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TERAHERTZ FREE ELECTRON MASER BASED ON EXCITATION OF A TALBOT-TYPE SUPER-MODE IN AN OVERSIZED MICROWAVE SYSTEM *

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Abstract

We propose an electron maser operating in the THz frequency range and based on the excitation of not a fixed transverse mode of the operating cavity but of a supermode formed by a fixed set of the transverse modes. This allows selective excitation of a THz operating wave in an oversized microwave system fed by a high-current relativistic electron beam.

INTRODUCTION

A natural problem arising in the case of realization of a THz electron oscillator with a high-current relativistic electron beam is an inevitable use of an oversized microwave system, which characteristic transverse size significantly exceeds the wavelength of the operating wave [1,2]. In this situation, it becomes difficult (and even just impossible starting from a certain limit) to provide selective excitation of a chosen transverse mode of the operating cavity. First of all, the selectivity of the feedback is a serious problem. A typical configuration of the electron-wave interaction region in an auto-oscillator is a piece of a waveguide terminated at the input/output ends by two mirrors providing the feedback; for instance, Bragg-type mirrors can be used to provide effective reflection of far-from-cut-off waves (Fig. 1a). Naturally, in the case of a far-from-cut-off wave excited in an oversized waveguide, it is difficult to create a reflector providing reflection of only one transverse mode [2]. Second, it is difficult to ensure the single-mode electron-wave interaction in an oversized system, as in these situations many transverse modes are close to resonance with electrons [3].

In this work, we propose a different concept of selective excitation of a THz operating wave in a high-power relativistic electron maser with an oversized microwave system fed by a high-current relativistic electron beam. Our basic idea is to give up working in a fixed transverse mode. Instead, we propose to work in a supermode, which is formed by a fixed spectrum of several transverse modes of an oversized waveguide (Fig. 1 b). The spectrum of the partial transverse modes is determined by both electronic and electrodynamic factors. First, effective interaction between the supermode formed by these partial modes and the electron beam should be provided. This means that in the middle part of the waveguide the supermode field should have a maximum at the point of injection of the electron beam into the transverse cross-section of the waveguide. Second, the supermode should possess a high Q-factor. Therefore, at the input/output ends of the waveguide, the transverse structure of the supermode should be very specific; namely, the field should be present only close to the mirrors to provide almost complete reflection of the supermode back to the cavity (Fig. 1b).

To organize such a super-mode, we propose a simple approach, which is based on the use of Talbot's effect [1,4,5], namely, periodic reproduction of the transverse structure of a multi-mode wave field in an oversized waveguide. We propose to use this approach to fix a high-Q operating supermode in a simple microwave system consisting of a waveguide terminated by two simple mirrors (Fig. 1 b). On the basis of a multi-mode set of self-consistent equations of the electron-wave interaction we demonstrate the possibility of selective self-excitation of the supermode. The "proper" transverse-axial structure of the high-Q supermode is formed at the small-signal stage of the gain guiding during several trips of the wave over the cavity. In fact, this effect (formation of a high-Q supermode by a set of several partial transverse modes) is analogous to the effect of mode locking known in the physics of quantum lasers [6].

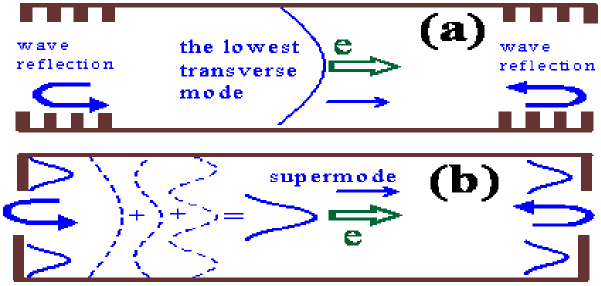

Figure 1: (a) Traditional scheme of excitation of a far-from-cutoff travelling wave in the cavity formed by a piece of waveguide with two input/output mirrors (Bragg-type mirrors are shown as an example). (b) Excitation of a supermode formed by a set of several transverse modes.

THE SIMPLEST 2-D MODEL: DEMONSTRATION OF THE BASIC IDEA

Let us consider a waveguide with a characteristic transverse size much bigger than the wavelength of the operating wave, $D \gg \lambda$ (Fig. 2). Any symmetrical transverse distribution of the wave field in this waveguide is reproduced $E(x,z+L)=E(x,z)$ with the period $L=D^2/\lambda$. This phenomenon of repetition of the transverse wave structure is well known in the literature as the Talbot effect [4] and widely used for designing various microwave systems [1,5].

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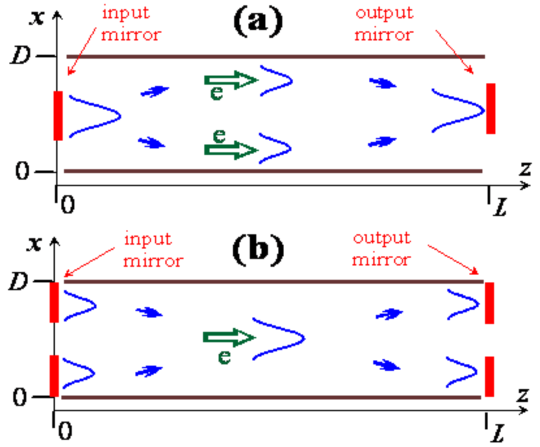

Figure 2: Two configurations of Talbot-type cavities. (a) The supermode field is focused in the center of the cross-section in the regions of the input/output mirrors and interacts with a tubular electron beam. (b) The supermode field is focused in the center of the waveguide cross-section in the middle of the waveguide, $z=L/2$, and interacts with a close-to-axis electron beam.

