Theory and Simulation of FELS with Planar, Helical, and Elliptical Undulators

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OUTLINE

- The Resonance Condition is obtained for an elliptical undulator

- The *JJ*-Factor is obtained in an elliptical undulator
  - Measures the effect of the lower beat wave → degrades the interaction
    - Suppressed due to the symmetry of a helical undulator
  - The *JJ*-factor varies smoothly: planar → elliptical → helical
  - It is important to determine the *JJ*-factor in order to:
    - Generalize the parameterization á la Ming Xie
    - Parameterize orbit-averaged simulations

- Simulations are performed using an elliptical undulator model
  - 1D simulations using PUFFIN
  - 3D simulations using MINERVA

THE RESONANCE CONDITION

- The orbits in a 1D elliptical undulator

$$\mathbf{B}_w = B_w\left(\hat{\boldsymbol{e}}_x \cos k_w z + u_e \hat{\boldsymbol{e}}_y \sin k_w z\right)$$

$$\mathbf{v}_\perp = -\frac{cK}{\gamma}\left(u_e \hat{\boldsymbol{e}}_x \cos k_w z + \hat{\boldsymbol{e}}_y \sin k_w z\right)$$

$$\frac{\upsilon_z^2}{c^2} = \frac{\upsilon_\parallel^2}{c^2} + \left(1 - u_e^2\right)\frac{K^2}{2\gamma^2}\cos 2k_w z$$

Ellipticity

$u_e = 1$: Helical Undulator

$u_e = 0$: Planar Undulator

- From energy conservation

$$\frac{\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2}{c^2} = 1 - \frac{1}{\gamma^2} \longrightarrow \frac{\upsilon_\parallel^2}{c^2} = 1 - \frac{1 + (1 + u_e^2)K^2/2}{\gamma^2}$$

- The Resonance Condition

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left[1 + \left(1 + u_e^2\right)\frac{K^2}{2}\right]$$

TO OBTAIN THE *JJ*-FACTOR

- The electron position is given by

$$z \cong \upsilon_{\|} t + \left(1 - u_e^2\right) \frac{K^2}{8\gamma^2 k_w} \sin 2k_w \upsilon_{\|} t$$

- We consider the interaction with an elliptically polarized wave

$$\delta\mathbf{E} = \delta\hat{E}\left[u_e \hat{\mathbf{e}}_x \sin\left(kz - \omega t\right) + \hat{\mathbf{e}}_y \cos\left(kz - \omega t\right)\right]$$

- Based upon Poynting's Theorem, we write

Upper Beat (Ponderomtive) Wave **Lower Beat Wave**

$$\mathbf{v}\cdot\delta\mathbf{E} = -\frac{cK}{4i\gamma}\delta\hat{E}\left\{\left(1+u_e^2\right)\exp\left[i\left(k+k_w\right)z - i\omega t\right] - \left(1-u_e^2\right)\exp\left[i\left(k-k_w\right)z - i\omega t\right]\right\} + c.c.$$

Vanishes when $u_e = 1$ for a helical undulator

- For the upper beat (ponderomotive) wave

$$\exp\left[i\left(k+k_w\right)z-i\omega t\right]=\exp\left[i\left(k+k_w\right)\upsilon_{\parallel}t-i\omega t\right]\sum_{n=-\infty}^{\infty}J_n\left(b\right)\exp\left(2ink_w\upsilon_{\parallel}t\right)$$

$$\approx\exp\left[i\left(k+k_w\right)\upsilon_{\parallel}t-i\omega t\right]J_0\left(b\right)$$

