Dynamics of superradiant emission by a prebunched e-beam and its spontaneous emission self-interaction

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Spectral formalism for finite time excitations

$$\{\tilde{\mathbf{E}}_q(\mathbf{r}), \tilde{\mathbf{H}}_q(\mathbf{r})\} = \{\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp), \tilde{\boldsymbol{\mathcal{H}}}_q(\mathbf{r}_\perp)\} e^{ik_{qz}z}$$

$$\breve{\mathbf{E}}(\mathbf{r}, \omega) = \sum_q \breve{C}_q(z, \omega) \tilde{\mathbf{E}}_q(\mathbf{r})$$

$$\breve{\mathbf{H}}(\mathbf{r}, \omega) = \sum_q \breve{C}_q(z, \omega) \tilde{\mathbf{H}}_q(\mathbf{r})$$

$$\frac{d\breve{C}_q(z, \omega)}{dz} = \frac{-1}{4\mathcal{P}_q} \int \breve{\mathbf{J}}(\mathbf{r}, \omega) \cdot \tilde{\mathbf{E}}_q^*(\mathbf{r}) d^2\mathbf{r}_\perp$$

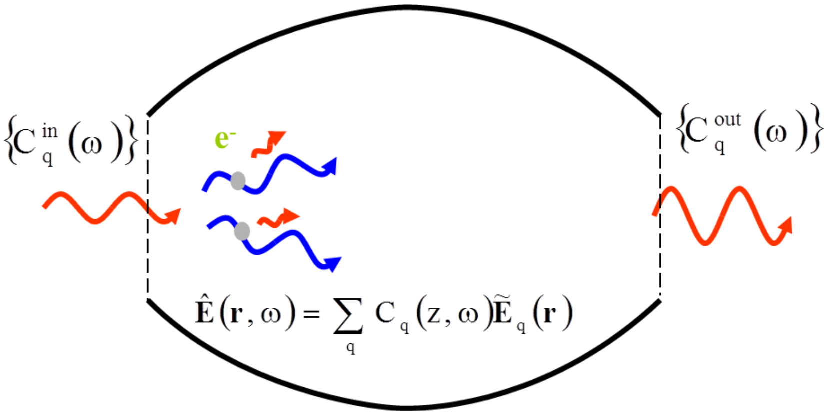
$$\mathcal{P}_q = \frac{1}{2} Re \iint (\tilde{\boldsymbol{\mathcal{E}}}_q \times \tilde{\boldsymbol{\mathcal{H}}}_q) \cdot \hat{\mathbf{e}}_z d^2\mathbf{r}_\perp = \frac{|\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp = 0)|^2}{2Z_q} A_{em\,q},$$

$$\frac{dW}{d\omega} = \frac{2}{\pi} \sum_q \mathcal{P}_q |\breve{C}_q(\omega)|^2$$

Representing the energy per frequency unit.

$$\breve{C}_q^{\mathrm{out}}(\omega) - \breve{C}_q^{\mathrm{in}}(0, \omega) = -\frac{1}{4\mathcal{P}_q} \int \breve{\mathbf{J}}(\mathbf{r}, \omega) \cdot \tilde{\mathbf{E}}_q^*(\mathbf{r}) dV,$$

Formulation of Radiation mode Expansion - cont.



Point particles current

$$J(\mathbf{r},t)=\sum_{j=1}^{N}-e\mathbf{v}_j(t)\delta(\mathbf{r}-\mathbf{r}_j(t))$$

$$\check{C}_q^{out}(\omega)-\check{C}_q^{in}(\omega)\equiv\sum_{j=1}^{N}\Delta\check{C}_{qj}(\omega)=-\frac{1}{4\mathcal{P}_q}\sum_{j=1}^{N}\Delta\check{\mathcal{W}}_{qj}$$

$$\Delta\check{\mathcal{W}}_{qj}=-e\int_{-\infty}^{\infty}\mathbf{v}_j(t)\cdot\tilde{\mathbf{E}}_q^*(\mathbf{r}_j(t))e^{i\omega t}dt$$

Split into a spontaneous (independent of the presence of radiation field) and stimulated (field dependent) part, but we shall neglect the last part for now:

$$\Delta\check{\mathcal{W}}_{qj}=\Delta\check{\mathcal{W}}_{qj}^{0}+\Delta\check{\mathcal{W}}_{qj}^{st}.$$

