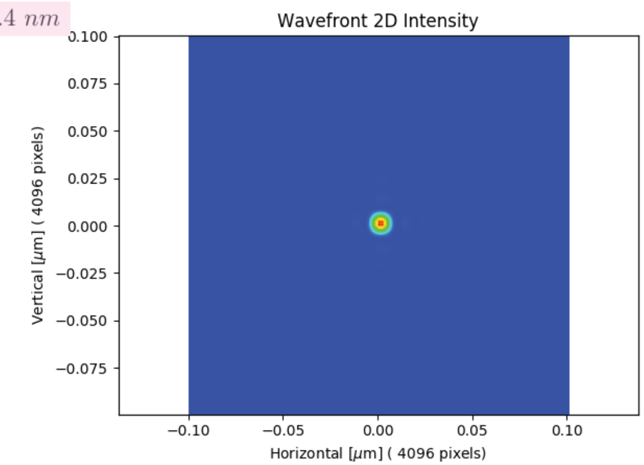

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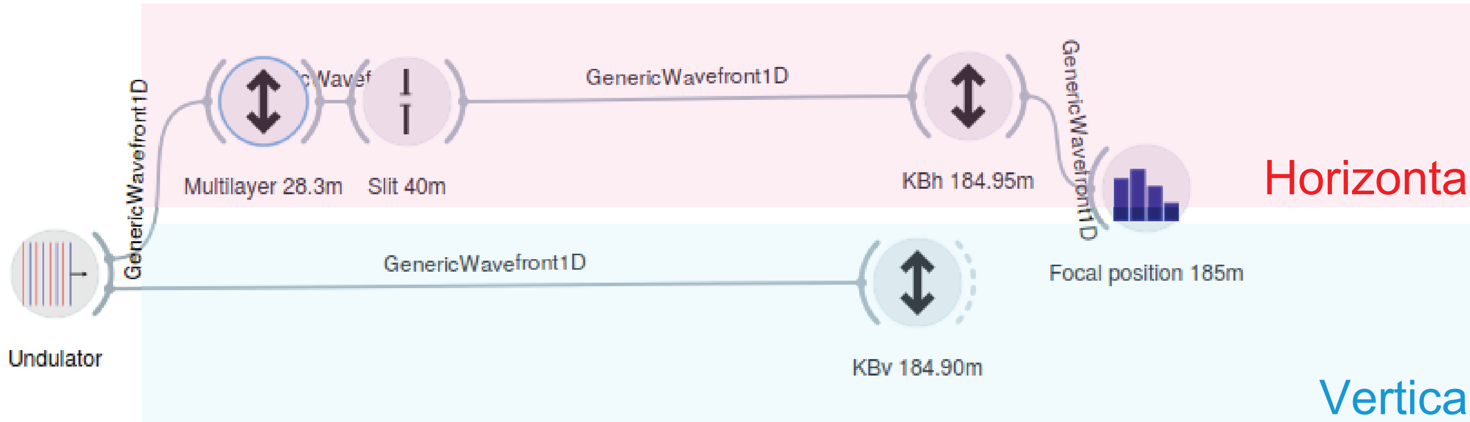

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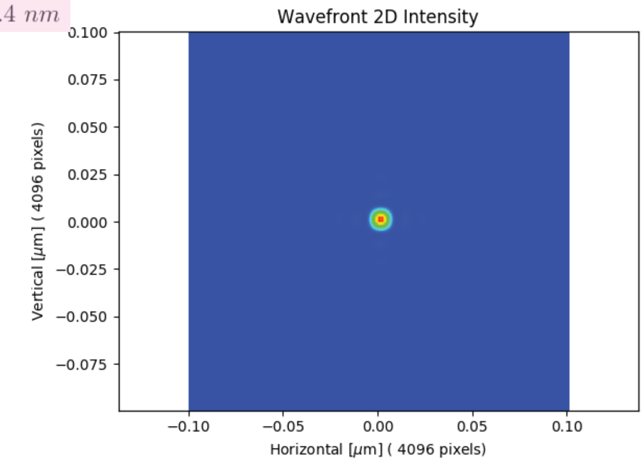
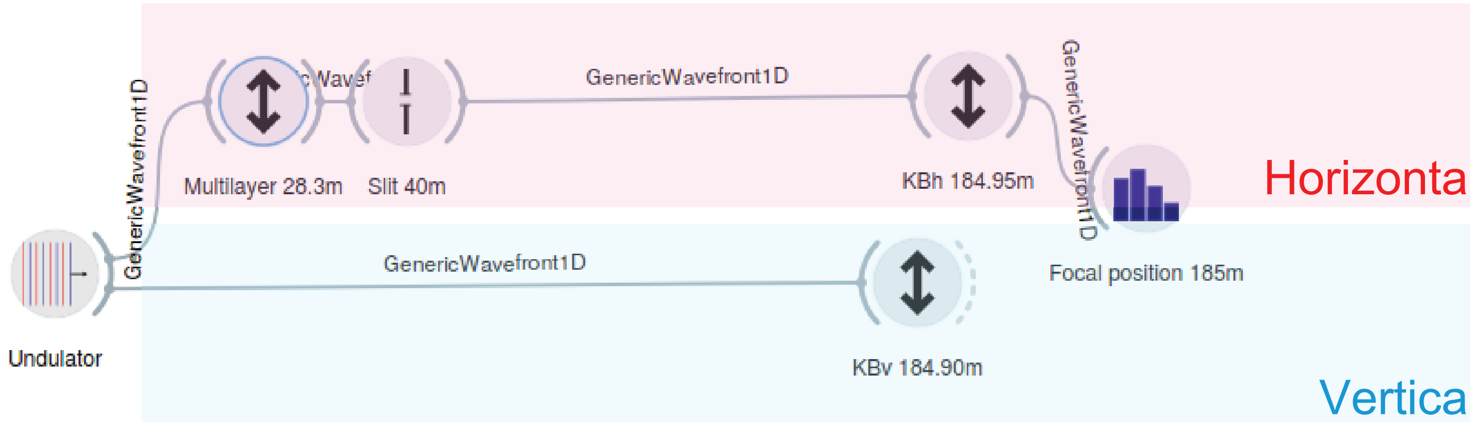


2) Diffraction limited (mirror clipping)