- For the lower beat wave

$$\exp\left[i\left(k-k_w\right)z-i\omega t\right]=\exp\left[i\left(k+k_w\right)\upsilon_{\parallel}t-i\omega t\right]\sum_{n=-\infty}^{\infty}J_{n+1}\left(b\right)\exp\left(2ink_w\upsilon_{\parallel}t\right)$$

$$\approx\exp\left[i\left(k+k_w\right)\upsilon_{\parallel}t-i\omega t\right]J_1\left(b\right)$$

$$b=\frac{\left(1-u_e^2\right)K^2/4}{1+\left(1+u_e^2\right)K^2/2}$$

$u_e = 1$ → $b=0$ Helical Undulator

$u_e = 0$ → $b=\frac{K^2/4}{1+K^2/2}$ Planar Undulator

THE *JJ*-FACTOR

- As a result, the source term in Poynting's theorem becomes

$$\mathbf{v}\cdot\delta\mathbf{E}\cong-\frac{cK}{4i\gamma}\left(1+u_e^2\right)\delta\hat{E}\exp\left[i\left(k+k_w\right)\upsilon_{\|}t-i\omega t\right]\left[J_0\left(b\right)-\frac{\left(1-u_e^2\right)}{\left(1+u_e^2\right)}J_1\left(b\right)\right]+c.c.$$

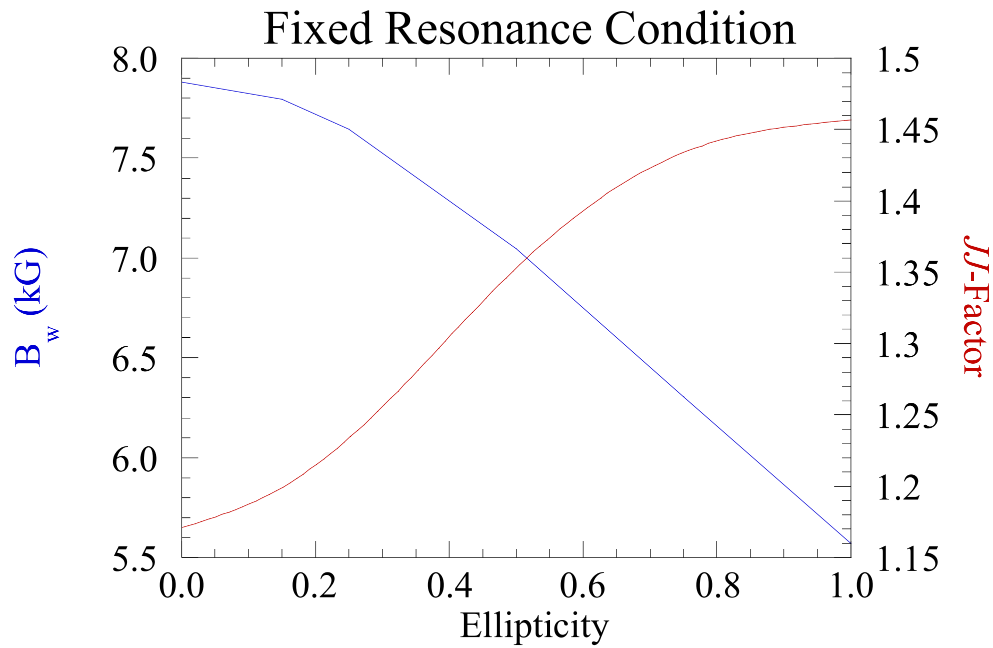
$$JJ=\sqrt{1+u_e^2}\,\frac{K}{\sqrt{2}}\left[J_0(b)-\frac{1-u_e^2}{1+u_e^2}J_1(b)\right]$$

