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PHASE SHIFTER APPLICATION IN DOUBLE UNDULATOR CONFIGURATION OF HEPS

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Abstract

For over 6 meters long straight-section of HEPS, collinear double-cryogenic permanent magnet undulator(CPMU) is designed for high energy photon users to achieve higher brightness. Angular and spatial profiles of radiation produced by the double undulator configuration have been derived analytically. The efficiency of phase shifter on improving the brightness of double-CPMU is therefore evaluated with the beam energy spread and emittance are taken into account. Optimized beta-functions of electron beam are obtained.

INTRODUCTION

In the first phase of HEPS construction, a total of 14 ID-based beamlines are required for constructed, of which 7 are based on in-vacuum undulators [1]. In order to satisfy the requirement of high-energy users, the phase error of these Ids should be reduced to below 2-3 ° especially for the application of harmonics higher than 9th. Therefore, the maximum length of in-vacuum undulator has to be less than 3 meters due to the limitations of the current manufacturing process. This leads to the necessity of installing two undulators in series on one 6 meters long straight section. In this case, if is it necessary to install an additional phase shifter between the two undulators and its effects on the radiation performance when considering the real beam parameters has become a significant problem should be investigated.

An intuitive view of this issue is that it does necessary for the phase matching between the two undulators. coherence effect will increase the on-axis radiation intensity to 4 times higher that of single undulator for maximum which equivalent to an undulator with the total length doubled or reduce it to zero for minimum in the case of phase mismatch. The effects of emittance and energy spread has been ignored yet which cause this view divorced from reality. When taking the emittance and energy spread into account, a view is that the beam energy spread will seriously undermine the coherence condition between the two undulator radiations with increasing of energy spread, especially for high-energy hard X-rays. A characteristic periods number is defined by $N_{c,n} = \frac{1}{5n}\left(\frac{\sigma_\gamma}{\gamma}\right)^{-1}$ and equivalent periods number is defined by $N_{ep}=2N+\varphi/2\pi$. Where φ is the phase slip between the two undulators. Coherence effects could clearly observed only when $N_{c,n} \gg N_{ep}$ [2] or the intensity distribution presents a geometrical superposition of intensity from two independent source points at the center of both undulators in the opposite case. The phase shifter has little effect in this case. However, it will see the conclusion is just opposite according to the work of this article later. Phase shifter is remain indispensable even if considering the effects of beam parameters.

To specify the performance of a synchrotron radiation (SR) source, photon flux density in the 4D phase space i.e. brilliance is the most common figure of merit. In general case, brilliance should be first calculated by the method of Wigner function [3] and then convoluted with the electron beam distribution in phase space to include the effects of emittance and energy spread. A widely used model to calculate the radiation brilliance from a single undulator is Gaussian approximation in the case of Gaussian electron beam distribution [4] which could help to simplify this calculation process. The only difference should be considered is that energy deviation of electrons will change the phase slip between the two undulators. Moreover, in most practical cases, it is sufficient to use on-axis brilliance to evaluated the SR performance. Therefore, we only calculate the on-axis brilliance in this paper.

RADIATION MODEL OF DOUBLE UNDULATOR CONFIGURATION

To start with the calculation of the spectra of the combination of two undulators with the phase shifter between them analytically, we illustrate the whole structure in Fig. 1 [5]:

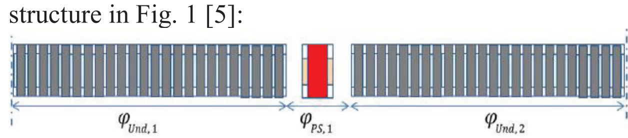

Figure 1: structure of the double undulator configuration.

