UNDULATOR PHASE MATCHING FOR THE EUROPEAN XFEL

Y. Li, J. Pflueger, European XFEL GmbH, Hamburg, Germany

Abstract
The undulator system in the European XFEL is mainly comprised 5-m long undulator segments and 1.1 m long intersections in between. In intersections the electron velocity is faster than it inside an undulator and the optical phase is detuned. The detune effect is also from the undulator fringe field where electron longitudinal speed also deviates from the oscillation condition. The total detune effect is compensated by a magnetic device called phase shifter, which is correspondingly set for a specific undulator gap. In this paper we introduce the method to set the phase shifter gap for each K parameter according to the measured magnetic field.

INTRODUCTION
High gain free electron lasers (FELs) using the principle of Self-Amplified Spontaneous Emission (SASE) are so far the only way to generate FEL radiation in the hard X-ray range [1-5]. For VUV radiation alternatives such as harmonic generation exist generate radiation which lead to increased stability [6]. However, aside from such differences both require long undulator systems with lengths from tens of meters up to about 200 meters depending on the radiation wavelength, electron beam and undulator parameters. Such a long undulator system cannot be built as a continuous device. For practical reasons of manufacturing it must be segmented into lengths around typically 5m maximum.

For the FEL process the interruption of the undulator implies a problem: The longitudinal speed of the electrons in the intersection is different to that inside the undulator and therefore the optical phase matching between laser field and electron motion is perturbed. In a fixed gap undulator system such as FLASH [1,2]or LCLS I [3] phase matching can be obtained by choosing the intersection length and tuning the end fields of the undulator segments properly. The phase matching in a tunable system is more complicated since the phase mismatch in the intersection changes with undulator gap. With a phase shifter, a small magnetic chicane, an additional delay is induced in the electron orbit. By properly selecting the phase shifter strength and hence the delay the optical phase can be matched at any gap [7].

The European X-ray free electron laser (XFEL) facility [5] is a large project driven by a 1.8 kilometer long superconducting linac. The SASE FEL is used throughout the European XFEL. An electrons beam is accelerated up to maximum of 17.5 GeV. Then it is guided through the undulator system to generate high quality soft and hard X-rays. The radiation wavelength can be changed by tuning the end fields of the undulator system such as FLASH [1,2] or LCLS I [3] phase matching can be obtained by choosing the intersection length and tuning the end fields of the undulator segments properly. The phase matching in a tunable system is more complicated since the phase mismatch in the intersection changes with undulator gap. With a phase shifter, a small magnetic chicane, an additional delay is induced in the electron orbit. By properly selecting the phase shifter strength and hence the delay the optical phase can be matched at any gap [7].

Figure 1 illustrates two undulator cells. Each cell is subdivided into four regions: On going from left to right the beginning of a cell is chosen in the field free region in the very left before the undulator. The region from at the beginning of the cell to the beginning of undulator bulk poles, including the drift space and the undulator fringe field, is called entrance fringe. The phase advance in this part is $\phi_{\text{Entr}}$. The periodic field region inside the undulator is called bulk field. The phase advance over this region is $\phi_{\text{Bulk}}$. Ideally at the first harmonic the phase advance in this region is $2\pi$ per period. Similar to the entrance fringe the region from the end of bulk field to the beginning of phase shifter is called exit fringe with the phase advance $\phi_{\text{Exit}}$. In the field free region after the exit fringe the phase shifter is placed. The phase advance over the phase shifter is $\phi_{\text{PS}}$. Since the phase shifter has very low fringe fields [7], it does not interfere with the undula-
tor and the spatial extension is very close to its physical length of 230mm only. The region after the phase shifter is again field free. Accordingly the phase advance over the first undulator cell can be written as

\[ \varphi \equiv \frac{2\pi}{\lambda} \left( \frac{1}{2} \int_{0}^{L_{1y}} (1 + i k z_{2}) d z_{2} \right) \]  

(1)

where \( k = \frac{2\pi}{\lambda_{\text{Rad}}} \) is the wave number, \( \lambda_{\text{Rad}} \) the radiation wavelength, \( c \) is the speed of light, \( e \) the electron charge and \( y \) the kinetic energy in units of the electron rest mass. \( I_{1y} \) is the 1st field integral of \( B_{y} \). The argument of the exponential function in Eq. (1) is the optical phase function [15]:

\[ \varphi(z_{1}) = \int_{0}^{z_{1}} \frac{k}{2\pi} (1 + i k z_{2}) d z_{2} \]  