- Helical Undulator

$$JJ=K$$

- Planar Undulator

$$JJ=\frac{K}{\sqrt{2}}\left[J_0(b)-J_1(b)\right]$$



SIMULATION CODES

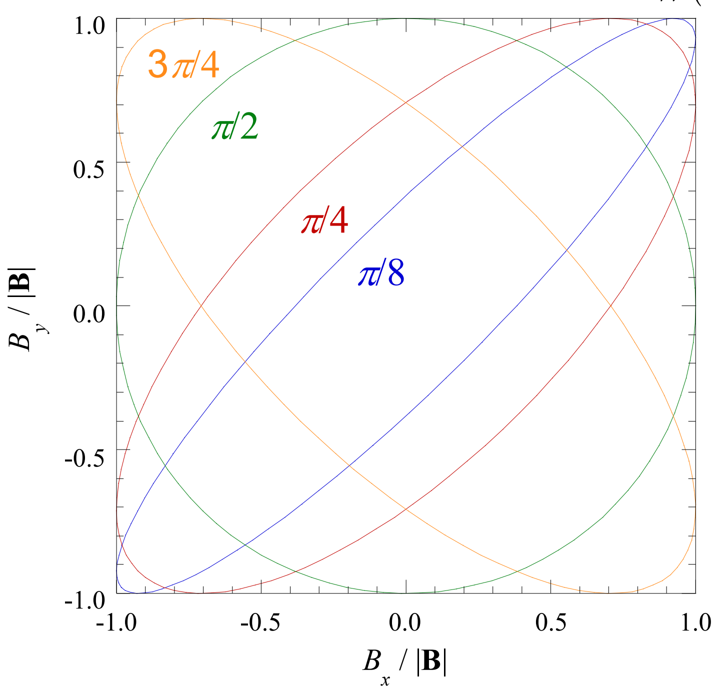
- **PUFFIN** – 1D & 3D, time-dependent, unaveraged numerical simulation code
  - L.T. Campbell & B.W.J. McNeil, Phys. Plasmas **19**, 093119 (2012)
  - 1D simulation of elliptical undulator
  - Results compared with orbit-averaged simulation using the resonance condition and *JJ*-factor
- **MINERVA** – 3D, time-dependent, SVEA code with full 3D orbit treatment (*i.e.*, no wiggler-average)
  - H. Freund, P. van der Slot, D. Grimminck, I. Setya, and P. Falgari, New J. Phys. **19**, 023020 (2017).
  - Using an APPLE-II undulator model

- Neither code needs an explicit statement of either the resonance condition or the *JJ*-factor since they are implicitly included in the particle dynamics

APPLE-II UNDULATOR MODEL

We model an APPLE-II undulator using a super-position of two crossed planar undulators with a relative phase shift ϕ

$$\mathbf{B}_w(\mathbf{x}) = B_w(z)\left[\sin(k_w z+\phi) - \frac{\cos(k_w z+\phi)}{k_w B_w}\frac{dB_w}{dz}\right]\hat{\boldsymbol{e}}_x \cosh(k_w x) + B_w(z)\left[\sin(k_w z) - \frac{\cos(k_w z)}{k_w B_w}\frac{dB_w}{dz}\right]\hat{\boldsymbol{e}}_y \cosh(k_w y) + B_w(z)\hat{\boldsymbol{e}}_z\left[\sinh(k_w x)\cos(k_w z+\phi) + \sinh(k_w y)\cos(k_w z)\right]$$



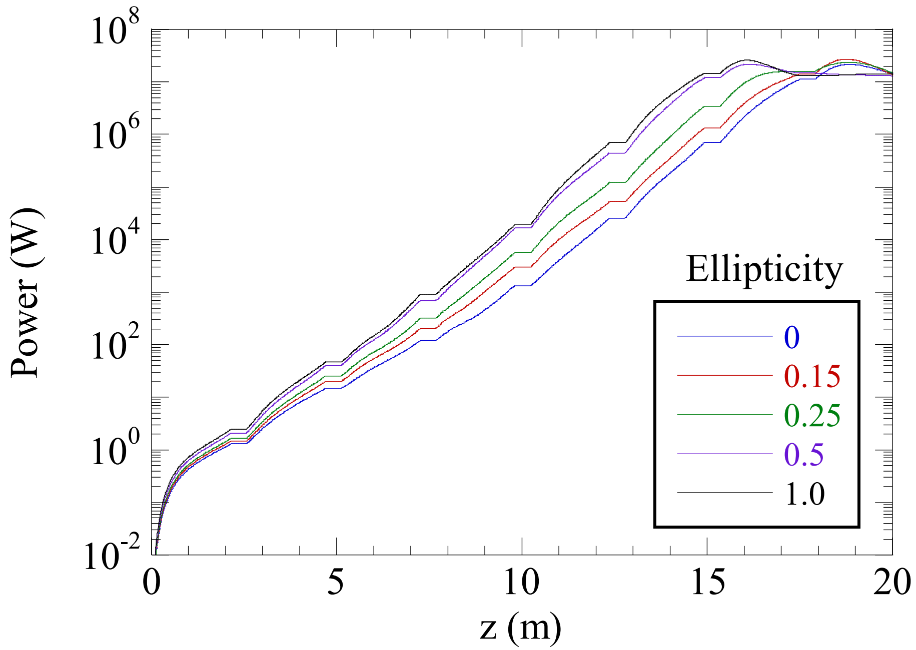
The ellipticity is given by:

