UNAVERAGED MODELLING OF A LWFA DRIVEN FEL

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Abstract

Preliminary simulations of a Laser Wake Field Accelerator driven FEL are presented using the 3D unaveraged, broad bandwidth FEL simulation code Puffin. The radius of the matched low-emittance electron beam suggests that the FEL interaction will be strongly affected by radiation diffraction. Parameter scaling and comparison between 3D and equivalent 1D simulations confirm this. The Puffin 1D simulations indicate that the energy spread conditions for FEL lasing are met, even without any beam phase space manipulation prior to injection into the undulator. The large diffraction in the Puffin 3D simulations creates boundary problems that will need to be overcome before further progress is made.

INTRODUCTION

With several linac driven X-ray FEL's currently in operation or construction around the world, there is much interest in the next generation of FEL facilities. The plasma based Laser Wakefield Accelerators (LWFA's) are a promising driver for future FEL facilities. Due to their large acceleration gradients compared to conventional RF-linacs, their compact size could dramatically reduce facility costs.

No plasma accelerator driven FEL has yet reported successful lasing primarily as the beam energy spreads from these accelerators are too large by approximately an order of magnitude. Some other plasma accelerator schemes exist that may promise an improvement in beam quality, but these are yet to be realised. A pragmatic approach was taken in a study [1], which considered a design that uses beams currently available from LWFA's to enable modest FEL gains to generate power levels measurably above the spontaneous power to be observed. A cryogenic undulator design with a small undulator period and large on-axis magnetic field was used. This design is chosen to maximise the FEL ρ parameter for a given beam and thus relax the energy spread requirement in the FEL $\sigma_\gamma/\gamma \lesssim \rho$. A chicane to stretch the beam before insertion into the undulator was utilized to both increase the beam length with respect to the cooperation length, and so increase the interaction length between radiation and electrons, and to reduce the localised energy spread. Genesis [2] simulations predicted a modest gain of ~ 6 over the spontaneous emission without bunch stretching, and a gain of $\sim 10^3$ when the stretching was optimized.

In averaged FEL simulation codes where the Slowly Varying Envelope Approximation (SVEA) is applied, the electron beam and radiation fields are modelled by a series of phase space 'slices' within which periodic boundary conditions are applied. Modelling the FEL interaction and electron phase space evolution with beams that are short, have correlated energy spreads (chirps) etc, such as those generated by plasma accelerators, can therefore be problematic. The 3D FEL simulation code Puffin [3] does not perform the Slowly Varying Envelope Approximation or averaging of the electron or radiation parameters and so may be better suited to such circumstances. Furthermore, the effects of dispersion in short beams, either from the chirp or from energy spread induced from the FEL interaction, may be more dramatic than equivalent effects in a longer pulse.

In the following the 3D parameters and likely consequences on the FEL interaction for a plasma accelerator driven FEL interaction are discussed. Puffin simulations of the FEL interaction using the parameter set of [1] are then presented, first in 1D, showing the requirement on the beam energy spread is satisfied. A 3D simulation is also presented. However, the small matched electron beam radius results in a large radiation diffraction which, for the field sampling size used, cannot be modeled properly.

PARAMETERS

Much information can be gained regarding a potential FEL interaction by calculating the scaled parameters that describe the interaction and comparing against a known set of limits which the scaled parameters must meet before good FEL lasing action can occur. Several works have derived these criteria over a period as summarised in [4, 5].

The physical parameters of the study of [1], in the scaled parameters of Puffin [3], are shown in Table 1. In [1], the electron beam was matched in one transverse direction to the natural focusing of the planar undulator. However, the Puffin model assumes a uniform focusing in *both* transverse directions. This results, in the case of the planar wiggler, in the beam being over-focused when using the same betatron wavelength. In the following, the beam is focused to give a matched beam of radius equal to the rms radius used in [1], giving the same diffraction length. This gives an incorrect betatron wavelength, but it is assumed this will

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have negligible effect in this case since the betatron wavelength is very large compared to a gain length in both cases.

In calculating these parameters, gaussian distributions have been used throughout for all variables.

It is seen that the FEL parameter ρ is sufficiently large that the energy spread $\sigma_\gamma/\gamma_r < \rho$ which indicates that for these parameters the effect of energy spread alone should not stop an FEL interaction occurring.

One can immediately see that the scaled emittance for the beam $\bar{\epsilon} \equiv \epsilon/(\lambda_r/4\pi) \ll 1$. To achieve good FEL amplification $\bar{\epsilon} \lesssim 1$ [3, 6], so that the value here is rather small. Although this small emittance does mean a small effective energy spread due to the betatron motion of the electrons, the Rayleigh range is quite short with respect to the gain length, as seen from the value of the scaled Rayleigh range $\bar{z}_R = z_R/l_g$. This is also confirmed by the relatively small value of the diffraction parameter B [6]. One can therefore expect to see degradation of the FEL interaction due to significant radiation diffraction in a gain length.

