NUMERICAL STUDY OF CHERENKOV RADIATION FROM THIN SILICA AEROGEL*

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Abstract

Vavilov-Cherenkov radiation, usually just Cherenkov radiation (CR), is commonly used in high energy charged particle and cosmic rays detectors. We have studied CR emitted from very low refractive index material such as silica-aerogel and found it may be useful tool for electron beam diagnostics since the opening angle (Cherenkov angle) is small, then the CR can be transported onto a detector located far from the radiator. We have prepared a thin (1 mm thick) hydrophobic silica-aerogel having refractive index of 1.05 that has been developed at Chiba University. Since the intensity of CR is much stronger than that of optical transition radiation, the CR is a better light source for low intensity beam diagnostics. In order to apply the CR to measurement of bunch length of electron beams, we have investigated properties of CR by numerical simulation study based on the Liénard-Wiechert potentials. In addition, possibility of intense THz source is also discussed.

CHERENKOV RADIATION

Frank-Tamm theory [1] explaining properties of the Vavilov-Cherenkov radiation [2,3] without charged particle (de-)acceleration is based on the assertion that a charge moving uniformly in a dielectric medium with the velocity faster than the velocity of light in the medium radiates spherical electromagnetic waves from each point of its trajectory, it is the so-called “Cherenkov ring”. Although the CR property seemed to be mostly understood because Tamm’s first consideration was in 1939, it is however very interesting that we can find some theoretical works regarding “Tamm Problem” arising from an instantaneous acceleration and deceleration of a charge at the beginning and termination of its motion [4,5].

The opening angle, Cherenkov angle θ_C, is in general characterized by the refractive index n and the particle velocity β as

$$\cos\theta_C = 1/(n\beta)\,. \tag{1}$$

Therefore, the radiation does not occur when $n\beta < 1$. Photon numbers of CR between the wavelengths λ_1 and λ_2 is given as [1]

$$N_{\text{photon}} = 2\pi\alpha z\left|\frac{1}{\lambda 1}-\frac{1}{\lambda 2}\right|\sin^2\theta_C\,, \tag{2}$$

where α and z are the fine structure constant and the radiator thickness, respectively. Although common e-beam diagnostics, transition radiation is often employed, the photon number of it is poorly small. Since CR is a significant effect, the study has probed the potential ability of CR for beam diagnostics. For example, yield of 500-nm photons with 1% bandwidth from relativistic electron is ~0.5 per electron for a 1-mm thick radiator having $n = 1.5$.

NUMERICAL EVALUATION

We start our analysis with the well-established Liénard–Wiechert potential [6]

$$\frac{dI}{d\omega d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\pi^2 c(\omega)}\omega^2 \times\left|\int \mathbf{n}\times\left(\mathbf{n}\times\beta(\omega)\right)\mathrm{e}^{\left[i\omega\left(t'-\frac{\mathbf{n}\cdot\mathbf{r}(t')}{c(\omega)}\right)\right]}dt'\right|^2 \tag{3}$$

where $c(\omega)$ is the speed of light in the medium and $\beta(\omega)$ is a particle speed with respect to $c(\omega)$. A vector **n** denotes a unit vector to the observing point from the particle located at $\mathbf{r}(t')$. Assuming a charge travelling in a straight line at a velocity and the velocity of light in the medium is

$$c(\omega) = 1/\sqrt{\varepsilon(\omega)\mu_0}\,, \tag{4}$$

then eq. (3) becomes

$$\frac{dI}{d\omega d\Omega} = \frac{\mu_0}{4\pi}\frac{e^2}{4\pi^2 c(\omega)}\omega^2 v^2\sin^2\theta\left|\int e^{i\omega\left(1-\beta(\omega)\cos\theta\right)}dt'\right|^2. \tag{5}$$

After some mathematical treatment, it can be shown that

$$\int e^{i\omega\left(1-\vec{\beta}(\omega)\cos\theta\right)}dt' = 2\pi\delta\left[\omega\left(1-\vec{\beta}(\omega)\theta\right)\right]. \tag{6}$$

Thus, we obtain

$$\frac{dI}{d\omega d\Omega} = \frac{\mu_0}{4\pi}\frac{e^2\omega^2}{4\pi^2 c(\omega)}\sin^2\theta\left(\frac{\sin\alpha}{\alpha}\right)^2(dz)^2\,, \tag{7}$$

where

$$\alpha = \frac{1}{2}\left[1-\vec{\beta}(\omega)\cos\theta\right]\omega\Delta t\,, \tag{8}$$

and Δt denotes the time passing through the medium.