Point particles current - cont.

$$\Delta \breve{\mathcal{W}}^0_{qj} = \Delta \breve{\mathcal{W}}^0_{qe} e^{i\omega t_{0j}}$$

$$\Delta \breve{\mathcal{W}}^0_{qe} = -e \int_{-\infty}^{\infty} v^0_e(t) \cdot \tilde{\mathbf{E}}^*_q(r^0_e(t)) e^{i\omega t} dt.$$

$$\breve{C}^{out}_q(\omega) = \breve{C}^{in}_q(\omega) + \Delta \breve{C}^0_{qe}(\omega) \sum_{j=1}^{N} e^{i\omega t_{0j}} + \left[\sum_{j=1}^{N} \Delta \breve{C}^{st}_{qj} \right]$$

$$\frac{dW_q}{d\omega} = \frac{2}{\pi} \mathcal{P}_q \left\{ \left| \breve{C}^{in}_q(\omega) \right|^2 + \right.$$

$$\left| \Delta C^{(0)}_{qe}(\omega) \right|^2 \left| \sum_{j=1}^{N} e^{i\omega t_{oj}} \right|^2 + \left[\breve{C}^{in\,*}_q(\omega) \Delta C^{(0)}_{qe}(\omega) \sum_{j=1}^{N} e^{i\omega t_{oj}} + c.c. \right] + ..$$

Point particles current - cont.

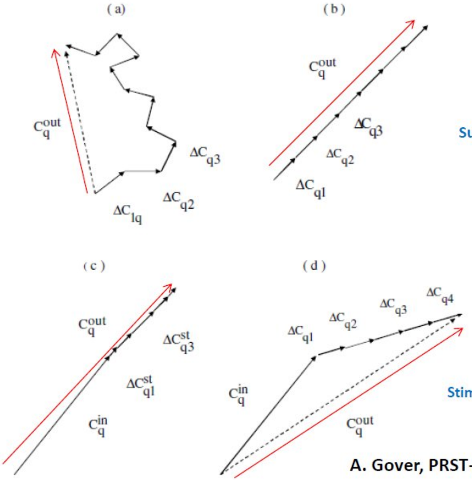
$$\frac{dW_q}{d\omega} = \left(\frac{dW_q}{d\omega}\right)_{in} + \left(\frac{dW_q}{d\omega}\right)_{sp/SR} + \left(\frac{dW_q}{d\omega}\right)_{ST-SR}$$

Spontaneous Emission

Superradiance

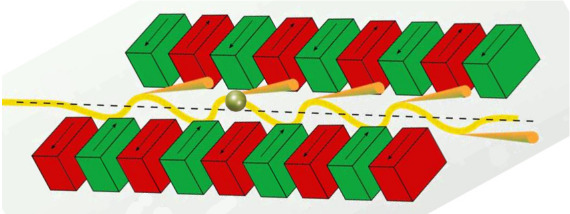
Stimulated Emission (Laser Amplifier)

Stimulated – Super radiance



A. Gover, PRST-AB 8, (030701) 2005

Example: Single bunch in undulator

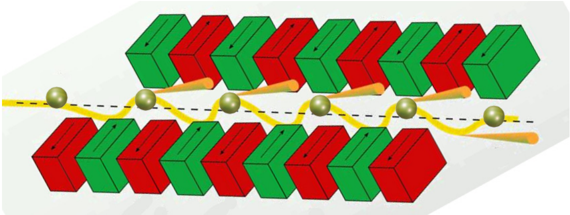


$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N^2 e^2 Z_q}{16\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)^2 \frac{L_w^2}{A_{em}} \mathrm{sinc}^2(\theta L_w/2)$$

$$\left(\frac{dW_q}{d\omega}\right)_{ST-SR} = |\check{C}_q^{in}(\omega)| \frac{Ne}{2\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)\sqrt{\frac{2Z_q\mathcal{P}_q}{A_{em\,q}}} L_w$$
$$\mathrm{sinc}(\theta L_w/2)\cos(\varphi_{qb0} - \theta L_w/2)$$

φ_{qb0} is the phase difference between the radiation field phase $\varphi_q(0)$ and the bunching current phase φ_{b0} at the entrance to the wiggler.