We propose to use the Talbot effect as a way to create an oversized microwave system of an electron maser that provides a high Q-factor for a supermode forming several transverse modes. Let us image that at the input of the cavity the field of this supermode is concentrated in a quite narrow section close to the center of the waveguide cross section (Fig. 2 a). If the length of the waveguide corresponds to Talbot's effect, then this transverse profile of the total wave field is reproduced at the output. This wave field can be completely reflected by a simple output mirror that has a metal surface only about the non-zero field area of the supermode. The counter propagation of the reflected wave back to the input mirror is completely analogous to the direct propagation of the supermode. Therefore, the input mirror, which is similar to the output one, completely reflects the counter wave into the forward wave and, thus, closes the feedback circuit.

Another scheme of a Talbot-type cavity can be formed by "shifting" the scheme shown in Fig. 2 a along the z-axis by half the Talbot length, $L/2$. In this case, the supermode forms two wave beams at the input and the output of the cavity (Fig. 2 b). In the middle of the cavity, these two beams are transformed into one beam located in the middle of the cavity cross-section. Again, the electron beam injected in the center of the cross section is easily separated from the input/output mirrors placed near the walls of the waveguide.

On the basis of a multi-mode set of self-consistent equations of the electron-wave interaction we demonstrate the possibility of the selective self-excitation of the supermode. As the first step, we use the simplest planar 2-D model (Fig. 2 b) with equidistant spectrum of transverse modes. The excitation process starts with small random noises in the electron beam. After the first trip of the wave packet through the cavity, several modes being close to the resonance with electrons are excited with different amplitudes and phases (Fig. 3 a). The biggest amplitude belongs to the transverse mode with n=3, as this mode is the closest to the resonance. As a result, the transverse structure of the wave field in the point before the output mirror is close to the structure of this mode with n=3. Naturally, the output mirror is almost transparent for this wave field, and about 2/3 of the wave power escapes from the cavity. However, the situation changes radically at the next several trips of the wave, when the transverse structure of a high-Q supermode is established (Figs. 3 b). In the small-signal regime of the electron-wave interaction, we see the supermode formed by a set of mainly five symmetrical transverse modes. The input mirror reflects this supermode almost perfectly (the power transition coefficient is under 1%). As for the output mirror, in the small-signal regime it reflects only about 85% of the power of this supermode back to the cavity. This is due to the effect of the electron–wave interaction on both the spectrum of the excited partial modes and their axial

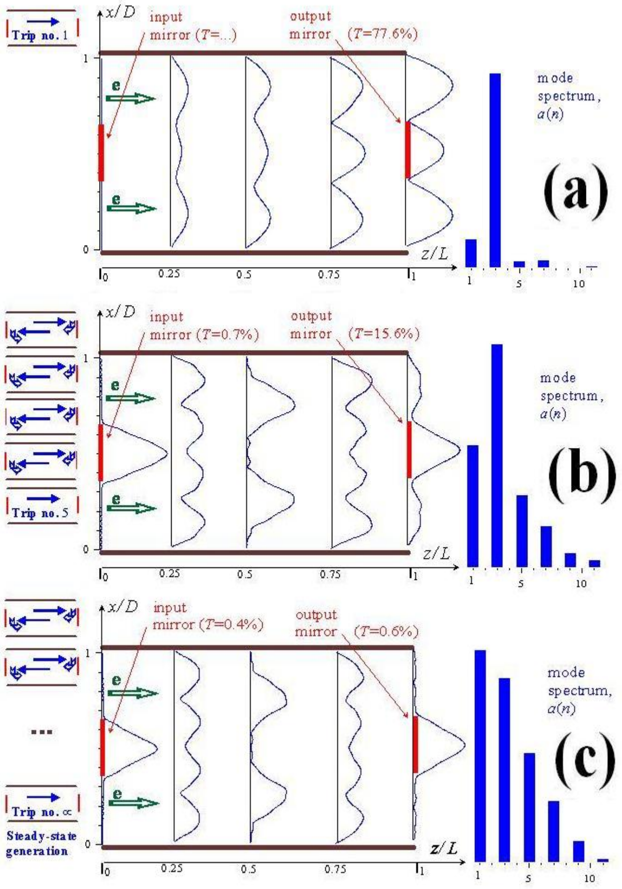

Figure 3: Evolution of the spatial-temporal structure of the RF wave excited in the Talbot-type cavity. Transverse structures of the wave field in different cross-sections of the cavity and spectra of the excited transverse modes in the small-signal regime of the auto-oscillator excitation after the first (a), fifth (b) , as well as at the steady-state regime of the stable operation (c).

structure; this disturbs slightly the ideal Talbot's reproduction of the transverse structure of the total field.