3) Slope errors

ID16A U18 L=1.4m @17.225 keV



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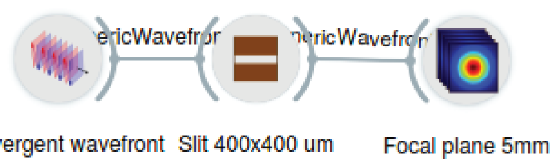
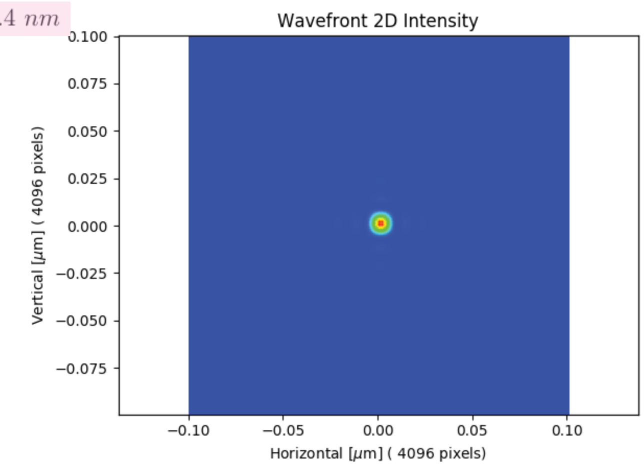
EBS Focal Plane	10.2 nm	5.4 nm

2) Diffraction limited (mirror clipping)

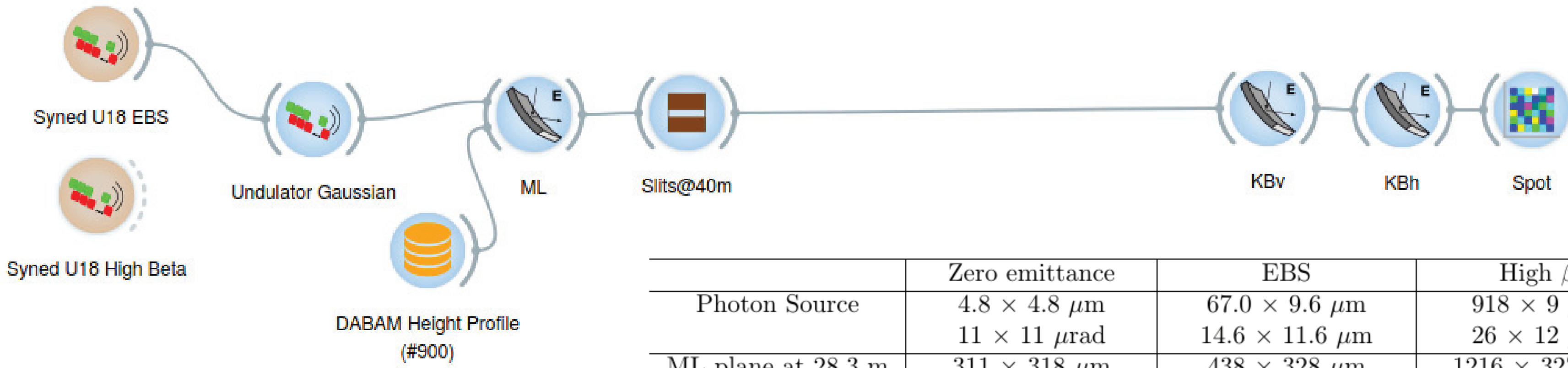




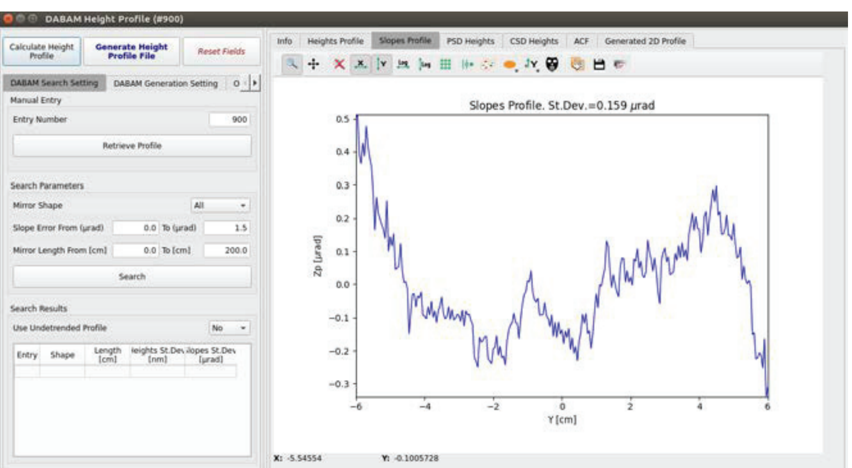
3) Slope errors

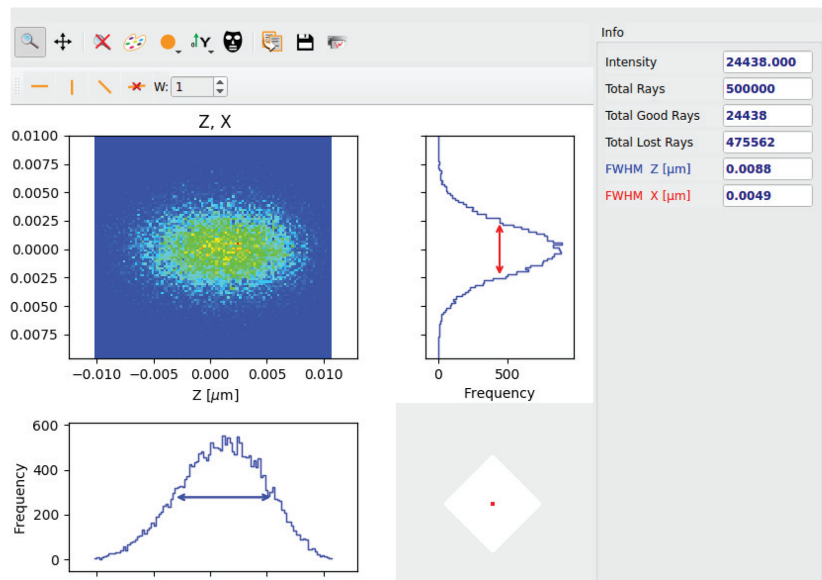
J. C. da Silva et al.: https://doi.org/10.1364/OPTICA.4.000492

RAY TRACING



	Zero emittance	EBS	High β
Photon Source	$4.8 \times 4.8\ \mu m$ $11 \times 11\ \mu rad$	$67.0 \times 9.6\ \mu m$ $14.6 \times 11.6\ \mu m$	$918 \times 9\ \mu m$ $26 \times 12\ \mu m$
ML plane at 28.3 m	$311 \times 318\ \mu m$	$438 \times 328\ \mu m$	$1216 \times 327\ \mu m$
Slit at 40 m	$3 \times 431\ \mu m$ (100%)	$28 \times 445\ \mu m$ (95%)	(11%)
KBv	(38%)	(35%)	(4.3%)
KBh	(6.9%)	(4.9%)	(0.44%)
Focal plane at 185m	1×2.4 nm (6.9%)	8.8×4.9 nm (4.8%)	15×5 nm (0.44%)





COHERENT FRACTION ID16A U18 L=1.4m @17.225 keV

https://arxiv.org/abs/1801.07542

Coherent fraction values (in %) calculated as occupation of the first coherent mode using the COMSYL software (CF^C), and compared with approximated expressions CF^E, CF^{EA} and CF^K (see text) for an undulator U18 L=1.4m at 17.225 keV.