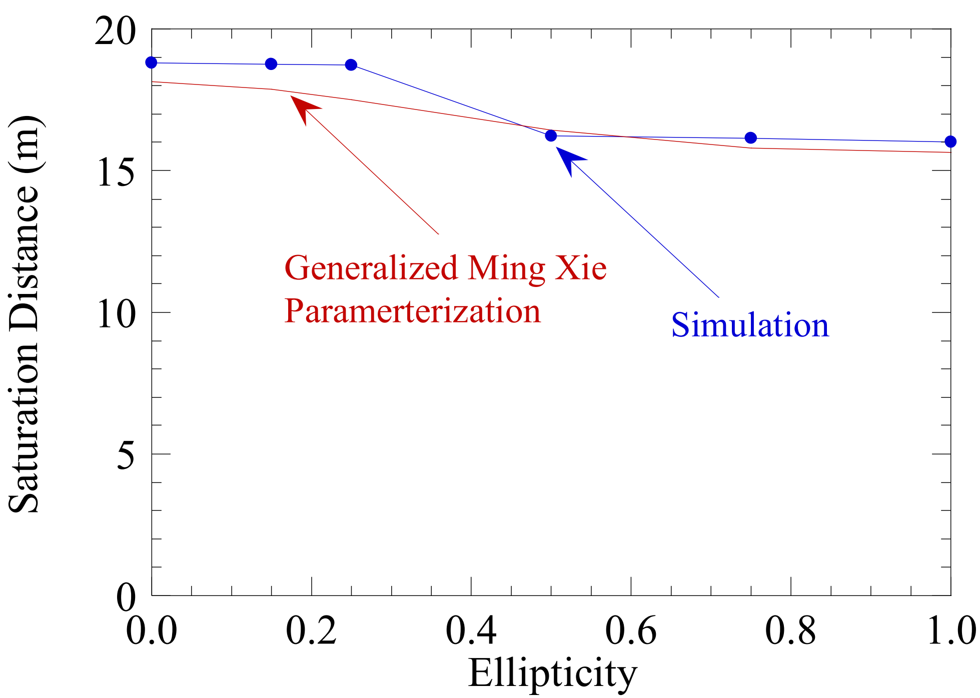
$$u_e = \frac{1-\cos\phi}{1+\cos\phi} \qquad 0 \le \phi < \pi/2$$

$$u_e = \frac{1+\cos\phi}{1-\cos\phi} \qquad \pi/2 \le \phi < \pi$$

MINERVA SIMULATIONS

- MINERVA simulations for SPARC-like parameters show a decrease in the gain length and increase in the saturation distance with decreasing ellipticity – as expected.