The supermode structure stays constant during the small-signal stage of the excitation of this auto-oscillator. The transition to a nonlinear regime of the stable steady-state generation leads to a slight change in the structure of the supermode due to the reduced influence of the electron-wave interaction. As a result, the Talbot's reproduction of the transverse structure of the supermode field is almost perfect, and the power transition factor of the output mirror is reduced from 15% (in the small-signal regime) down to 0.6% (Fig. 3 c). Thus, a very high-Q supermode is excited; the total losses of the power of the supermode at the input/output mirrors are about 1%. One more result is that in the steady-state regime the mode with $n = 3$ closest to the resonance ceases to be dominant. This means that the spectrum of the supermode in the steady-state regime is determined basically by the parameters of the microwave system. The predictability of the spectrum of partial transverse modes of the waveguide forming the supermode is an important factor from the point of view of organization of the radiation output from the cavity.

HIGH-POWER THz FEL

We have used this approach for a design of a THz Free-Electron Maser fed by a relativistic high-current electron beam. We study the possibility to use a relativistic electron beam produced from modern high-current accelerator (5-10 MeV / 2-10 kA / 200 ns) to excite a Talbot-type supermode at frequencies close to 2 THz. For a 7 MeV beam considered in simulations, the undulator period of 6 cm corresponds to the electron-wave resonance in the required frequency range.

In this design, we have modified the supermode configuration shown in Fig. 1 a. Namely, it is possible to reduce twice the cavity length by using the output mirror providing the almost total reflection of the wave packet (Fig. 4). The Talbot's effect takes place for the equidistant spectrum of the transverse modes. In a waveguide with circular cross section, the spectrum is close-to-equidistant only for quite high transverse modes, and the "halved" Talbot's supermode is formed by these modes at the cavity length $L = 2R^2/\lambda$.

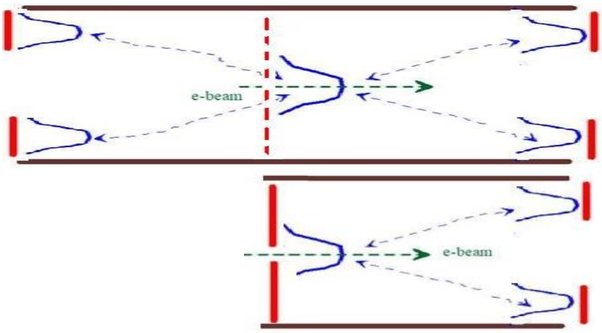

Figure 4: Transition from the Talbot-type cavity shown in Fig. 2 b to the "halved" cavity.

Naturally, the use of high operating transverse modes results in a weak electron-wave interaction. However, we have found that at a slightly shorter length, $L = 1.9R^2/\lambda$, the "halved" Talbot's effectapproximately takes place for a supermode formed basically by low ($TE_{1,n}$ n=1, …, 7) transverse modes.

In simulations (Fig. 5), we used this supermode in a cavity with the radius ~ 1 cm and the length ~ 1 m. The parameters of the system is chosen so that the closest-to-resonance transverse mode is $TE_{1,4}$. The supermode possess a quite low diffraction Q-factor, so that ~50% of the supermode filed passes the output mirror and forms the output radiation. The calculated efficiency of this FEM is at the level of 4-6% at electron currents 2-4 kA; this corresponds to the output power 0.6-1.7 GW at the frequency close to 2 THz.

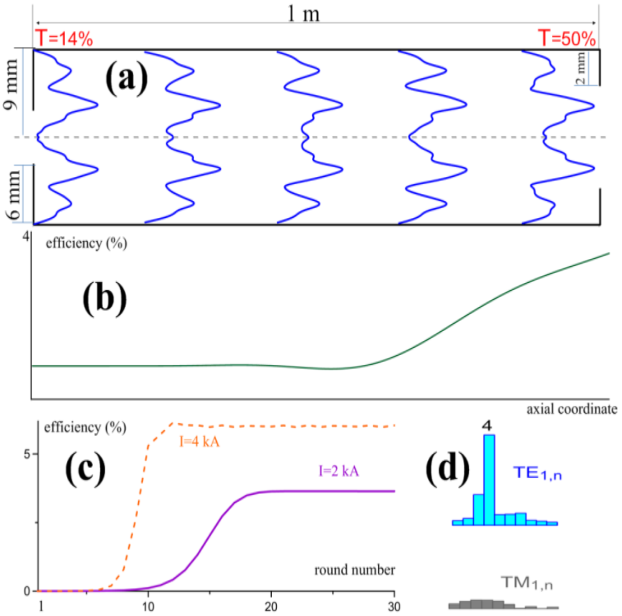

Figure 5: Relativistic high-current 2 THz FEM. (a) Operating cavity and transverse structures of the super-mode in various cross-sections of the cavity. (b) electron efficiency versus the axial coordinate in the steady-state regime. (c) Electron efficiency versus the wave trip number at electron currents 2 kA and 4 kA. (d) Spectrum of the supermode at the output of the cavity.

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