Storage ring	CF^C	CF^E	CF^{EA}	CF^K
ESRF (EBS)	2.8	3.22	3.08	0.88
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	Photon beam divergence	Photon beam size	Photon phase space volume	Coherent Fraction
Kim				
Elleaume				

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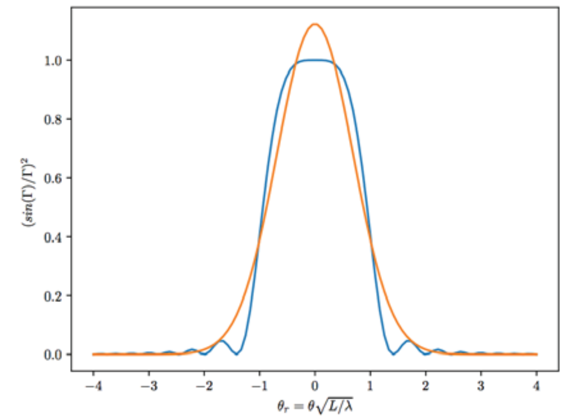

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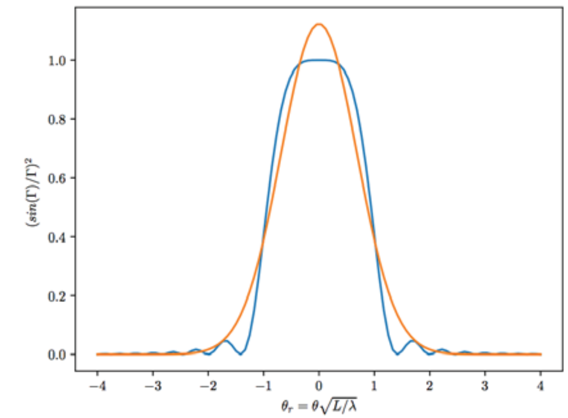

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Kim		$\sigma_r = \frac{1}{4\pi}\sqrt{2\lambda L}$		$CF^K = \frac{\left(\frac{\lambda}{4\pi}\right)^2}{\sqrt{\left(\sigma_x^2+\frac{\lambda L}{8\pi^2}\right)\left(\sigma_y^2+\frac{\lambda L}{8\pi^2}\right)\left(\sigma_{x'}^2+\frac{\lambda}{2L}\right)\left(\sigma_{x'}^2+\frac{\lambda}{2L}\right)}}$
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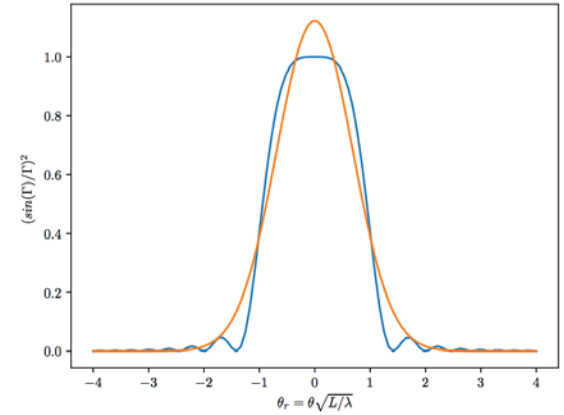

	Photon beam divergence $\sigma_{r'}$	Photon beam size σ_r	Photon phase space volume	Coherent Fraction
Kim		$\sigma_r = \frac{1}{4\pi}\sqrt{2\lambda L}$		$CF^K = \frac{\left(\frac{\lambda}{4\pi}\right)^2}{\sqrt{\left(\sigma_x^2 + \frac{\lambda L}{8\pi^2}\right)\left(\sigma_y^2 + \frac{\lambda L}{8\pi^2}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)}}$
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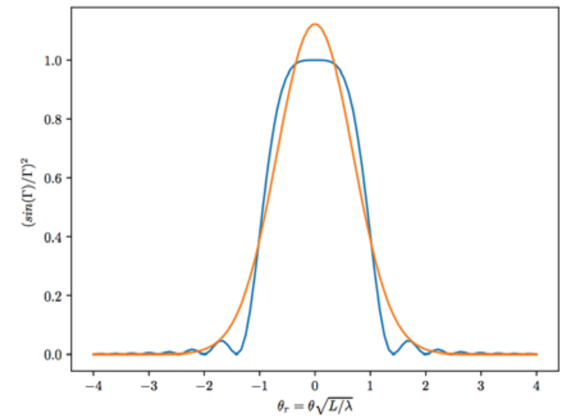

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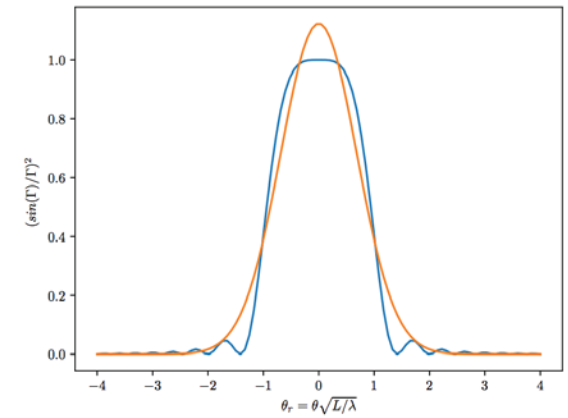

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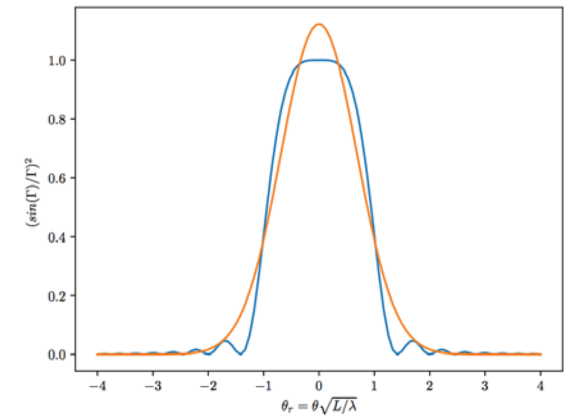

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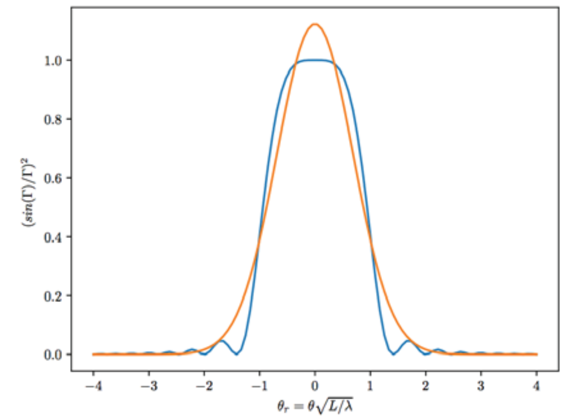