Where $\varphi_{und,1}$ and $\varphi_{und,2}$ represent to the phase slip in each undulator. φ_{ps} is the phase slip between the two undulator. We ignore the front-ends of the two undulators for it only cause an additional phase slip which contain in φ_{ps}. The radiation field then is expressed by the sum of the two complex field emitted from both undulators as

$$E_{Double}(\omega,\theta,t) = \left[1 + e^{i(\varphi_{Und,1}+\varphi_{PS})}\right] E_{Single}(\omega,\theta,t),$$

Where E_{single} denote the field emitted from a single undulator. And the on-axis radiation intensity is written as

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$$I_{Double}(\delta)=\left|e^{i\left[(1+\delta)n\varphi+\frac{2Nn\pi}{1+\delta}\right]}+1\right|^2 I_{Single}(\delta)$$

$$\propto 2\{1+\cos[(1+\delta)n\varphi-2Nn\pi\delta]\}Sinc^2(Nn\pi\delta).$$

Where $\delta=\Delta\omega/\omega_n$ is regarded as the detune factor. It is important to note that $\Delta\omega$ is the offset of the nth harmonic energy due to the electron energy deviation. It is differ from an arbitrary energy offset compare with the reference harmonic energy. N is the undulator periods number and φ represent the phase slippage between the two undulator without electron energy deviation.

On-Axis Angluar Flux Density

If we assumed the distribution of the energy spread is Gaussian with the RMS σ_ε, it caused the δ obeys the Gaussian distribution with the RMS $2\sigma_\varepsilon$. The on-axis angluar flux density is then a convolution shown as below.

$$I_{total}=\frac{1}{2\sqrt{2\pi}\sigma_\varepsilon}\int_{-\infty}^{\infty}e^{-\frac{\delta^2}{8\sigma_\varepsilon}}I_{Double}(\delta)d\delta$$

$$=\frac{1}{\sqrt{2\pi}\sigma_\varepsilon}(I_1+I_2)$$

$$I_1=\int_{-\infty}^{\infty}e^{-\frac{\delta^2}{8\sigma_\varepsilon}}Sinc^2(Nn\pi\delta)d\delta$$

$$I_2=\int_{-\infty}^{\infty}e^{-\frac{\delta^2}{8\sigma_\varepsilon}}\cos[(1+\delta)n\varphi-2Nn\pi\delta]Sinc^2(Nn\pi\delta)d\delta$$

It is indicated that I_{total} can be divided into two parts. The first part is the intrinsic angluar flux density emitted from each undulator and the second part is the contribution of coherence relate to the phase slippage φ. Integrate these two parts respectively.

There is an approximation in second integration is that

$$Sinc^2(Nn\pi\delta)\approx e^{-\frac{(Nn\pi\delta)^2}{2}}.$$

Figure 2 shows the effect of energy spread on on-axis angular flux density for different harmonics in the case of φ=2π. Where I_0 is the on-axis angular flux density of single undulator without energy spread. The total flux density is thus normalized by I_0. It is obviously that the flux density of high harmonics behaves more sensitive to the energy spread.

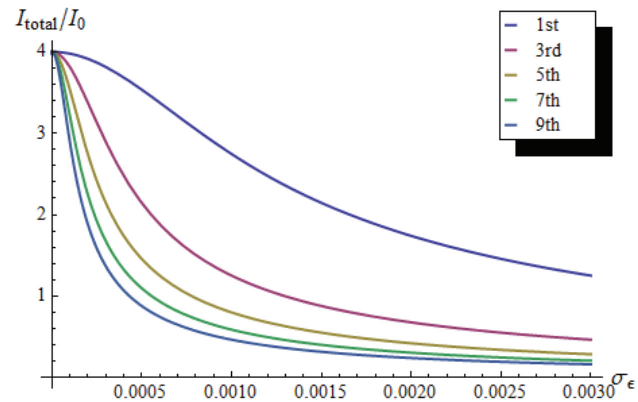

Figure 2: on-axis angular flux density of different harmonics vary with energy spread. We choose N=100 for the only undulator parameter during the calculation.

