FELs and high-energy electron cooling

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Content

And so, my fellow FELers, ask not what storage ring can do for FELs;

Ask what FELs can do for your storage rings!

And so, my fellow Americans, ask not what your country can do for you; ask what you can do for your country.

Accelerators at Collider Accelerator Department at BNL

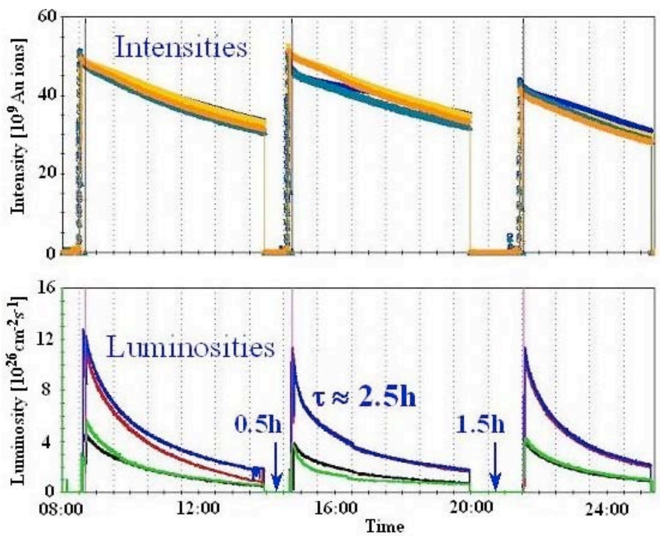
FEL-07

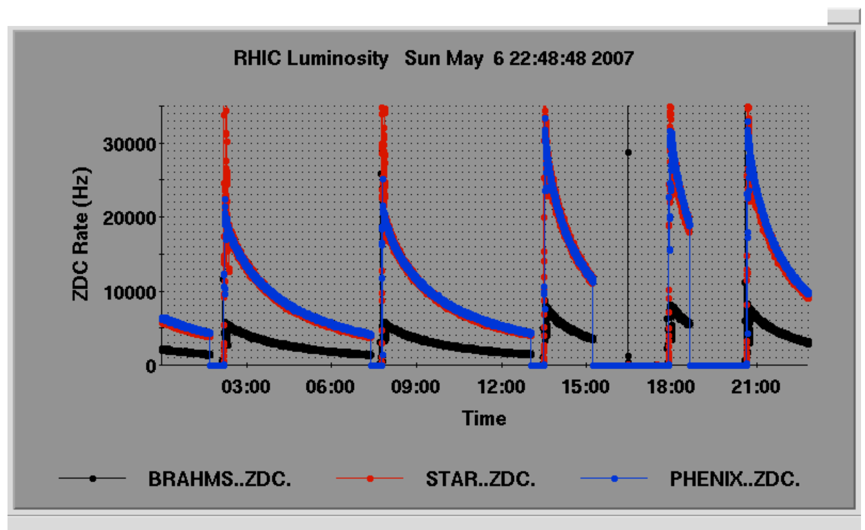
Measure of Performance

- In Colliders - Luminosity, L [cm^{-2} sec^{-1}]

$$\dot{N}_{events} = \sigma_{A \to B} \cdot L \qquad L = \frac{f_{coll} \cdot N_1 \cdot N_2}{4\pi\beta^* \varepsilon}$$

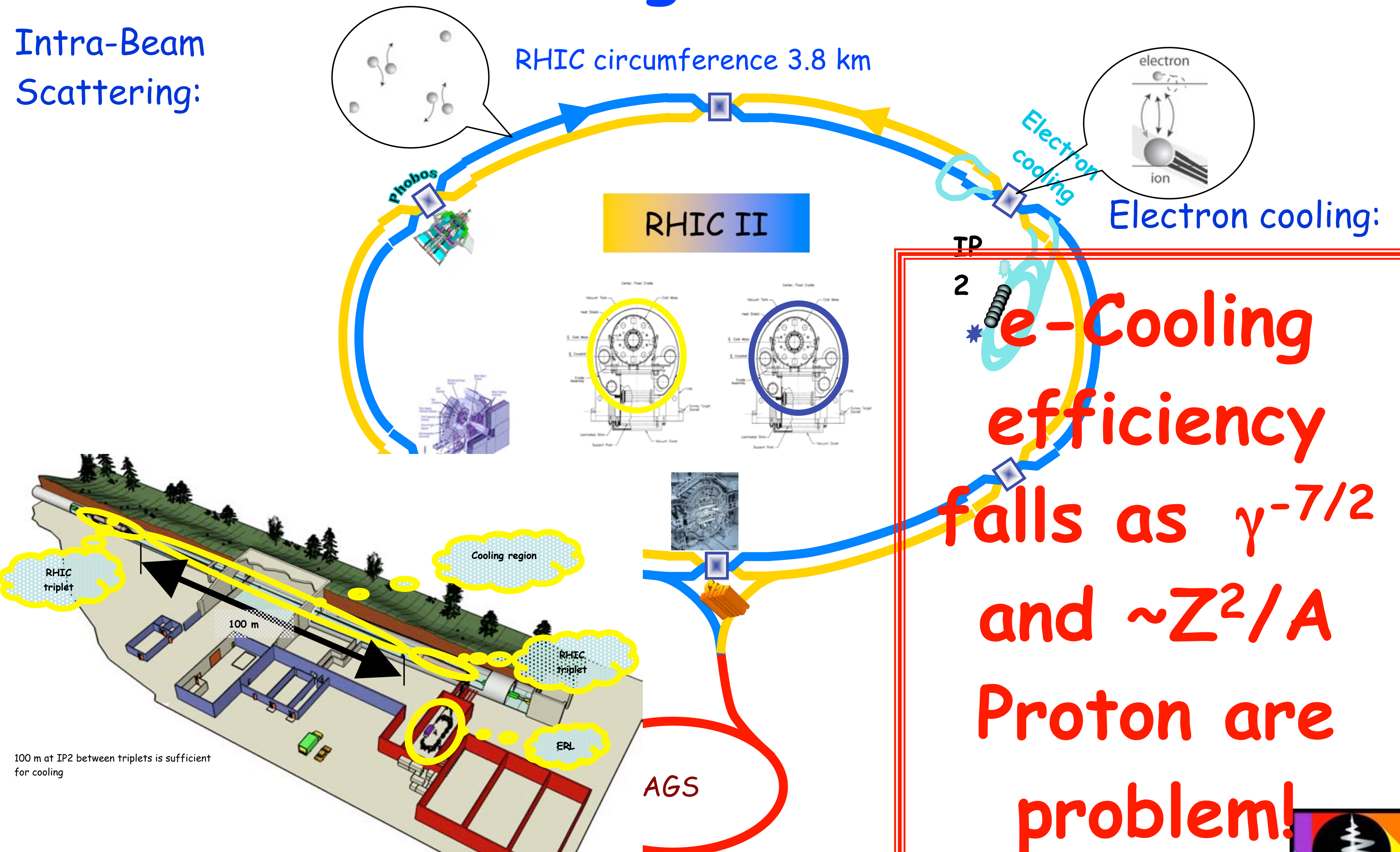
- Main sources of luminosity reduction - emittance growth and loss of particles