(2)

For convenience a normalized form of \( A_{n} \), \( A_{n} \), is used. In terms of the optical phase \( \varphi(z_{1}) \) and the 1st field integral Eq. (1) is rewritten:

\[ A_{n} = \int_{0}^{L_{1y}} (1 + i k z_{2}) d z_{2} \]  

(3)

\( A_{n} \) is complex. \( A_{n} \), for two undulator segments is the vector sum of two complex numbers. Using the normalized form is written as:

\[ A_{n,\text{sum}} = A_{n,1} + e^{i(\varphi_{\text{und},1} + \varphi_{PS})} A_{n,2} \]  

(4)

where \( A_{n,1} \) and \( A_{n,2} \) denote \( A_{n} \) of the first and the second undulator, respectively. Both undulators and the corresponding radiation are similar but not identical. Each can be expressed in complex polar coordinates as:

\[ A_{n} = |A_{n}|e^{i\psi} \]  

(5)

\( \psi \) is the phase of the radiation complex, \( A_{n} \), and must not be confused with the optical phase \( \varphi \).

The total \( A_{n} \) of two undulators is the complex sum of two. Eq (4) is then rewritten as:

\[ A_{n,\text{sum}} = |A_{n}|e^{i\psi_{1}} + |A_{n}|e^{i\psi_{2}} \cdot e^{i(\varphi_{\text{und},1} + \varphi_{PS})} \]  

(6)

The maximum for \( |A_{n,\text{sum}}| \) is obtained if the condition:

\[ \psi_{1} = \varphi_{\text{und},1} + \varphi_{PS} + \psi_{2} + 2\pi n \]  

(7)

is fulfilled where \( n \) is an integer. Eq. (7) is the criterion for calculating the phase matching. Figure 2 gives an illustrative description. \( A_{n} \) of the two undulators are plotted in the complex plane. If Eq. (7) is fulfilled the resulting \( A_{n} \) is longest if \( A_{n,1} \) and \( A_{n,2} \) are collinear.

![Figure 1: Definition of the different field regions in two sample undulator cells.](Image)

![Figure 2: Sum of the \( A_{n} \) of two undulators. The length of \( A_{n} \) is maximum if the condition \( \psi_{1} = \varphi_{\text{und},1} + \varphi_{PS} + \psi_{2} + 2\pi n \) is satisfied.](Image)
MATCHING RESULTS

Comparison of $K$ for Two Undulators

A SASE undulator system such as SASE1 or SASE2 of the European XFEL comprises 35 undulator segments and 34 phase shifters, which have to be matched together by applying the methods derived in the last section. For all these components accurate magnetic measurement data are available: For all undulators there are accurate high resolution field maps.

For demonstration of the method two undulator segments, the U40-X005 and the U40-X006 and one phase shifter, the PS073, are used. The $K$-parameter of the two undulators is set from 1.5 up to 3.9 with a step size of 0.2. The gap for each segment was fitted to match the desired $K$-parameter. Fig. 3 illustrates the required gap of the two segments as a function of $K$. The blue curve shows the difference. It is seen that there is an almost constant difference of about 0.15 mm with a very slight variation with gap. It is due to differences in the mechanics, encoder initialization and magnet structure and shows the need of individual gap adjustment. This is a quite representative for other undulator segments as well.

**Figure 3:** Gap vs. $K$-parameter for the U40-X005, black squares, and U40-X006, red circles. The difference is shown by the blue triangles.

Phase Matching using the $A_n$

The condition for proper phase matching is defined by Eq. (7). The real and imaginary part of the normalized $A_n$ using Eq (3) and the phase $\psi$, can be calculated. Figure 4 demonstrates the phase matching using the hodograph representation.