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Hereafter we have used a refractive index $n = 1.05$ for the radiator medium and an electron energy of 30 MeV for numerical evaluations. Spatial distributions of the Cherenkov ring from single electron calculated for the wavelengths of 3 μm and 30 μm is shown in Fig. 1. The observed angular range is –0.5 rad to +0.5 rad in both horizontal and vertical planes while the Cherenkov angle is 0.31 rad (17.75°). It should be noted that the calculated angular distributions of air CR using presented parameters in Figs. 2 and 4 of Ref. [5] mostly corresponds to the exact solutions given in the article. As one notices that the Cherenkov ring of longer wavelength is getting fat. It is because of laxly interference condition.

The phenomenon is much more clearly seen in Fig. 2. In addition, since the intensity of CR is inversely proportional to the square of the wavelength, even the shorter bunch length such as 100 fs, coherent enhancement of the radiation intensity is not much impressive (see Fig. 3).

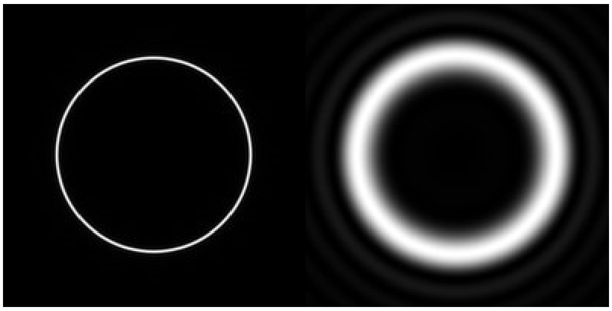

Figure 1: Calculated Cherenkov rings for the wavelengths of 3 μm (left) and 30 μm (right).

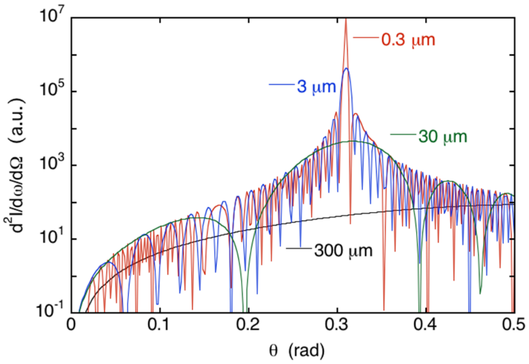

Figure 2: Angular distributions of CR for various wavelengths.

BUNCH LENGTH MEASUREMENT

As indicated in Fig. 4, observable bunch length is possibly observed as

$$\sigma_{\text{observed}} = \sqrt{\left(\sigma_L \cos\theta_C\right)^2 + \left(\sigma_T \sin\theta_C\right)^2}\ . \tag{9}$$

Note the radiator thickness does not affect the temporal structure ideally.

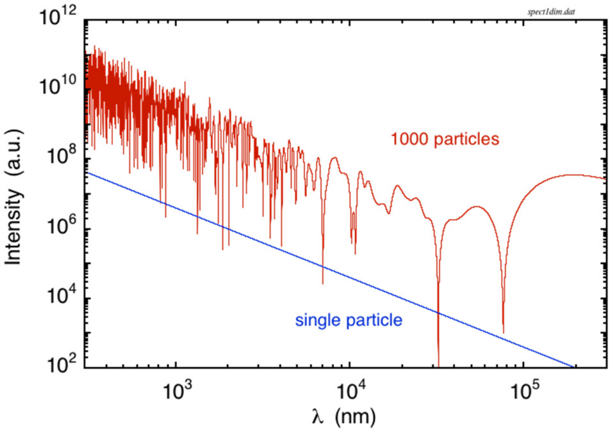

Figure 3: Calculated CR intensities. For the 1000 particle simulation, the bunch length employed is 100 fs (rms).

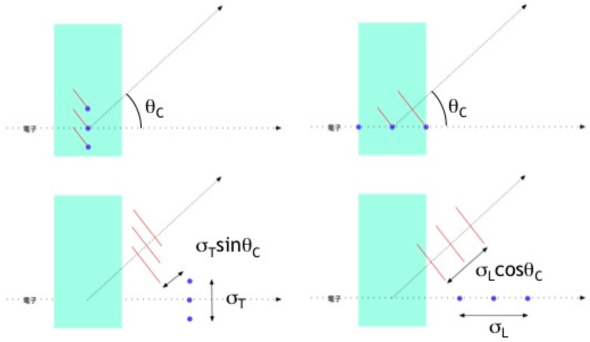

Figure 4: Schematics for bunch length measurement.

By using the approximation $n = 1+\delta$, we obtain

$$\cos\theta_C = \frac{1}{n\beta} \approx 1 - \frac{\theta_C{}^2}{2} = \frac{1}{1+\delta} \approx 1-\delta\,, \tag{10}$$

and

$$\sin\theta_C = \sqrt{1-\cos^2\theta_C} \approx \sqrt{1-\left(1-\delta\right)^2} \approx \sqrt{2\delta}\ . \tag{11}$$

Thus, the effect of the transverse beam size on the observed bunch length is expected to be:

$$\frac{\sigma_T \sin\theta_C}{\sigma_L \cos\theta_C} = \frac{\sigma_T \sqrt{2\delta}}{\sigma_L\left(1-\delta\right)}\ . \tag{12}$$

If we use a radiator of $n = 1.05$, a ratio of transverse size and bunch length in observed bunch length is ~0.33. It can be concluded that the transverse beam size must be focused onto three times of the bunch length at least.