Electron cooling and IBS in RHIC

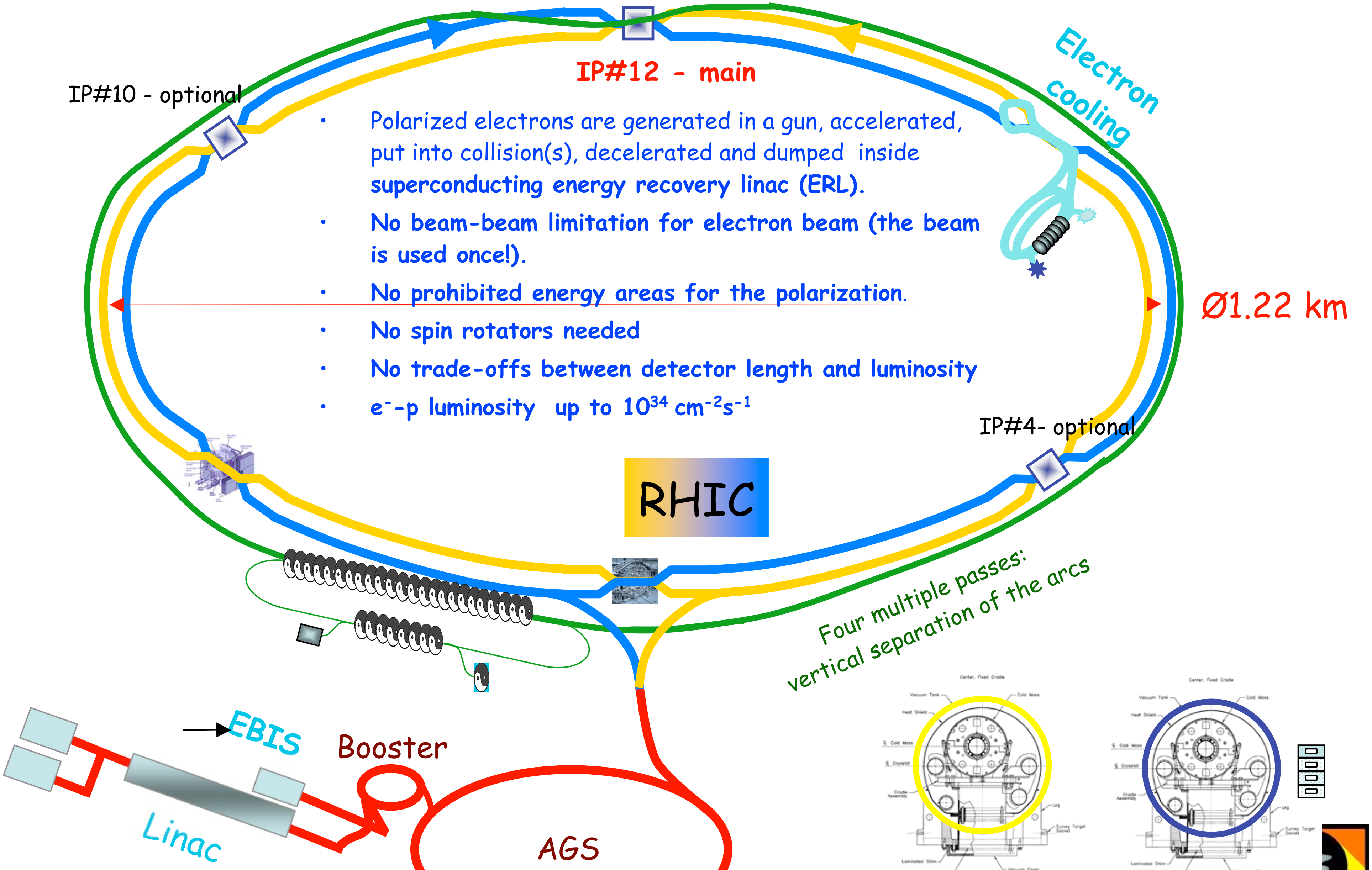
Intra-Beam Scattering:



Electron cooling:

e-Cooling efficiency falls as $\gamma^{-7/2}$ and $\sim Z^2/A$ Proton are problem!

Linac-Ring Design based on 5-20^{+} GeV ERL



- Polarized electrons are generated in a gun, accelerated, put into collision(s), decelerated and dumped inside **superconducting energy recovery linac (ERL).**
- **No beam-beam limitation for electron beam (the beam is used once!).**
- **No prohibited energy areas for the polarization.**
- **No spin rotators needed**
- **No trade-offs between detector length and luminosity**
- **e^--p luminosity up to 10^{34} cm^{-2}s^{-1}**

CERN – Large Hadron Collider (LHC)

Time	2007-
Circumference [km]	26.7
Energy [GeV]	7000 p 580000 Pb
Particles	p-p Pb-Pb
Peak luminosity [$10^{30}cm^{-2}s^{-1}$]	**10000 (design)**

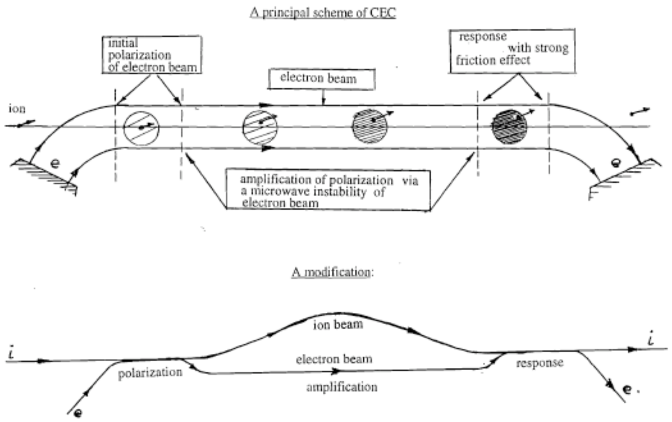
Cooling LHC - is it possible to even dream about?
It is just 10^{10} times harder that cooling antiprotons in Fermilab recycler; 10^{8} times harder than cooling Au ions in RHIC

FEL-07

History of idea:

coherent electron cooling was suggested by Yaroslav Derbenev about 26 years ago

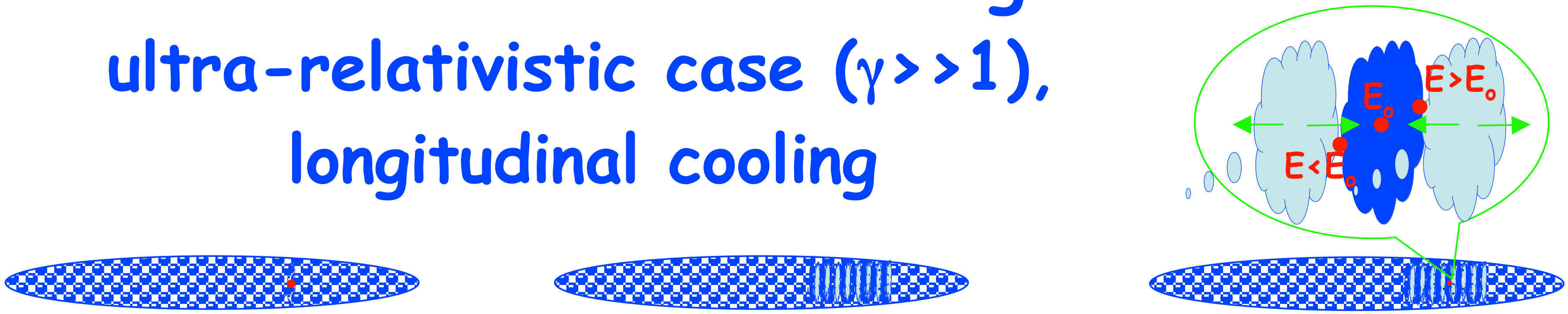
- Y.S. Derbenev, Proceedings of the 7th National Accelerator Conference, V. 1, p. 269, (Dubna, Oct. 1980)
- Coherent electron cooling, Ya. S. Derbenev, Randall Laboratory of Physics, University of Michigan, MI, USA, UM HE 91-28, August 7, 1991
- Ya.S.Derbenev, Electron-stochastic cooling, DESY , Hamburg, Germany, 1995 ……….



<u>Q: What changed in last 25 years?</u>

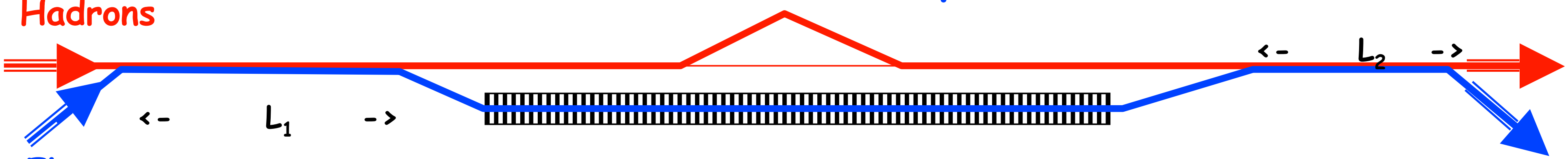
A: Accelerator technology caught up with the idea - high gain amplification at optical (μm and nm) wavelengths became reality

Coherent electron cooling: ultra-relativistic case ($\gamma>>1$), longitudinal cooling