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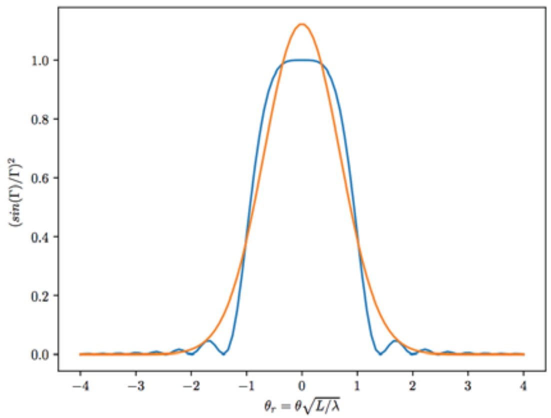

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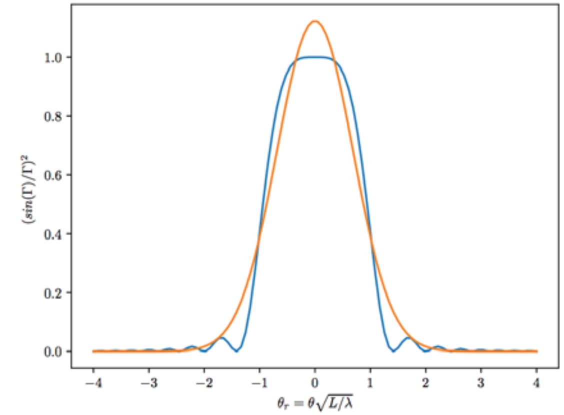

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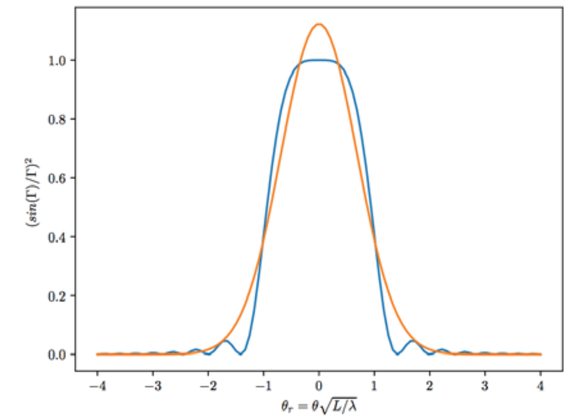
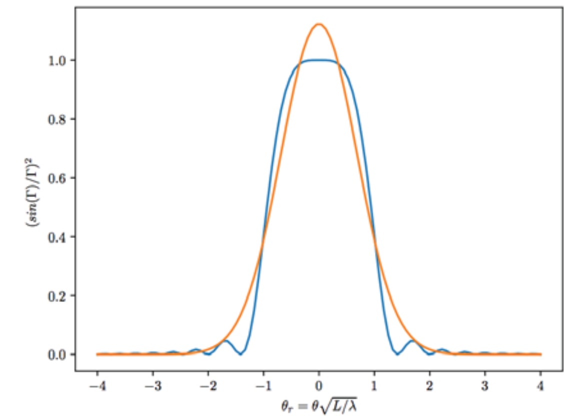


	Photon beam divergence $\sigma_{r'}$	Photon beam size σ_r	Photon phase space volume $\sigma_r \sigma_{r'}$	Coherent Fraction
Kim	$\sqrt{\frac{\lambda}{2L}}$	$\sigma_r = \frac{1}{4\pi}\sqrt{2\lambda L}$	$\frac{\lambda}{4\pi}$	$CF^K = \frac{\left(\frac{\lambda}{4\pi}\right)^2}{\sqrt{\left(\sigma_x^2 + \frac{\lambda L}{8\pi^2}\right)\left(\sigma_y^2 + \frac{\lambda L}{8\pi^2}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)}}$
Elleaume	$0.69\sqrt{\frac{\lambda}{L}} \sim \sqrt{\frac{\lambda}{2L}}$	$\frac{2.740}{4\pi}\sqrt{\lambda L}$	$\frac{1.89\lambda}{4\pi} \sim \frac{\lambda}{2\pi}$	$CF^{EA} = \frac{\left(\frac{\lambda}{2\pi}\right)^2}{\sqrt{\left(\sigma_x^2 + \frac{\lambda L}{2\pi^2}\right)\left(\sigma_y^2 + \frac{\lambda L}{2\pi^2}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)}}$

https://arxiv.org/abs/1801.07542

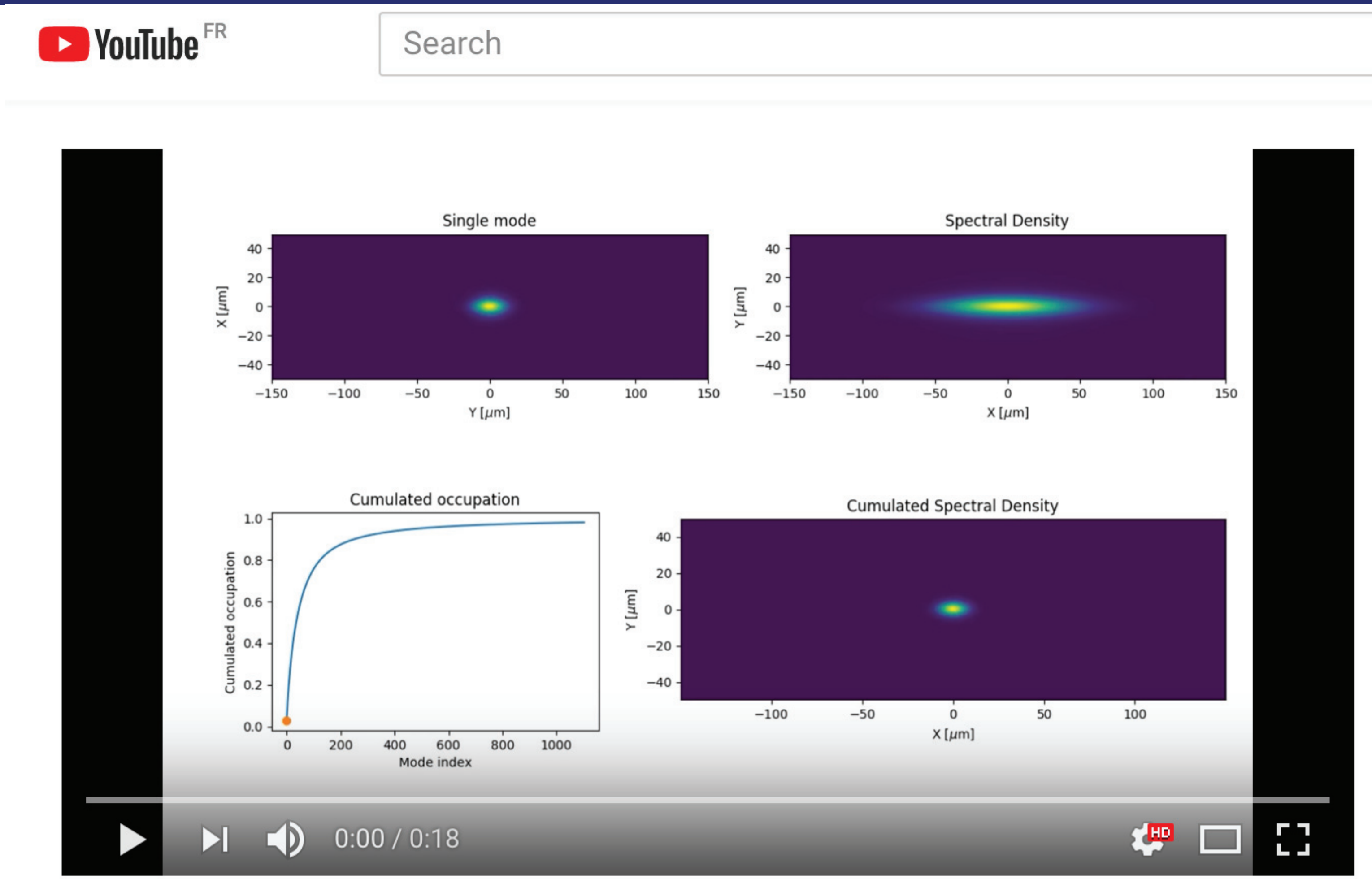
Coherent fraction values (in %) calculated as occupation of the first coherent mode using the COMSYL software (CF^C), and compared with approximated expressions CF^E, CF^{EA} and CF^K (see text) for an undulator U18 L=1.4m at 17.225 keV.

