Orbital Angular Momentum from SASE

J.F Morgan [1,2], B.W.J. McNeil [1,2], B.D. Muratori [2,3], P. Williams [2,3], A.Wolski [2,4]

1 SUPA, Department of Physics, University of Strathclyde, Glasgow, G4 0NG
2 The Cockcroft Institute Daresbury Laboratory, Daresbury, Warrington, WA4 4AD
3 ASTeC, STFC Daresbury Laboratory, Warrington, WA4 4AD, UK
4 University of Liverpool, Liverpool, UK

Introduction

Light beams with helical phase fronts are of interest as these beams carry orbital angular momentum, OAM. The phase front is characterised by, $e^{il\phi}$, where ϕ is the azimuthal coordinate and l is the OAM index. This property of light has been shown to have various applications: for example, in imaging systems and microscopic optical tweezers [1]. Using free electron lasers, FELs, to create these beams allows OAM light to be produced at shorter wavelengths and higher intensities than before. Current methods which produce OAM in a FEL use seed lasers. Output is restricted by the quality of seeds available. This causes difficulty at very short wavelengths as a seed may not be available at the required wavelength and the intensity of the seed must be large to overcome the initial shot noise in the beam. It would be useful, instead, for the initial seed for amplification to come from the shot noise in the electron beam itself. This work looks at the feasibility of just this, generating OAM through suppression of the Gaussian mode.

Theory

Mode Competition

Electrons enter the undulator with random phases due to shot noise in the electron gun. This incoherent radiation can be described by a superposition of the orthogonal Laguerre-Gaussian beams, LG_{pl},

$$E(r,\phi) = \sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty} a_{pl} LG_{pl}$$

a_{pl} is the initial mode amplitude. Although all modes have a small initial amplitude, the Gaussian mode has the smallest gain length and dominates FEL interaction, suppressing the higher order modes.

Suppression of the Gaussian Mode

Phase shifts can be used to disrupt the interaction between the electrons and a radiation mode. Examination of the transverse phase profile of the OAM modes indicates that a rotational shift, $\Delta\phi_r$, of the electron beam results in a relative phase shift between the electrons and the transverse modes of $l\Delta\phi_r$. When coupled with a longitudinal shift causing a phase change, $\Delta\theta$, the total phase change for the different modes is,

$$\Delta\psi_l = \Delta\theta + l\Delta\phi_r$$

Through careful selection of $\Delta\theta$ and $\Delta\phi_r$, different relative phase changes between the electrons and OAM modes are achieved. If successive repetition of the shifts causes the exponential gain of the Gaussian mode to be disrupted then a dominant OAM mode will self-select for amplification.

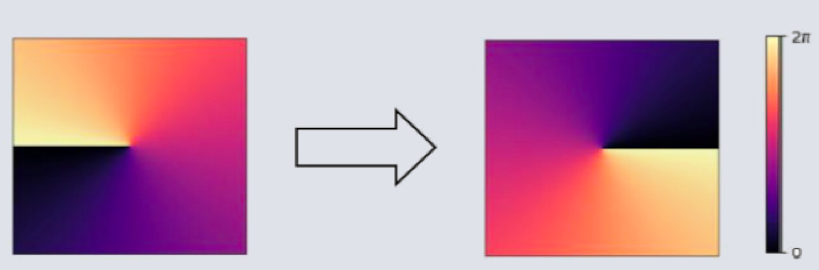


Figure 2: Rotation of l = 1 OAM beam. This π/2 rotation causes a π/2 phase shift.

Initial Results

The FEL is modelled using the FEL simulation code Puffin [2]. Presented, is the result when the electron beam is shifted longitudinally and rotated according to the rotation matrix,

$$R(\phi_r) = \begin{pmatrix} \cos\phi_r & 0 & -\sin\phi_r & 0 \\ 0 & \cos\phi_r & 0 & -\sin\phi_r \\ \sin\phi_r & 0 & \cos\phi_r & 0 \\ 0 & \sin\phi_r & 0 & \cos\phi_r \end{pmatrix}$$

which acts on the phase space vector (x, p_x, y, p_y).