Most versatile option

Hadrons



Electrons

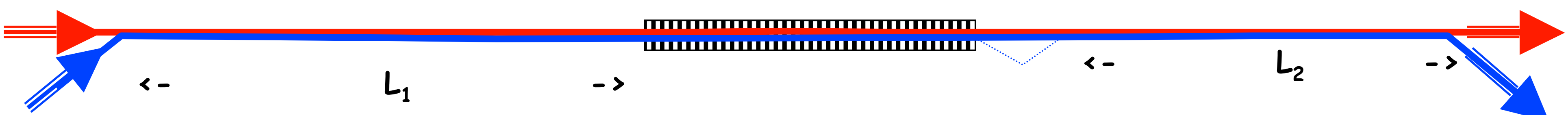
Modulator:region 1
about a quarter of
plasma oscillation

Longitudinal dispersion for hadrons

Kicker: region 2

Amplifier of the e-beam modulation via SASE FEL

Hadrons

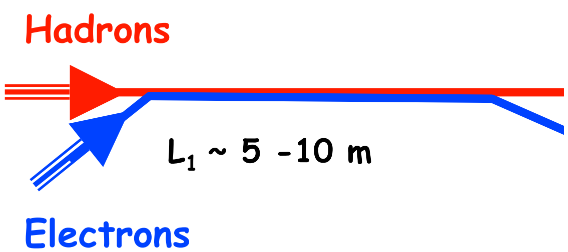


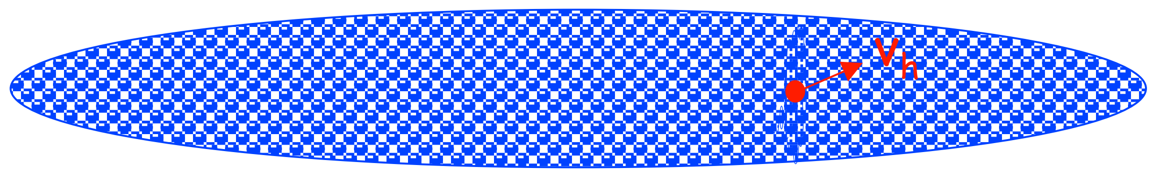
Electrons

Most economical option

Modulator: Interaction region 1
Length: about a quarter of plasma oscillation

$$\omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m_e}}$$





$$r_{//,\,lab} \propto \frac{1}{\gamma}\frac{\sigma_E}{E}\sqrt{\frac{\beta\varepsilon_n}{2}\frac{I_A}{I_e}} \qquad r_{//,lab}(.1\%) \propto 7\cdot 10^{-5}[m]/\gamma$$

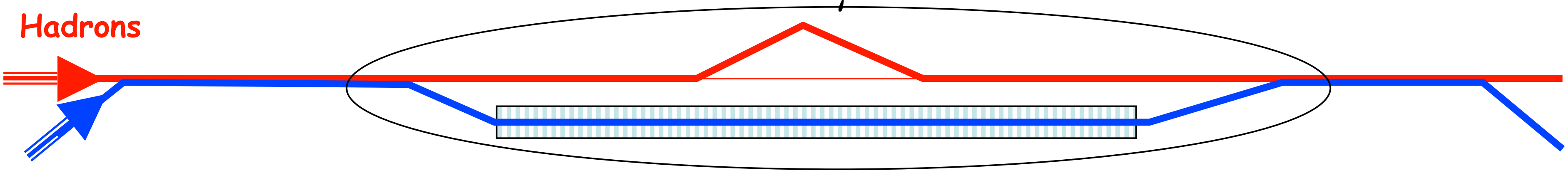
$$r_\perp \propto \varepsilon_n\sqrt{\frac{\gamma\cdot I_A}{2I_e}} \qquad r_\perp \sim 0.3mm$$

Each hadron generates modulation in the electron
Density with total charge of about minus charge of the hadron, Z

Longitudinal dispersion for hadrons, time of flight depends on its energy: $(T-T_o)\, v_o = -D\,(E-E_o)/E_o$

$$D = D_{free} + D_{chicane};\ D_{free} = \frac{L}{\gamma^2};\ \ D_{chicane} = l_{chicane} \cdot \theta^2$$

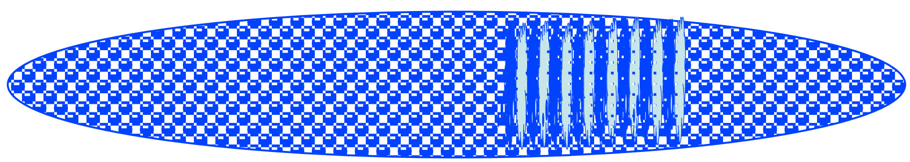
Hadrons



Electrons

Amplifier of the e-beam modulation- high gain FEL

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1 + a_w^2\right) \qquad L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt{3}} \qquad L_G = L_{Go}(1+\Lambda)$$



Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{gain}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength λ. Maximum gain for the electron density of HG FEL is ~ 10^3.

$$v_{group} = (c + 2v_{//})/3 = c\left(1 - \frac{1 + a_w^2}{3\gamma^2}\right) = c\left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right) = v_{hadrons} + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right)$$

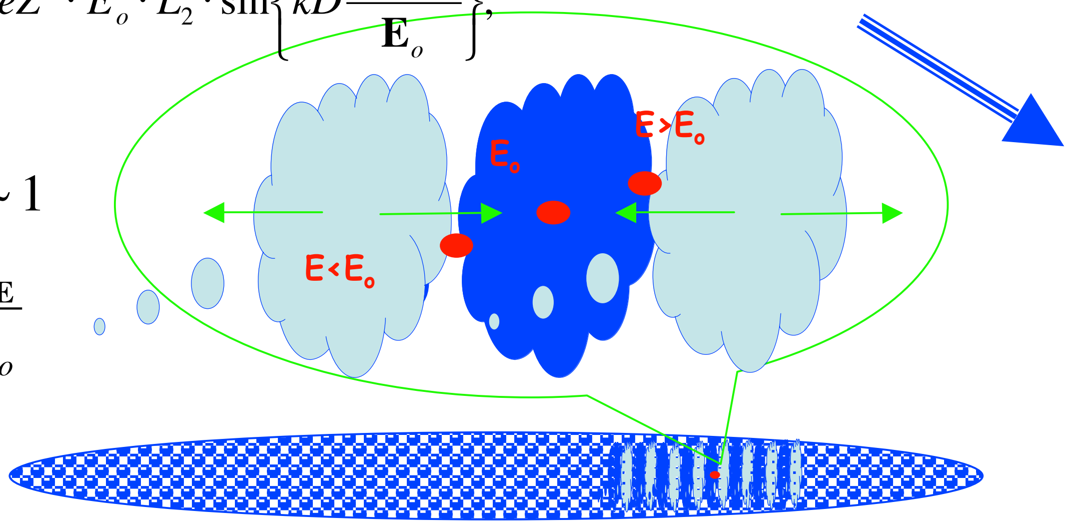
Kicker: Interaction region 2

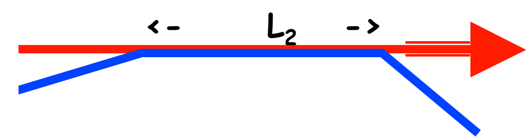
A hadron with central energy (E$_o$) phased with the hill where longitudinal electric field is zero, a hadron with higher energy (E>E$_o$) arrives earlier and is decelerated, while hadron with lower energy (E<E$_o$) arrives later and is accelerated by the collective field of electrons

$$\Delta \mathbf{E} = -eZ^2 \cdot E_o \cdot L_2 \cdot \sin\left\{ kD \frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o} \right\};$$

$$kD\sigma_\varepsilon \sim 1$$

$$\sigma_\varepsilon = \frac{\sigma_{\mathbf{E}}}{\mathbf{E}_o}$$





$$J_{CEC} = -\frac{\Delta \mathbf{E}}{\mathbf{E}-\mathbf{E}_o} \approx \frac{e \cdot E_o \cdot L_2}{\gamma_o m_p c^2 \cdot \sigma_\varepsilon} \cdot \frac{Z^2}{A}$$

Overlap effect

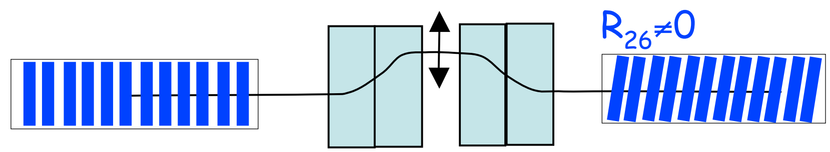
Electron bunches are usually much shorter that the hadron bunches and cooling time for the entire bunch is proportional to the bunch-lengths ratios

$$J_{bunch} = J_{CEC} \frac{\sigma_{\tau,e}}{\sigma_{\tau,h}}$$

Transverse cooling

- Transverse cooling can be obtained by using coupling with longitudinal motion via transverse dispersion
- Sharing of cooling decrements is similar to sum of decrements theorem for synchrotron radiation damping, I.e. decrement longitudinal cooling can be split into appropriate portions to cool both transversely and longitudinally: $J_s+J_h+J_v=J_{CEC}$