Storage ring	CF^C	CF^E	CF^{EA}	CF^K
ESRF (EBS)	2.8	3.22	3.08	0.88
ESRF (Highβ)	0.13	0.14	0.14	0.04



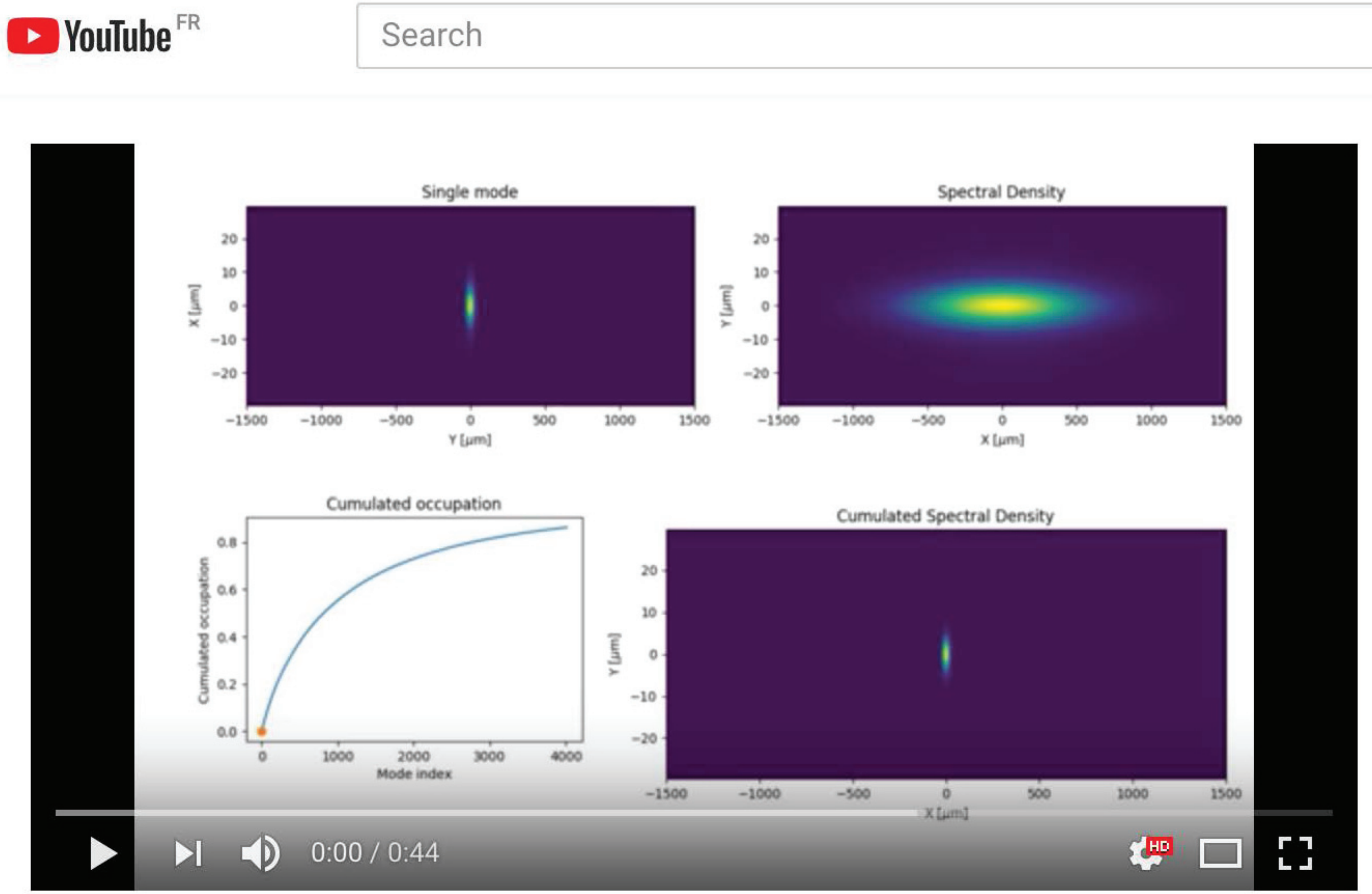
	Photon beam divergence $\sigma_{r'}$	Photon beam size σ_r	Photon phase space volume $\sigma_r \sigma_{r'}$	Coherent Fraction
Kim	$\sqrt{\frac{\lambda}{2L}}$	$\sigma_r = \frac{1}{4\pi}\sqrt{2\lambda L}$	$\frac{\lambda}{4\pi}$	$CF^K = \frac{\left(\frac{\lambda}{4\pi}\right)^2}{\sqrt{\left(\sigma_x^2 + \frac{\lambda L}{8\pi^2}\right)\left(\sigma_y^2 + \frac{\lambda L}{8\pi^2}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)}}$
Elleaume	$0.69\sqrt{\frac{\lambda}{L}} \sim \sqrt{\frac{\lambda}{2L}}$	$\frac{2.740}{4\pi}\sqrt{\lambda L}$	$\frac{1.89\lambda}{4\pi} \sim \frac{\lambda}{2\pi}$	$CF^{EA} = \frac{\left(\frac{\lambda}{2\pi}\right)^2}{\sqrt{\left(\sigma_x^2 + \frac{\lambda L}{2\pi^2}\right)\left(\sigma_y^2 + \frac{\lambda L}{2\pi^2}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)\left(\sigma_{x'}^2 + \frac{\lambda}{2L}\right)}}$