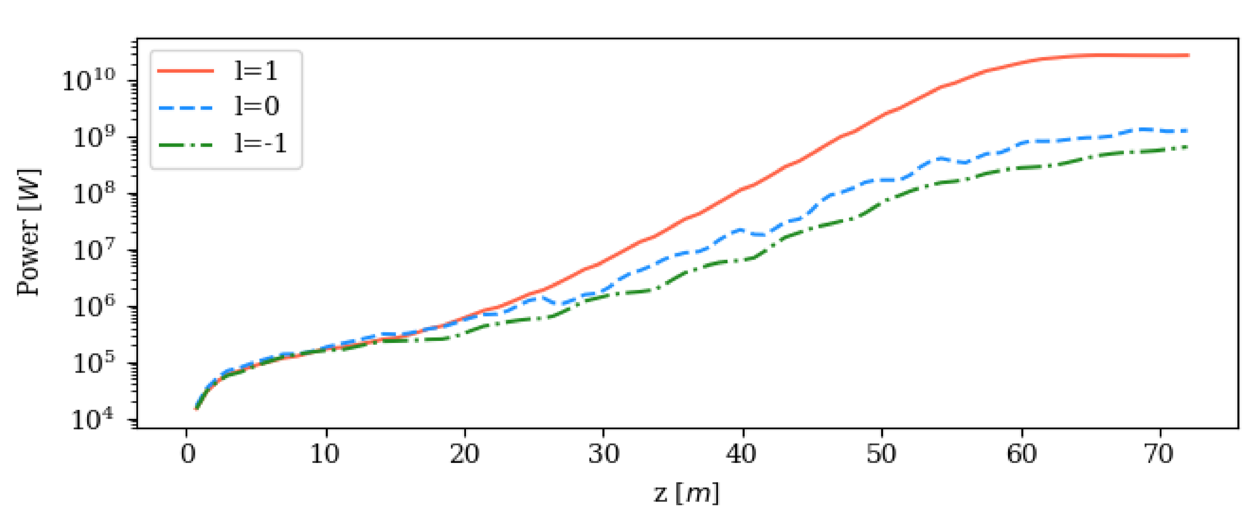


Figure 2: Time-averaged power decomposition of the radiation into Laguerre-Gaussian modes when shifts are imposed between undulator modules.

$\Delta\theta$	$\Delta\phi_r$	$\Delta\psi_0$	$\Delta\psi_1$
π	π	π	2π
π/2	3π/2	π/2	2π
3π/2	π/2	3π/2	2π

Table 1: Shifts between undulator modules a l = 1 mode.

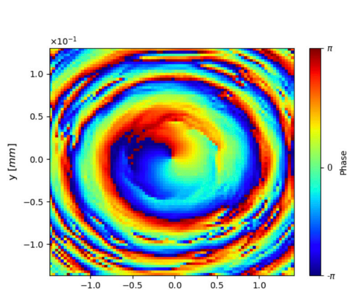


Figure 3: Phase of the radiation when shifts are imposed between undulator modules.

Beamline for Rotation of an Electron Beam

A system to rotate a beam with the transfer matrix, $R(\phi_r)$, can be constructed from two sets of five quadrupoles, with each set having the same drift and quadrupole strength. Each set of quadrupoles is arranged symmetrically, such that their transfer matrix, M,