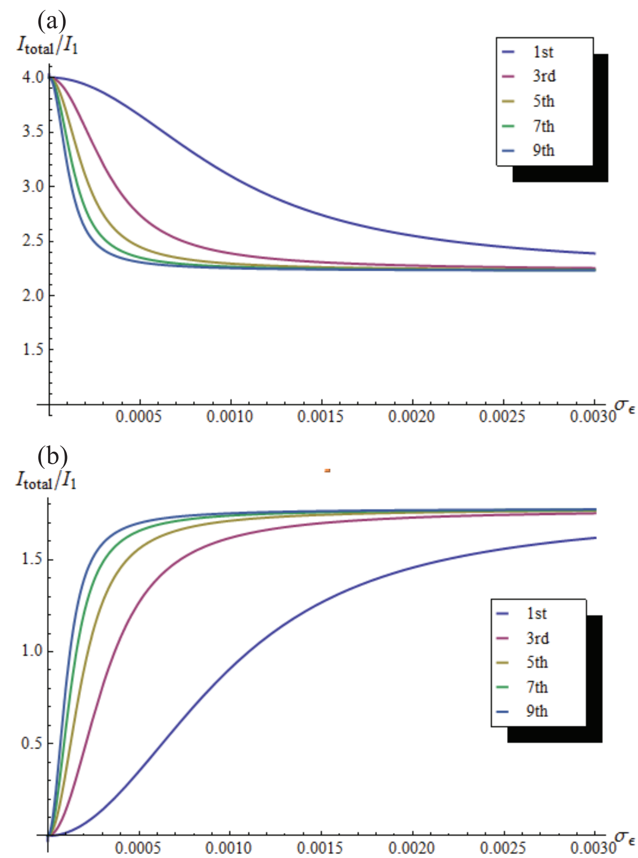

Figure 3: (a) gain of on-axis angular flux density of different harmonics vary with energy spread on the condition of phase matched. (b) gain of on-axis angular flux density of different harmonics vary with energy spread on the condition of phase mismatched.(N=100 in both case).

We introduce a gain factor defined by I_{total}/I_1 in order to tell if the phase shifter is necessary more intuitively. If the radiation from the two undulators was completely incoherence, the gain factor would be equal to 2. That means the total flux density is the sum of that from two undulators simply i.e. the phase shifter has little effects on the gain. It is clearly shown in Fig. 3 that even at the range of large energy spread the gain factor is not tend to

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2 for both cases of phase matched and mismatched. It has at least 0.5-0.6 times of I_1 for difference between the two case which indicated the significance of phase matching by the phase shifter even taking the beam energy spread into account.

Angular Distribution of Photon Flux Density

We next investigate the angular flux distribution of double undulator configuration. We only interesting about the photon energy equals to the resonance energy without any deviation. In this case, the detune factor δ can be rewrite as a function of electron energy γ and observe angle θ given by

$$\delta(\theta,\gamma)=\frac{\omega_n(\gamma_0,0)-\omega_n(\gamma,\theta)}{\omega_n(\gamma,\theta)}$$

$$\omega_n(\gamma,\theta)=\frac{2\gamma^2}{\left(1+k^2/2+\gamma^2\theta^2\right)}\frac{2\pi c}{\lambda_u},$$

where λ_u is the period length of undulator and γ_0 represent to the energy without any offset. If we assume the offset of the electron energy $\Delta\gamma \ll \gamma_0$ it could be expressed the detune factor $\delta(\Delta\gamma,\theta)$ by

$$\delta(\Delta\gamma,\theta)=\frac{\gamma_0^2\theta^2}{1+k^2/2}-2\frac{\Delta\gamma}{\gamma_0} \qquad [6]$$

Substituted this expression into I_{double} derived above, angular flux distribution of nth harmonic is able to obtain analytically and shown as

$$I_{total}=\frac{2}{\sqrt{2\pi}\sigma_\varepsilon}\int_{-\infty}^{\infty}e^{-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}}d\varepsilon$$
$$\{1+\cos[(1+\delta(\varepsilon,\theta))n\varphi-2Nn\pi\delta(\varepsilon,\theta)]\}Sinc^2(Nn\pi\delta(\varepsilon,\theta))$$
$$=2(I_1+I_2),$$

where $\varepsilon=\Delta\gamma/\gamma 0$ is the energy deviation. Integrate I1 and I2 respectively as before. The approximation of sinc function is still available in the integration.