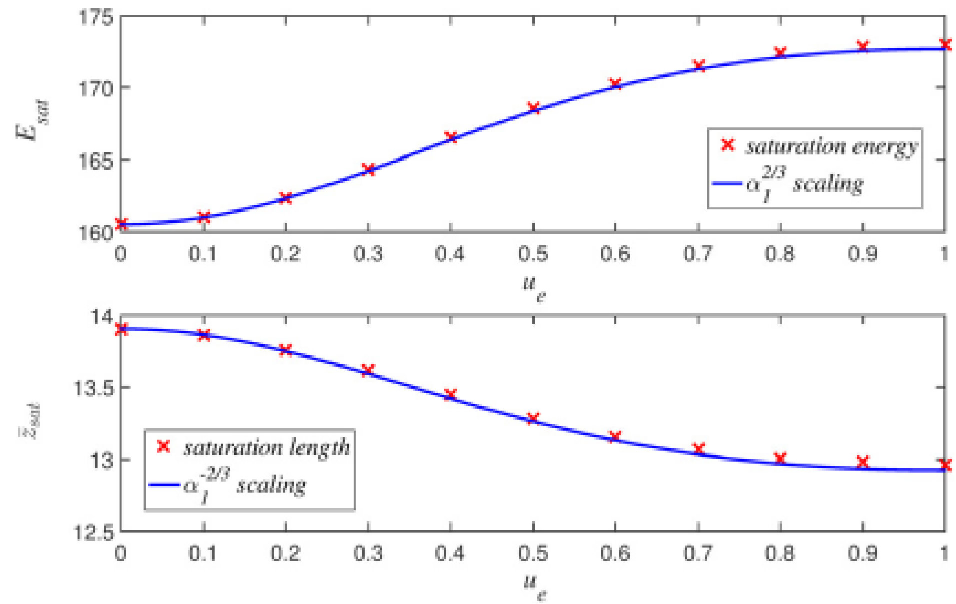
- MINERVA is in reasonable agreement with the Ming Xie formulae that have been generalized to include the new resonant wavelength and *JJ*-factor.

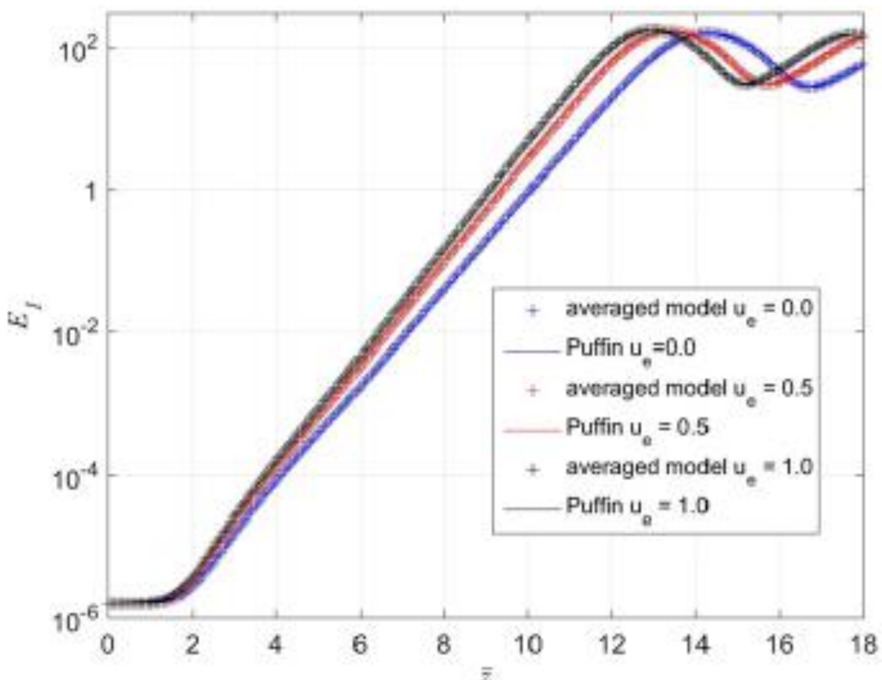
H. Freund, P. van der Slot, D. Grimminck, I. Setya, and P. Falgari, New J. Phys. 19, 023020 (2017)

PUFFIN SIMULATIONS

- Comparison between PUFFIN and the predicted scaling of the saturation energy and distance show good agreement

$$\alpha = J_0(b) - \frac{1-u_e^2}{1+u_e^2} J_1(b)$$





- Comparisons between PUFFIN and the averaged simulation show good agreement
- Saturation distance increases with the ellipticity as expected and seen in MINERVA