Looking at wavelengths short enough such as visible region, the radiator thickness affects the width of the Cherenkov ring Γ. It can be expressed by a convolution of radiator thickness and beam transverse size. Using the radiator thickness $2t$ and the beam size projection onto propagation axis σ (= $\sigma_T \cos\theta_C$), we obtain

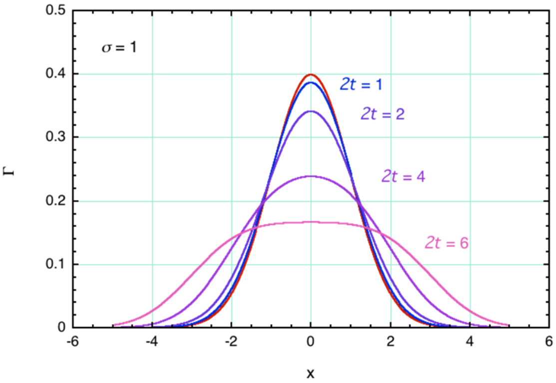

Figure 5: Calculated width of the Cherenkov ring normalized to the transverse beam size.

$$\Gamma = \frac{1}{2t\sqrt{\pi}} \times \left(\sum_n \frac{(-1)^n \left[\frac{1}{\sqrt{2}\sigma}(x+t) \right]^{2n+1}}{n!(2n+1)} - \sum_n \frac{(-1)^n \left[\frac{1}{\sqrt{2}\sigma}(x-t) \right]^{2n+1}}{n!(2n+1)} \right) . \quad (13)$$

We can see the effect of the radiator thickness if the beam size is approximately three times larger than the radiator thickness.

TEST EXPERIMENT

A silica aerogel film produced by hydro-phobic treatment for which the refractive index does not change in the vacuum was provided by Chiba University [7,8]. In a test experiment, we used a light extraction system shown in Fig. 6 (upper), and a photograph of the Cherenkov ring is shown in Fig. 6 (lower).

A Cherenkov ring is clearly seen. In order to suppress the Cherenkov angle for light transportation, we employed the minimum refractive index silica aerogel available at the moment ($n = 1.05$).

SUMMARY OF FUTURE PROSPECTS

We have studied properties of the Cherenkov radiation from a thin silica aerogel by numerical evaluation. An accelerator test facility, t-ACTS at Tohoku University is capable of providing 100-fsec electron pulses with a beam energy of 30 MeV [9] while employing a velocity-bunching scheme [10]. A thin silica aerogel with water-free hydrophobic treatment is used as Cherenkov radiator. A complete Cherenkov ring emitted from the 1-mm-thick silica aerogel having low refractive index was already observed. Various experiments with Cherenkov light may start soon. Currently, we are developing a specially designed axicon-mirror system to transport the Cherenkov radiation for long distance.

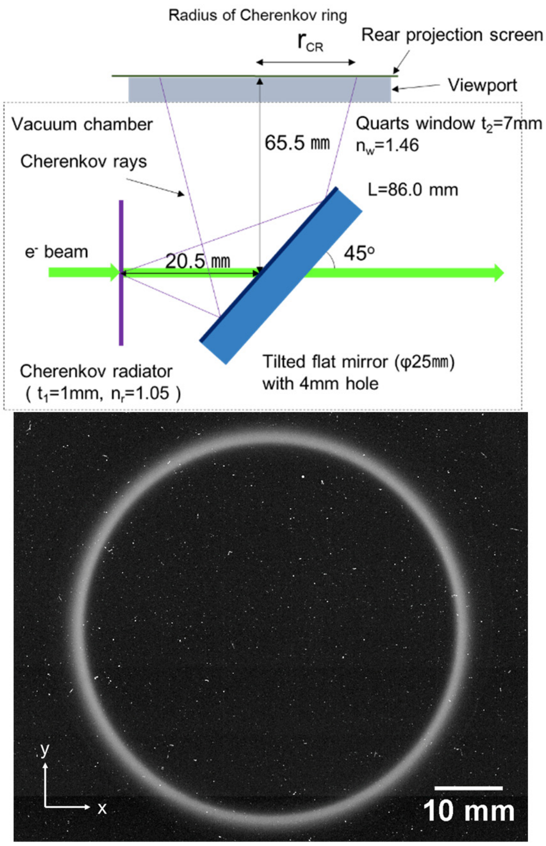

Figure 6: Apparatus of a test experiment (upper) and observed Cherenkov ring (lower) using a 50 MeV electron beam.

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