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the fronts of the charged planes



$$\delta z = -R_{26}\cdot x$$

$$\Delta \mathbf{E} = -eZ^2\cdot E_o\cdot L_2\cdot$$

$$\sin\left\{k\left(D\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o}+R_{16}x'-R_{26}x+R_{36}y'+R_{46}y\right)\right\};$$

$$\Delta x = -\eta\cdot eZ^2\cdot E_o\cdot L_2\cdot kR_{26}x+....$$

$$J_x(\max)\cong\frac{\eta\sigma_\varepsilon}{\sigma_x}J_{CEC}$$

FEL-07

Estimation of maximum force

- Assume that in 1/4 of plasma oscillation in region 1, at the end of which a clamp of electrons with charge -Ze is formed
- Assume that longitudinal extend of the electron clamp is well within $\lambda_o /2\pi$; in this case the amplitude of the charge density modulation is equal -e.
- Assume maximum gain in SASE FEL* is $G \sim 10^3$
- Assume that electron beam is wider than $2\gamma_o\lambda_o$- it allows using 1D field
- Length of the region 2 is equal to beta-function of hadron beam

Charge modulation can be estimated as -G*Ze; the cross-section of the electron beam is simple combination of transverse beta-function, normalized emittance and relativistic factor (frame independent):

$$A_\perp = 2\pi\beta_\perp\varepsilon_n/\gamma_o$$

i.e. the charge density in CM frame can be written as

$$\rho = \frac{k}{2\gamma_o}\frac{G\cdot Z\cdot e}{A}\cdot\sin(kz/2\gamma_o)$$

$$divE \cong kE_z/2\gamma_o = 4\pi\rho;$$

$$E_z = Z\cdot E_o\cdot\sin(kz/2\gamma_o); \qquad E_o = \frac{2G\cdot e}{\beta_\perp\varepsilon_n}\gamma_o$$

FEL-07

Estimation of maximum force - continued

$$A_\perp = 2\pi\beta_\perp\varepsilon_n / \gamma_o$$

Longitudinal electric field is the same in the lab and CM frames

CM frame

$$\rho = \frac{k}{2\gamma_o}\frac{G\cdot Z\cdot e}{A}\cdot\sin(kz/2\gamma_o)$$

$$divE \cong kE_z/2\gamma_o = 4\pi\rho;$$

$$E_z = Z\cdot E_o\cdot\sin(kz/2\gamma_o); \quad E_o = \frac{2G\cdot e}{\beta_\perp\varepsilon_n}\gamma_o$$

$$J_{SEC} = 2G\cdot\frac{r_p}{\sigma_\varepsilon\varepsilon_n}\cdot\frac{L_2}{\beta_\perp}\cdot\frac{Z^2}{A}$$

Note that damping decrement does not depend on the energy of particles !

Tevatron ? LHC ?

Effects of the surrounding particles

Each charged particle causes generation of an electric filed wave-packet proportional to its charge and synchronized with its initial position in the bunch

$$E_z = \sum_{i,hadrons} Z \cdot E_o(v_o t - z + z_j) \cdot \sin k(v_o t - z + z_i) - \sum_{j,electrons} E_o(v_o t - z + z_j) \cdot \sin k(v_o t - z + z_j)$$

Evolution of the RMS value: resembles stochastic cooling!

Best cooling rate achievable is ~ 1/Ñ, Ñ is effective number of hadrons in coherent sample ($N_c\lambda$); cooling "faster" will only

$$\frac{d\sigma_E^2}{dn} = -2\Delta \frac{kD}{\mathbf{E}_o}\sigma_E^2 + \frac{1}{2}\Delta^2 \tilde{N}$$

$$\Delta = eZ^2 \cdot L_2 \cdot E_o; \tilde{N} = \tilde{N}_h + \tilde{N}_e / Z^2$$

$$\frac{\sigma_E^2}{\mathbf{E}^2{}_o} = \frac{1}{4kD} \cdot \frac{\Delta}{\mathbf{E}_o} \cdot \tilde{N}$$

$$J_{CEC} = \frac{\Delta}{2\sigma_E} = \frac{2}{\tilde{N}}\left(kD\sigma_\varepsilon\right) \sim \frac{1}{\tilde{N}}$$

FEL-07

Cooling of hadron beams

Machine	Species	Energy GeV/n	Synchrotron radiation, hrs	Electron cooling, hrs	CEC, hrs
RHIC	Au	100	20,961 ∞	~ 1	0.03
RHIC	protons	250	40,246 ∞	> 30	0.8
LHC	protons	450	48,489 ∞	> 1,600	0.95
LHC	protons	7,000	13/26	∞ ∞	< 2

Test using BNL R&D ERL: Au ions in RHIC with 40 GeV/n, L_{cooler} < 15 m

N per bunch	$2\ 10^9$	Z, A	79, 197
Energy Au, GeV/n	40	γ	42.63
RMS bunch length, nsec	2.5	Relative energy spread	0.1%
Emittance norm, μm	2.5	$\beta_\perp$, m	10
Energy e^-, MeV	21.79	Peak current, A	50
Charge per bunch, nC	5	Bunch length, nsec	0.1
Emittance norm, μm	3	Relative energy spread	0.1%
$\beta_\perp$, m	10	L_1 (lab frame) ,m	4.8
ω_{pe}, CM, Hz	$4.18\ 10^9$	Number of plasma oscillations	0.25
$\lambda_{D\perp}$, μm	112	$\lambda_{D\|\|}$, μm	2.6
λ_{FEL}, μm	18	λ_w, cm	5
a_w	0.555	L_{Go}, m	0.36
Amplitude gain =100, L_w , m	5.4 (-> 6.5)	L_{G3D}, m	0.50
L_2 (lab frame) ,m	3	Cooling time, local, minimum	0.26 minutes
N_{turns}, Ñ, 5% BW	$1.2\ 10^5$> $1\ 10^6$	Cooling time, beam, min	8.1 minutes

Au ions in RHIC with 100 GeV/n, L_{cooler} ~ 20 m

N per bunch	$2\ 10^9$	Z, A	79, 197
Energy Au, GeV/n	100	γ	106.58
RMS bunch length, nsec	1	Relative energy spread	0.1%
Emittance norm, μm	2.5	$\beta_\perp$, m	5
Energy e^-, MeV	54.5	Peak current, A	50
Charge per bunch, nC	5	Bunch length, nsec	0.1
Emittance norm, μm	3	Relative energy spread	0.1%
$\beta_\perp$, m	10	L_1 (lab frame) ,m	8.5
ω_{pe}, CM, Hz	$5.9\ 10^9$	Number of plasma oscillations	0.25
$\lambda_{D\perp}$, μm	78	$\lambda_{D\|\|}$, μm	0.75
λ_{FEL}, μm	**3**	**λ_w, cm**	**5**
a_w	**0.603**	**L_{Go}, m**	**0.5**
Amplitude gain =200, L_w , m	**8.11 (-> 9)**	**L_{G3D}, m**	**0.77**
L_2 (lab frame) ,m	5	**Cooling time, local, minimum**	**0.08 minutes**
$N_{min\ turns}$ or Ñ in 5% BW	$6\ 10^5$> $2\ 10^5$	**Cooling time, beam, min**	**1.93 minutes**