COMSYL EBS SOURCE (CF 0.028)



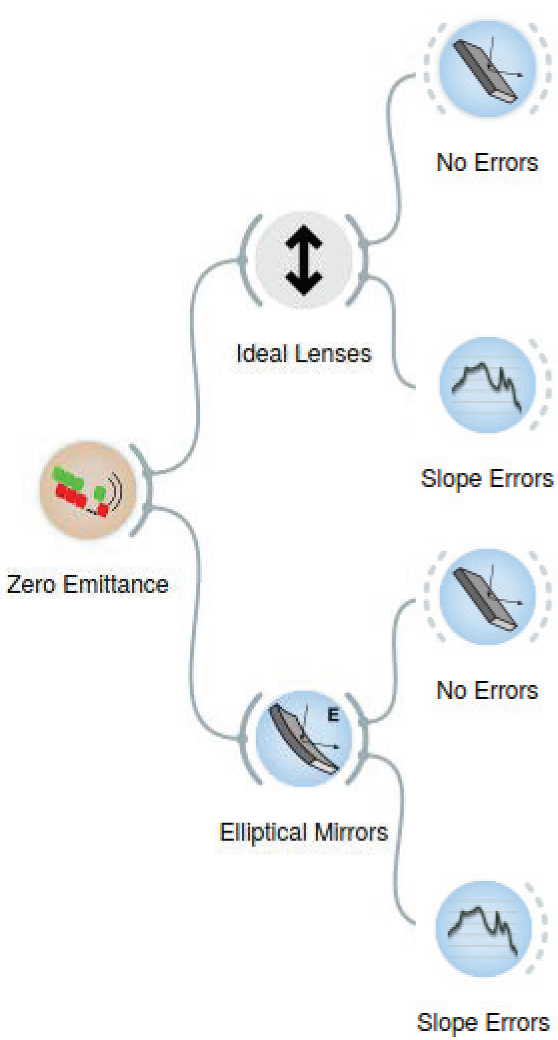
Coherent modes of synchrotron radiation for EBS

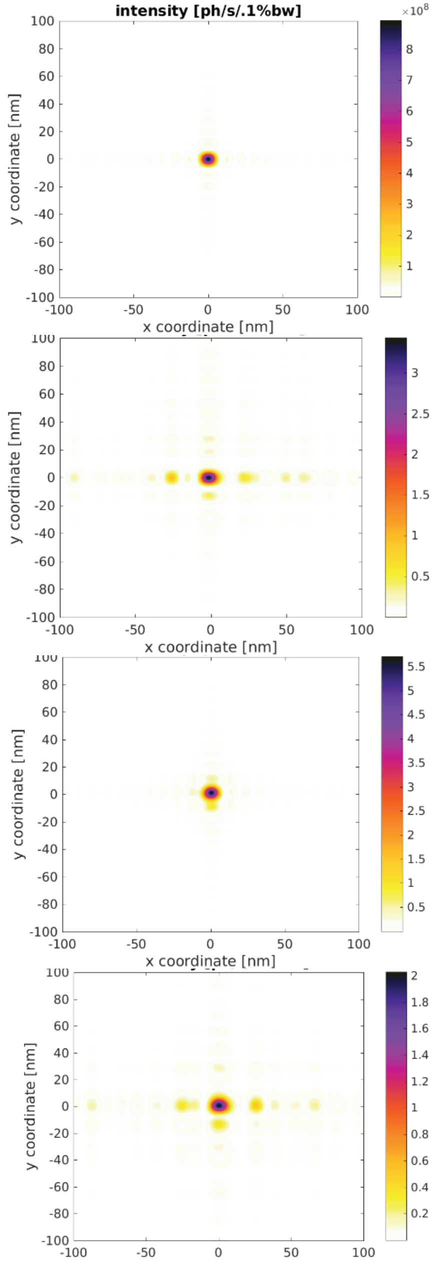
COMSYL HIGH BETA SOURCE (CF 0.0013)

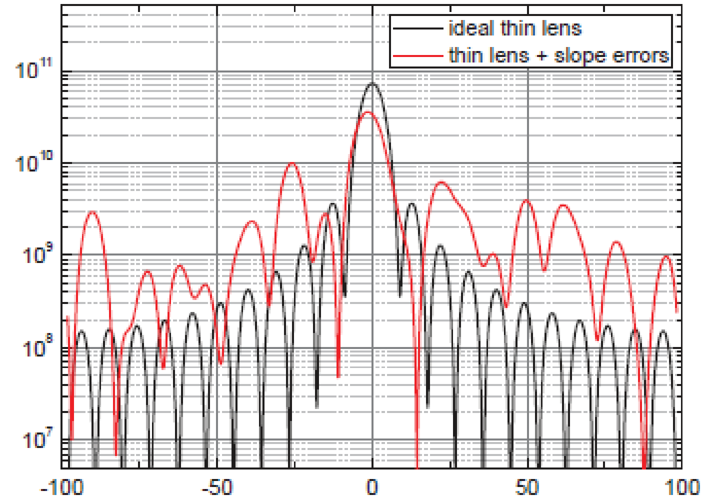


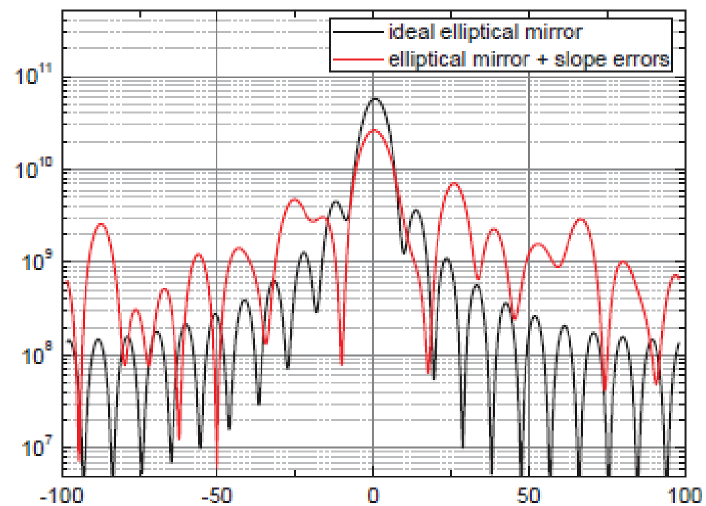
Coherent modes of synchrotron radiation for ESRF-High beta