Example: Finite train of N_M tight bunches in undulator



$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N^2 e^2 Z_q}{16\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)^2 \frac{L_w^2}{A_{em}}\left|M_M(\omega)\right|^2 \mathrm{sinc}^2(\theta L_w/2)$$

$$\left(\frac{dW_q}{d\omega}\right)_{ST-SR} = |\check{C}_q^{in}(\omega)|\frac{Ne}{2\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)\sqrt{\frac{2Z_q\mathcal{P}_q}{A_{em\,q}}}L_w \left|M_M(\omega)\right| \mathrm{sinc}(\theta L_w/2)\cos(\varphi_{qb0} - \theta L_w/2)$$

$$M_M(\omega) = \frac{\sin(N_M\pi\omega/\omega_b)}{N_M\sin(\pi\omega/\omega_b)},$$

Single frequency formalism for infinite time excitations

$$\{\tilde{\mathbf{E}}_q(\mathbf{r}), \tilde{\mathbf{H}}_q(\mathbf{r})\} = \{\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp), \tilde{\boldsymbol{\mathcal{H}}}_q(\mathbf{r}_\perp)\} e^{ik_{qz}z}$$

$$\tilde{\mathbf{E}}(\mathbf{r}) = \sum_q \tilde{C}_q(z,\omega)\tilde{\mathbf{E}}_q(\mathbf{r})$$

$$\tilde{\mathbf{H}}(\mathbf{r}) = \sum_q \tilde{C}_q(z,\omega)\tilde{\mathbf{H}}_q(\mathbf{r})$$

$$\mathcal{P}_q = \frac{1}{2} Re \iint (\tilde{\boldsymbol{\mathcal{E}}}_q \times \tilde{\boldsymbol{\mathcal{H}}}_q) \cdot \hat{e}_z d^2\mathbf{r}_\perp = \frac{|\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp = 0)|^2}{2Z_q} A_{em\,q},$$

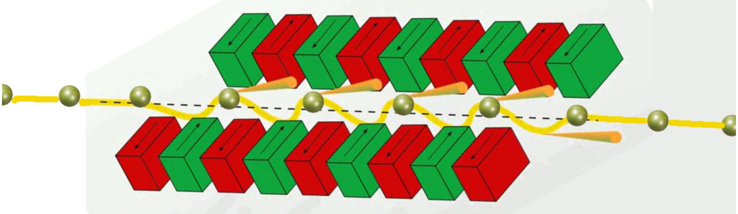
$$\frac{d\tilde{C}_q(z)}{dz} = \frac{-1}{4\mathcal{P}_q} \int \tilde{\mathbf{J}}(\mathbf{r}) \cdot \tilde{\mathbf{E}}_q^*(\mathbf{r}) d^2\mathbf{r}_\perp$$

$$P = \sum_q \mathcal{P}_q |\tilde{C}_q(\omega)|^2$$

Representing the radiated power.

$$\tilde{C}_q^{\text{out}}(\omega) - \tilde{C}_q^{\text{in}}(0) = -\frac{1}{4\mathcal{P}_q} \int \tilde{\mathbf{J}}(\mathbf{r}) \cdot \tilde{\mathbf{E}}_q^*(\mathbf{r}) dV,$$

Example: Infinite train of tight bunches in undulator



$$P_{SR} = \frac{1}{32} Z_q \frac{N^2 e^2 \omega_0^2 |\tilde{\beta}_w|^2}{\pi^2 \beta_z^2} \frac{L_w^2}{A_{em\,q}} \,\mathrm{sinc}^2(\theta L_w/2)$$

$$P_{ST-SR} = \frac{1}{4} |\tilde{C}_q(0)| \frac{N e \omega_0 |\tilde{\beta}_w|}{\pi \beta_z} \sqrt{\frac{2 Z_q \mathcal{P}_q}{A_{em\,q}}} L_w \cos(\varphi_{qb0} - \theta L_w/2)\, \mathrm{sinc}(\theta L_w/2)$$