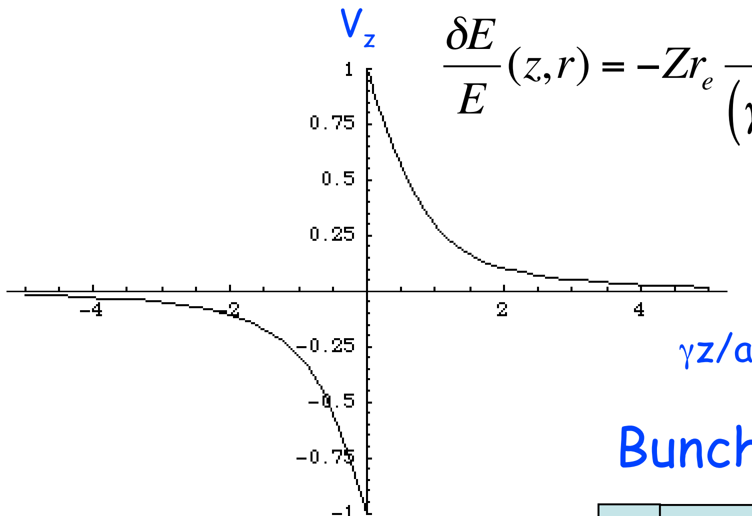
250 GeV polarized protons in RHIC, L_{cooler} ~

N per bunch	$2\ 10^{11}$	Z, A	1, 1
Energy Au, GeV/n	250	γ	266.45
RMS bunch length, nsec	1	Relative energy spread	0.04%
Emittance norm, μm	2.5	$\beta_{\perp}$, m	10
Energy e^-, MeV	136.16	Peak current, A	100
Charge per bunch, nC	5	Bunch length, nsec	0.2
Emittance norm, μm	3	Relative energy spread	0.04%
$\beta_{\perp}$, m	10	L_1 (lab frame) ,m	30
ω_{pe}, CM, Hz	$4.19\ 10^9$	Number of plasma oscillations	0.25
$\lambda_{D\perp}$, μm	1004	$\lambda_{D\|\|}$, μm	0.17
λ_{FEL}, μm	**0.5**	**λ_w, cm**	**5**
a_w	**0.648**	**L_{Go}, m**	**0.87**
Amplitude gain =100, L_w , m	**13 (-> 15)**	**L_{G3D}, m**	**1.22**
L_2 (lab frame) ,m	10	**Cooling time, local, min**	**1.96**
$N_{min\ turns}$ or $\tilde{N}$ in 10% BW	$6.7\ 10^6 > 5.9\ 10^6$	**Cooling time, beam, min**	**49.2**

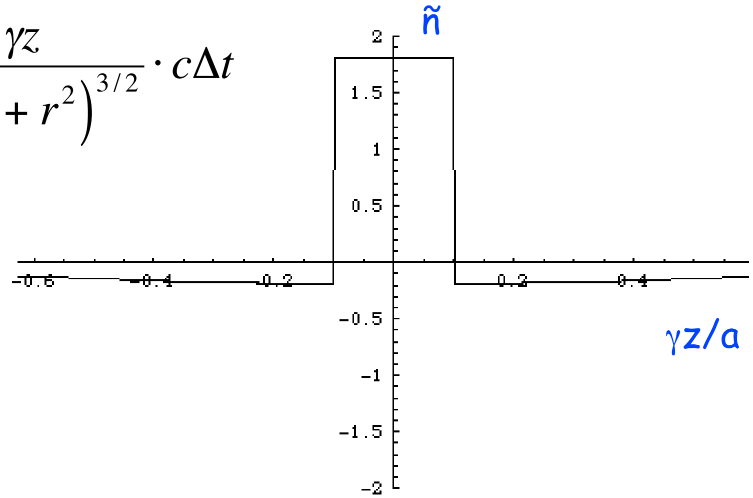
7 TeV protons in LHC: 430 m !

N per bunch	$1.15\ 10^{11}$	Z, A	1, 1
Energy Au, GeV/n	7000	γ	7460
RMS bunch length, nsec	0.25	Relative energy spread	0.0113%
Emittance norm, μm	3.5	$\beta_\perp$, m	50
Energy e^-, MeV	3,812	Peak current, A	1000
Charge per bunch, nC	5	Bunch length, nsec	0.05
Emittance norm, μm	3	Relative energy spread	0.1%
$\beta_\perp$, m	50	L_1 (lab frame) ,m	**300 !!!!!!**
ω_{pe}, CM, Hz	$4.19\ 10^9$	Number of plasma oscillations	0.125
$\lambda_{D\perp}$, mm	3.7	$\lambda_{D\|\|}$, μm	0.17
λ_{FEL}, μm	**0.01**	**λ_w, cm**	**5**
a_w	**4.61**	**L_{Go}, m**	**1**
Amplitude gain =500, L_w , m	**24 (-> 30)**	**L_{G3D}, m**	**1.86**
L_2 (lab frame) ,m	**50**	**Cooling time, local, min**	**1.4 minutes**
$N_{min\ turns}$ or Ñ in 10% BW	$8\ 10^6 \gg 2\ 10^6$	**Cooling time, beam**	**90 minutes**

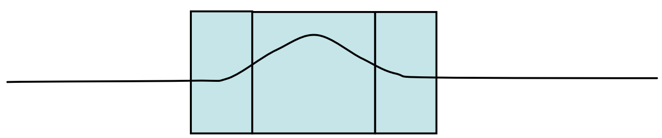
Velocity map & buncher

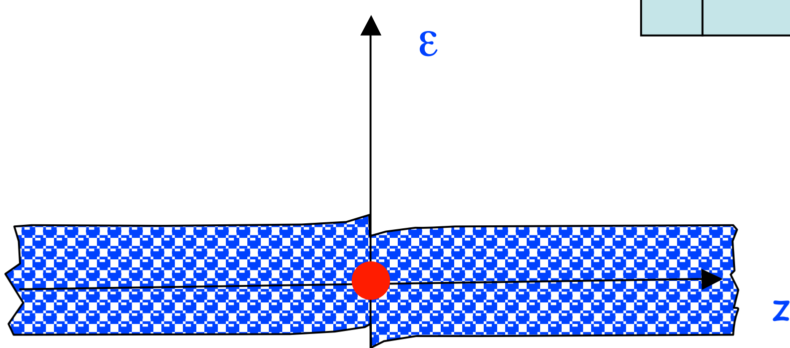


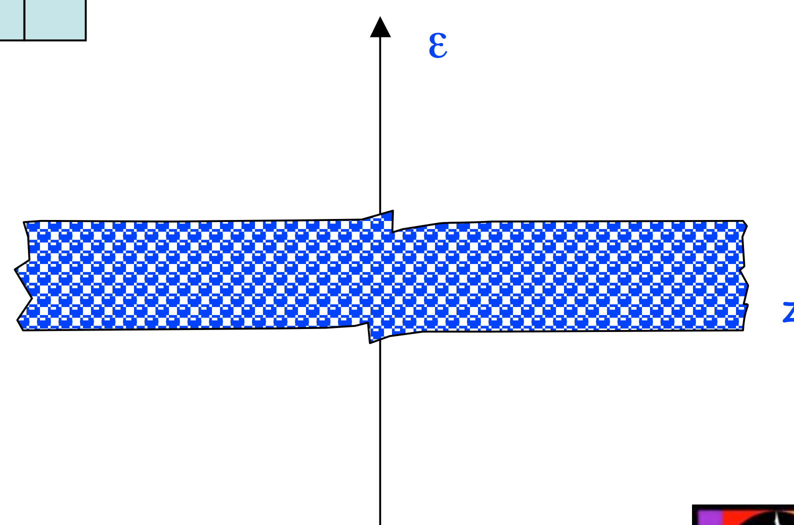
$$\frac{\delta E}{E}(z,r) = -Zr_e \frac{\gamma z}{\left(\gamma^2 z^2 + r^2\right)^{3/2}} \cdot c\Delta t$$



Buncher







$$\left\langle \frac{\delta E}{E} \right\rangle \cong -2Z \frac{r_e}{a^2} \cdot \frac{L_{pol}}{\gamma} \cdot \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2/\gamma^2 + z^2}} \right)$$

7 TeV protons in LHC: advanced ERL/CEC

N per bunch	$1.15\ 10^{11}$	Z, A	1, 1
Energy Au, GeV/n	7000	γ	7460
RMS bunch length, nsec	0.25	Relative energy spread	0.0113%
Emittance norm, μm	3.5	$\beta_{\perp}$, m	50
Energy e^-, MeV	3,812	Peak current, A	100
Charge per bunch train, nC	**10**	Bunch length, nsec	0.05
Emittance norm, μm	4	Relative energy spread	0.01%
$\beta_{\perp}$, m	50	L_1 (lab frame) ,m	**~100 (buncher)**
ω_{pe}, CM, Hz	$4.19\ 10^9$	Number of plasma oscillations	0.125
$\lambda_{D\perp}$, mm	3.7	$\lambda_{D\|\|}$, μm	0.17
λ_{FEL}, μm	**0.01**	**λ_w, cm**	**5**
a_w	**4.61**	**L_{Go}, m**	**2.74**
Amplitude gain =500, L_w , m	**21.1 (-> 10)**	**L_{G3D}, m**	**3.39**
L_2 (lab frame) ,m	50	**Cooling time, local, min**	**~3 minutes**
$N_{min\ turns}$ or Ñ in 10% BW	$2\ 10^6 \gg 0.4\ 10^6$	**Cooling time, beam**	**~20 minutes**