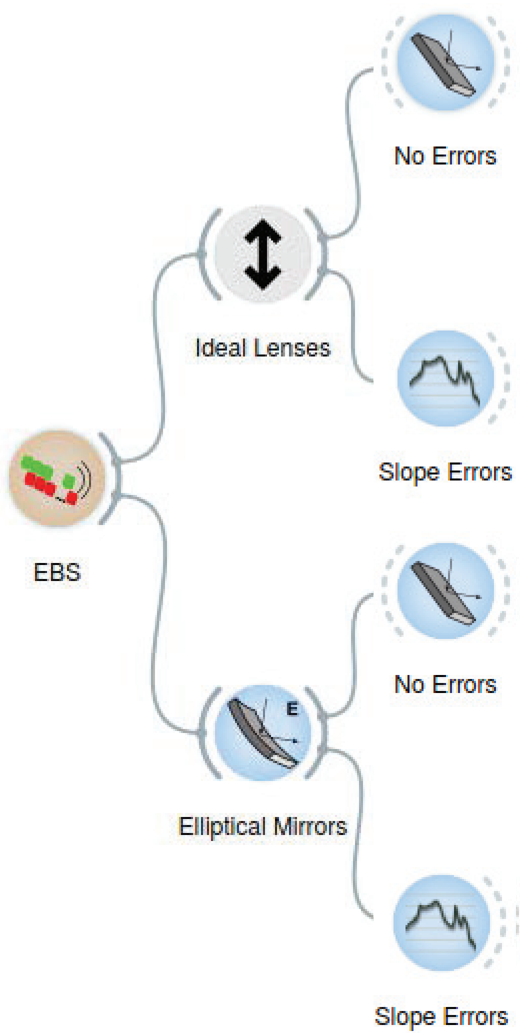
SRW – ZERO EMITTANCE



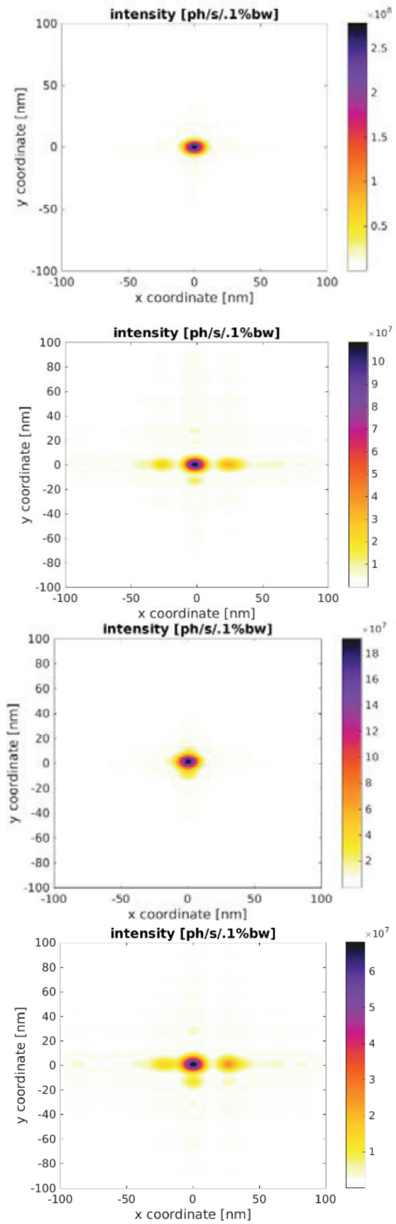




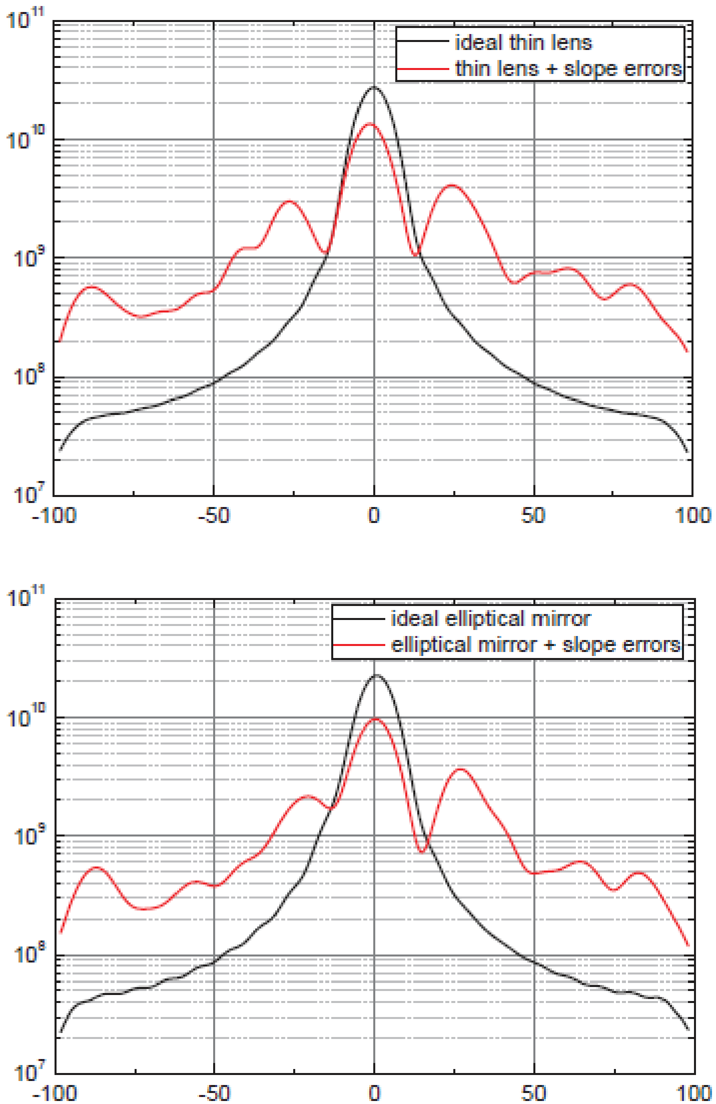




No Errors
Ideal Lenses
Slope Errors
EBS
No Errors
Elliptical Mirrors
Slope Errors