Dynamics of a periodically bunched beam interacting with radiation field

Power of the electron bunches

$$N_b m c^2 \frac{d\gamma}{dt} = Q_b \mathbf{v} \cdot \mathbf{E}(\mathbf{r}, t),$$

combined with the excitation equation

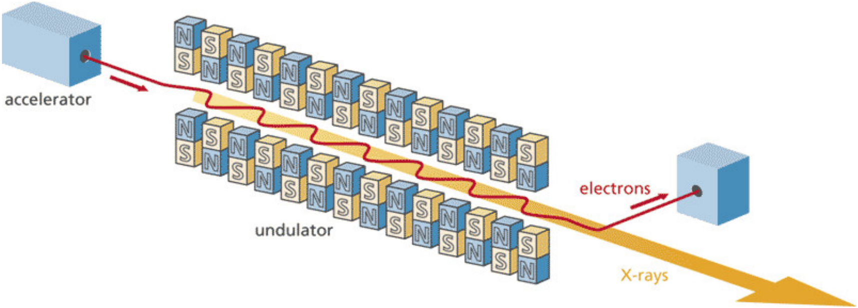
$$\frac{d\tilde{C}_q}{dz} = \frac{-1}{4\mathcal{P}_q} \int \tilde{\mathbf{J}} \cdot \tilde{\mathbf{E}}_q^* d^2\mathbf{r}_\perp.$$

using the definition

$$\psi \equiv -[\varphi_b(z) - \varphi_q(z) - \pi/2] = -\int_0^z \theta(z')dz' + \psi(0),$$

Results in Pendulum equation

Pendulum equation - uniform wiggler

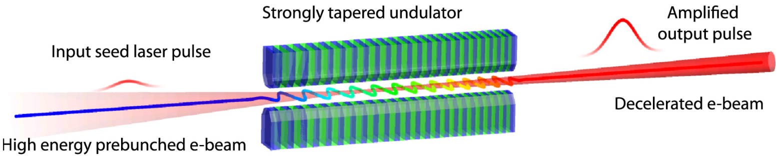


$$\frac{d|\tilde{C}_q|}{dz} = B \sin\psi,$$

$$\frac{d\delta\gamma}{dz} = -\frac{\beta_{zr}^3 \gamma_{zr}^2 \gamma_r}{k_0} K_s^2(z) \sin\psi,$$

$$\frac{d\psi}{dz} = \frac{k_0}{\beta_{zr}^3 \gamma_{zr}^2 \gamma_r} \delta\gamma + \frac{B}{|\tilde{C}_q|} \cos\psi$$

Pendulum equation - tapered wiggler



$$\frac{d|\tilde{C}_q|}{dz} = B \sin\psi,$$

$$\frac{d\delta\gamma}{dz} = -\frac{\beta_{zr}^3 \gamma_{zr}^2 \gamma_r}{k_0} K_s^2(z)[\sin\psi - \sin\psi_r],$$

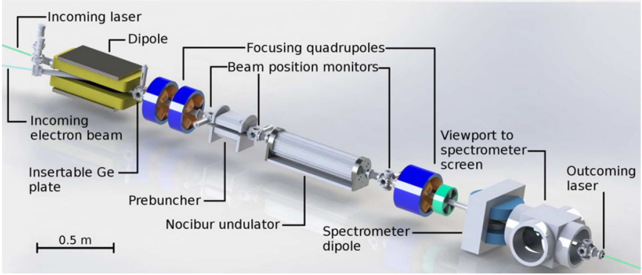
where $0 < \psi_r < \pi/2$.

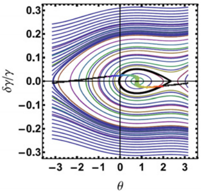
$$\frac{d\psi}{dz} = \frac{k_0}{\beta_{zr}^3 \gamma_{zr}^2 \gamma_r} \delta\gamma + \frac{B}{|\tilde{C}_q|} \cos\psi$$

N.M. Kroll, P.L. Morton, M.N. Rosenbluth, IEEE J. Quant. Electron., VOL. QE-17, NO. 8, AUGUST 1981