The abscissa represents the real and the y-axis the imaginary part in arbitrary units. Two cases representing the maximum and minimum $K$ values, 3.9 and 1.7, are chosen for demonstration. Going along a perfect undulator the complex evolves along a straight line starting at 0. For each $K$ value, two conditions called ‘matched’ and ‘anti-matched’ are shown. ‘Matched’ fulfills the phase matching condition, Eq. (7) i.e. $2n\pi$. For the ‘Anti-matched’ condition the phase delay between the undulators is $(2n+1)\pi$. Results are shown by the left and right plots, respectively. The start angle is different to various $K$ value and it depends on the entrance ending field. It is seen that in the matched condition the $A_n$ of two the undulators, U40-X005 and U40-X006, have the same in length and point in exactly the same direction and the total length of the radiation from two undulators is twice the length of a single one and reaches the maximum intensity as illustrated in Figure 2. In contrast as seen by the right plot in the anti-matched condition $A_n$ of the two undulators have again the same length but reverse direction. Therefore the total length is zero. It should be emphasized that this applies to the forward direction only. The effect of residual field errors is seen by some small wiggles on the lines and resulting small deviations from perfect straightness.

**Figure 4:** Hodograph of the real and imaginary part of the radiation $A_n$ for two $K$ values. The left plot corresponds matched phasing with $2n\pi$ phase difference. The right plot corresponds anti-matched phasing with $(2n+1)\pi$ phase difference leading to zero amplitude in forward direction.

Small differences in the ending fields as well as small individual differences in the phase shifter require strict matching of undulators and phase shifters. Figure 5 shows the required phase shifter gap as a function of the undulator $K$-parameter. The phase shifter can be operated on different phase numbers, $\nu$, [7]. In order to have a sufficiently large tuning range over the whole operational $K$ harmonic number $\nu = 14$ or larger needs to be used. Fig. 5 shows the results. Each curve corresponds to a specific harmonic number. It is seen that the larger the harmonic number the larger the $K$ range. However at large $K$ the space between the curves gets smaller and the phase gets more sensitive to the phase shifter gap.

**Figure 5:** The required gap of phase shifter PS073 placed between U40-X005 and U40-X006 as a function of the $K$-
parameter. The individual curves correspond to different phase numbers. Continuous tuning over the whole K-range requires harmonic numbers $t_{14}$.

**Phase Matching by Pole**

Phase matching described so far is the most general treatment which based on the complex $A_n$ of the two undulators calculated from their measured fields. Under the assumption that the bulk structure has zero phase error there is a simple alternative using Eq. (12). Only the end field contributes. In reality the problem arises how to select the boundary between bulk and end fields. The applicability of this simplification was again tested using the undulators U40-X005 and U40-X006 and the phase shifter PS073.

![Figure 6: Phase compensation as function of the pole number where the bulk field starts.](image)

Figure 6 shows the required phase compensation of the phase shifter as function of the pole number where the bulk structure starts. It is seen that depending on the start the phase varies significantly from about 50 to 120 degrees with an RMS value of 12.9 degrees. There is no hard criterion to select a specific pole as the start pole for the bulk field. In the bulk structure there is a systematic deviation from a $2\pi$ phase advance per period. The explanation is that on all European XFEL undulators small parabolic girder deformations have been observed, which result from changing magnetic forces but they are well within the specifications. Therefore different extensions of the bulk field leads to different phase matching requirements [10]. Since the curve in Figure 6 has some symmetry averaging can be used. The average of the requested phase in Figure 6 is $90^\circ$ and is used to calculate the phase shifter gap. Now, using Eq. (12) phase shifter gap settings in full analogy to Fig. 5 can be calculated.

Both methods provide comparable results. Instead of reproducing curves such as in Figure 5 a quantitative analysis is given in Figure 7. The black curve shows the difference of the phase shifter gap, the blue curve the phase difference of phase shifter.

![Figure 7: Comparison between the results of two matching criteria. The black curve shows the difference of the required phase shifter gap, the blue curve the phase difference of phase shifter.](image)

**CONCLUSION**

In this paper the theoretical basis for the proper matching the optical phase of different tuneable undulator segments with the help of phase shifters are worked out. It is used in large distributed undulator systems for SASE FELs. Two matching methods, based on the undulator on-axis radiation and the optical phase, are derived and compared. We prove to the undulator with identical bulk field these two methods are equivalent.

The matching results for two undulator segments in SASE1 are shown as the example. The Gap-K curves for $2\pi$ phase matching are illustrated and compared.

**REFERENCES**