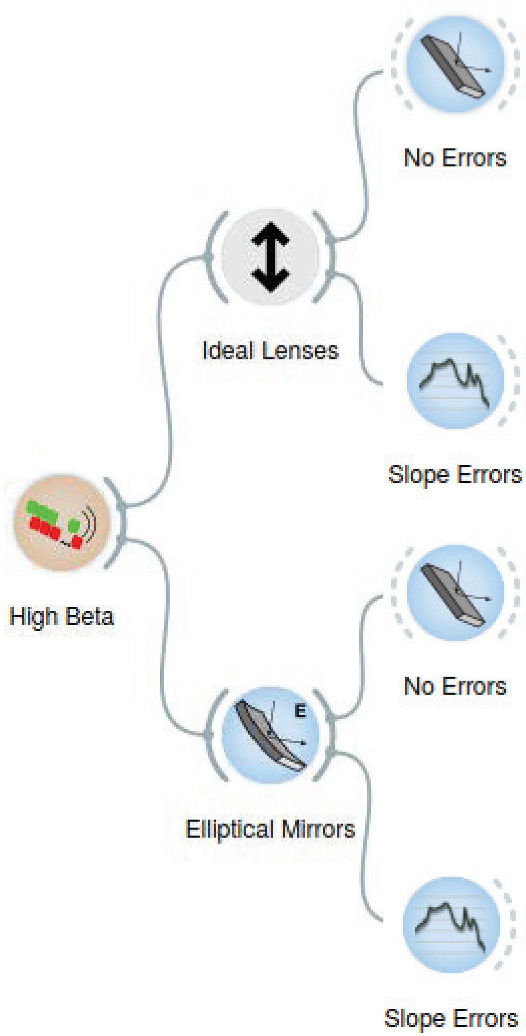


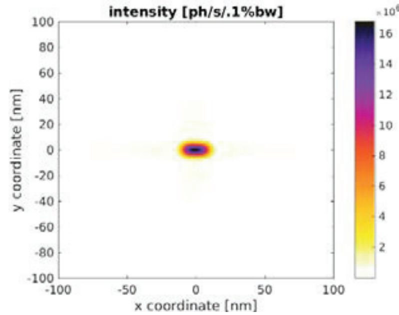
intensity [ph/s/.1%bw]
y coordinate [nm]
x coordinate [nm]

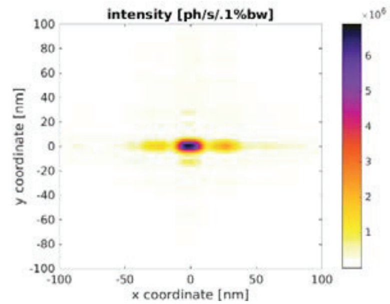


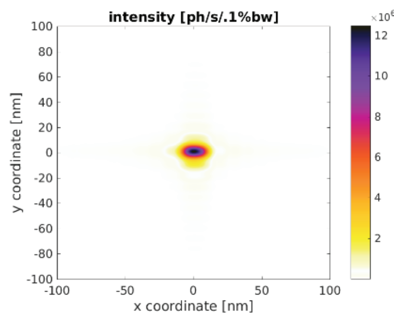
ideal thin lens
thin lens + slope errors
ideal elliptical mirror
elliptical mirror + slope errors

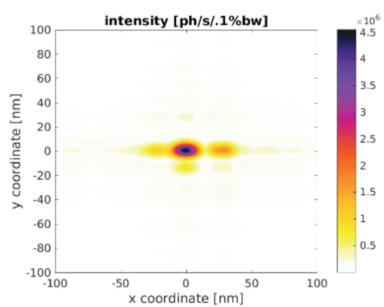
SRW – HIGH BETA

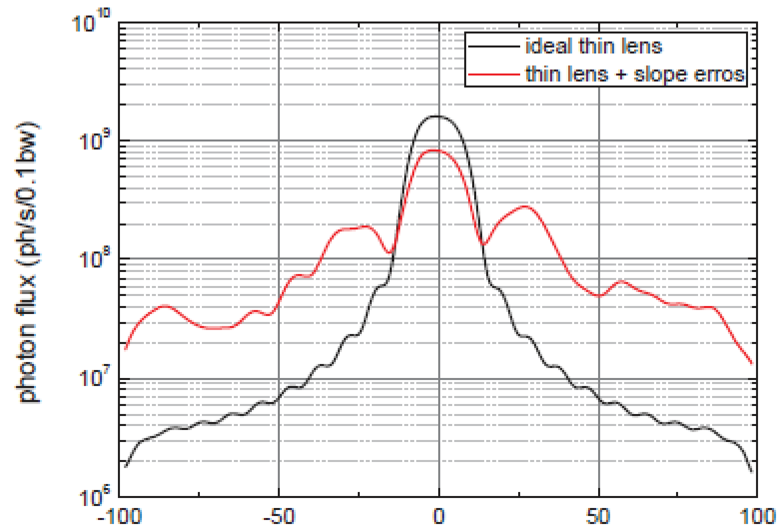


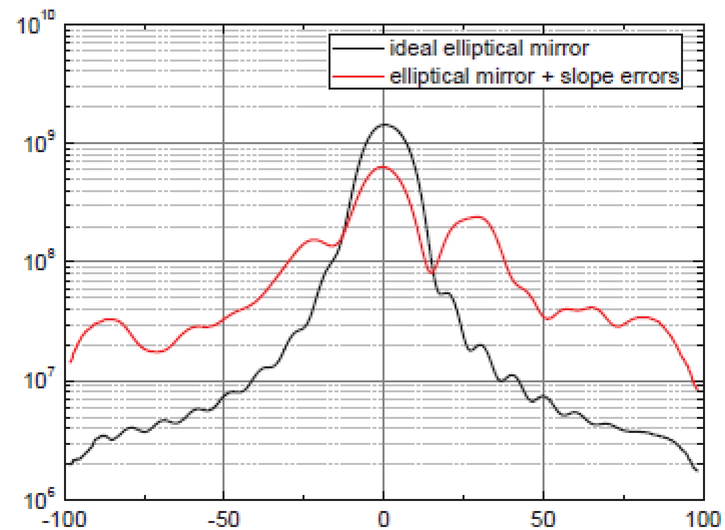












SUMMARY AND CONCLUSIONS

- **The synchrotron beam emission is due to a collaborative effect of the electrons in a bunch that are responsible of the partial coherence of the beam.**
- **Zero emittance rings are really “diffraction limited” providing a single coherence mode. Upgrade storage ring emission must be treated as partial coherence.**
- **For storage rings emission all coherence properties can be deduced from the Cross Spectral Density. Its storage and propagation is usually unmanagleable by present computers.**
- **COMSYL introduces a new accurate coherent mode decomposition that:**
  - Provides a method of effective storage of CSD
  - Introduces the new concept of “mode spectrum” that quickly summarizes the main coherence properties at a given point of the beamline
  - Computes accurately the coherent fraction
  - Allows to use known propagation methods to propagate modes
  - Permits computing coherent properties of the beam at *any* point of the beamline
- **Applications for a simplified coherence beamline and a nanofocusing ultimate beamline (ID16A) are discussed**

THANKS!

Mark Glass

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Julio Cesar Da Silva

Ray Barrett

Thank you!