Figure 4 shows the photon flux density distribution of observe angle at different harmonic energies. Figure 5 shows the comparison of the analytic results with the SPECTRA result. Both the results of the first and the third harmonic agree well.

It is also to see that energy spread extend the angular distribution range of radiation central cone according to Fig. 6. The RMS angular divergence can be derived from the angular flux distribution analytical as

$$\sigma_{r'}^2=\frac{\int_{-\infty}^{\infty}\theta^2 I_{total}(\theta,\sigma)d\theta}{\int_{-\infty}^{\infty}I_{total}(\theta,\sigma)d\theta}=\frac{DI_{total}}{TI_{total}}$$

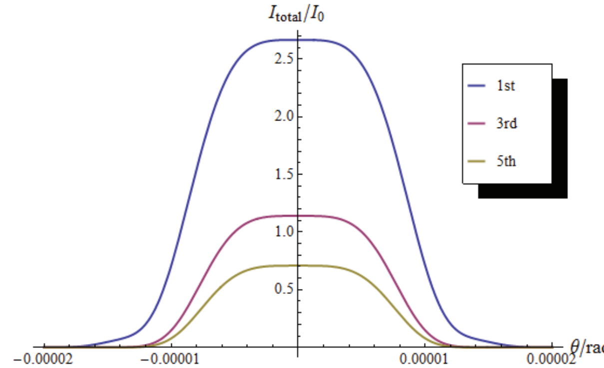
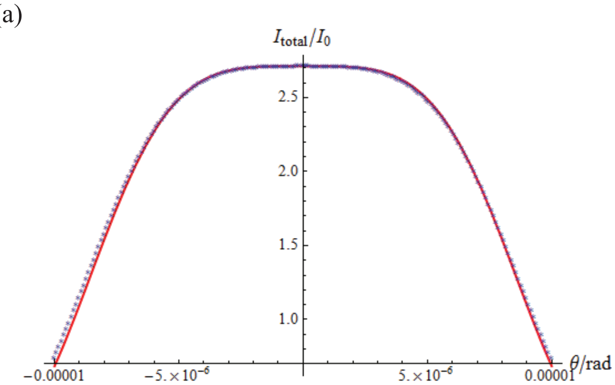


Figure 4: photon flux density distribution of observe angle at different on-axis harmonic energies. Parameters we chosen in this calculation are electron energy e equals to 6Gev, energy spread σε=0.1%, N=100, φ=2π, k=2.1.

(a)



(b)

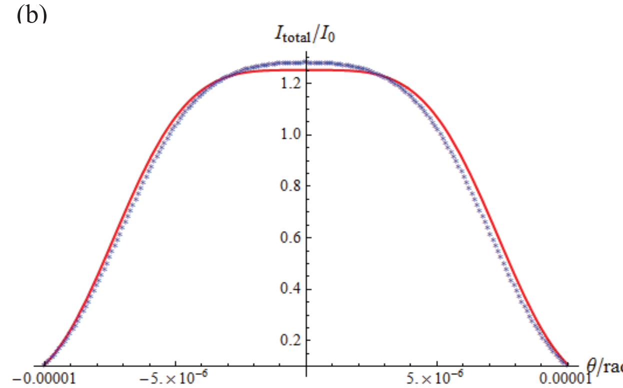


Figure 5: comparison of analytical results of angular flux density distribution with the SPECTRA result. where 5(a)for the first harmonic and 5(b)for the third harmonic. The dash line represent to the numerical results calculated by SPECTRA and the red line represent to the analytic result.

Substitutes the expression of I_{total} into the form above, the RMS angular divergence is obtained. Note that in the case of phase mismatched, the angular distribution of angular flux density is like a ring, only in the case of phase matched it makes the expressions above meaningful. The result of RMS angular divergence is shown in the Fig. 7.