Conclusions

- Coherent electron cooling is very promising method for high energy hadron colliders
- It takes full advantage of high gain FELs based on high brightness ERLs
- Proof of principle experiment of cooling Au ions in RHIC at ~ 40 GeV/n is feasible with existing R&D ERL (oper. starts in 2009)
- Cooling 100 GeV/n ions and 250 GeV protons in RHIC seems to be straight forward
- Cooling protons in LCH at 7 TeV seems to be possible, but may require slightly more elaborate scheme (buncher, etc.)
- Question of possible shot-noise suppression in electron beam is very interesting and should be further studied

FEL formulae: following Ming Xie

$$L_G = L_{Go}(1+\Lambda)$$

$$L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt{3}}$$

$$\rho = \frac{1}{\gamma}\sqrt[3]{\frac{I_e}{16 I_A}\left(\frac{K_w JJ}{k_w \sigma_r}\right)^2};$$

$$JJ = \left[\frac{1}{J_o(\xi) - J_o(\xi)}\right]; \; \xi = K_w^{\,2} / \left(4 + 2K_w^{\,2}\right)$$

$$\eta_d = \frac{1}{2\sqrt{3}\hat{\sigma}_x^2} = \frac{L_{G0}}{2k_1\sigma_x^2} \text{ (diffraction parameter)},$$

$$\eta_\varepsilon = \frac{2}{\sqrt{3}}\hat{k}_\beta^2\hat{\sigma}_x^2$$

$$= k_\beta L_{G0}\frac{\varepsilon}{\lambda_1/(4\pi)} \text{ (angular spread parameter)},$$

$$\eta_\gamma = \frac{\hat{\sigma}_\eta}{\sqrt{3}} = 4\pi\frac{L_{G0}}{\lambda_u}\sigma_\eta \text{ (energy spread parameter)},$$

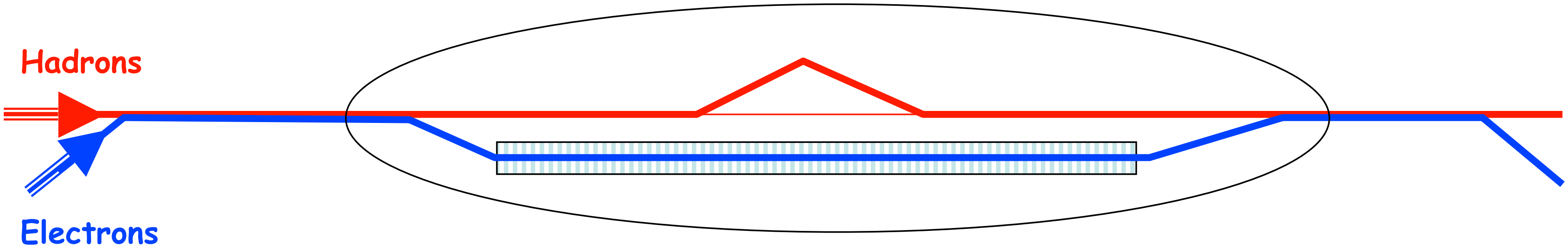
$$\Lambda = a_1\eta_d^{a_2} + a_3\eta_\varepsilon^{a_4} + a_5\eta_\gamma^{a_6} + a_7\eta_\varepsilon^{a_8}\eta_\gamma^{a_9} + a_{10}\eta_d^{a_{11}}\eta_\gamma^{a_{12}}$$
$$+ a_{13}\eta_d^{a_{14}}\eta_\varepsilon^{a_{15}} + a_{16}\eta_d^{a_{17}}\eta_\varepsilon^{a_{18}}\eta_\gamma^{a_{19}}, \quad (80)$$

where the fitting coefficients are

$$a_1 = 0.45, \quad a_2 = 0.57, \quad a_3 = 0.55, \quad a_4 =$$
$$a_5 = 3, \quad a_6 = 2, \quad a_7 = 0.35, \quad a_8 = 2.9,$$
$$a_9 = 2.4, \quad a_{10} = 51, \quad a_{11} = 0.95, \quad a_{12} = 3,$$
$$a_{13} = 5.4, \quad a_{14} = 0.7, \quad a_{15} = 1.9, \quad a_{16} = 1140,$$
$$a_{17} = 2.2, \quad a_{18} = 2.9, \quad a_{19} = 3.2. \quad (81)$$

Longitudinal dispersion for hadrons, time of flight depends on its energy: $(T-T_o)\, v_o = -D\, (E-E_o)/E_o$

$$D = D_{free} + D_{chicane};\ D_{free} = \frac{L}{\gamma^2};\ \ D_{chicane} = l_{chicane} \cdot \theta^2$$

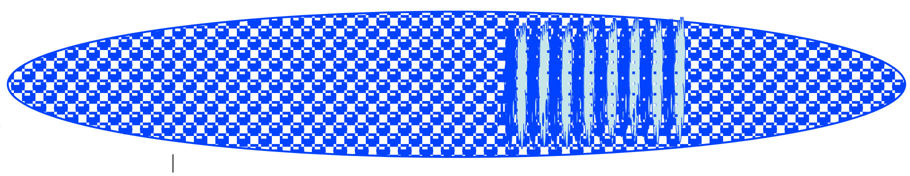


$$\rho = \left(\frac{a_u}{4\gamma}\frac{\Omega_p}{\omega_u}\right)^{2/3};$$

Amplifier of the e-beam modulation- SASE FEL

$$L_{G0} = \frac{\lambda_u}{4\pi\sqrt{3}\rho}.$$

$$L_G = \frac{\lambda_u}{8(\mathrm{Im}\,\mu_0)\pi\rho} \equiv L_{G0}(1+\Lambda),$$



$$\Lambda = a_1\eta_d^{a_2} + a_3\eta_\varepsilon^{a_4} + a_5\eta_\gamma^{a_6} + a_7\eta_\varepsilon^{a_8}\eta_\gamma^{a_9} + a_{10}\eta_d^{a_{11}}\eta_\gamma^{a_{12}} + a_{13}\eta_d^{a_{14}}\eta_\varepsilon^{a_{15}} + a_{16}\eta_d^{a_{17}}\eta_\varepsilon^{a_{18}}\eta_\gamma^{a_{19}}, \quad (80)$$

where the fitting coefficients are

$$a_1 = 0.45,\quad a_2 = 0.57,\quad a_3 = 0.55,\quad a_4 = 1.6,\quad a_5 = 3,\quad a_6 = 2,\quad a_7 = 0.35,\quad a_8 = 2.9,\quad a_9 = 2.4,\quad a_{10} = 51,\quad a_{11} = 0.95,\quad a_{12} = 3,\quad a_{13} = 5.4,\quad a_{14} = 0.7,\quad a_{15} = 1.9,\quad a_{16} = 1140,\quad a_{17} = 2.2,\quad a_{18} = 2.9,\quad a_{19} = 3.2. \quad (81)$$

Electron density modulation is amplified in SASE FEL and made into a train with duration of N alternating hills (high density) and valleys (low density) with period of FEL wavelength $\lambda_o = \lambda_w(1+a_w^2)/2\gamma^2$; $N_c \sim L_{gain}/\lambda_w$
Maximum gain of the electron density of SASE FEL is ~ 10^3.

$$\lambda = \frac{\lambda_w}{2\gamma_{//}^2};\quad \gamma_{//}^2 = \gamma^2/(1+a_u^2)$$

$$v_{group} = (c + 2v_{//})/3 = c\left(1 - \frac{1}{3\gamma_{//}^2}\right) = c\left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}\left(1 - a_u^2\right)$$

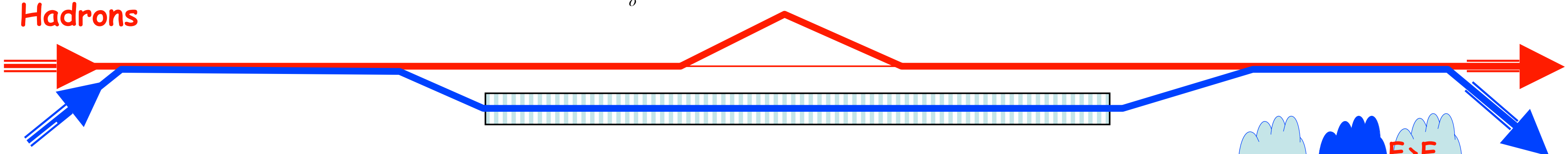
Sketch of coherent electron cooling (CEC): ultra-relativistic case ($\gamma >> 1$); longitudinal cooling - continued

Let's assume that longitudinal electric filed exited by the amplified modulation in the electron beam is (E(z) is a smooth envelope of the wave-packet and particle With central (equilibrium) energy arrives at z-vt=0 : (note that the field is normalized for unit charge!)