For the parameters of Table 1, the electron bunch is relatively short with respect to the cooperation length $l_c = \lambda_r/4\pi\rho \approx 1.7\mu$m. This reduces the radiation coupling to the electrons as it slips out of the bunch after only a few gain lengths. This effect can mitigated and was investigated in [1], by 'stretching' the chirped electron bunch in a chicane prior to injection into the FEL undulator. This also has the effect of reducing both the electron energy chirp and the slice energy spread of the bunch. However, the stretching also reduces the electron current, so increasing the gain length and therefore the diffraction in the gain length, $\bar{z}_r$. All simulations presented here are for the 'un-stretched' parameters of Table 1.

The energy chirp of the bunch will cause the bunch to disperse and stretch as it propagates along the undulator. This may affect the FEL process, e.g. by stretching out the electron microbunching from the resonant wavelength as the interaction progresses. Short electron bunches may also generate Coherent Spontaneous Emission that may subsequently be amplified [7]. Such effects cannot be modeled in an averaged simulation code. As a gaussian current profile is used here, it is not expected that CSE will play a significant role in the startup of the interaction. However, this can change significantly for differently shaped pulses and if there is wavelength-scale structure in the electron current distribution.

Presently, Puffin does not model the effects of space charge, which may become significant with larger FEL ρ-parameters.

1D SIMULATIONS

Using the parameters of Table 1, Puffin was used in 1D mode to simulate the FEL interaction of the chirped bunch. In Fig. 1 the scaled power, relative electron energy and spectral power are plotted in the scaled temporal frame $\bar{z}_2$ and frequency ω/ω_r respectively. The scaled temporal frame is a window that travels with the radiation, so that the 'head' of the electron bunch is to the left of the plot

Table 1: Parameters used in simulations. The bracketed term for $\bar{z}_R$ and B is the value used in the Puffin simulation (see text.)

$\bar{a}_w$	2.3
λ_w	1.5cm
γ_r	600
σ_γ/γ_r	0.005
λ_r	134nm
pulse length σ_z	0.5μm
ρ	0.0223
k_β	1.12×10^{-2}
η	2.5×10^{-8}
$\bar{\epsilon}$	3.12×10^{-2}
$\bar{z}_R$	0.398
B	0.71

and the bunch slips left to right in the window on propagating through the FEL undulator. The Scaled Power as plotted is not the usual time-averaged power, i.e. a power envelope, but rather the instantaneous power of an oscillatory linearly polarised field (the fast oscillations cannot be resolved in the plot). Note that in the 1D case, $|A_\perp|^2$ is the on-axis *intensity* so that the Scaled Power, as plotted, is $\pi\bar{\sigma}_x^2|A_\perp|^2$, where $\bar{\sigma}_x = \bar{\sigma}_y = 0.126$ is the rms radius of the beam in the scaled transverse units $\bar{x}, \bar{y}$. As the 1D scaled intensity at saturation for a cold bunch of good quality is $|A_\perp|^2 \sim 1$ [8], the Scaled Power from such a bunch at saturation will be $\sim \pi\bar{\sigma}_x^2 \approx 0.05$.

Detail of the electron phase space clearly shows that the electrons are being modulated at the radiation wavelength (of period ≈ 0.27 in $\bar{z}_2$) towards the head of the bunch $34 < \bar{z}_2 < 39$ and are becoming bunched and losing energy to the radiation around the centre of the bunch $40 < \bar{z}_2 < 43$ where the current is maximum.

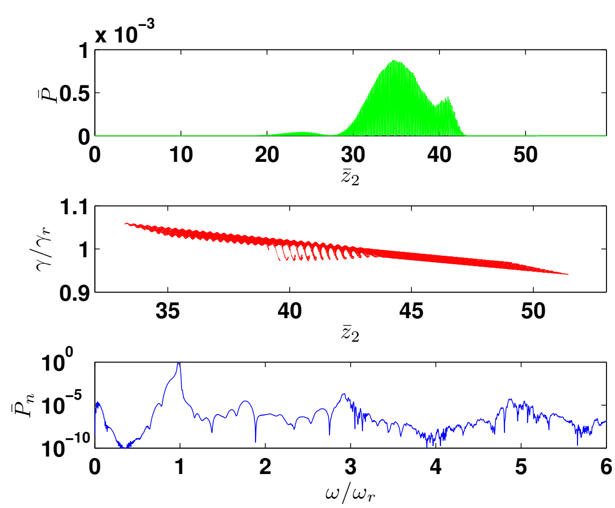

Figure 1: Scaled Power (top), electron phase space (middle) and scaled spectral power (bottom) from a 1D simulation of the parameters in Table 1. (Note the different scale in $\bar{z}_2$ for the plot of electron phase-space).