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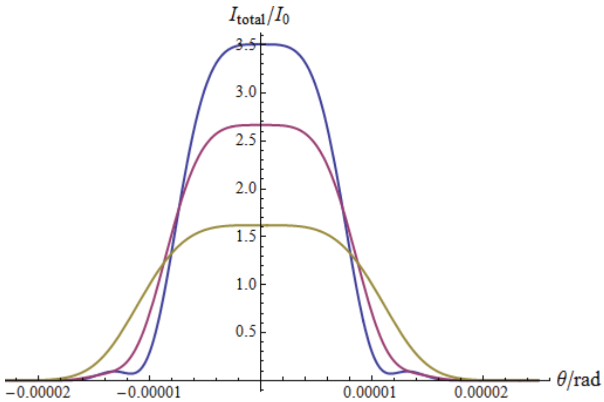

Figure 6: angular flux distributions with different energy spreads of first harmonic. Where Blue line, red line and yellow line represent to σε=0.05%, σε=0.1%, σε=0.2% respectively. Other parameters used in the calculation are the same with before.

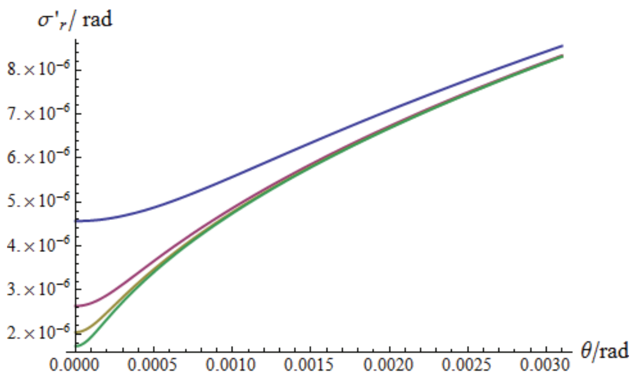

Figure 7: RMS angular divergences of different harmonics vary with energy spread. The condition of calculation is same as before.

Brilliance and Optimized Beta-Functions

Phase shifter works on the condition of phase matched in most cases for improving the brilliance. We also calculate the brilliance on this condition. For the angular distribution of photon density is near Gaussian, it is appropriate to use the well-know expression of brilliance in the Gaussian approximation as [4]

$$B = \frac{F}{4\pi^2 \Sigma_x \Sigma'_x \Sigma_y \Sigma'_y}$$

$$= \frac{F}{4\pi^2\left[\sigma_x^2 + \sigma_r^2(\sigma_\varepsilon)\right]\left[\sigma'^2_x + \sigma'^2_r(\sigma_\varepsilon)\right]\left[\sigma_y^2 + \sigma_r^2(\sigma_\varepsilon)\right]\left[\sigma'^2_y + \sigma'^2_r(\sigma_\varepsilon)\right]}$$

where

$$F = 2\pi\sigma'^2_r I_{total}.$$

In order to calculate the brilliance, it is necessary to obtain the expression of photo source size which should obtain by the Fourier transform at the source points in general. We make a simplification treatment in this paper as shown below [7]:

$$\sigma_r(\sigma_\varepsilon) = Qs(\sigma_\varepsilon)\sigma_r(0)$$

$$\sigma_r(0) = \frac{\lambda_n}{4\sigma'_r(0)}$$

$$Qs(x) = 4\left[\frac{1}{2I_1(x/4) + 2I_2(x/4)}\right]^{2/3}$$

Assuming the electron and the photon beam waists are both located to the middle of the straight section. The optimized beta-function is then obtain by

$$\beta_0 = \frac{\sigma_r(\sigma_\varepsilon)}{\sigma'_r(\sigma_\varepsilon)}.$$

Figure 8 shows the brilliance tune-curve of CPMU18 which has the undulator parameters same as before. The beam parameters are chosen according to the latest HEPS lattice scheme which εx=34.2pm, βx=1.9m, εy=4pm, βy=2.2m, σε=0.106% and current I=0.2A. We compare the brilliance of double undulator configuration with the single undulator which the length is doubled. It is clearly to see the beam parameters effects on double undulator configuration is more significant than the latter. In Fig. 9 brilliance of CPMU18 calculated under two different lattice schemes of HEPS are compared. The brilliance of double undulator configuration is more sensitive to the energy spread than to the emittance especially for the high harmonics. It is indicated in Fig. 10 that the optimized beta-function the no more a constant but a function of the energy spread and harmonic number when take the energy spread into account.