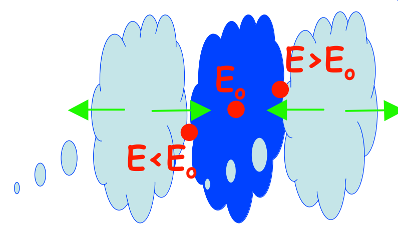
$$E_z = Z \cdot E_o(v_o t - z) \cdot \sin k(v_o t - z); \quad k = \frac{2\pi}{\lambda_o}$$

In this case the energy change in the region 2 is given by (note that L2 should be smaller than 1/4 of plasma oscillation)

$$\Delta \mathbf{E} = -eZ^2 \cdot E_o \cdot L_2 \cdot \sin\left\{ kD \frac{\mathbf{E} - \mathbf{E}_o}{\mathbf{E}_o} \right\};$$



Selecting $kD\sigma_{\mathrm{E}}/\mathbf{E}_o \sim 1$ gives cooling for $\{-\pi\sigma_E, \pi\sigma_E\}$ energy range, and for small deviation the decrement of the cooling will be (γ is the relativistic factory of hadron, m_p is the rest mass of the proton and A is the atomic number of hadron (ion),)



$$J_{SEC} = -\frac{1}{2}\frac{\Delta \mathbf{E}}{\mathbf{E} - \mathbf{E}_o} \approx \frac{1}{2}\frac{eZ^2 \cdot E_o \cdot L_2}{\sigma_{\mathrm{E}}} = \frac{1}{2}\frac{e \cdot E_o \cdot L_2}{\gamma_o m_p c^2 \cdot \sigma_\varepsilon} \cdot \frac{Z^2}{A}; \quad \sigma_\varepsilon = \frac{\sigma_{\mathrm{E}}}{\mathbf{E}_o}$$

Note1 : 1/2 in JCEC comes from the fact there are synchrotron oscillations

Note2 : cooling strength of the CEC is proportional to Z^2/A, i.e. is strongly enhanced for ions (Z^2/A = 31.7 for $^{79}Au_{197}$)

Sketch of coherent electron cooling (CEC): ultra-relativistic case ($\gamma \gg 1$); longitudinal cooling - continued

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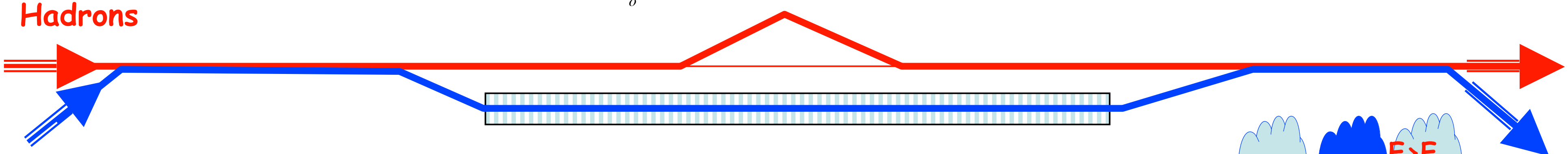

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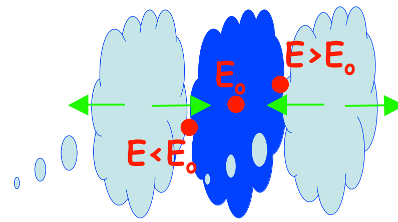

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Estimations for maximum a RHIC case: stochastic limits

One can assume that SASE FEL bandwidth is one of the best demonstrated: VISA at ATF at BNL had gain length of ~ 15 cm with wiggler period ~ 2 cm. Hence 10% for the SASE FEL gain bandwidth is about maximum achievable - let's use it. Scaling is trivial anyway…. Note that length of SASE FEL is only few meters long!

In addition to regular kicks from self-exited field hadron will experience random fields excited by surrounding hadrons and also electrons, with total number of effective particles: N_h is number of hadrons with charge in the Z in the bunch:

$$\tilde{N} = N_h \frac{N_c \lambda_o}{\sqrt{2\pi}\sigma_{zh}} + \frac{1}{Z} N_e \frac{N_c \lambda_o}{\sqrt{2\pi}\sigma_{ze}}$$

For RHIC case these numbers are for the above electron beam, 2E11 protons per bunch of 2E9 Au ions per bunch are:
(note that contribution form electrons is significant!)

$$\tilde{N} = \left\{ \frac{2.7 \cdot 10^7 - protons}{3.3 \cdot 10^5 - Au\ \ ions} \right\}$$

This is known limit for stochastic cooling that $\sim 1/\tilde{N}$ is the best achievable cooling rate (otherwise beam heating exceeds the cooling). This gives 350 seconds for protons and 4 second for Au in RHIC. We will need to split the damping 3-ways.

It means also that SASE FEL Gain can be ~ 100 (~ 5 gain lengths), which removes completely question of saturation.

Decay

$$\frac{dp}{dt} = -\alpha p \quad \Rightarrow \quad p = p_o e^{-\alpha t}$$

Damped Oscillator

$$\frac{dx}{dt} = p; \quad \frac{dp}{dt} = -\omega^2 x - \alpha p$$

$$\Downarrow$$

$$\frac{d^2 p}{dt^2} = -\omega^2 p - \alpha \frac{dp}{dt}$$

$$\boxed{\begin{gathered} p = p_o e^{-\chi t} \cos(\Omega t + \varphi) \\ \chi = \alpha/2; \quad \Omega^2 = \omega^2 - \chi^2 \end{gathered}}$$

Diffusion for a damped oscillator

$$X = \begin{bmatrix} x \\ \hline p \end{bmatrix};\; X_{n+1} = e^{-\chi} \cdot M \cdot X_n + \delta X; \quad M \cdot Y = Ye^{i\mu}; \quad \delta X = \begin{bmatrix} \tilde{x} \\ \tilde{p} \end{bmatrix};\; Y = \begin{bmatrix} w \\ w' + i/w \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ \hline p \end{bmatrix} = \frac{1}{2}\left\{aY + a^* Y^*\right\} \equiv \mathrm{Re}\left\{aY\right\};\; Y^{*T} SY = -2i \Rightarrow a = iY^{*T} SX$$

$$\left|a_{n+1}\right|^2 = \left|Y^{*T} SX\right|^2 = \left|a_n e^{i\varphi} + Y^{*T} S\delta X_{rand}\right|^2 =$$

$$e^{-2\chi}\left|a_n\right|^2 + 2e^{-\chi}\,\mathrm{Re}\, a_n e^{i\varphi}\left(Y^T S\delta X_{rand}\right) + \left|Y^{*T} S\delta X_{rand}\right|^2$$

Diffusion for a damped oscillator - continued

$$|a_{n+1}|^2 - |a_n|^2 = (e^{-2\chi} - 1)|a_n|^2 + 2e^{-\chi}\,\mathrm{Re}\, a_n e^{i\varphi}\left(Y^T S\delta X_{rand}\right) + \left|Y^{*T} S\delta X_{rand}\right|^2$$

$$\left\langle 2\,\mathrm{Re}\, a_n e^{i\varphi}\left(Y^T S\delta X_{rand}\right)\right\rangle = 0;$$

$$Y^{*T} S\delta X_{rand} = \left[w, w' - i/w\right]\cdot\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\cdot\begin{bmatrix}\tilde{x}\\ \tilde{p}\end{bmatrix} = w\tilde{p} - \tilde{x}w' + i\tilde{x}/w;$$

$$\left\langle \Delta|a|^2\right\rangle = -2\chi\left\langle |a|^2\right\rangle + \left\langle \delta a_{rand}{}^2\right\rangle; \quad \left\langle \delta a_{rand}{}^2\right\rangle = \left\langle \left(w\tilde{p} - \tilde{x}w'\right)^2 + \frac{\tilde{x}^2}{w^2}\right\rangle$$