$$M = Q_1 D_A Q_2 D_B Q_3 D_B Q_2 D_A Q_1$$

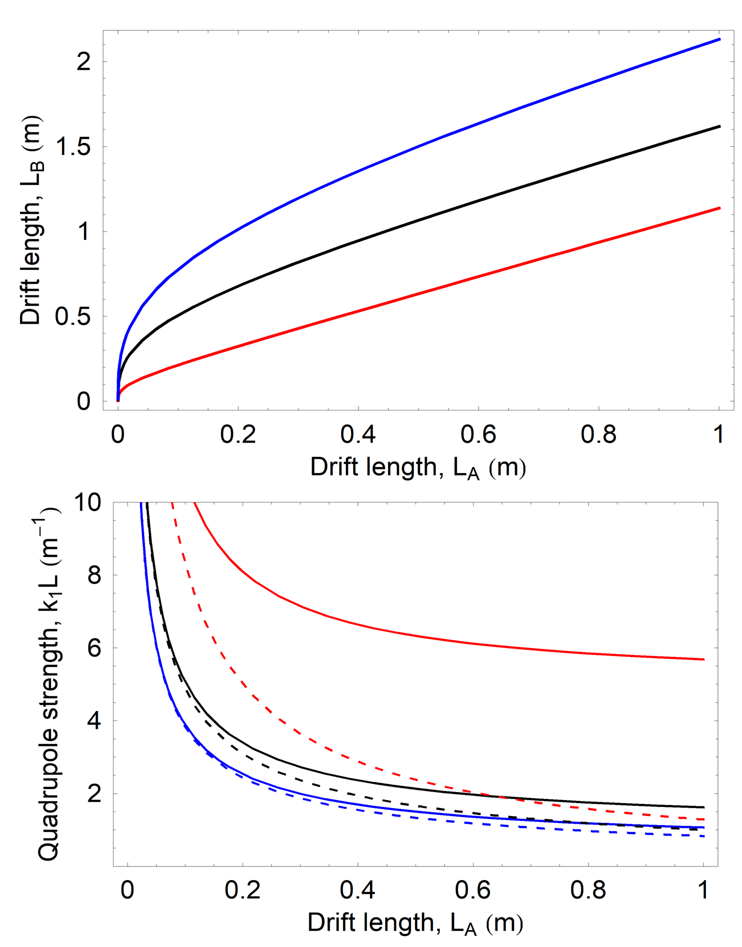
gives a phase advance through angles μ and $\mu + \pi$ in the transverse and horizontal spaces respectively. Q_n is the transfer matrix for a quadrupole of focusing strength $k_1 L_n$ and $D_{A(B)}$ is the transfer matrix for a drift of length $L_{A(B)}$

To achieve the desired rotation, ϕ_r, the first set of quadrupoles is tilted by an angle $\phi_r/2 + \pi/4$ around the longitudinal axis; the second set is tilted by an angle $-\pi/4$. Physical solutions are found for $\mu = \pi/2$ giving the focusing strengths and length as,

$$k_1 L_1 = \frac{L_B \xi}{L_A^2 - L_B^2} \qquad k_1 L_2 = -k_1 L_3 = \frac{\xi}{L_B} \qquad L_B = \frac{2}{3}\left(\eta + \frac{1}{2} + \frac{1}{\eta}\right) L_A$$

where

$$\xi^2 = 1 + \frac{L_B}{L_A}, \; \eta^3 = \frac{27}{16}\frac{\beta^2}{L_A^2}\left(1 + \sqrt{1 - \frac{32}{27}\frac{L_A^2}{\beta^2}}\right) - 1$$



Example Beamline :

For - L_A = 0.35

$\mu = \pi/2$

$\beta = 1m$

Total Beam Length $\approx 5m$

Max quadupole strength = $2.53m^{-1}$

Figure 4: Drift length L_B(top) and quadrupole strength (bottom) as functions of drift length L_A for the transformation M. The blue, black and red lines correspond to $\beta = 2\,m$, $1\,m$ and $0.2\,m$ respectively. The solid line (Bottom) shows the magnitude $k_1 L_1$ and the dashed line shows $k_1 L_2$.

Discussion

Initial trials of the rotation beamline have highlighted the following problems.

- If there is a large amount of diffraction between undulator modules diminishes the interaction between the radiation and the electrons and the Gaussian mode if not supressed.
- Practical concerns arise from the length of the rotation beamline which more than doubles the total length of the undulator sections.
- The electrons change in longitudinal phase (z) position over the beamline is different for different electrons due to variations in the transverse components in the momentum. This leads to a debuncing of the electron beam.

References

[1] Yao, A. M. and Padgett, M. J., Adv. Opt. Photon. **3**, 161 (2011)
[2] Campbell, L.T. and McNeil, B.W.J., Phys. Plasmas. **19**, 093119 (2012).