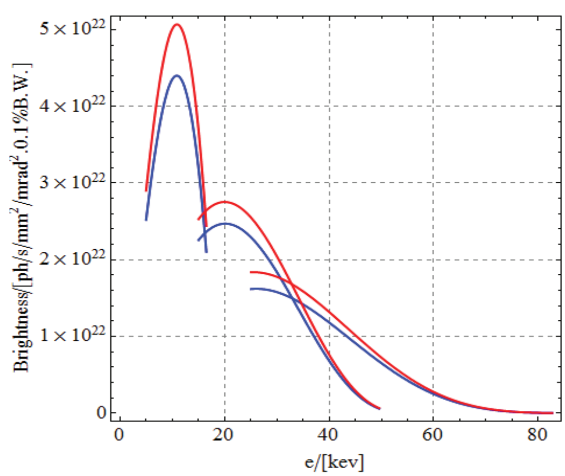

Figure 8: brilliance tune-curve of CPMU18. Where εx=34.2pm, βx=1.9m, εy=4pm, βy=2.2m and current I=0.2A. the remain parameters are the same as before. The red line represent to the brilliance of single undulator with the length doubled. The blue line is the brilliance of double undulator configuration.

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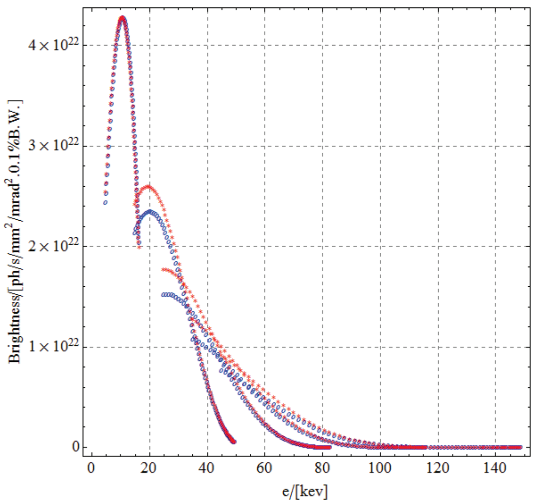

Figure 9: brilliance comparison of CPMU18 between the two different parameter schemes. The only two differences of parameters in these two schemes are emittance and energy spread. The blue points refer to the scheme with emittance εx=34.2pm and energy spread σε=0.106% while the red points refer to the scheme with emittance εx=40pm and energy spread σε=0.081%. Any other parameters are the same as that in Figure 9.

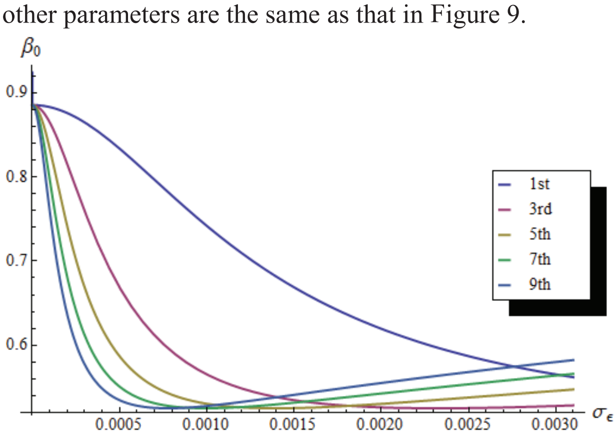

Figure 10: optimized beta-function vary with the energy spread at different harmonic number.

SUMMARY

An analytic expression of the on-axis brilliance and angular flux density distribution are derived. The analysis above indicates that the double undulator configuration with a phase shifter in the middle appears more sensitive to the energy spread than a single undulator which makes us to pay more concern about the energy spread during the lattice design of the storage ring. However, energy spread can not undermine the coherence thoroughly yet i.e. phase shifter is necessary in any cases. The optimized beta-function is then no longer a constant but a function of energy spread and harmonic number.

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