A. Gover, Phys. Rev. ST-AB 8, 030701 (2005)

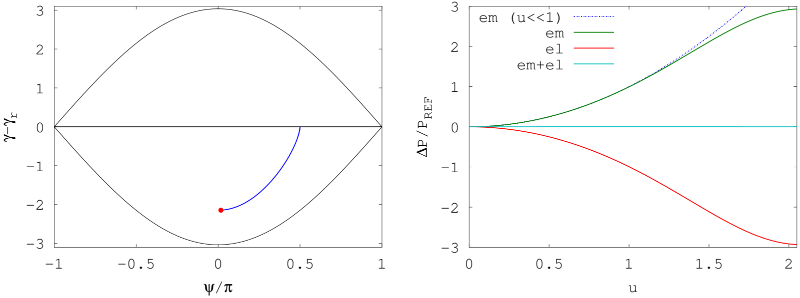
Parameters based on Nocibur experiment (but using tight bunches)





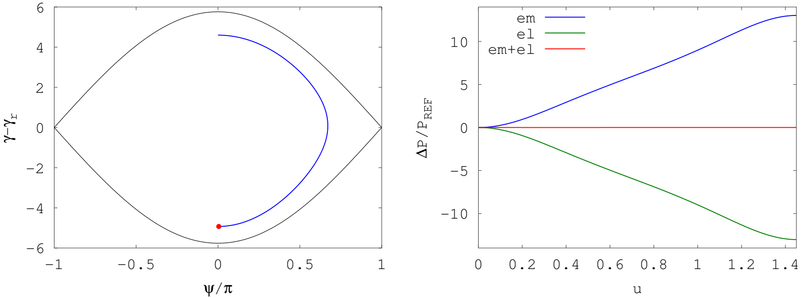
N. Sudar, P. Musumeci, J. Duris, and I. Gadjev, PRL 117, 174801 (2016)

Results - uniform wiggler, self-interaction, superradiance



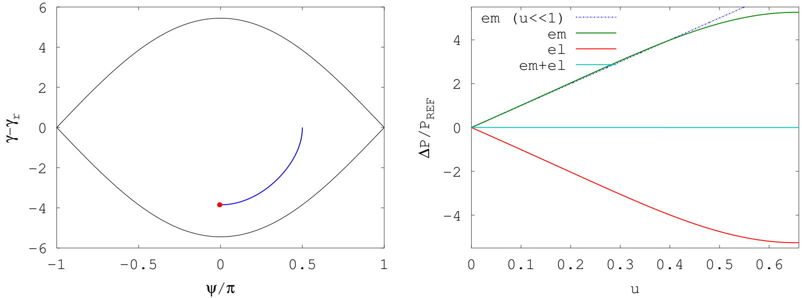
Video_6.avi

Results - uniform wiggler, stimulated superradiance, maximum energy extraction



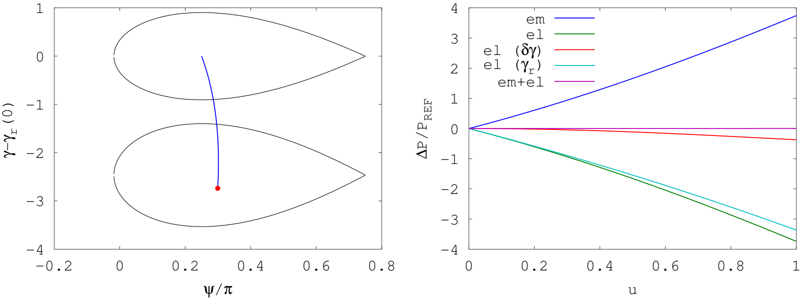
Video_2.avi

Results - uniform wiggler, stimulated superradiance, maximum gain



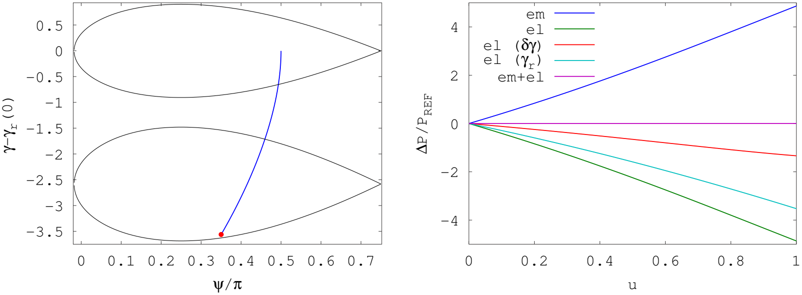
Video_3.avi

Results - tapered wiggler, initial phase ψ_r



Video_4.avi

Results - tapered wiggler, initial phase $\pi/2$



Video_5.avi

Thanks for your attention