J.R. Henderson, L.T. Campbell, H.P. Freund, B.W.J. McNeil, New J. Phys. 18, 062003 (2016)

SUMMARY

- We have developed analytic formulae for the resonant wavelength and the *JJ*-Factor
    - Generalization of Ming Xie's parameterization (MINERVA, 3D)
    - Used in 1D orbit-averaged simulation (PUFFIN, 1D)

- PUFFIN and MINERVA simulations illustrate some essential features associated with elliptical undulators
    - Dependence of the resonance condition and interaction strength on the undulator field strength

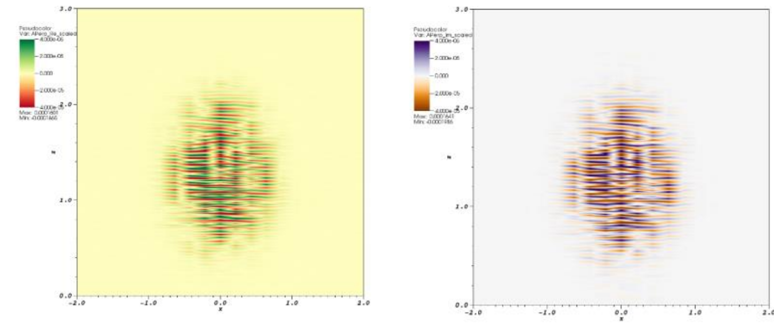
- This work represents a beginning in the theory and simulation of elliptical polarization in FELs

SUPPLEMENTAL VIEWGRAPHS

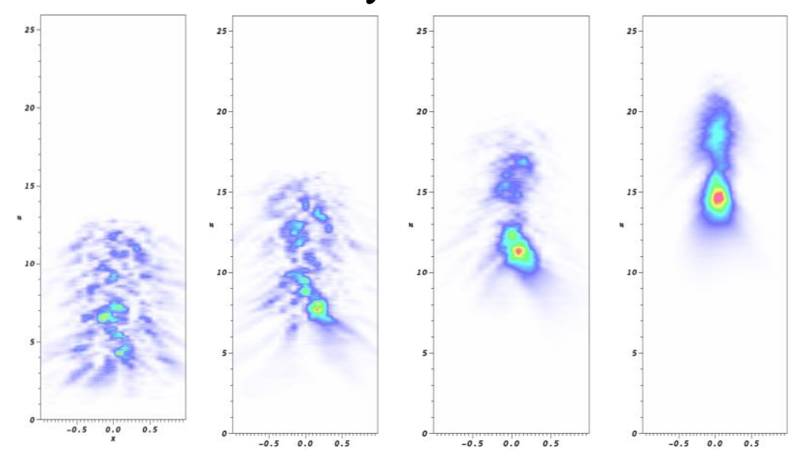
THE PUFFIN SIMULATION CODE

- No slicing
- No period averaging
- Non-SVEA
- Neglects backwards wave, space-charge
- Models broadband emission
- Includes:
  - variably polarized undulators,
  - variable undulator tunings,
  - tapering,
  - beam energy oscillations,
  - Undulator lattices with chicanes and quads using point transforms

Full *x* and *y*–polarized fields:



Intensity evolution:



THE MINERVA SIMULATION CODE

- Fully 3-D with Time-Dependence
- E&M fields treated using the polychromatic SVEA approximation
  - Multi-slice time-dependent and/or polychromatic physics
  - Modal decomposition of the fields
  - Amplifier (MOPA)/Oscillator (linked to OPC)/SASE/OK/HGHG
- Particle dynamics are treated from first principles (not KMR)
  - Harmonics & sidebands implicitly included in orbit dynamics
- Additional Features/Capabilities
  - Wiggler models
    - Parabolic-Pole-Face, Flat-Pole-Face, Canted-Pole, Helical
    - APPLE-II model describes planar, helical, elliptical symmetry
    - Import from a field map
  - Quadrupole & Dipole Field Models
  - Non-Gaussian distributions (TBD)
  - Import Phase Space
  - Restart Capability