Stationary values

$$\left\langle \Delta|a|^2\right\rangle = 0 \;\Rightarrow \sigma_a^2 = \left\langle |a|^2\right\rangle = \frac{\left\langle \delta a_{rand}{}^2\right\rangle}{2\chi};$$

$$\sigma_x^2 = \left\langle x(s)^2\right\rangle = w(s)^2\,\frac{\left\langle \left(w\tilde{p} - \tilde{x}w'\right)^2 + \tilde{x}^2/w^2\right\rangle}{2\chi}$$

$$\sigma_p^2 = \left\langle p(s)^2\right\rangle = \left(w'(s)^2 + 1/w(s)^2\right)\frac{\left\langle \left(w\tilde{p} - \tilde{x}w'\right)^2 + \tilde{x}^2/w^2\right\rangle}{2\chi}$$

Synchrotron oscillations (or standard oscillator)

$$Y = \begin{bmatrix} 1/\sqrt{\Omega} \\ i\sqrt{\Omega} \end{bmatrix}; \quad \tilde{t} = \delta t = 0 \qquad\qquad \left\langle \delta a_{rand}{}^{2} \right\rangle = \Omega \left\langle \tilde{\varepsilon}^{2} \right\rangle$$

Stationary values

$$\left\langle \Delta |a|^{2} \right\rangle = 0 \;\; \Rightarrow \sigma_a^2 = \left\langle |a|^2 \right\rangle = \frac{\Omega \left\langle \tilde{\varepsilon}^2 \right\rangle}{2\chi};$$

$$\sigma_\tau^2 = \left\langle \tau^2 \right\rangle = \Omega^2 \frac{\left\langle \tilde{\varepsilon}^2 \right\rangle}{2\chi}; \;\; \sigma_\varepsilon^2 = \left\langle \varepsilon^2 \right\rangle = \frac{\left\langle \tilde{\varepsilon}^2 \right\rangle}{2\chi}$$

Estimations for maximum a RHIC case: lab and CM systems as needed

Hadrons				
E/n	2.509878E+11	eV		
f rev	7.81963E+04	Hz		
F beam	9.38355E+06			
γ	267.5		**250**	GeV
Emmitance, m rad	9.34579E-09	m		
β	**6**	m		
Area	3.52328E-07	m^2		
Z	**1**			
A	**1**			
$\sigma E/E$	1.00E-03			
σ_E	2.51E+08	eV		

Dynamic Debye length (z-direction) for protons is 10 times monger that that of electron (relative energy spread of protons is 10x of electrons), but still it is 2 10^{-7} m, I.e. much shorter than the FEL wavelength of 1 micron.

Electrons		
Energy	1.367E+08	eV
Charge	1.000E+01	nC
Bunch length	0.1	nsec
Peak current	1.000E+02	A
Emittance, norm	3.000E-06	m rad
β	**3**	m
σ	1.83E-04	m
Area	2.114E-07	m^2
Area	**2.114E-03**	**cm^2**
n_e, density CM	3.682E+16	m^{-3}
ωe, CM	1.08246E+10	Hz
λe, CM	1.74015E-01	m
λe, lab frame	46.5	m
$\lambda/4$, lab frame	11.6	m
v_t	4.90323E+06	m/sec
r_t	**7.115E-04**	m
σ_ε	1.00000E-04	
v_//	2.99792E+04	m/sec
rD_//, CM	**4.35E-06**	m
rD_//, Lab frame	**1.63E-08**	m
Wavelength	1.00E-06	m

Phase advance	9.42E-01	
Field	2.21E+01	V/m
Length of interaction	**6**	m
ΔE	1.32E+02	eV
N_turns	1.90E+06	
Time	**24.25**	**sec**
	0.40	**mins**
	0.001	**hours**

Amplification	**1.00E+03**	
Charge	4.80E-07	ESU
Density	2.27E-04	ESU/cm
Field	2.86E-03	Gs
	8.57E-01	V/cm
	8.57E+01	V/m
Longgitudinal dispersion	1.50E-04	m
length of chicane	10	m
angle	0.003872983	rad

i.e. damping time for CEC is in seconds for protons and in sub-seconds for Au ions

Effects of the surrounding particles

Each charged particle causes generation of a electric filed wave-packet proportional to its charge and synchronized with its initial position in the bunch

constant

$$E_z = \sum_{i,hadrons} Z \cdot E_o(v_o t - z + z_j) \cdot \sin k(v_o t - z + z_i) - \sum_{j,electrons} E_o(v_o t - z + z_j) \cdot \sin k(v_o t - z + z_j)$$

$$\boxed{\Delta \mathbf{E}_k \equiv \frac{d(\mathbf{E} - \mathbf{E}_o)}{dn} = -eZ \cdot L_2 \cdot \left\{ Z \sum_{k,hadrons} E_o(z_i - z_k + \xi) \cdot \sin k(z_i - z_k + \xi) - \sum_{j,electrons} E_o(z_j - z_k + \xi) \cdot \sin k(z_j - z_k + \xi) - \right\}; \quad \xi = D\frac{\mathbf{E} - \mathbf{E}_o}{\mathbf{E}_o} + v_o t - z + z_k}$$

Standard way of treating it is to look on the evolution of the RMS value:

$$\frac{d\left\langle (\mathbf{E}_k - \mathbf{E}_o)^2 \right\rangle}{dn} = \left\langle 2\Delta \mathbf{E}_k (\mathbf{E}_k - \mathbf{E}_o) + \Delta \mathbf{E}_k^{\;2} \right\rangle; \quad \xi = \xi_o + D\frac{\mathbf{E}_k - \mathbf{E}_o}{\mathbf{E}_o}$$

$$\left\langle \Delta \mathbf{E}_k (\mathbf{E}_k - \mathbf{E}_o) \right\rangle = -eZ^2 \cdot L_2 \cdot \left\langle (\mathbf{E}_k - \mathbf{E}_o) E_o(\xi) \cdot \sin k(\xi_o) \right\rangle \cong -eZ^2 \cdot L_2 \cdot \frac{kD}{\mathbf{E}_o} \cdot E_o \left\langle (\mathbf{E}_k - \mathbf{E}_o)^2 \right\rangle \cdot \cos k\xi_o \quad \Rightarrow \cos k\xi_o = 1$$

$$\left\langle \Delta \mathbf{E}_k^{\;2} \right\rangle = \left\langle \sum_{hadrons} \left\{ eZ^2 \cdot L_2 \cdot E_o(\xi + z_k - z_i) \right\}^2 \cdot \sin^2 k(z_i - z_k + \xi) + \sum_{electrons} \left\{ eZ^2 \cdot L_2 \cdot E_o(\xi + z_k - z_j) \right\}^2 \cdot \sin^2 k(z_j - z_k + \xi) \right\rangle =$$

$$\frac{1}{2}\left\langle (eZ^2 \cdot L_2)^2 \sum_{hadrons} E_o(\xi + z_k - z_i)^2 + (eZ^2 \cdot L_2)^2 \sum_{electrons} E_o(\xi + z_k - z_j)^2 \right\rangle = \frac{(eZ^2 \cdot L_2 \cdot E_o)^2}{2} \left\{ Z^2 \cdot \tilde{N}_h + \tilde{N}_e \right\}$$

$$\tilde{N}_h = \sum_{hadrons} E_o(\xi + z_k - z_i)^2 / E_o^{\;2}; \tilde{N}_e = \sum_{electrons} E_o(\xi + z_k - z_j)^2 / E_o^{\;2};$$

$$\frac{d\sigma_E^2}{dn} = -2\Delta \frac{D}{\mathbf{E}_o}\sigma_E^2 + \frac{1}{2}\Delta^2(\tilde{N}_h + \tilde{N}_e / Z^2);$$

$$\Delta = eZ^2 \cdot L_2 \cdot E_o;$$

$$\frac{\sigma_E^2}{\mathbf{E}^2_o} = \frac{1}{4kD} \cdot \frac{\Delta}{\mathbf{E}_o} \cdot (\tilde{N}_h + \tilde{N}_e / Z^2)$$