Due to the energy chirp, the beam is seen (not shown here) to stretch as it propagates through the undulator. However, from the discussions above, it is expected, but not verified here, that 3D diffraction effects will account for most of the differences between the 1D and 3D simulations presented in the following section.

Note that the radiation power appears to have saturated with a peak at $\bar{z}_2 \approx 35$ which has propagated through and beyond the peak of the electron bunch current located at $\bar{z}_2 \approx 42$. From the spectrum, most of the power is about the resonant frequency at $\omega/\omega_r = 1$.

3D SIMULATIONS

The same parameters of the previous 1D simulation are now used in a 3D simulation that includes the diffraction effects. As noted in the introduction, the method of focusing of the electron beam in Puffin here gives a longer betatron wavelength than that of a planar undulator, although it is anticipated that the effect of this will be negligible compared to the diffraction.

Puffin uses a split-step method [3] to simulate diffraction and radiation generation from the electron beam self-consistently. The diffraction step of the integration is nominally set to occur every undulator period. However, due to the relatively large diffraction experienced in this simulation it was found necessary to carry out a diffractive step 12 times per undulator period.

The preliminary 3D results equivalent to the 1D Fig. 1 are plotted in Fig. 2. As previously, the scaled instantaneous power is plotted which here is the numerically integrated intensity over the transverse plane: $\int |A_\perp(\bar{x},\bar{y},\bar{z}_2,\bar{z})|^2 d\bar{x}d\bar{y}$. Note that diffraction 'smears out' the fast oscillatory behaviour previously observed in the 1D simulation - the power at different frequencies diffract at different rates.

The introduction of the radiation diffraction appears to have significantly limited the gain, reducing the scaled power by a factor $\sim 10^2$ from the 1D simulation. Nevertheless, this power is $\sim 4-5$ orders of magnitude over the spontaneous emission power and significantly greater than the factor of ~ 5 of the averaged simulations of [1].

However, the radiation power is at a lower frequency than the usual resonant FEL frequency, and is broadband. Examination of the intensity in the transverse plane, see Figures 3 and 4, shows the simulation is completely dominated by diffraction. Puffin solves difraction in Fourier space, so the boundary conditions are cyclic when solving the diffraction. The low frequency emission results in a shorter diffraction length, and the transverse boundaries automatically setup to accomodate a more conventional FEL have proven insufficient to model the extreme diffraction occuring in this case. Figure 3 shows a reasonable transverse intensity distribution at the center of the pulse in the longitudinal axis, but in Figure 4, further towards the head of the pulse, one observes interference patterns caused by the radiation propagating through the periodic boundary conditions.

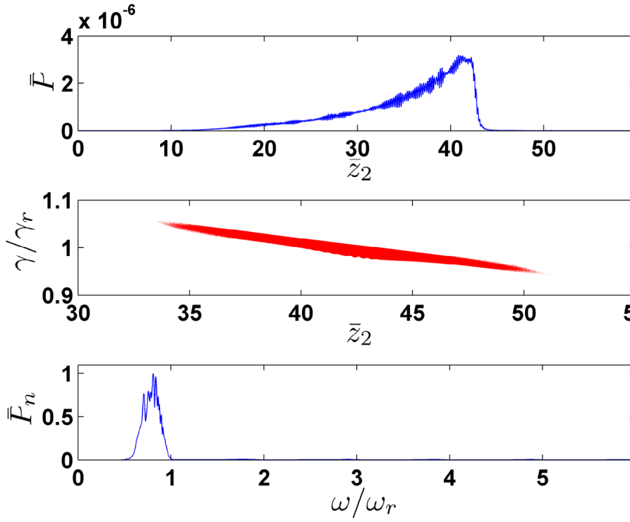

Figure 2: As Fig. 1 but for 3D Puffin simulation with the parameters of Table 1.

It is unclear at this stage whether the lower frequency peak of the power spectrum results from errors introduced by the incorrectly modeled diffraction or from other physical effects. Note that the unphysical transverse behaviour occurs towards the head of the electron bunch, where there is less charge - there is a guiding effect before the radiation propagates away from the higher current region of the bunch and strongly diffracts. It cannot be ruled out that these effects are causing large errors in other aspects of the simulation. Therefore, to model these diffraction effects properly, either the area of the transverse field model must be significantly, possibly prohibitively, increased. Alternatively, absorbing or transparent boundary conditions can be introduced, however, these cannot conserve total energy in the simulation.

CONCLUSIONS

Driven by experimental progress, the simulation of potential plasma-based accelerator FELs is an important developing field. The beams from these accelerators can differ substantially from their linac counterparts in terms of their peak current, bunch duration, emittance and their correlated and uncorrelated energy spreads. While averaged FEL simulation codes are able to model linac-driven FEL interactions in excellent agreement with experiment, they may be less suitable for plasma-based accelerator FELs. The discretisation and averaging of the radiation and electron beam parameters over the resonant radiation wavelength make a mobile electron phase space, coherent spontaneous and broad bandwidth emission difficult or impossible to model correctly.

The results presented in this paper are a first step in the unaveraged 3D modeling of a LWFA driven FEL using the

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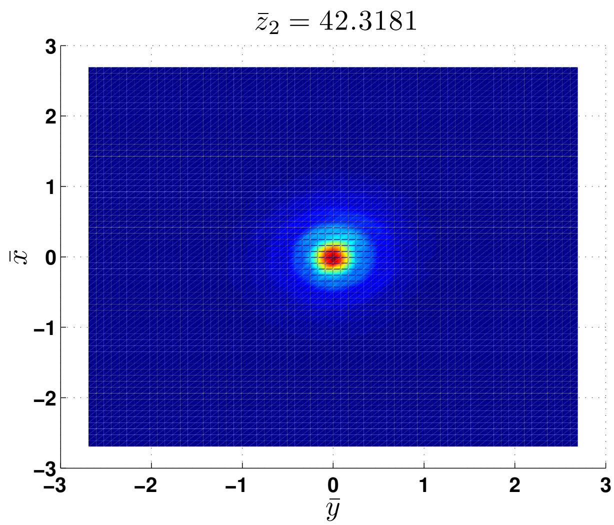

Figure 3: Transverse scaled intensity at $\bar{z}_2 \approx 43.5$ from 3D Puffin simulation with the parameters of Table 1.

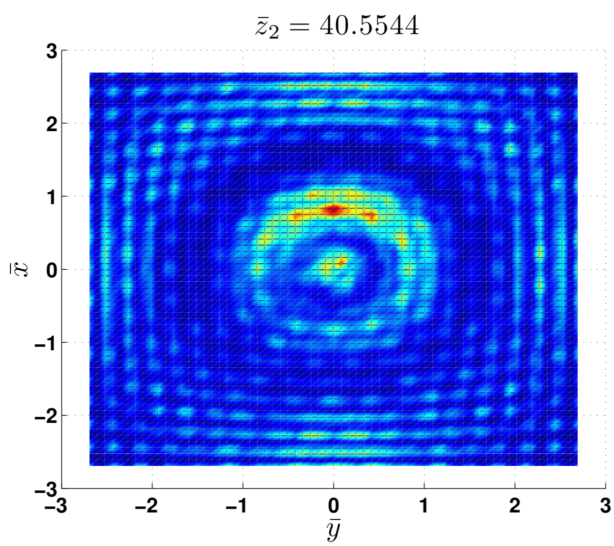

Figure 4: Transverse scaled intensity at $\bar{z}_2 \approx 37.6$ from 3D Puffin simulation with the parameters of Table 1.

Puffin simulation code. The 3D results are seen to be not correct due to problems modeling large diffraction effects, and are presented only as 'work-in-progress'.

The 1D simulations indicate that the energy spread condition for an FEL is satisfied without the need for a chicane to stretch the beam so reducing the slice energy spread.

The 3D simulation indicates that the system exhibits excessive diffraction for the model parameters. The large diffraction regime is challenging for Puffin to model correctly. While the longitudinal power profile and beam phase space seem reasonable, it is not possible, in light of the the diffraction issues, to make any claims for the validity of these aspects. In previous simulations performed with Puffin, with parameters found in more conventional FELs where diffraction lengths are closer to being optimised, a low frequency filter applied in the transverse plane was sufficient to prevent unphysical reflections from radiation propagating to the transverse boundaries. Further simulations are needed either using this method, or other boundary condition methods as discussed above, to solve this large diffraction regime correctly.

To mitigate diffraction effects on the FEL interaction, an unmatched beam of larger radius at the start of the interaction could be used. An increased beam radius will of course reduce the ρ parameter, so tighten the energy spread requirement, and will also introduce a beam radius oscillation. Further research into balancing and optimising these effects is required.

Further simulations will in future also be performed for different electron beam stretching factors prior to injection into the FEL undulator, to check the gain enhancement as observed in the simulations of [1]. The modeling would also benefit from more realistic current profiles than the gaussian. For the relatively short electron bunches used here, CSE may dominate spontaneous emission leading to a quicker FEL start-up [7]. Such a process may also give a better temporal coherence and greater output intensity .

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